

# A STACKELBERG GAME-THEORETIC TRADE CREDIT MODEL WITH CREDIT RESPONSE TIME LENGTH THROUGH CREDIT FUNCTION

\*Peter E. Ezimadu and Sophia O. Ezimadu

Department of Mathematics, Delta State University, Abraka, Nigeria

\*Corresponding Author Email Address: [peterezimadu@yahoo.com](mailto:peterezimadu@yahoo.com)

## ABSTRACT

The nature and effect of credit period CP has been one of the major focuses and concerns in modelling TC settings. Only very little has been done on the distance in time between credit periods of channel members, which can be referred to as credit response time length. This work considers a supplier-retailer supply channel in which the supplier provides trade credit to the retailer, and allows a delayed repayment time. The retailer in-turn allows a delayed repayment time, and also aids sales by engaging in product promotion. It models the channel members' payoff using credit function, and explores two channel structures: a situation involving TC, and another without TC. It obtains a closed-form optimal promotion effort, credit period and Stackelberg equilibrium solutions for the payoffs. The work examines the effect of the supplier's CP and credit response time length on promotion and payoffs. It shows that while large credit period provision from the supplier will translate into large promotion effort, a large credit response time length – early repayment – requires a large promotion effort. It further shows that both players' payoffs continuously increase with credit period, with the supplier benefiting more from the increase. On the other hand while the supplier's payoff continuously reduces with increasing credit response time length, the reverse is the case with the retailer. Being the channel leader, the supplier's interest, can lead to making policies that can constrain the retailer to delay repayment until his optimal profit is achieved. In general the players perform better with credit scenario than no-credit scenario. Further, the supplier can opt for very early repayment provided he has a favourable agreement which will earn him larger payoff than what obtains in the other scenarios since the channel payoff in this situation is larger.

**Keywords:** Trade Credit, Stackelberg Game, Supply Channel, Product Promotion, Credit Response Time Length, Credit Period, Credit Function

## INTRODUCTION

Papers on credit periods CP usually consider 30 days agreements (Klapper *et al.*, 2012; Ferrando & Mulier, 2013), however, there are variations by countries, types of firms or industries and type of transaction. There are situations where trade credit TC periods can be less than 30 days, or as much as 6 months (Ng *et al.*, 1999; Faichamps *et al.*, 1995; Fabbri & Klapper, 2016). It has been shown that shorter credit allowable periods are given for soft goods (Klapper *et al.*, 2012; Ng *et al.*, 1999). According to Costello (2013), allowable periods range from 8 days for perishable goods to 90days for complex products. This work focuses on the length and distance between credit periods and how they affect individual

channel members' performance as well as the channel performance.

The length of CP is very fundamental in TC, and a lot of studies have made inroads trying to addressing the relationship between TC provision and other market variables. Bai (2009) examined a situation where each member of a supply chain makes decision with information being asymmetric. The author averred that the supplier can use credit period contract as a means of persuading the retailer to open up on his selling cost. Based on deteriorating items, Wu *et al.* (2014) considered credit period and lot size for two-level credit financing with expiration date, and showed that optimal credit and cycle time are unique. Mahata (2015) proposed an EOQ model in which the supplier gives credit to the retailer, and the retailer gives TC to the buyer. The author determined the retailer's optimal replenishment policy by modeling his inventory as a profit maximization problem, and showed that the optimal CP which the retailer allows the product buyer and the cycle time are unique. In a consideration of integrated production with imperfect limits and where customer demands from a number of retailers depend on stock, Manna *et al.* (2017) developed a manufacturer-retailer-consumer supply chain model in which the retailer is allowed TC period by the manufacturer, and the retailer allows the customers TC period in order to stimulate demand. Considering a situation where product with a maximum life existence or usefulness deteriorates over time, Patel *et al.* (2018) developed an inventory situation where the supplier allow the retailer credit period, and the retailer extends the allowed credit period to the customer. In an examination of the retailer's trade credit TC approach and ordering policy in a supplier-retailer setting Bi *et al.* (2021) considered a situation where the supplier provides TC to the retailer, and the retailer provides downstream TC to the end-users. In their consideration the relationship between the upstream and downstream CP is uncertain. Studying a situation where a seller provides order-linked credit to the buyer, where demand depends on both price and CP, Tsao *et al.* (2022) formulated and EOQ model with product deterioration using discounted cash flow. They determined the selling price, the CP and the product replenishment cycle simultaneously. Other much recent trade credit studies are Lefebvre (2023) which examined the duration of credit repayment involving small and medium-sized enterprises in the EU. Also, Tabash *et al.* (2023) studied the effect of government policies on trade credit engagements.

Game-theoretic modeling approach has been very useful in supply chain studies such as TC models. Wu *et al.* (2021) used Stackelberg game to consider a TC channel setting in which the channel members are concerned about avoiding of minimizing losses. They compared their developed game model with a newsvendor model, and observed that the risk-averse parameter,

sharing strategy and TC are affected by tolerating risk. Zhang *et al.* (2021) considered the effect of demand on provision of TC, and studied how a channel performs where the credit is linked to the demand for supplier's product. Considering a supplier-Stackelberg game setting, a retailer-Stackelberg game setting, and an integrated supply setting, they averred that if credit is provided, then the setting in which the supplier is the game leader performs better than the others. Zhou *et al.* (2022) studied a manufacturer-distributor-retail TC supply channel with early payment. The distributor is constrained by capital. Silaghi and Moraux (2022) considered TC using a theoretical setting, and noted that it can be used for channel coordination. They determined both Nash and Stackelberg solutions, and observed that wholesale price, procurement within the channel and TC provision can be affected by placing a limit TC maturity date. Hovelaque *et al.* (2022) developed a Stackelberg supplier-retailer-bank game model in which the bank or the supplier is considered to be the leader of the supply channel and the retailer is taken to be the follower. They considered a non-cooperative channel in which demand for the product is sensitive price to. Maiti and Giri (2015) considered a manufacturing situation where a producer uses his consumed product as raw material for the manufacture of his new product. They considered a Nash game as well as a Stackelberg game among a number of channel settings. Jin and Wang (2020) considered a channel in which the members are constrained by financial limitations, and used a Stackelberg game to demonstrate that such financial constraints leads to giving of small amount of trade credit to the retailer. Wu *et al.* (2018) used Stackelberg game to study a supply channel involving competing unequal retailers with inventory replenishment. Much recently, Emtehani *et al.* (2023) modelled a multi-leader trade credit Stackelberg game. Yang *et al.* (2023) considered a Stackelberg game in a supply channel in which the supplier decides on the wholesale price. Jana and Mondal (2024) considered a Stackelberg game in which a

supplier provides retailers with product in a market setting where demand depends on the duration of credit. Chu *et al.* (2024) considered a trade credit situation involving bargaining game. This work examines a trade credit supply channel involving a supplier and a retailer. It uses game theory to model a situation where the supplier who plays the role of the channel Stackelberg leader provides credit goods to the retailer and allow him a delayed repayment period. The retailer in turn also allows the consumer a delayed repayment period, and engages in product promotion. The work will compare two scenarios: a decentralized channel situation involving the provision of trade credit, and a channel setting without credit provision. The paper will consider the effect of the credit response time length – the difference in time between two credit periods, the manufacturer's credit period on the promotion effort and the players' payoffs and channel payoff. It is pertinent to note that the nature and effect of CP is a major subject of discussion in modelling TC settings. However, only very little has been done on the effect of credit response time length on important decision variables and payoffs. This is the centre of this work.

**MATERIALS AND METHODS**

**The Model**

This paper considers a TC situation in which the supplier provides credit  $\psi$  to the retailer instead of direct financial assistance. The retailer is expected to repay this credit gesture by the time  $t_s$ . The retailer engages in product promotion with an effort  $\varphi_R$  as a motivational strategy to sell the supplier's product. He also provides credit to his end-users allowing a credit repayment time  $t_R$ . We note that Figure 1 illustrates the flow of credit from supplier through the retailer to the consumer, and the cash flow repayments in reverse order. Figure 2 shows the timeline of credit repayment.

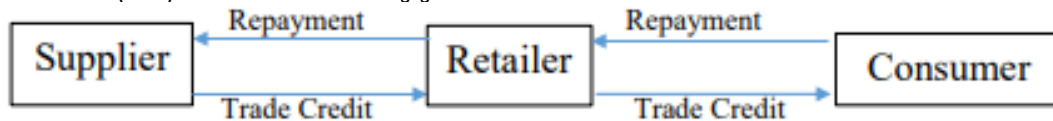


Figure 1. Flow of Credit and Cash Repayment

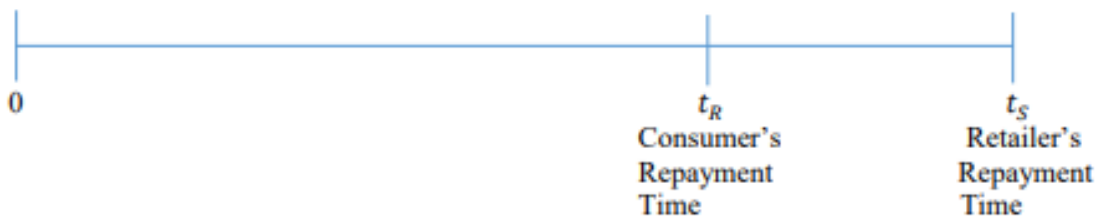


Figure 2. Timeline of Credit Repayment

**Notations**

We employ the following notations:

- $M_R$  Retailer's margin
- $M_S$  Supplier's margin
- $\varphi_R$  Retailer's promotion effort
- $\beta$  Promotion effectiveness
- $t_R$  Retailer's credit repayment time to the end-user

- $t_S$  Supplier's credit repayment time to the retailer
- $W_R$  Retailer's payoff function
- $W_S$  Supplier's Payoff function

**Promotion Function**

Advertising is largely used as a persuasive mechanism on a given audience towards taking action regarding sales or services. On the

other hand, sales promotion is employed as a temporary marketing campaign strategy by a firm, usually to arouse interest and product/service demand. Thus, these two strategies are usually employed to boost sales. However, while advertising can be employed for a relatively long time, promotion is a short term approach. Thus considering the close relationship between these two concepts we adopt the advertising-sales function

$$f(\varphi_R) = \beta\sqrt{\varphi_R} \quad (1)$$

as our promotion-sales function, where  $\beta$  is the promotion effectiveness. A form of this function has been employed by Xie and Wei (2009) and Ezimadu (2019a). A similar trend can be found in Ezimadu and Nwozo (2018) in a dynamic setting, Ezimadu and Nwozo (2019) in a dynamic situation with retail competition, and He *et al.* (2009) in a stochastic setting. This is in tandem with the commonly observed saturation effect exhibited as diminishing returns on advertising, and promotion by extension, in the short term (Simon and Arndt (1980), Karray and Zaccour (2006), Ezimadu (2019b)).

### Credit Function

We consider a refinement of Ezimadu-Ezimadu credit function (Ezimadu and Ezimadu 2022). This refinement considers the distance in time between two channel members' credit period  $t_S - t_R$ . We note that a large  $t_S - t_R$  implies early repayment while a small value implies a much later repayment. It is natural for the supplier to give credit to his retailer for early repayment, and reduce it as repayment time prolongs. Thus we have that

$$\psi \propto t_S - t_R. \quad (2)$$

To increase revenue and payoff, the supplier would prefer a large margin. This can negatively affect the retailer. Thus as a motivation he would want to increase his credit to the retailer with increase in margin. That is

$$\psi \propto M_S. \quad (3)$$

The retailer's promotion effort is fundamental to the sale of the supplier's product. Thus to motivate the retailer, the supplier will let his credit increase with retailer's promotion effort. As such we have that

$$\psi \propto \sqrt{\varphi_R}. \quad (4)$$

Thus from (1), (2), (3) and (4) we have that

$$\begin{aligned} \psi(M_S, \varphi_R, t_S - t_R) \\ = \gamma M_S \beta \sqrt{\varphi_R} (t_S - t_R), \end{aligned} \quad (5)$$

where  $\gamma$  is the constant of proportionality. We note that credit function was also employed by Ezimadu and Ezimadu (2023) in a Stackelberg game setting.

## RESULTS

### Credit Scenario Stackelberg Equilibrium

Each player's payoff is given by

$$\begin{aligned} \text{Payoff} = \text{Price Margin} \times \text{Product Demand} \\ - \text{Expenditure} + \text{Credit Received} \\ - \text{Credit Given}. \end{aligned}$$

Thus we have that the retailer and the supplier's payoffs are given by

$$\begin{aligned} \max_{\varphi_R > 0} W_R = M_R f(\varphi_R) - \varphi_R \\ + \psi(M_S, \varphi_R, t_S - t_R) \end{aligned}$$

and

$$\begin{aligned} \max_{t_S \geq t_R > k} W_S = M_S f(\varphi_R) \\ - \psi(M_S, \varphi_R, t_S - t_R) \end{aligned}$$

respectively. That is.

$$\begin{aligned} \max_{\varphi_R > 0} W_R = M_R \beta \sqrt{\varphi_R} - \varphi_R \\ + \gamma M_S \beta \sqrt{\varphi_R} (t_S - t_R) \end{aligned} \quad (6)$$

and

$$\begin{aligned} \max_{t_S \geq t_R > k} W_S = M_S \beta \sqrt{\varphi_R} \\ - \gamma M_S \beta \sqrt{\varphi_R} (t_S - t_R). \end{aligned} \quad (7)$$

Now, maximizing (6) with respect to  $\varphi_R$  we have

$$\frac{\partial W_R}{\partial \varphi_R} = \frac{M_R \beta}{2\sqrt{\varphi_R}} - 1 + \frac{\gamma M_S \beta (t_S - t_R)}{2\sqrt{\varphi_R}} = 0$$

implying

$$\varphi_R = \left[ \frac{\beta(M_R + \gamma M_S(t_S - t_R))}{2} \right]^2. \quad (8)$$

Rearranging (7) we have

$$\begin{aligned} \max_{t_S \geq t_R > k} W_S = [M_S \beta \\ - \gamma M_S \beta (t_S - t_R)] \sqrt{\varphi_R}. \end{aligned} \quad (9)$$

Using (8) in (9) we have

$$\begin{aligned} \max_{t_S \geq t_R > k} W_S \\ = \beta [M_S \\ - \gamma M_S (t_S - t_R)] \left[ \frac{\beta(M_R + \gamma M_S(t_S - t_R))}{2} \right]. \end{aligned} \quad (10)$$

Maximizing we have

$$\begin{aligned} \frac{\partial W_S}{\partial t_S} = \frac{\beta^2}{2} \{ [M_S - \gamma M_S t_S + \gamma M_S t_R] (\gamma M_S) \\ + [M_R + \gamma M_S t_S - \gamma M_S t_R] (-\gamma M_S) \} \\ = 0, \end{aligned}$$

implying

$$t_S = \frac{2\gamma M_S t_R - M_R + M_S}{2\gamma M_S}. \quad (11)$$

Rearranging (6) we have

$$\begin{aligned} \max_{\varphi_R > 0} W_R \\ = [M_R \beta + \gamma M_S \beta (t_S - t_R)] \sqrt{\varphi_R} - \varphi_R. \end{aligned} \quad (12)$$

Using (8) and (11) in (12) we have

$$\begin{aligned} W_R = [M_R \beta + \gamma M_S \beta (t_S - t_R)] \\ - t_R \left[ \frac{\beta(M_R + \gamma M_S(t_S - t_R))}{2} \right] \\ - \left[ \frac{\beta(M_R + \gamma M_S(t_S - t_R))}{2} \right]^2 \\ = \frac{\beta^2 [M_R + \gamma M_S(t_S - t_R)]^2}{4} \end{aligned} \quad (13)$$

$$\begin{aligned} = \frac{1}{4} \beta^2 \left[ M_R + \gamma M_S \left( \frac{2\gamma M_S t_R - M_R + M_S}{2\gamma M_S} \right) \right]^2 \\ - \gamma M_S t_R \\ = \frac{1}{16} \beta^2 (M_R + M_S)^2. \end{aligned} \quad (14)$$

We observe that (10) can be expressed as

$$\begin{aligned} & \max_{t_S \geq t_R > k} W_S \\ & = [\beta M_S - \gamma \beta M_S t_S \\ & + \gamma \beta M_S t_R] \left[ \frac{\beta M_R + \beta \gamma M_S t_S - \beta \gamma M_S t_R}{2} \right]. \end{aligned} \quad (15)$$

Using (11) in (15) we have

$$= \frac{1}{2} \left[ \frac{\beta (M_S + M_R)}{2} \right]^2. \quad (16)$$

Let  $C_{RL} = t_S - t_R$  be the credit response time length. This means that a large  $C_{RL}$  implies that the distance between allowable payment time  $t_S$  and actual payment time  $t_R$  is large. This implies early payment. On the other hand a small  $C_{RL}$  means late payment by the retailer. Thus (13) becomes

$$= \frac{\beta^2 [M_R + \gamma M_S C_{RL}]^2}{4}, \quad (17)$$

and (10) becomes

$$\begin{aligned} & \max_{t_S \geq t_R > k} W_S \\ & = \beta [M_S - \gamma M_S C_{RL}] \left[ \frac{\beta (M_R + \gamma M_S C_{RL})}{2} \right]. \end{aligned} \quad (18)$$

Now, maximizing we have

$$\frac{\partial W_S}{\partial C_{RL}} = \frac{\beta^2 (M_S - \gamma M_S C_{RL})(\gamma M_S C_{RL} + M_R)}{2 C_{RL}} = 0,$$

implying

$$\frac{C_{LR}}{1} = \frac{1}{\gamma} \quad (19)$$

or

$$C_{LR} = -\frac{M_R}{\gamma M_S}. \quad (20)$$

Since  $C_{LR} \leq 0$  we have that (19) is the appropriate for  $C_{LR}$ .

### No-Credit Equilibrium

In the absence of credit we have that (6) can be expressed as

$$\begin{aligned} \max_{\varphi_R > 0} W_R & = M_R \beta \sqrt{\varphi_R} \\ & - \varphi_R, \end{aligned} \quad (21)$$

so that

$$\frac{\partial W_R}{\partial \varphi_R} = \frac{M_R \beta}{2\sqrt{\varphi_R}} - 1 = 0$$

implying that

$$\varphi_R = \left( \frac{\beta M_R}{2} \right)^2. \quad (22)$$

Further, using (22) in (7) for a situation where there is no provision of credit we have that

$$W_S = \frac{\beta^2 M_R M_S}{2}. \quad (23)$$

Using (22) in (21) we have

$$W_R = \frac{1}{4} \beta^2 M_R^2. \quad (24)$$

### DISCUSSION

This work is based on a Stackelberg game in which the supplier is the channel leader, while the retailer is the follower. Based on the first-mover's advantage we have that  $M_S > M_R$ . We let  $M_S = 200$  and  $M_R = 180$ .  $\beta$  is the promotion effectiveness which

indicates the rate of response to promotion, so that  $\beta \in [0,1]$ . Thus we let  $\beta = 0.2$ . We let the retailer's credit period to be  $t_R = 80$ , and to minimize the possible effect of credit on the payoffs we let  $\gamma = 0.01$ .

### Effect of the Supplier's Credit Period and Credit Response Time Length on Promotion Effort

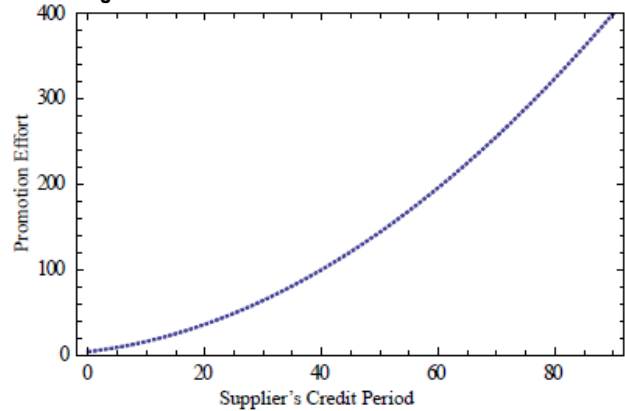


Figure 3: Illustration of the Effect of the Supplier's Credit Period on Promotion Effort

From Figure 3 we observe that as the supplier's credit period increases, the promotion effort also increases. We note that increasing the supplier's credit period implies late repayment which goes with low credit support from the supplier to the retailer. In this situation, despite the low credit, he is constrained to increase promotion effort, because of the increase in the length of the credit period which in a sense suggests elongation of the sales period which implies (that is, leads to) increase in promotion effort.

Figure 4 shows that as the credit response length increases, and tends to the supplier's credit period  $t_S$ , that is, as repayment gets more and more earlier which is the same as saying that as early repayment time increases, the promotion effort also increases. That is, the more earlier repayment is made by the retailer, the more he engages in promotion. This is because early repayment will require more effort to equally ensure early sale to guarantee his revenue, and hence payoff.

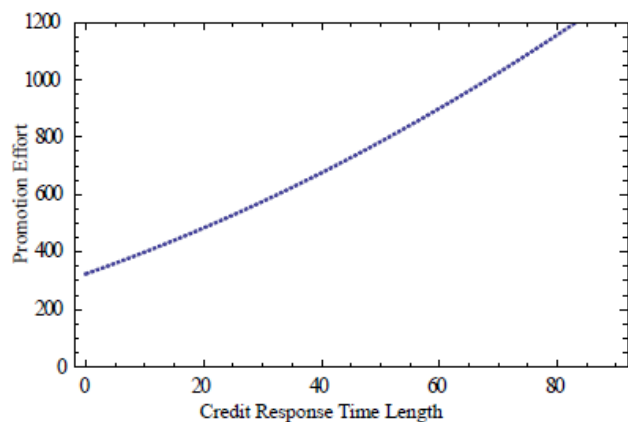
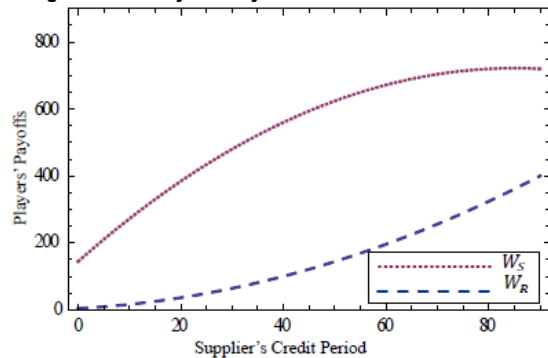


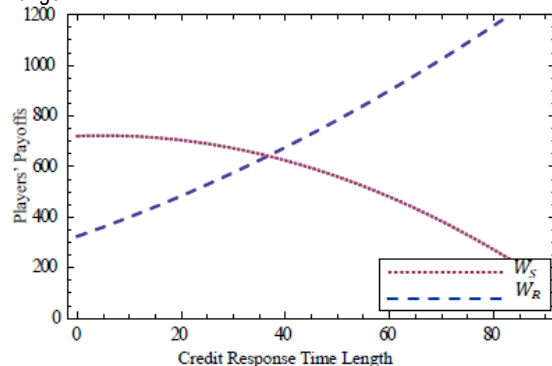
Figure 4: Illustration of the Effect of Credit Response Time Length on Promotion Effort

**Effect of Supplier's Credit Period and Credit Response Time Length on the Players' Payoff**



**Figure 5:** Illustration of the Effect of Supplier's Credit Period on Players' Payoff

Clearly, the plots in Figure 5 indicate that increase in the supplier's allowable credit period to the retailer results in increase in the both channel members' payoff. However, while the retailer's payoff appears to exhibit continuous increase, the supplier's payoff exhibits diminishing marginal returns which will apparently become clear diminishing return as his allowable credit gets elongated beyond the normal time. The implication is that being in-charge of the allowable period, the supplier is better positioned to dictate terms to the retailer, and should utilize his first-mover's advantage to ensure that his allowable credit period is within his favourable range.



**Figure 6:** Illustration of the Effect of Credit Response Time Length on Players' Payoff

From the plots in Figure 6 we have that as the credit response time length increases, the supplier's payoff which appears larger than that of the retailer exhibits a downward trend. That is, for every increase in the credit response length (which implies earlier repayment), the supplier's payoff reduces because sales time is short which may imply low patronage. On the other hand, the retailer's payoff increases with the credit response length. This clearly follows from Figure 4 where the promotion increases with the credit response length. This increase in promotion resulting from increase in the credit response length leads to continuous increase in the retailer's payoff.

We note that a competitively dominant supplier would want a situation where

$$W_S > W_R.$$

We note that at the point where both payoffs are equal we have that

$$\beta[M_S - \gamma M_S C_{RL}] \left[ \frac{\beta(M_R + \gamma M_S C_{RL})}{2} \right] = \frac{\beta^2 [M_R + \gamma M_S C_{RL}]^2}{4}$$

implying

$$C_{RL} = \frac{M_S - 2M_R}{3\gamma M_S} + \frac{M_R + M_S}{3\gamma M_S},$$

so that

$$C_{RL} = -\frac{M_R}{\gamma M_S} < 0$$

or

$$C_{RL} = \frac{2M_S - M_R}{3\gamma M_S}$$

By the first-mover's advantage  $M_S > M_R$ .

Therefore

$$C_{RL} = \frac{2M_S - M_R}{3\gamma M_S}$$

$> 0$

$$(25)$$

is more appropriate.

Thus for  $W_S > W_R$  we must have that the choice of  $C_{RL}$  must be less than that obtained in (25) above.

### Effect of the Various Scenarios on the Payoffs

**Table 1:** A Comparison of the Payoffs for the Various Scenarios

Scenario s Payoffs	No- Credit Scenari o ( $t_S$ = $t_R$ = 0)	Optima l Trade Credit ( $t_S, t_R$ > 0)	Optimal Credit Respons e Length ( $t_S -$ $t_R \neq 0$ )	Extreme Credit Respons e Length ( $t_S$ $\neq 0, t_R$ = 0)
$W_R$	324	361	361	1225
$W_S$	720	722	722	210
<b>Channel Payoff</b>	1044	1083	1083	1435

From Table 1 we observe that the players' payoffs are larger with credit provision (which is the case of optimal  $C_{RL}$ ) than where there is no credit. We also note a similar trend for the entire channel payoff. However, we observe a different trend for very large credit response time length. In this case, while the retailer's payoff is very large, the supplier's payoff is much smaller compared to that of the retailer and his payoffs in the other scenarios. The implication is that where it is possible, the supplier should adopt credit provision instead of a no-credit. Further he can opt for very early repayment provided he has a favourable agreement which will earn him larger returns than what obtains in the other scenario since the channel payoff in this situation is larger.

### Conclusion

This work considered a TC supply channel involving a supplier who is the Stackelberg leader and a retailer who sells only the supplier's brand in a product class. The work used a credit function which is based on the supplier's credit period and the time distance between the two players' credit periods which we called the credit response time length to model a situation in which the supplier provides credit to the retailer allowing a delayed repayment time. The retailer in-



turn allows the end-user a delayed repayment time while also engaging in product promotion. The paper obtained the retailer's optimal promotion effort, and the supplier's optimal credit period. It also obtained a closed-form Stackelberg equilibrium solution for the payoffs for the credit and no-credit scenario.

The work shows that the retailer increases promotion effort as the supplier's credit period increases. It observed that the promotion effort increases with the credit response time length. The payoffs increase with the supplier's credit period, with supplier benefiting more from his extension of credit period. It observed that while early repayment is beneficial to the retail, it is less beneficial to the supplier. The reverse is the case with late repayment. Thus to ensure a better payoff the supplier can insist on a sharing contract that guarantees late repayment or better still focus on optimal length.

The work examined a supplier-Stackelberg bilateral monopoly TC supply channel with retailer as follower. An improvement can use a modification of the model to study a situation where a distributor is incorporated into a manufacturer-retailer channel, with the manufacturer and distributor providing trade credit to the retailer as was considered by Ezimadu (2016) with the provision of subsidy. Another extension can consider a multiple suppliers-multiple retailers channel where all the players engage in promotion. These extensions can provide more understanding on trade credit.

## REFERENCES

- Bai, S.-z. (2009). Research on credit period contract under the uncertain cost information of retailer. 2009 Chinese Control and Decision Conference, Guilin, China, 3131-3135. doi: 10.1109/CCDC.2009.5192922
- Bi, G., Wang, P., Wang, D. & Yin, Y. (2021). Optimal credit period and ordering policy with credit-dependent demand under two-level trade credit. *International Journal of Production Economics*, 242, Article 108311.
- Chu, L. Y., Yang, S. A. and Yao, J. (2024). Trade Credit and Operational Investment: A Bargaining Perspective. SSRN <https://ssrn.com/abstract=4727605>  
<http://dx.doi.org/10.2139/ssrn.4727605>
- Costello, A. (2013). *The determinants and economic consequences of trade credit policy*. [http://leeds-faculty.colorado.edu/zeyun/workshop13-14/files/CostelloTC\(October2013\).pdf](http://leeds-faculty.colorado.edu/zeyun/workshop13-14/files/CostelloTC(October2013).pdf).
- Emtehani, F., Nahavandi, N. & Rafiei, F. M. (2023). Trade credit financing for supply chain coordination under financial challenges: a multi-leader-follower game approach. *Financial Innovation*, 9(6). <https://doi.org/10.1186/s40854-022-00401-1>
- Ezimadu, P. E. & Nwozo, C. R. (2018). Modeling Dynamic Cooperative Advertising in a Decentralized Channel. *Yugoslav Journal of Operations Research*, 28(4): 539-566. <https://yujor.fon.bg.ac.rs/index.php/yujor/article/view/45>
- Ezimadu P. E. & Nwozo C. R. (2019). A manufacturer-retailers dynamic cooperative advertising with retail competition. *International Journal of Operational Research*, 34(1): 104-143.
- Ezimadu, P. E. (2016) Cooperative Advertising in a Manufacturer-Distributor-Retailer Supply Chain. *Transactions of the Nigerian Association of Mathematical Physics*, 2:205-216. <http://e.nampjournals.org/product-info.php?pid3141.html>
- Ezimadu, P. E. (2019a). A Game-Theoretic Cooperative Advertising Model: The Feasibility of the Distributor's Involvement in a Manufacturer-Distributor-Retailer Channel. *FUW Trends in Science & Technology Journal*, 4(2): 416-421. <http://www.ftstjournal.com/Digital%20Library/42%20Article%2019.php>
- Ezimadu, P. E. (2019b). Modelling Subsidy as a Cooperative Advertising Channel Coordination Mechanism. *Nigerian Journal of Basic and Applied Science*, 27(2): 127-135. <http://dx.doi.org/10.4314/njbas.v27i2.17>  
<http://www.njbas-udus.com/index.php?fulltxt=49480&fulltxtj=97&fulltxtp=97-1558109741.pdf>
- Ezimadu, P. E. & Ezimadu, S. O. (2022). A Game-Theoretic Credit Period and Promotion Model in a Supplier-Retailer Channel. *Asian Research Journal of Mathematics*, 18(11): 351-361. <https://journalarjom.com/index.php/ARJOM/article/view/609> <https://doi.org/10.9734/arjom/2022/v18i11609>
- Ezimadu, S. O. & Ezimadu, P. E. (2023). A Three-Level Game-Theoretic Trade Credit Model in a Supply Channel. *Journal of Advances in Mathematics and Computer Science*, 38(7): 146-159.
- Ferrando, A. & Mulier, K. (2013). Do firms use the trade credit channel to manage growth? *Journal of Banking & Finance*, 37(8): 3035-3046.
- Fabbri, D. & Klapper, L. F. (2016). Bargaining power and trade credit. *Journal of Corporate Finance*, 41: 66-80.
- Fafchamps, M., Pender, J. & Robinson, E. (1995). *Enterprise finance in Zimbabwe*. RPED Case Study Series. World Bank, Washington DC.
- He, X., Prasad, A. & Sethi, S. P. (2009). Cooperative Advertising and Pricing in a Dynamic Stochastic Supply Chain: Feedback Stackelberg Strategies. *Production and Operations Management*, 18(1): 78-94. <https://onlinelibrary.wiley.com/doi/10.1111/j.1937-5956.2009.01006.x>
- Hovelaque, V., Viviani, J.-L. & Mansoor, M. A. (2022). Trade and bank credit in a non-cooperative chain with a price-sensitive demand. *International Journal of Production Research*, 60(5): 1553-1568.
- Jana, R. K. & Mondal, R. (2024). A Supplier-Stackelberg Game Model under Two Levels of Trade Credit Considering Default-Risk. *International Journal of Supply and Operation Management*, 11(3): 334-350. [http://www.ijom.com/article\\_2939.html](http://www.ijom.com/article_2939.html)
- Jin, W. & Wang, C. (2020). Modeling and simulation for supply chain finance under uncertain environment. *Technological and Economic Development of Economy*, 26(4): 725-750. <https://doi.org/10.3846/tede.2020.12054>
- Karray, S. & Zaccour, G. (2006). Could co-op advertising be a manufacturer's counterstrategy to store brands? *Journal of Business Research*, 59: 1008-1015.
- Klapper, L., Laeven, L. & Rajan, R. (2012). Trade credit contracts. *Review of Financial Studies*, 25(3): 838-867.
- Lefebvre, V. (2023). Trade credit, payment duration, and SMEs' growth in the European Union. *International Entrepreneurship and Management Journal*, 19: 1313-1340. <https://doi.org/10.1007/s11365-023-00871-4>

- Ng, C. K., Smith, J. K. & Smith, R. L. (1999). Evidence on the determinants of credit terms used in interfirm trade. *The Journal of Finance*, 54(3): 1109-1129.
- Mahata, G.C. (2015). Retailer's optimal credit period and cycle time in a supply chain for deteriorating items with up-stream and down-stream trade credit. *Journal of Industrial Engineering International*, 11:353-366.
- Maiti, T. & Giri, B. C. (2015). A closed loop supply chain under retail price and product quality dependent demand. *Journal of Manufacturing Systems*, 25(3): 624-637.
- Manna, A. K., Das, B., Dey, J. K. & Mondal, S. K. (2017). Two layers green supply chain imperfect production inventory model under bi-level credit period. *Review of Applied Management Studies*, 15(2): 124-142.
- Patel, D. G., Shah, N. H., & Shah, D. B. (2018). Optimal Credit Period and Lot-Size for Deteriorating Items with Fixed Life Time and Trade Credits, *Handbook of Research on Promoting Business Process Improvement through Inventory Control Techniques*, pp14. DOI: 10.4018/978-1-5225-3232-3.ch003
- Silaghi, F. & Moraux, F. (2022). Trade credit contracts: Design and regulation. *European Journal of Operational Research*, 296(3): 980-992. <https://www.sciencedirect.com/science/article/pii/S0377221721003660>
- Simon, J. L. & Arndt, J. (1980). The shape of the advertising function. *Journal of Advertising Research*, 20: 11–28.
- Tabash, M. I., Farooq, U., Hamouri, B. & Kumar, A. (2023). Effect of country governance on trade credit activities: Empirical evidence from Pakistan. *Cogent Economics & Finance*, 11(2). <https://doi.org/10.1080/23322039.2023.2233315>
- Tsao, Y-C., Fauziah, H-A., Vu, T-L. & Masrurroh, N.A. (2022). Optimal pricing, ordering, and credit period policies for deteriorating products under order-linked trade credit. *Journal of Industrial and Management Optimization*, 18(6): 4151-4182.
- Wu, C., Liu, X. & Li, A. (2021). A loss-averse retailer–supplier supply chain model under trade credit in a supplier-Stackelberg game. *Mathematics and Computers in Simulation*, 182: 353-365. <https://www.sciencedirect.com/science/article/pii/S0378475420303803>
- Wu, D., Zhang, B. & Baron, O (2018). A trade credit model with asymmetric competing retailers. *Production and Operations Management*, 28(1): 206-231. <https://doi.org/10.1111/poms.12882>
- Wu, J., Ouyang, L-Y., Cardenas-Barron, L. E. & Goyal, S. K. (2014). Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing. *European Journal of Operational Research*, 237(3): 898-908.
- Xie, J. & Wei, J. (2009). Coordination advertising and pricing in a manufacturer-retailer channel. *European Journal of Operational Research*, 197:785-791. <https://www.sciencedirect.com/science/article/pii/S0377221708004992>
- Yang, Y., Liu, J. & Hu, T. (2023). Capital allocation and pricing decisions under trade credit with time-sensitive stochastic demand. *Transportation Research Part E: Logistics and Transportation Review*, 173, p.103093. <https://doi.org/10.1016/j.tre.2023.103093>
- Zhang, C., You, M. & Han, G. (2021). Integrated supply chain decisions with credit-linked demand: A Stackelberg approach. *Scientia Iranica*, 28(2): 927-949.
- Zhou, W., Hu, Y., Lin, T. & Ding, Z. (2022). Trade Credit and Buyer Financing. *Journal of Systems Science and Systems Engineering*, 31(3): 288-312. <https://doi.org/10.1007/s11518-022-5530-z>