Relationship between the Angle of Repose and Angle of Internal Friction for Agricultural Granular Materials:

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Abstract

The angles of repose and internal friction are important parameters which determine the flow characteristics of agricultural granular materials, such as flour, maize, beans, wheat, sorghum and rice. The angles have a big influence on the design of flow and storage structures of agricultural materials such as hoppers, silos, bunkers and bins because they determine whether the flow will be smooth or not, and whether the bursting forces in relation to vertical forces will be great or not. An apparatus for determination of the angle of repose was specifically designed for the purpose. The angles of repose for three different grains, namely maize, sorghum and rice at four moisture content levels, namely 10, 15, 20 and 25%wb were determined using the designed apparatus. A tri-axial compression machine was used to determine the angles of internal friction for the same grains and same moisture contents. The data obtained were fed into SAS statistical software for step-wise regression analysis. A model of the form $\Phi_r = 22.35 + 0.013\Phi^2_i + 0.019[MC]^2$ *where* $\Phi_r =$ *the angle of repose,* $\Phi_I =$ *angle of internal friction and MC = percent moisture content on wet basis was established, and used to predict the angle of repose for the tested grains with high accuracy* $(R^2 = 0.97)$. *It was concluded that the angle of repose was consistently higher than the angle of internal friction for all the granular materials tested, and the angle of internal friction was highest for maize, followed by rice and finally sorghum.*

Keywords: Angle of repose, angle of internal friction, granular materials, triaxial compression machine, moisture content

Introduction

Frictional properties of agricultural granular materials are important in the design of equipment for solid flow and structures for storage of these materials. For example, in predicting the lateral pressure on a retaining wall in storage bins, or design of bins and hoppers for gravity or forced flow, the coefficient of friction between granular materials is needed as a design parameter .

If the friction between particle to particle is high, the heap formed by a pile of such a granular material is sharp, making a steep cone, and vice versa. The angle with the horizontal at which the materials will stand when piled is termed the angle of repose, while the angle of friction appearing under the effect of friction between individual grains of a material is termed the angle of internal friction (Mohsenin, 1986).

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An important flow property of solids known as critical flow factor for conical hoppers is dependent on the angle of friction between the solids and the wall of the hopper, the hopper slope , the outlet diameter, the angle of internal friction of the solid as well as the solids' size distribution and moisture content (Boumans, 1984). The angle of internal friction also features in the design of shallow bins, and this is well elaborated by the Rankine equation (Mohsenin, 19986). An improperly designed bin may cause obstruction of flow, erratic flow, development of dead zones resulting in degradation of the solid, segregation and several other problems and these problems will be eliminated if and only if the design specifications will considered properly (Mohsenin 1986; Boumans 1984). Also the concept of the angle of internal friction is important in determination of pressure distribution in compression chambers.

Lorenzen, 1957 (quoted by Mohsenin, 1986), reported that the design of deep bins and other similar storage structures, the pressure ratio, $k = \sigma_3 / \sigma_1$, referred to as the ratio of lateral pressure, σ_3 , to vertical pressure, σ_1, at a given point in the material is needed, which can also be found from the angle of internal friction as seen in equation 1:

$$k = \frac{1 - \sin\Phi_i}{1 + \sin\Phi_i}$$

Knowing the value of k, the horizontal pressure against the wall can be estimated for any given vertical pressure. The vertical pressure causes a column action while the lateral pressure causes a bending action (bursting force) on the wall. However the value of k is not constant but varies with the type of material and the geometry of the bin, as well as depth, friction, cohesion properties and moisture content of the materials. The influence of these various factors on the pressure ratio is best illustrated by the well- known Janssen's equation given for lateral pressure in deep bins as presented by Mohsenin (1986).

The presence of moisture on the rubbing surfaces may cause an increase in friction due to increasing adhesion, and therefore, influences the friction coefficient.

Several methods have been used to determine the angle of repose (Fowler and Wyatt, 1960 ;quoted by Mohsenin, 1986) (Figure 1). The an-

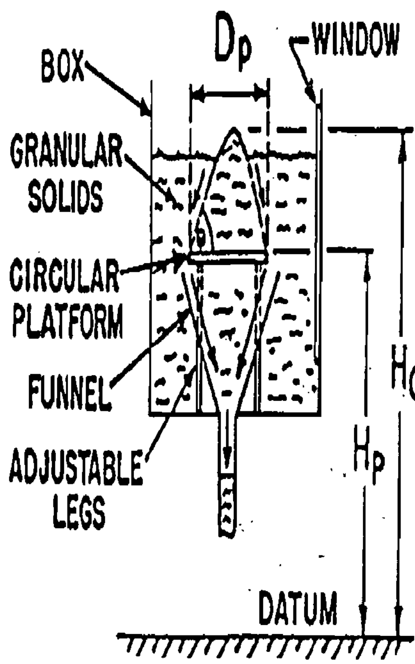

Figure 1: Schematic representation of the apparatus used for determination of angle of repose (Source : Mohsein, 1986)

gle of internal friction can be measured by using, among other methods, a triaxial compression machine (Stewart, 1968, quoted by Mohsenin, 1986) (Figure 2). Some scientists have used the value of the angle of repose in place of the angle of internal friction, and this has led to introduc-

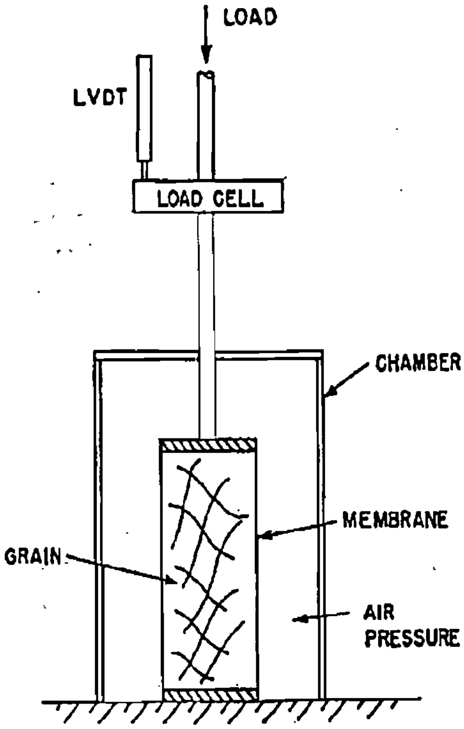

Figure 2: Schematic representation of the triaxial testing machine used for determination of the angle of internal friction (Source: Mohsein, 1986)

tion of errors in engineering calculations. Lorenzen, (1957) attempted to relate the angle of internal friction, Φ_i and the angle of repose Φ_r, with the hope that a simple test of repose angle determination would yield the value of angle of internal friction, from which *k* could be determined. His results showed that the values of the two angles ran almost parallel to each other at various moisture contents but no simple relationship existed where by the angle of internal friction could be estimated from the angle of repose, with reasonable accuracy. Others (Sitkei and Elsevier, 1986) reported that there is no published relationship between the angle of repose and the angle of internal friction for agricultural granular materials. Therefore, the objectives of this study were: a) to determine the angle of repose for selected grains at pre-determined moisture contents b) to determine the angle of internal friction for the selected grains and moisture contents and c) to develop a mathematical model that can predict the relationship between the angle of repose and angle of internal friction for selected agricultural granular materials at various moisture contents.

Methodology

Preparation of the samples for the experiment

Three types of grains, namely maize, sorghum and rice were selected for the study, and the samples were purchased from a local market, in Morogoro, Tanzania. The samples were sieved to obtain uniform grains, because variation in particle size would influence the two angles. The moisture contents of the samples were determined using a moisture meter (Wagtech model, Berkshire, England) which was first calibrated against the conventional oven method. The moisture contents for all the grains were found to be between 10 and 12% wb, and they were therefore dried to a starting moisture content of 10 % wb for uniformity. Four moisture content levels, namely 10, 15, 20 and 25% wb were selected for the study. Therefore, grains had to be conditioned to the respective moisture contents by adding an appropriate amount of moisture which was calculated according to the formula:

$$MC = \frac{W_m}{W_m + W_s} \times 100$$

where MC is the moisture content (wb), W_m is the mass of the moisture and W_s is the mass of the dry solids. The conditioning of the grains was done by measuring the required amount of the sample, putting the sample in a plastic bag, adding the calculated amount of water and shaking vigorously. The contents of the bag were allowed to come to equilibrium for a minimum of about 4 hours when the moisture content of the grains was re-checked by the moisture meter.

Determination of the angle of repose

The Fowler and Wyatt (1960) apparatus (Fig. 1) was fabricated in the Department of Agricultural Engineering of the Sokoine University, and used to determine the angles of repose. The angles for all the three grain samples at the four moisture contents were calculated from equation 3:

$$\Phi_r = \tan^{-1} \frac{2(H_c - H_p)}{D_p}$$

where H_c, H_p and D_p are, respectively, the height of the cone, height of the platform and diameter of the platform. The whole experiment was replicated three times.

Determination o f the angle of internal friction

A triaxial compression machine (Model Franz Geissinger, Nuklear - Technik - Geo Techkin, West Germany) was used for the study. The machine comprises the chamber into which the sample is introduced (Figure 2)

The granular material was enclosed in a thin rubber membrane to seal it from the chamber fluid. The essential qualities of the membrane should be that it exerts the minimum restraint to the sample and also prevents any leakages both from the chamber into the sample and vice versa.

The cylinder was then confined by lateral pressure, σ_3, in the cell fluid. The source of the cell pressure was a liquid column and pressurized gas by using mechanical pump. Additional stress (compressive) was applied to the top of the sample cylinder by a movable piston and the difference between the pressure on the top and side surface of the sample cylinder generate shear stresses on different planes. The compressive pressure and vertical deformation of the cylinder at failure were automatically recorded by a computer which was interfaced to the triaxial machine through an IEEE (interface of the Institute of Electrical and Electronic Engineers).

The compressive pressure was added to the cell confining pressure in order to calculate the average vertical stress applied to the sample. The area, however changed during a compression test owing to the shortening and widening of cylindrical sample (termed barrelling). Therefore, to correct for the changes in area, the following equations were used:

$$\sigma_1 = \frac{P}{A} + \sigma_3$$

$$A = \frac{A_0}{1-\varepsilon}$$

$$\varepsilon = \frac{\delta l}{l_0}$$

Where: σ_1 = major principle stress; σ_3 = minor principle stress; P = axial compressive force; A_0 = initial area of the sample; A = the sample cylinder cross section area at any time and ε, = vertical strain of the sample. The whole experiment was replicated three times and the data obtained were fed into SAS for step-wise regression.

Results and discussion

The results reveal that the values of the angles of repose and internal friction were affected by the moisture content. They both increased with increase in the moisture content of the grains. The angle of internal friction was especially sensitive to moisture content above 15%, the reason being that for moisture contents in excess of this value, water became available to wet the friction surfaces, there by increasing the frictional force. At moisture content below this value, the values of angle of internal friction showed a gradual increase. The angle of repose was more sensitive to moisture content than the angle of internal friction. The increase of moisture content strongly influenced the angle of repose from the moisture content of 10% to 25%, consistent with Mohsenin, (1986), who reported that from this trend, the angle could reach even 70° if the moisture content increased to more than 50% wb. In this study, it was observed that the angle of repose was consistently higher than the angle of internal friction for all the samples tested.

The results show that, the moisture content also influenced the cohesive pressure between individual grains but not strongly. The cohesive force is seen to increase as the moisture content increases, At low moisture content the maize grains tend to be cohesionless because the tangent line in Mohr's circles (which were drawn from the data obtained in this study) passes very near to original, but as the moisture content increases the value also tends to increase.

Maize grains were found to have highest values of internal friction followed by rice and lastly sorghum. From this trend it can be concluded that the angle of internal friction was also affected by particle size, because particles for maize are largest, followed by rice and lastly sorghum.

The step-wise regression obtained by statistical package, SAS, yielded an equation of the form:

$$\Phi_r = 22.35 + 0.013\Phi^2{}_i + 0.019[MC]^2$$

Where:

Φ_r = is the angle of repose;

Φ_i = angle of internal friction;

MC is the percent moisture content on wet basis.

The equation was used to predict the angle of repose from the angle of internal friction and moisture content between 10 and 25% wb for the tested samples. As seen in Figure 3, the model predicted the angle of repose very well for all the samples tested (R^2 = 0.97). The model also gave a good prediction of the angle of repose for individual grains, namely maize, rice and sorghum (Figures 4 to 6).

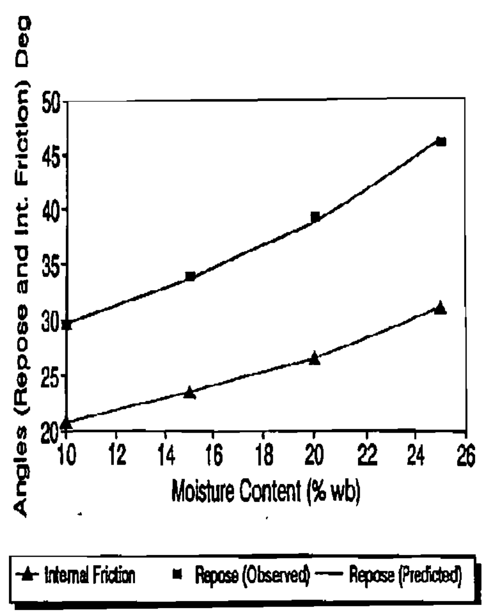

Figure 3: Angele of repose as a function of angle of internal friction and moisture content for maize, rice and Sorghum Each point is a mean of 9 observations

Model validation

In order to test the validity of any model, the model results must be compared with numerical data independently derived from experiments or observations of the environment, other than those used to formulate the model itself (Donigian

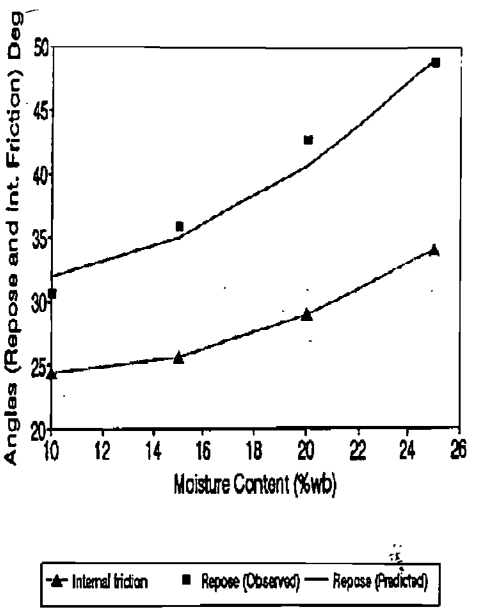

Figure 4: Angle of repose as a function of angle of internal friction and moisture content for maize grain. Each point is a mean of 3

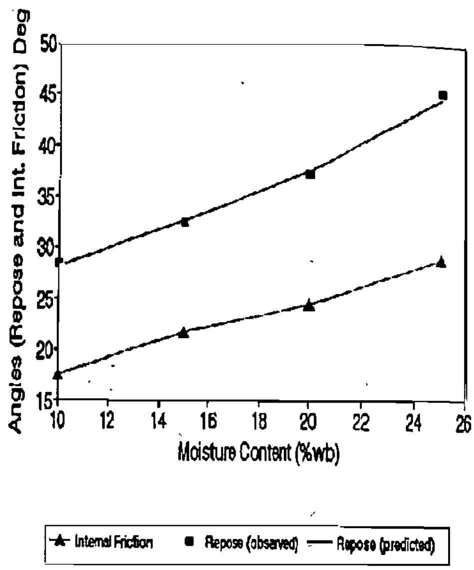

Figure 6: Angle of repose as a function of angle of internal friction and moisture content for rice grain. Each point is a mean of 3

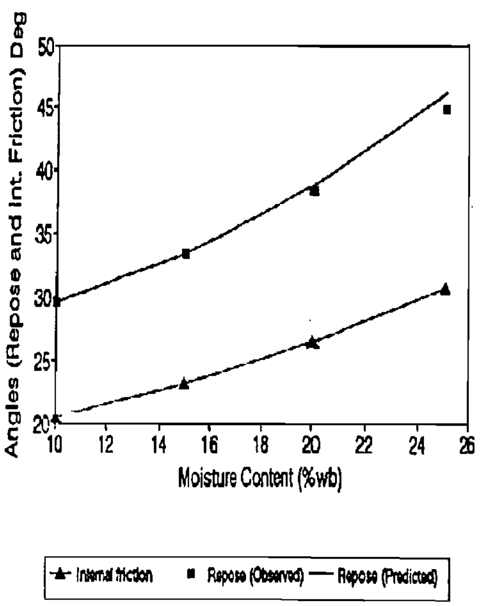

Figure 5: Angle of repose as a function of angle of interna friction and moisture content for maize. Each point is a mean of 3 observations

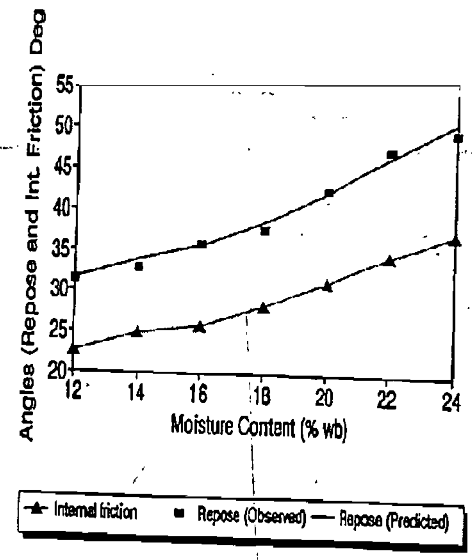

Figure 7: Angle of repose as a function of angle of internal friction and moisture content for data of wheat obtained from literature (Source: Sitkei, 1986)

and Rao, 1989). Therefore, data for the angle of repose, angle of internal friction and moisture content of wheat were obtained from literature (Sitkei, 1986) and used to validate the model. The model prediction of the angle of repose for the wheat (Figure 7) was as good as the prediction for maize, rice and wheat; which proves there is a definite relationship between the angle of repose and the angle of internal friction. With this finding, the statement by Lorenzen (1957), that there is no simple relationship between the two angles, is untrue.

Conclusions

a) It has been proved that a definite relationship, given by equation (7), does exist between the angle of repose and the angle of internal friction for the granular materials tested.
b) Both the angle of repose and internal friction increased with increase in moisture content, but the angle of repose was always higher than the angle of internal friction.
c) The angle of internal friction was largest for maize, followed by rice and finally by sorghum.

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