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Full Length Research Article

A ROBUST EVALUATION OF THE EXPONENTIAL-LOGARITHMIC DISTRIBUTION AND ITS APPLICATION TO TRADE CREDIT PERIOD DATA FOR PERISHABLE GOODS (A CASE STUDY OF AGBOR FRUIT MARKET, AGBOR, DELTA STATE)

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ABSTRACT
In this work, the Exponential-Logarithmic Distribution (ELD) was used to analyze real-life data, and some properties of the distribution were discussed with the help of the Mathematica software. Simulation study was also carried out for different values of the parameters of some distributions with the aid of the mathematical package, and observations were recorded. Comparison between the theoretical moments and the raw moment shows that the Exponential-Logarithmic distribution is a good fit for the data. The method of maximum likelihood estimation was used (mle) to estimate the model parameters. The score function could not be solved directly since it is a non-linear system of equation and the Newton Raphson's iterative method was used for the numerical computation of the parameter estimates. This was achieved with the use of the mass, stat4, nacopula and fitdistrplus packages in R software. The model was applied to a real-life problem on the length of time (trade credit period) it takes a retailer to pay back goods bought on credit from the supplier before he is debt free in Ika-south local government area, Delta state and prediction was made with the aid of the survival and the hazard function. At time zero, the probability of "survival" (that is the probability that a retailer remains a debtor immediately after the purchase is 1.0). This is the same as saying that 100% of retailers remain debtors immediately after purchase. Now, the median survival is approximately 26 hours. Thus, the length of time (credit period) from after purchase that half of the retailers receive trade credit funding is 26 hours that is to say, half of them will "survive" trade credit funding after 26 hours.

Keywords: trade credit period, exponential-logarithmic distribution (ELD), survival function, hazard function, supplier

INTRODUCTION
Trade credit is a short-term financing that exist in almost all businesses. Trade credit may be considered as a substitute for money between a supplier and a retailer in a supply chain. Olushola and Olushola (2012), asserted that there had been little study on the use of trade credit as a means of funding in Nigeria. Paul et al. (2021), defined trade credit period as the time frame between when an order is made and when payment is received.
On trade credit provision, Ezimadu and Ezimadu (2023b), analyzed a Stackelberg game-theoretic retailer-manufacturer trade credit period ratio to ascertain the amount of credit the retailer can receive from the supplier, they also considered a refinement of the Ezimadu and Ezimadu credit function [Ezimadu and Ezimadu (2022)], which was modified by Ezimadu and Ezimadu (2023c) and their result showed that the promotion effort increases with the supplier's credit period.
On a three-level game-theoretic trade credit model in a supply chain, Ezimadu and Ezimadu (2023a), used promotion effort, price margins and credit period dependent credit function, and they observed that the distributor involvement is very crucial to the payoffs.
On the other hand, survival analysis can be defined as a collection of statistical methods that focus on duration and timing until an event occurs. It examines how long it will take an event of interest to occur, like from being single to getting married, from birth to death, from patient with terminal disease to death, from the time of admission into an institution, to the time of graduation and so on. Since it deals with time analysis to the occurrence of an event, time here becomes a very important aspect of these models used to analyze them and it can be in different units such as seconds, hours, days, weeks, months or years. The time it will take before an event occurs is known as 'The survival time'. It is the time duration that an organism 'survives' the given time.
Survival time of organisms, structures, materials, is very important in biological and engineering sciences. This is a significant part of a study and is keen to modelling the lifetime data by a failure distribution. The exponential, Rayleigh, and Weibull distributions are the most frequently used distributions in survival analysis and life testing. These distributions have a number of desired properties and good physical explanations. However, the exponential distribution has a constant failure rate (due to its one parameter $h(t) = \lambda$) while the Rayleigh distribution has increasing failure rate. The Weibull distribution generalizes both distributions due to the fact that it has increasing, constant and decreasing failure rates. The Weibull distribution is able to model fatigue failure and was successfully used to describe vacuum tube failure and ball bearing failure (Gleser, 1989).

Compounding distributions to model lifetime data have been introduced by mixing a continuous lifetime and power series distributions. Esmaeili and Niaparast (2022), introduced the exponentiated half logistics-power series which was obtained by compounding the exponentiated half-logistic distribution with power series distributions. Also, Altun et al., (2022) obtained a new one-parameter discrete distribution by compounding the Poisson and xgamma distributions. Shakhatreh et al., (2021) obtained a novel class of distributions called the compound inverse Lindley power series distributions by compounding the inverse Lindley distribution and power series distributions. Silva and Cordeiro (2015), obtained a new distribution known as the Burr XII power

series distribution by compounding the Burr XII distribution and the power series distribution. The exponential geometric distribution (EGD), exponential Poisson distribution (EPD) and exponential logarithmic distributions (ELD) were introduced and studied by Adamidis and Loukas (1998), Kus (2007) and Tahmasbi and Rezaei (2008), respectively. Recently, Chahkandi and Ganjali (2009) introduced the exponential power series (EPS) distributions.
The cumulative distribution function (cdf), probability density function (pdf), survival and hazard functions of a distribution are often used in modeling Lifetime data, this is due to the fact that they give insight into the way the different parameters affect the behavior of the functions, especially the hazard function (failure rate).
A number of models are used to analyze survival data. These models include parametric, non-parametric and semi- parametric.
Parametric models assume that the underlying distribution of the survival times follow certain known probability distribution. Examples of popular distribution include the Weibull, lognormal, gamma and so on.
In non-parametric models, the Kaplan Meier method is widely used to estimate the parameters of the models and it is used to show the graph of the probability of survival as a function of time.
Semi-Parametric models make fewer assumptions than the typical parametric models. A good example is the Cox regression model which is used for modeling data. It provides useful information regarding the relationship of the hazard function.
There are some existing literatures that considers decreasing failure rate. These distributions are proposed by the concept of mixing (compounding) two or more well-known distributions. These distributions are categorized as those that may have caused some wind of improvement in a particular system of study.
If the functions of these distributions are monotone, then they have interesting properties for lifetime distributions. Several authors have reported cases where the hazard function decreases with time. Some striking examples are that of business mortality discussed by Lomax (1954), failure in semiconductors from various lots or in the air-conditioning equipment of a fleet of Boeing 720 aircraft's by Proschan (1963). Also, Saunders and Myhre (1983), discussed failure rate in the life of integrated circuit modules.
Generally, a population is likely to exhibit decreasing failure rate when its behavior over time is categorized by "work hardening" (in engineering terms) or "immunity" (in biological terms); at times the broader term "infant mortality" is used to denote the decreasing failure rate phenomenon.
Subsequent improvement of survival with time may occur by means of physical alterations that produced self-development, or due to population heterogeneity. Certainly, Proschan (1963), provided that the decreasing failure rate stuff is inherent to mixtures of distributions with constant failure rate. McNolty et al., (2012), also discussed some other properties of exponential mixtures. Gleser (1989), established the converse for any gamma distribution with shape parameter less than one. In addition, Gurland and Sethuramm (1994), gave some examples showing that such results may also embrace mixtures of distributions with speedily increasing failure rate.
Kumar and Satheenthar (2021), developed a new class of lifetime distribution having decreasing failure rate by compounding the exponential distribution with the positive hyper-Poisson distribution. A mixture of truncated geometric distribution and exponential distribution with decreasing failure rate was introduced by Adamidis and Loukas (2005). The exponential-Poisson (EP) distribution was proposed by Kus (2007), and was later generalized by Hemmati et al., (2011) using Wiebull distribution. Also, Preda et al., (2011) modified it as a particular case of the modified exponential-Poisson distribution.
Also, the exponential-logarithmic distribution was discussed by Tahmasbi and Rezaei (2008). Silva et al., (2010), proposed a new distribution with decreasing, increasing and upside-down bathtub failure rate. Chahkandi and Ganjali (2009), proposed a two-parameter distribution with decreasing failure rate by mixing power-series distribution.
A Weibull power series class of distributions with Poisson distribution was presented by Morais and Barreto-Souza (2011). Morais (2009), in a master degree thesis presented a class of generalized beta distributions, pareto power series and Weibull power series. Lately, Alkarni and Oraby (2012), and Alkarni (2012), discussed a class of truncated Poisson and logarithmic distributions with any continuous lifetime distribution.
The Exponential-Weibull distribution was obtained by Mudholkar and Srivastava (1993), to extend the generalized exponential distribution. This distribution was also studied by Mudholkar et al., (1995), Mudholkar and Hutson (1996) and Nassar and Eissa (2003). Barreto-Souza and Cribari-Neto (2009) introduced the generalized exponential-Poisson distribution which extends the exponential-Poisson distribution in similar way that the generalized exponential distribution extends the exponential distribution. Pappas et al., (2014) introduced the generalized exponential logarithmic distribution which extends the exponential-logarithmic distribution.
Alkarni and Oraby (2012), investigated a new lifetime class of truncated Poisson distribution and a lifetime distribution that exhibits decreasing failure rate and they named it 'A new life class', because it generalizes other distribution which have been studied in literature, they focused on lifetime distributions with nice forms and shapes and it was observed that since this new class is not part of the exponential family, its moment doesn't have a simple form.
Alkarni (2012), presented a new lifetime distribution with decreasing failure rate by compounding truncated binomial with any continuous lifetime distribution, its properties were discussed, its maximum likelihood estimate was obtained through the Expectation Maximization (EM) algorithm and in other to obtain the asymptotic covariance matrix, the Fisher information matrix was derived.
Asgharzadeh et al., (2014), introduced a family of continuous lifetime distribution by compounding a distribution that is continuous and the Poisson-Lindley with hazard function also decreasing, its properties were considered and the method of maximum likelihood estimates was used for the estimation of its parameters.
Hassan et al., (2015), proposed a new class of continuous lifetime distributions, this new class complements the Poisson-Lindley distribution as proposed by Asgharzadeh et al., (2014), It is derived by compounding the maximum of an independent and identically distributed (iid) random variables with the Poisson-Lindley distribution. The properties of this distribution were discussed and the estimates of the parameters were obtained through the method of maximum likelihood estimates.

MATERIALS AND METHODS

The Exponential-Logarithmic Distribution (ELD)

Tahmasbi et al., (2008) derived the exponential-logarithmic

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distribution (ELD) using the probability density function (PDF) of the exponential distribution and the probability mass function (PMF) of the logarithmic distribution.

In this work, we show the derivation of the Exponential-Logarithmic Distribution, (ELD), its Moments, Survival Function, Hazard Function and Estimation of its parameters using the Maximum Likelihood Estimate (MLE) method.

The probability density function of the exponential distribution

Let $v_1, v_2, \ldots, v_z$ be a random sample from an exponential distribution with probability density function (pdf) as follows:

$$f_v(v,\beta) = \beta e^{-\beta v}, \qquad \beta > 0, \qquad (1)$$

where $1/\beta$ is the scale parameter.

The probability mass function of the logarithmic distribution

Let z be a random variable from a logarithmic distribution with probability mass function (PMF) given by

$$P_{(z)}(z,p) = \frac{(1-q)^z}{-z\ln q}, \quad z = 1,2,\ldots \qquad 0 < q < 1 \qquad (2)$$

where q is the shape parameter.

The Compounding of the ELD

Let the random variables v and z be independent and identically distributed (iid),

v_i denotes the failure time of the ith system and z is the total number of functioning systems. In other words, the lifetime is represented by the $v's$ and each $z's$ defects are detected only after causing failure.

Let T denote the lifetime of z where,

$$T = \min(v_1, v_2, \ldots v_z)$$

By definition, the cumulative density function $F(t)$ with probability density function $f(t)$ is given by

$$F(t) = \Pr(T \le t)$$

Then, the conditional cumulative distribution of T given z is

$$F(T|z) = Pr(T \le t|z) = 1 - Pr(T \le t|z) = 1 - \Pr(v_1 > t_1, v_2 > t_2, \ldots, v_z > t_z) = 1 - Pr^{(z)}(t|z) = 1 - [1 - F_v(v;\beta)]^z \qquad (3)$$

Recall, from (1) that

$$f_v(v,\beta) = \beta e^{-\beta v}$$

then,

$$F_V(v,\beta) = \int_0^t f_v(v,\beta)dv = \int_0^t \beta e^{-\beta v}\,dv = [-e^{-\beta v}]_0^t = 1 - e^{-\beta t}.$$

Thus, we have that

$$F(T|z) = 1 - \left[1 - \left(1 - e^{-\beta t}\right)\right]^z = 1 - \left[e^{-\beta t z}\right]$$

$$= 1 - e^{-\beta t z}, \qquad z = 1,2,\ldots$$

Hence, the conditional probability density function of T given z is

$$f(t|z) = \frac{d}{dt}F(T|z) = \frac{d}{dt}\left(1 - e^{-\beta t z}\right) = \beta z e^{-\beta t z}$$

The joint pdf of T and z is obtained as

$$f_{t,z}(t,z;q,\beta) = \frac{(1-q)^z}{-zInq}\beta z e^{-\beta t z} = \frac{(1-q)^z}{-Inq}\beta e^{-\beta t z} = \frac{-\beta}{Inq}\sum_{z=1}^{\infty}\{(1-q)e^{-\beta t}\}^z$$

but

$$\sum_{z=1}^{\infty}\left[(1-q)e^{-\beta t}\right]^z = \frac{(1-q)e^{-\beta t}}{1-(1-q)e^{-\beta t}} \qquad (4)$$

It is observed that the pdf of ELD is monotone decreasing with modal value $\frac{\beta(1-q)}{q\ln q}$ at $t = 0$ and also leads to zero as t tends to infinity.

As q tends to 1, the ELD tends to exponential distribution with parameter β as will be seen in the simulation study.

RESULTS

The PDF of the ELD

The probability density function is obtained by the integration of equation (4)

$$f_T(t;q,\beta) = -\frac{1}{Inq} * \frac{\beta(1-q)e^{-\beta t}}{1-(1-q)e^{-\beta t}} \qquad (5)$$

The CDF of the ELD

The cumulative distribution function (CDF) as derived by Tahmasbi et al (2008) is given as

$$F_T = 1 - \frac{In(1-(1-q)e^{-\beta t}}{Inq} \qquad (6)$$

The Survival Function of the ELD

The survival function is given as

$$S(t) = 1 - F(t)$$

$$= 1 - \left\{1 - \frac{\ln\left(1-(1-q)e^{-\beta t}\right)}{\ln q}\right\}$$

$$= 1 - \left[\frac{\ln q - \ln\left[1-(1-q)e^{-\beta t}\right]}{\ln q}\right]$$

$$= \frac{\ln q - \ln q + \ln\left[1-(1-q)e^{-\beta t}\right]}{\ln q}$$

$$= \frac{\ln\left[1-(1-q)e^{-\beta t}\right]}{\ln q} \qquad (7)$$

The Hazard Function of the ELD

The Hazard Function is

$$h(t) = \frac{f(t)}{S(t)} = \frac{\frac{\beta}{\ln q}\left[\frac{(1-q)e^{-\beta t}}{1-(1-q)e^{-\beta t}}\right]}{\frac{\ln\left[1-(1-q)e^{-\beta t}\right]}{\ln q}}$$

$$= -\frac{\beta(1-q)e^{-\beta t}}{(1-(1-q)e^{-\beta t})\ln(1-(1-q)e^{-\beta t})} \qquad (8)$$

The ith Moment

The ith moment about the origin of a continuous random variable T, denoted μ_i is defined as

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$$\mu_i = E(T^i) = \int_{-\infty}^{\infty} t^i f(t)dt, \quad i = 0,1,2,\ldots \tag{9}$$

so that the moment of the ELD model is

$$\mu_i = E(T^i) = \int_0^{\infty} (t^i) - \frac{1}{Inq} * \frac{\beta(1-q)e^{-\beta t}}{1-(1-q)e^{-\beta t}} \tag{10}$$

The ith moment as evaluated by the means of the mathematica software for the ELD as in (11)

$$\mu_i = E(T^i; q, \beta) = -\frac{i!\,polylog(i+1, 1-q)}{\beta^i Inq}, \quad i = 1,2,\ldots \tag{11}$$

Where polylog(.) is a polylogarithmic function and it is defined as follows (Lewin,1981):

$$polylog(a,z) = \sum_{k=1}^{\infty} \frac{z^k}{k^2} \tag{12}$$

From the above, the first six moments can be written as

$$\mu_1 = E(T; q, \beta) = -\frac{polylog(2, 1-q)}{\beta^1 Inq}$$

$$\mu_2 = E(T^2; q, \beta) = -\frac{2polylog(3, 1-q)}{\beta^2 Inq}$$

$$\mu_3 = E(T^3; q, \beta) = -\frac{6polylog(4, 1-q)}{\beta^3 Inq}$$

$$\mu_4 = E(T^4; q, \beta) = -\frac{24polylog(5, 1-q)}{\beta^4 Inq}$$

$$\mu_5 = E(T^5; q, \beta) = -\frac{120polylog(6, 1-q)}{\beta^5 Inq}$$

$$\mu_6 = E(T^6; q, \beta) = -\frac{720polylog(7, 1-q)}{\beta^6 Inq}$$

The moment related measures are given as follows:

The Mean {E(T)}

The mean of the ELD is given by

$$\mu = \mu_1 = E(T) = -\frac{polylog(2, 1-q)}{\beta Inq} \tag{13}$$

The Variance (σ^2)

The variance of the ELD is given by

$$\sigma^2 = \mu_2 - \mu^2$$

$$= -\frac{2polylog(3, 1-q)}{\beta^2 Inq} - \left\{-\frac{polylog(2, 1-q)}{\beta^1 Inq}\right\}^2$$

$$Var(T) = -\frac{2polylog(3, 1-q)}{\beta^2 Inq} - \frac{polylog^2(2, 1-q)}{\beta^2 In^2 q} \tag{14}$$

The Standard Deviation

The standard deviation is the square root of the variance, that is,

$$SD = \sqrt{\mu_2 - \mu^2} \tag{15}$$

The Pearson's Coefficient of Skewness

$$\frac{3(\bar{X} - Md)}{s}, \tag{16}$$

where $\bar{X}$ is the mean, Md is the median and s is the standard deviation of the sample.

The Pearson's Coefficient of Kurtosis

$$\frac{\mu_4}{\sigma^4} - 3 \tag{17}$$

DISCUSSION

In this section, we consider the probability density function (pdf), survival function and hazard function of the exponential distribution, exponential-logarithmic distribution, exponential-Poisson distribution and the Poisson-exponential distribution. These were achieved using simulation. The aim is to consider the behavior of these density functions at different parameter levels, where x or t represents time. The values of the parameters used were randomly chosen.

The pdf of the exponential distribution with parameter λ is given by

$$f_x(x,\lambda) = \lambda e^{-\lambda x}, x > 0, \lambda > 0, \tag{18}$$

and the graph is show below in Figure 1

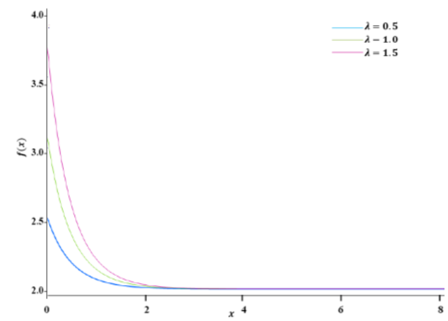

Figure 1 The pdf of the exponential distribution with parameter λ.

The hazard function of the exponential distribution is given by

$$H(x) = \lambda x \tag{19}$$

and the graph is given below in figure 2

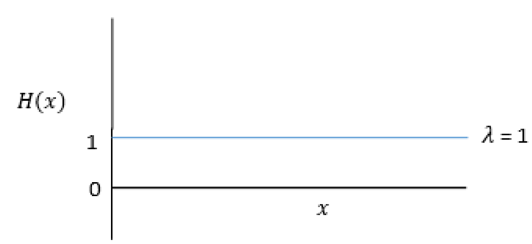

Figure 2 Hazard function of the exponential distribution

The hazard function of the Exponential- Logarithmic Distribution with parameters (q, β) is given by

$$h(x) = \frac{-\beta(1-q)e^{-\beta x}}{(1-(1-q)e^{-\beta x})\ln(1-(1-q)e^{-\beta x})} \tag{20}$$

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The graphs of $h(x)$ for the exponential-logarithmic distribution are given in Figure 3.3 and 3.4. Clearly, both graphs depict decreasing failure rate and its shape parameter q and scale parameter β resembles that of the EP and PED in long term.

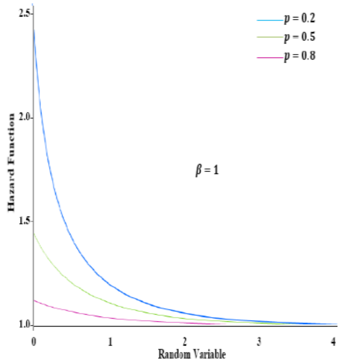

Figure 3 The graph of the hazard function of the exponential-logarithmic distribution for $\beta = 1,\ p = 0.2, 0.5, 0.8$

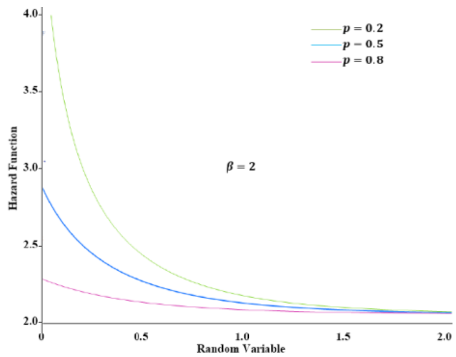

Figure 4 The graph of the hazard function of the exponential-logarithmic distribution for $\beta = 2,\ p = 0.2, 0.5, 0.8$.

The survival function of the exponential-logarithmic distribution with parameters (q, β) is given by

$$S(x) = \frac{In(1-(1-q)e^{-\beta x})}{Inq}$$

and the graphs are given below in figure 5a and figure 5b for different values of β

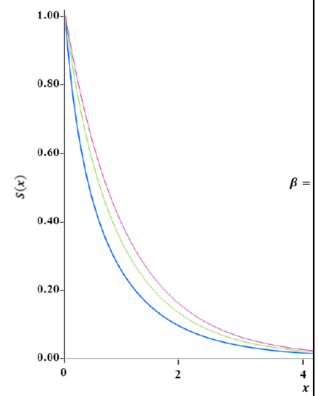

Fig (5a) Survival function graph of the exponential-logarithmic distribution

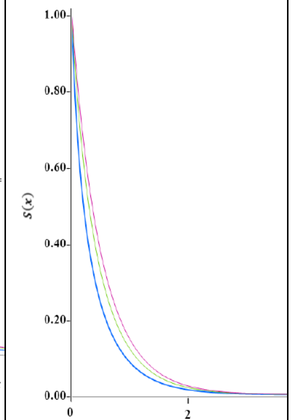

Fig (5b) Survival function graph of the E-L distribution

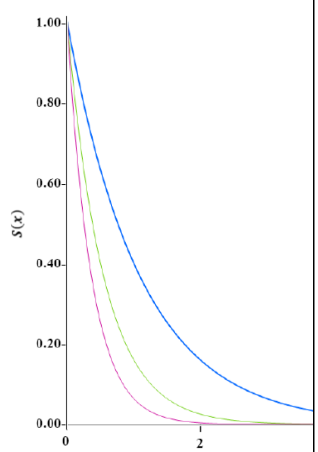

Fig (5c) Survival function graph of the exponential distribution

Simulation study for the exponential-logarithmic distribution

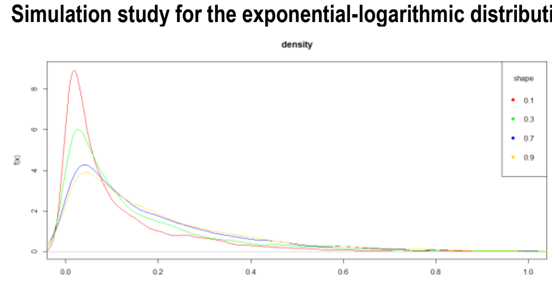

Figure 6 Simulation Study for the Exponential-Logarithmic Distribution (ELD).

Figure 6 shows the different shapes of the ELD when its scale

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parameter $\beta = 0.2$, we can see that as $q \to 1$, the ELD tends to the Exponential distribution.
The applicability of the ELD to real life data set is shown. ELD is fitted to the data to make prediction. The probability density function of the distribution is stated as;

$$f_T(t;q,\beta) = -\frac{1}{Inq} \times \frac{\beta(1-q)e^{-\beta t}}{1-(1-q)e^{-\beta t}}, \quad 0 < q < 1, \ \beta > 0. \quad (22)$$

The estimates of the parameters of the distribution are obtained through the method of maximum likelihood via Newton Raphson's iterative scheme.

Data Collection and Analysis
An uncensored data on retailers on perishable goods in Ika-south Local Government Area of Delta State between the year 2015 and 2020 was used. This was obtained by the administration of questionnaires, the data collected was particularly on how long they received trade credit in hours counting from when they made the credit purchase. The reasons for the disparity in hours could not be accounted for as it could be due to many factors. According to Cancho et al (2011), this kind of data set is a complementary risk data as it has no information about which factor was responsible for them being indebted to their suppliers, this failure can be masked from our own individual views.

A comprehensive information about a 108 data set collected is given below

9, 9, 13, 15, 214, 219, 23, 27, 331, 30, 32, 32, 33, 18, 19, 19, 21, 27, 44, 44, 44, 44, 35, 36, 4, 41, 42, 6, 54, 55, 57, 46, 47, 47, 48, 49, 68, 70, 71, 71, 57, 61, 62, 62, 63, 83, 87, 86, 71, 71, 71, 74, 77, 8, 98, 99, 107, 86, 88, 88, 89, 95, 95, 120, 125, 125, 109, 11, 11, 111, 112, 115, 142, 155, 154, 129, 13, 131, 133, 137, 139, 200, 207, 213, 173, 173, 181, 182, 189, 19, 385, 9, 36, 47, 61, 71, 88, 11, 131, 181, 23, 271, 284, 299, 15, 200, 19, 55

CREDIT PERIOD DURATION	FREQUENCY IN HOURS
0-50	42
51-100	32
101-150	15
151-200	10
201-250	4
251-300	3
301-350	1
350-400	1

Table 1 Summary Statistics of Frequency Distribution
The Frequency distribution table is used to show the number of values that fall in each of the classes. It also helps in the construction of a histogram.

Mean	88.76852
Median	70.5
Skewness	1.473081
Kurtosis	5.287748
Maximum	385
Minimum	4
Range	381

Table 2 Basic summary statistics for the length of time it takes a retailer to be debt free.
From Table 2, we observe that the distribution is positively or right skewed. Thus, the distribution points to the right due to the fact that its skewness value is greater than 0.
For the trade credit period data above, the distribution of the data has high peak since the kurtosis value is greater than 0, thus it suggests that it belongs to the exponential family.

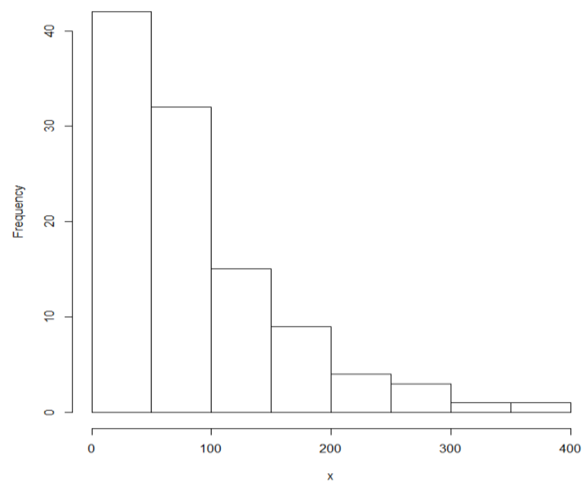

Figure 7 Histogram of Credit Period Duration in hours
Using the Histogram, a good choice of distribution for this model could be the Exponential-Logarithmic distribution since it belongs to the exponential family that is simple to interpret.

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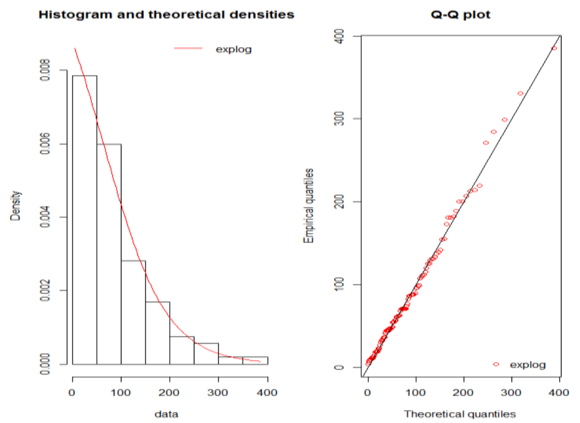

(a) (b)

Figure 8 Analysis for the Exponential-Logarithmic Distribution. (a) is the Histogram and the theoretical Density, while (b) is and Q-Q plot of the sample data and the estimated Exponential-Logarithmic Distribution.

The Q-Q plot is a plot for comparing two probability distributions, usually the sample distribution and the estimated theoretical distribution.

The Newton Raphson iterative method for numerical computation of the MLE of the above data set is used and the initial value is taken to be $\phi_0 = (q_0, \beta_0)' = (0.8, 1.5)'$. The estimate is shown below

$$\hat{q} = 0.2519 \quad \hat{\beta} = 0.0158$$

Table 3 Summary statistics of the sample moments and estimated theoretical moments

	Mean	Median	Variance	Standard Deviation	Skewness	Kurtosis
theoretical moments	88.518	69.3189	5669.71	75.297	2.0001	4.9887
Sample moments	88.769	70.5	5705.6	75.535	1.473081	5.287748

Consider the Diagram below:

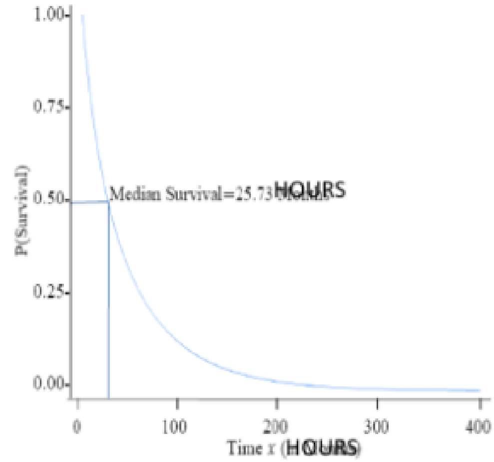

Figure 9 Survival Curve (Probability of Survival) showing how long it will take a retailer to be indebted to the supplier.

The vertical axis represents the probability of surviving. In this work, it represents the probability that “credit funding exists in the retailer’s life”. That is, the probability that a retailer receives credit funding from the supplier. It can also be considered as the proportion of trade credit funded retailers. The horizontal axis is used for time in days. To see this more clearly, let us consider the following:

At time zero, the probability of “survival” (that is the probability that a retailer remains indebted immediately after purchase is 1.0). This is the same as saying that 100% of retailers remain indebted immediately after purchase.

Now, the median survival is approximately 26 hours. Thus, the length of time from purchase that half of the retailers remain indebted is 26 hours that is to say, half of them will “survive” trade credit funding after 26 months.

It therefore follows that a retailer who is armed with this knowledge, on purchase should not worry so much if he is not debt free before the 26 hours after purchase.

Now, beyond this time, about half of the retailer is expected to be debt free. Thus, a sound retailer has a 50% chance of being debt free by this time.

Consider also, the hazard function as shown below

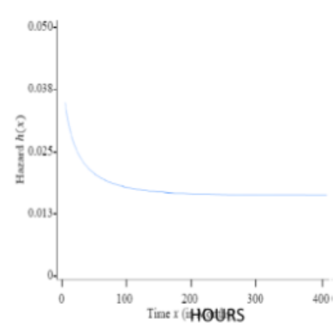

Figure 10 The complementary hazard function

Considering the hazard function, we observe that initially, it decreases monotonically. But as time progresses the rate of decrease reduces, and then eventually it becomes parallel to the horizontal axis. This implies that at the initial stage, the rate at which trade credit funding is failing is decreasing. In other-words, the rate at which retailers are becoming debt free is on the increase. But overtime, this rate of becoming debt free starts reducing. This suggests that it is easier for a retailer to be debt free at the initial stage than much later.
In other words, the number of retailers that are debt free at the initial stage is higher compared to a relatively later time. This is obvious from the asymptotic nature of the graph in the long run, this asymptotic nature means that overtime, trade credit funding will fail completely. That is being debt free will be possible for all.

Conclusion
In this work, the Exponential-Logarithmic Distribution (ELD) was used to analyze real life data, some properties of the distribution was examined, the hazard function was derived from the probability density function and also the survival function, the moments of the distribution with the help of the Mathematica software was obtained, simulation study was carried out for different values of the parameters of some distributions with the aid of the mathematical package, and observations were recorded. Comparison between the theoretical moments and the raw moment shows that the Exponential-Logarithmic distribution is a good fit for the data.
The method of maximum likelihood estimation was used (mle) to estimate the model parameters. The score function could not be solved directly since it's a non-linear system of equation the Newton Raphson's iterative method was used for the numerical computation of the parameter estimate this was achieved with the use of the mass, stat4, nacopula and fitdistrplus packages in R software. The model was applied to a real-life problem on the length of time (credit period) it takes a retailer to be debt free in Ika-south local government area, Delta state and prediction was made with the aid of the survival and the hazard function.

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