Science World Journal Vol. 19(No 4) 2024 https://dx.doi.org/10.4314/swj.v19i4.18
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

Full Length Research Article

MATHEMATICAL MODELLING AND SIMULATION OF EFFECTIVE PRODUCTION OF NON-ISOTHERMAL TRANSIENT HYDROGEN NATURAL GAS MIXTURE USING FVSM

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ABSTRACT

Mathematical modelling and simulation of effective production of non-isothermal transient Hydrogen Natural Gas mixture is an important phenomena in gas producing industries. Gas mixture production is a production of natural gas mixed with hydrogen which remain a problem to gas producing industries. During production transient pressure is usually generated by rapid and sudden closure of shut-off valve at wellhead and the flow environment. Many authors in gas producing industries consider gas as a single substances and does not consider the production of natural gas mixed with hydrogen. In this work a one-dimensional transient compressible model comprising conservation of mass, momentum and energy which incorporate hydrogen natural gas mixture equation with the properties of flow environment has been developed with the aim of determining the effect of the mixture on the flow parameters. The model was solved using flux vector splitting method (FVSM). The effect of body force on transient pressure and sound wave due to the well flow geometry including the effect of hydrogen mass ratio has also been discussed. The results obtained when considering isothermal flow in a well of $700\ ft$ deep using different wellbore diameter $0.073m$, $0.088m$ different time and different thermal conductivities shows good agreement with published results. For non-isothermal flow, the pressure and sound wave are observed to be increasing when the hydrogen mass ratio are increased. The heat exchange between the flowing fluid and the well flow environment increase the temperature of the flowing fluid. The result of this work reflect gas flow law and has provide a technical way forward for production of hydrogen natural gas mixture to all gas producing industries.

Keywords: Transient non-isothermal flow, Well Geometry, Hydrogen natural gas mixture, Flux Vector Splitting Method (FVSM).

INTRODUCTION

Gas production is the process of removing gas from the reservoir through the wellbore to the wellhead and subsequently to the end users. When gas is no single it consist of mixture of other gasses which make removal to be more difficult. In the past when a well is dying it is reactivated through injection of mixture of substance to boast back the life of the production well. Modelling and simulation of hydrogen natural gas mixture is therefore important to every gas producing industries. For this reason many authors working in gas producing well consider some parameters which necessarily affect the production and how there can be taken care off. In the olden days correlations and steady state approach are usually applied to gas production which does not give satisfactory results because the used applications failed to consider the natural gas mixture and transient aspect leading to premature closure of most gas wells. Attempt by existing models were done to correct these anomalies using simplified governing equations mostly ignoring the transient aspect of the flow environment due to gas mixture and environmental change. When modelling transient flow production of hydrogen natural gas mixture, problem always occur because of the dependency on both space and time, however the technique appears to be superior in representing actual gas well conditions. During gas mixture production, fluid experiences flow propagation which led to large pressure surges. Solution to such problem is best obtain using scheme which is unconditionally stable (flux vector splitting method). Bahbahni Najad and Shakeri, (2008) reported that the method best accommodate the flow propagation and has been applied in pipeline gas transportation. Gas producing well always encountered problems where authors have been developing models to find their solutions. One of the father of work in gas well was Kirkpatrick (1959). He presented a simple flowing fluid temperature and pressure gradient of injected mixture substance that can be used to predict gas lift valves at the injection depth. His work is worth mention because he was the first to consider the effect of injection valve in well flow problems. Many other authors such as Ramey, (1962) discussed exclusively the overall heat transfer mechanism in a wellbore. He pointed out experimentally that heat transfer in the wellbore is unsteady but there is a variation between the bottomhole temperature and the wellhead. Juiping *et al*., (2012) in an attempt to find solution to problem of gas well, developed couple systems of ordinary differential equations for pressure, temperature, density and velocity distribution in HPHT gas wells according to the conservation of mass, momentum and energy based on steady state assumption. The equations were simultaneously solved and four ordinary differential equations were obtained to account for the pressure, temperature, density and the velocity distribution according to the demand of fourth order Ranger Kuta method. Jiuping *et al,* (2013) revisited their work of (2012) and developed a couple system of partial differential equations for the variation of pressure, temperature, velocity and density at different time and depth in high pressure, high temperature well for two phase gas/liquid during flow in the producing well and applied splitting techniques with Eulerian Generalized Reiman Problems (GRP). Ramey, (2013) improved the work of Ramey, (1962) with further work on wellbore temperature where a model was developed for solution to transient heat transfer problem which involve heat conduction during production of hot fluids. The work reported that heat transfer to earth was unstable due to the effect of thermal resistance in the well bore radial convection. The solution also allowed the temperature estimate of fluids, tubing and casing as a function of depth and time. Zohra *et al.,* (2014) presented a model for transient phenomena in two phase homogeneous flow that account for both geometrical parameters of the pipe and mass fraction of the gas in the two-phase mixture flow which was solved using method of characteristics.

Mbaya and Amin, (2015), presented isothermal model for unsteady flow of gas in producing well without the energy equation where gas state equation was incorporated to determine the heat transfer in the producing gas well. Farhan *et al.,* (2019) applied ANSYS Fluent to determine the temperature of the flowing fluid and reported that as the hot fluid from bottom hole rises up the wellbore, its temperature is higher than the surrounding earth temperature which causes heat loss to the surroundings. They further reported that when flow rate of the fluid increases, more of the hotter fluid from the bottom displaces the colder fluid in the wellbore at any given point and therefore the temperature increases. Mbaya (2021) studied different type of closing laws on the pressure and the behaviour of the flow in the producing well at the instance of closure during production by developing a one-dimensional mathematical model for the closing function during gas production based on the principle of conservation laws. Mbaya and Hanafi (2021) presented a one-dimensional compressible model comprising conservation of mass and momentum and in cooperating gas state equation to investigate the behaviour of the heat exchange between fluid temperature, surrounding earth and the flow environment of the wellbore. Bo *et al*., (2021) reported that tubing leakage is one of the main reasons that cause annular transient pressure in HPHT gas wells. The work further stated that there exist a relationship between leakage rate and sustained annular transient pressure with fluid temperature distribution.

From the existing literatures enough work has been done on wellbore which is capable of producing fluid but no enough literature on production of hydrogen natural gas mixture. In this paper a one-dimensional transient compressible model comprising the conservation of mass, momentum, energy which incorporate hydrogen natural gas mixture equation and the flow environment has been developed and was solved using flux vector splitting method (FVSM)

MATERIALS AND METHOD

Gas well is connected with reservoir where the fluid is expected to be extracted. Flow in such environment is governed by equation of motion and for computational accuracy can be achieved when a suitable method is applied. In this research implicit flux vector splitting method (FVSM) is considered in solving the governing equation consisting of continuity, momentum, and energy.

ASUMPTIONS

In developing the governing equation assumptions were made based on the following:

a. The fluid is homogeneous mixture of hydrogen and natural gas and the flow is one dimensional compressible which include transient condition.
b. For the non-isothermal flow in the wellbore, the gas properties varied and not constant overtime due to disturbances at the wellhead.
c. Due to the flow environment in the wellbore, some properties, such as the mixture density, mass inflow, and velocity will change accordingly
d. The thermal conductivity of casing and cement are in infinite direction.

GOVERNING EQUATIONS

The governing equations consist of coupled system of non-linear hyperbolic partial differential equations which in-cooperate other parameters of the wellbore environment. Heat transfer mechanism and the thermal conductivities of the wellbore environment with the mixture ratio introduced as source term is considered.

$$\frac{\partial(\rho)}{\partial t}+\frac{\partial \dot{M}_x}{\partial x}=\dot{q} \qquad 1$$

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial\left(a^2\rho+\alpha\rho u^2\right)}{\partial x}= -\alpha\left(u\dot{q}\right)-\rho g\cos\theta+\frac{f\rho u|u|}{R} \qquad 2$$

$$\frac{\partial(\rho T_e)}{\partial t}+\frac{\partial(\rho u T_e)}{\partial x}= \frac{R\pi k_e r_{to}U_{to}\left(T_{fm}-T_e\right)}{\dot{q}C_p\left(k_e+r_{to}U_{to}f\left(t_D\right)\right)} \qquad 3$$

In equation (1) when the porosity is constant, $\dot{M}_x$ is replaced by ρu the mass flow rate $\dot{q}=\frac{2\rho_{Im}u_{Im}}{R}$, and ρ_{Im}, u_{Im} are respectively inflow density of the hydrogen natural gas mixture and its inflow velocity, U_{to} is overall heat transfer coefficient, T_e is undisturbed formation (initial) temperature, T_{fm} is the flowing mixture fluid temperature, $f\left(t_D\right)$ is dimensionless transient heat conduction time function for formation, this parameter enter into wellbore heat loss calculations because heat flow in the surrounding formation varies with time. Heat loss to these formations are large initially, but decreases with time as thermal resistance to the flow of heat builds up in the formation, r_{to} outer radius of tube (m), u local velocity, k_e thermal conductivity of earth, R radius of reservoir which is equal to wellbore radius, and a^2 sound wave propagation.

U_{to} and $f\left(t_D\right)$ are calculated as in (4) and (5

$$U_{to}=\left[\frac{r_{to}}{r_{ti}h_{fm}}+\frac{r_{to}\log\left(\frac{r_{to}}{r_{ti}}\right)}{k_{twb}}+\frac{r_{to}\log\left(\frac{r_{ins}}{r_{to}}\right)}{k_{ins}}+\frac{r_{to}}{r_{ins}\left(h_c+h_r\right)}+\frac{r_{to}\log\left(\frac{r_{co}}{r_{ci}}\right)}{k_{cas}}+\frac{r_{to}\log\left(\frac{r_h}{r_{co}}\right)}{k_{cem}}\right]^{-1} \qquad 4$$

$$f\left(t_D\right)=\begin{cases}1.128\sqrt{t_D}\left(1-0.3\sqrt{t_D}\right) & t_D<1.5\\ \left(0.4063+0.5\log t_D\right)\left(1+\frac{0.6}{t_D}\right) & t_D>1.5\end{cases} \qquad 5$$

where r_{ti} inner radius of tubing, r_{cem} radius of cement, r_{cas} radius of casing, r_w wellbore radius, k_{an} thermal conductivity of annulus, k_{cem} thermal conductivity of cement, h_r and h_c are heat transfer due to radiation and natural convection respectively while h_{fm} is hydrogen mixture fluid heat transfer mechanism, and $t_D = \frac{\alpha t}{r_{wb}^2}$, $\alpha = \frac{k_e}{\rho c_e}$, c_e is the heat capacity of the formation that acted on the flowing fluid temperature.

HYDROGEN NATURAL GAS MIXTURE EQUATION

We define the density of hydrogen and natural gas as:

$$\rho_h = \frac{m_h}{u_h} \qquad 6a$$

$$\rho_g = \frac{m_g}{u_g} \qquad 6b$$

where m_g, m_h, u_g and u_h are the mass of natural gas mixed with hydrogen and velocity of natural gas mixed with hydrogen, respectively. The hydrogen mass ratio will be used in determining the produced mixture density which is given as:

$$\dot{Q} = \frac{m_h}{m_h + m_g} \qquad 7$$

Now considering the definition of density of gas, the density of hydrogen-natural gas mixture is defined as:

$$\rho = \frac{M_m}{U_m} \qquad 8$$

Where $M_m = M_h + M_g$ and $U_m = U_h + U_g$ putting equation (6a,b) in equation (7) and (8), we obtain the density of hydrogen-natural gas mixture according to the mass ratio $\dot{Q}$ as:

$$\frac{1}{\rho} = \frac{m_h}{\rho_h\left(m_h + m_9\right)} + \frac{m_g}{\rho_g\left(m_h + m_g\right)} \qquad 9$$

Further simplification of equation (8) resulted to;

$$\rho = \left[\frac{\dot{Q}}{\rho_h} + \frac{\left(1-\dot{Q}\right)}{\rho_g}\right]^{-1} \qquad 10$$

Equation (10) is the density of hydrogen-natural gas mixture and is defined according to the mass ratio $\dot{Q}$ and the density of hydrogen and natural gas according to isentropic laws is written as:

$$\rho_h = \rho_{ho}\left(\frac{P}{P_o}\right)^{\frac{1}{h}} \qquad 11a$$

$$\rho_g = \rho_{go}\left(\frac{P}{P_o}\right)^{\frac{1}{h^r}} \qquad 11b$$

where each ρ_{ho} represents the initial density of hydrogen, and ρ_{go} represents initial density natural gas. P is the transient pressure of the produced mixture fluid, and P_o is reservoir pressure of the mixture fluid. The solution of the governing equations (1), (2) and (3) can be obtain numerically, the density of the hydrogen-natural gas mixture ρ must be expressed according to the gas pressure P. Now if (11a) and (11b) are substituted in (10), the expression of the average density of the mixture can be obtain as (12):

$$\rho = \left[\frac{\dot{Q}}{\rho_{ho}}\left(\frac{P}{P_o}\right)^{\frac{1}{h}} + \frac{\left(1-\dot{Q}\right)}{\rho_{go}}\left(\frac{P}{P_o}\right)^{\frac{1}{h^r}}\right]^{-1} \qquad 12$$

When considering compressible flow the sound propagation of the pressure wave is defined as:

$$a^2 = \left(\frac{\partial P}{\partial \rho}\right)_s, \quad a = \left(\frac{\partial P}{\partial \rho}\right)_s^{\frac{1}{2}} \qquad 13$$

In equation (13) the subscript s represent condition of constant entropy. Now taking the derivative of (12) with respect to P, and substituting into (13) result to the sound wave propagation equation as:

$$a = \sqrt{\left[\frac{\dot{Q}}{\rho_{ho}}\left(\frac{P}{P_o}\right)^{\frac{1}{h}} + \frac{\left(1-\dot{Q}\right)}{\rho_{ho}}\left(\frac{P}{P_o}\right)^{\frac{1}{h^r}}\right] \times \left[\frac{1}{\rho}\left(\frac{\dot{Q}}{n\rho_{ho}}\left(\frac{P}{P_o}\right)^{\frac{1}{h}} + \frac{\left(1-\dot{Q}\right)}{n^1\rho_{go}}\left(\frac{P}{P_o}\right)^{\frac{1}{h^r}}\right)\right]} \qquad 14$$

Wellhead Production Closing Wave Equation

During the production period the disturbances at the wellhead necessitate a closing valve function equation. The function is a Mathematical function that describes the wave variation of mixture gas flow as the wellhead valve closes. The derivation of equation (13) with respect to time (t) and distance (x) gives equation stating the relationship between pressure and density with the effect of sound wave. The equation can be written as

$$\frac{\partial \rho}{\partial x} - \frac{1}{a^2}\frac{\partial p}{\partial x} = 0, \qquad 15a$$

$$\frac{\partial \rho}{\partial x} - \frac{1}{a^2}\frac{\partial p}{\partial x} = 0 \qquad 15b$$

Putting equation (15a) and (15b) in the equation of motion yield the following equation

$$\frac{\partial \rho}{\partial t} + \rho a^2 \frac{\partial u}{\partial x} = 0, \qquad 16a$$

$$\frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{fu|u|}{2D} = 0 \qquad 16b$$

During production of gas the friction factor is sometimes neglected because the value is very small since it has a less viscosity. Now differentiating equation (16a) with respect to time (t) and distance (x) we obtained equation (17a) and (17b).

$$\frac{\partial^2 \rho}{\partial t^2} + \rho a^2 \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right) = 0, \qquad 17a$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\right)+\frac{1}{\rho}\frac{\partial^2 p}{\partial x^2}=0 \qquad 17b$$

Equation (17a) and (17b) are substituted in one-dimensional wave equation which best describes the pattern of closing valve function of (Provenzona 2011) at the wellhead. The equations are simplified as

$$\frac{1}{a^2}\frac{\partial^2\rho}{\partial t^2}+\rho\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right)=0, \qquad 18a$$

$$\frac{\partial^2\rho}{\partial t^2}+\rho\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\right)=0 \qquad 18b$$

The sound wave equations is solve numerically, in the absent of field data steady state solution is used as the initial condition (Zahoa and Adewumi, 1995) which are given as follows.

$$\frac{\partial u(x,0)}{\partial x}=\dot{q} \qquad 19a$$

$$\frac{\partial \rho u(x,0)}{\partial x}=-a^2\frac{\partial\rho}{\partial x}-\frac{f\rho u|u|}{2D} \qquad 19b$$

$$\frac{\partial(\rho uT_e)(x,0)}{\partial x}=\frac{R\pi k_e r_{to}U_{to}\left(T_f - T_e\right)}{\dot{q}C_p\left(k_e+r_{to}U_{to}f\left(t_D\right)\right)} \qquad 19c$$

Boundary Condition

The boundary conditions for the production of hydrogen natural gas mixture depend on the wellhead valve operational time and the types of valve closure at the wellhead. The boundary conditions at the bottom of the wellbore (initial point), $x=0$, are given by:

$$\rho(0,t)=\rho_0(t) \qquad 20a$$

$$\frac{\partial u(0,t)}{\partial x}=u_o(t), \qquad 20b$$

$$T(0,t)=T_o(t) \qquad 20c$$

where ρ_0, u_0 and T_0 is defined as density, velocity and temperature at the reservoir and bottom of the wellbore, respectively. The boundary conditions at the end point $x=L$ are:

$$\rho u(L,t)=\rho u_L(t), \qquad 21a$$

$$\frac{\partial u(L,t)}{\partial x}=u_L(t) \qquad 21b$$

$$\rho ue(L,t)=h_L(t) \qquad 21c$$

where ρu_L, u_L and h_L is defined as mass flux, velocity and heat flux at the wellhead with numerical initial conditions

Numerical Solution

Equations (1), (2) and (3) can be transform according to the demand of the flux vector splitting method as

$$\frac{\partial Q}{\partial t}+\frac{\partial E(Q)}{\partial x}=H(Q) \qquad 22$$

where

$$Q=\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}, \qquad 23a$$

$$E(Q)=\begin{bmatrix} q_2 \\ \frac{q_2^2}{q_1}+a^2 q_1 \\ \frac{q_2 q_3}{q_1} \end{bmatrix} \qquad 23b$$

$$H(Q)=\begin{bmatrix} \dot{q} \\ \frac{q_2 u_1}{\dot{q}}+q_1 g\cos\theta+\frac{f q_2^2}{2d q_1} \\ \frac{\pi k_e r U\left(T_f - T_e\right)}{\dot{q}C_p\left(k_e + rUf(t)\right)} \end{bmatrix} \qquad 23c$$

Where $q_1=\rho$, $q_2=\rho u$ and $q_3=\rho uT$

IMPLICITLY STEGER-WARMING FLUX VECTOR SPLITTING METHOD

The implicit Steger-Warming flux vector splitting method (FVSM) is the numerical method used to solve (1), (2) and (3). Equation (22) is discretized using finite difference method which resulted to a scheme in delta notation a $\left[\frac{\Delta t}{\Delta x}A_{i+1}^{n(-)}\right]\Delta Q_{I+1}-\left[\frac{\Delta t}{\Delta x}A_{i-1}^{n(+)}\right]\Delta Q_{i+1}+\left[I+\frac{\Delta t}{\Delta x}\left(A_i^{n(+)}-A_i^{n(-)}\right)\right]\Delta Q_i$

$$-\Delta t B_i^n \Delta Q_i=-\frac{\Delta t}{\Delta x}\left[E_i^{n(+)}-E_{i-1}^{n(-)}+E_{i+1}^{n(+)}-E_i^{n(-)}\right]+\Delta t H_i^n$$

24

where $A=\frac{\partial E(Q)}{\partial Q}$ and $B=\frac{\partial H(Q)}{\partial Q}$, and I is the identity matrix, A and B are the Jacobian matrices defined as:

$$A=\begin{bmatrix} 0 & 1 & 0 \\ -\alpha u^2+a^2 & 2\alpha u & 0 \\ -uT & T & u \end{bmatrix} \qquad 25a$$

$$B=\begin{bmatrix} \dot{q} & 0 & 0 \\ -g\cos\theta-\frac{fu^2}{2d} & \dot{q}+\frac{2fu}{2d} & 0 \\ 0 & -\frac{\pi k_e r_{to}U_{to}\left(T_f - T_e\right)}{\dot{q}C_p\left(k_e+r_{to}U_{to}f\left(t_D\right)\right)} & 0 \end{bmatrix} \qquad 25b$$

The flux vector *E* and the Jacobian matrix *A* are split in positive and negative as:

$$A^+=\frac{u}{2a}\begin{bmatrix} a-u & \frac{a+u}{u} & \frac{T(a-u)}{au} \\ -\left(u^2+a^2\right) & \frac{(u+a)^2}{u} & \frac{T\left(-u^2+2ua+a^2\right)}{au} \\ \frac{-a^2(u+a)}{T} & \frac{a^2(u-a)}{uT} & \frac{a(u+a)}{u} \end{bmatrix}, \qquad 26a$$

$$A^- = \frac{u}{2a}\begin{bmatrix} u-a & \frac{-(u-a)}{u} & \frac{T(u-a)}{au} \\ (u-a)^2 & \frac{-(u-a)^2}{u} & \frac{T(u-a)^2}{au} \\ \frac{a^2(u-a)}{T} & \frac{-a^2(u-a)}{uT} & \frac{a(u-a)}{u} \end{bmatrix} \quad 26b$$

RESULTS AND DISCUSSION

The model was compared with the work of Juiping *et al*. (2013) on temperature and pressure distribution during production. The results shows good agreement using two points in the wellbore of different radius. When the tube outside diameter is 0.073*m* and initial pressure is 40 *Mpa*, initial temperature of $80^o\ C$ is plotted as shown in Figure 1 at different depth. We observed that the temperature increases with the increasing depth of the well and keeping the depth constant, the temperature increases with the increasing time. Similarly when the depth is kept constant, the pressure is plotted as in Figure 2; it is also observed that as time increases pressure increases. We apply equation (14) to determine behaviour of sound during production at the wellhead and we observe that the sound wave remain slightly constant in the bottom of the wellbore but drop significantly towards wellhead as distance simultaneously increases with time during production figure 3.

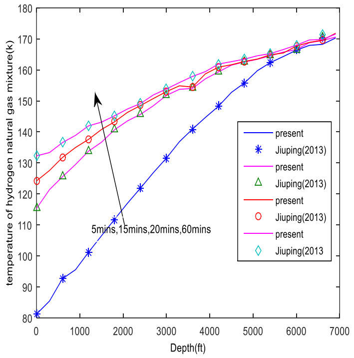

Figure 1: Temperature Compare

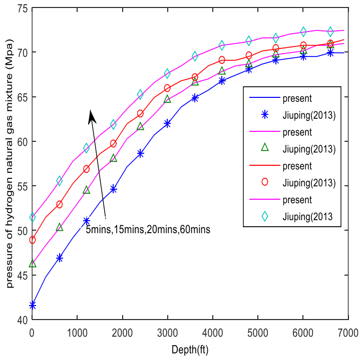

Figure 2: Pressure Compared

$$E^+ = \frac{1}{2}\begin{bmatrix} \rho(u+a) \\ \rho(u+a)^2 \\ \frac{\rho a^2(u+a)}{T} \end{bmatrix}, \quad E^- = \frac{1}{2}\begin{bmatrix} \rho(u-a) \\ \rho(u-a)^2 \\ \frac{\rho a^2(u-a)}{T} \end{bmatrix} \quad 27a,b$$

$$\Delta Q = Q^{n+1} - Q^n \quad 28$$

When (24) is applied to each grid point, a block tridiagonal systems of algebraic equations is obtained.

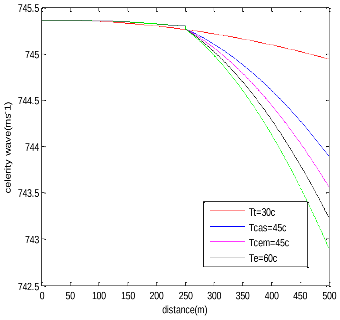

Figure 3: Behaviour of sound

We also, observed that density of the mixture fluid drops when temperature increases due to the temperature of the flow environment. The flow environment temperature when mixed up with flowing fluid temperature makes the density lighter which nesseciate it to drop as shown in figure 4. Similarly the mixture velocity is also observed, when the over roll heat transfer coefficient increases along the depth of the well the velocity also drop significantly as in figure 5. We have test the effect of sound wave on the non-isothermal flow of the gas mixture on two pionts in the wellbore. The two pionts in the wellbore are wellbore diameter $0.073m$, distance of $100m$ and a diameter of $0.088m$ at a distance $500m$ and observed that the sound wave at the upper part of the wellbore is less hard toward the well head than that of distance $500m$ deep in the wellbore as time increases figure 6 and 7.

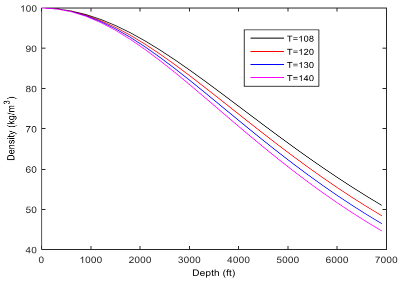

Figure 4: Mixture Density

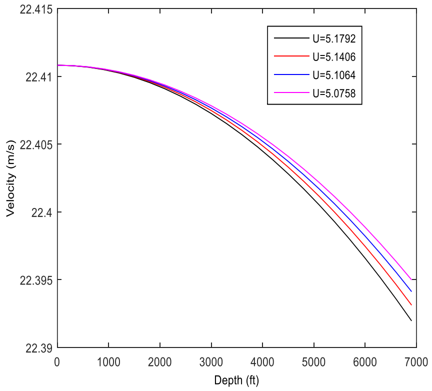

Figure 5: Mixture Velocity

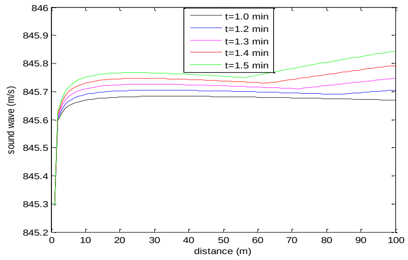

Figure 6: Effect of sound wave on non-isothermal 100m

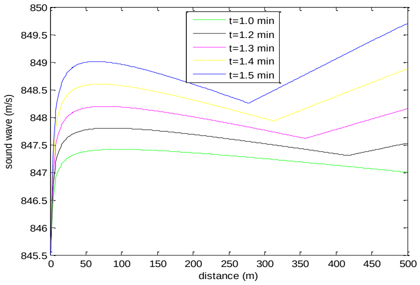

Figure 7: Effect of sound wave on non-isothermal 500m

Using equation (3) we tested the effect of thermal conductivities on the produced gas mixture. We observed that tubing temperature affect the mixture fluid temperature, it drop when flowing up to the wellhead as is in figure 8. Velocity decreases as the outer radius of wellbore increases. It was also observed that pipe radius has a great impact on the velocity gradient as in figure 9. We also observed that as the cement thermal conductivities acted on the mixture temperature, it was more stable at the bottom of the wellbore but drop faster toward the wellhead as in figure 10. The pressure distribution was also observed at the outer radius of the wellbore and simultaneously increases with the pressure figure 11.

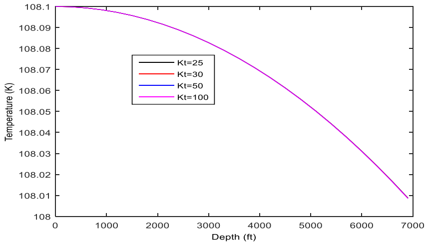

Figure 8: Effect of tubing temperature

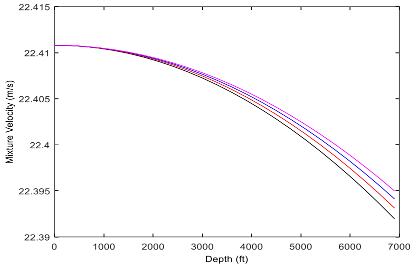

Figure 9: Effect of wellbore outer radius

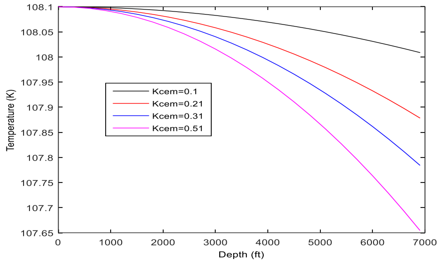

Figure 10: Cement thermal Conductivity

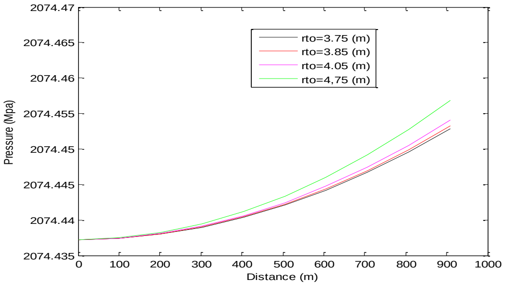

Figure 11: Pressure distribution with outer wellbore radius

Conclusion

We have developed a model for production of transient flow of hydrogen natural gas mixture and solved using Steger Warming Flux Vector Splitting Method. The model was first compared with the existing work of Juiping *et al*, (2013) for temperature and pressure distribution. Other parameters of the flow environment were tested and the results shows that all are affected by the production of the hydrogen-natural gas mixture. We also tested the effect of the sound wave on the production of hydrogen natural gas mixture and was observed that the sound was stronger at a distance of $100\,m$ than $500\,m$. We concluded that production of hydrogen natural gas mixture is difficult because the flowing mixture has an effect on all the flow parameters. We also recommended that the method be tested in the gas injection well.

REFERENCES

Behbahani Nejad and Bagheri, (2008), A MatlabSimulinks Library for Transient Flow Simulation of Gas Network" World Academy of Science Engineering and Technology.

Bo Zhong, Zhixiong Xu, Nu Zu and Hongtao Liu., (2021), Characteristics of Substained annular Pressure and High Temperature Gas Well Considering Multiple Leakage of Tubing String. Journal of Petroleum Science and Engineering, 196(1) 108083. SBN978-1-61399-7.

Farhan Lafta Rashid, Hasan Qahtan Hussein, Asaad Salim Bded (2019), Numerical Simulation of Fluid Flow and Heat Transfer in Wellbore, Journal of Advanced Research in Fluid Mechanics and Thermal Sciences Akademia Baru 60, Issue 1 - 60-70 ISBN 2289-7879

Jiuping Xu, Wu, Z., Wang, S., Qi, B., Chen, K., Li, X. and Zhao, X. (2013), A direct Eulurian GRP Schemes for the Prediction of Gas liquid two phase in HPHT transient gas wells. Journal of Abstru ct and Applied Analysi.,

Jiuping Xu, Wu, Z., Wang, S., Qi, B., Chen, K., Li, X. and Zhao, X. (2013), A direct Eulurian GRP Schemes for the Prediction of Gas liquid two phase in HPHT transient gas wells. Journal of Abstru ct and Applied Analysi.,

Jiuping Xu, Wu, Z., Wang, S., Qi, B., Chen, K., Li, X. and Zhao, X. (2012), Prediction of Temperature, Pressure, Density,Velocity Distribution in H-T-H-P Gas Wells. The Canadian Journal of Chemical Engineering

Kirkpatrick, C. V. (1959), Advances in Gas-lift Technology. Presented at Drilling and Production Practice, New York, New York, USA, 1 January. API-59-024.

Mbaya Jibrin H. and Jannaidu Hannafi (2021), Mathematical Modelling of Fluid Flow and Total Heat Transfer Process in Wellbore. Science World Journal Vol. 16 (No 2), ISSN: 1597-6343 (Online), Published by Faculty of Science, Kaduna State University.

Mbaya and Amin (2018), Numerical Modelling and Simulation of Transient Flow and Heat Transfer of Gas in a Producing Gas Well, American Scientific Publishers, Volume 24, Number 5, May 2018, pp. 3616-3621(6)

Mbaya (2021), Mathematical Modelling and Simulation of Closing Function of Water Hammer System during Gas Production, Science World Journal Vol. 16 (No 2) ISSN: 1597-6343 (Online), ISSN: 2756-391

Ramey, H. J. (1962), Wellbore Heat Transmission, Journal of Petroleum Technology and Science, vol. 225, pp. 427–435.

Ramey H. J., Guppy K. H., Heber C. L., (2013), Effect of Non-Darcy Flow on the Constant-Pressure Production of Fractured Wells. Society of Petroleum Engineers Journal 21(03):390-400