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Full Length Research Article

OPTIMAL SOLUTION TECHNIQUES FOR CONTROL PROBLEM OF EVOLUTION EQUATIONS

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ABSTRACT

A lot of optimal solution techniques exist in literature and have been employed to solve optimization problems. However, in this work, two optimal solution techniques namely the Steepest Descent Method (SDM) and the Extended Conjugate Gradient Method (ECGM) were applied to two evolution equations to determine which best approximates the system to its analytic solution. Comparisons were also carried out between the SDM and the ECGM for optimal solution and convergence. In doing so, the Liberty Basic Programming Language (LBPL) was employed to perform various numerical iterations on the respective algorithms of the techniques. It was discovered that the SDM proved to be a better technique in obtaining an optimal solution than the ECGM.

Keyword: Optimal Control, Extended Conjugate Gradient Method (ECGM), Steepest Des- cant Method (SDM), optimal solution, optimization.

INTRODUCTION

It is often the desire of problem solvers to quickly and easily locate the solution to their problem. However, they encounter challenges of either ill-posed problem or inadequate information to pave way for solving the problem. Most states are not adequately described by ordinary differential equations, but can be modeled by other forms of differential equations such as the delay differential equations, partial differential equations, integral equations or coupled ordinary and partial differential equations. Ibiejugba and Olugbara (2007). The Encyclopedia of Mathematics (2019) defined an Evolution equation as an equation which has the possibility of constructing its solution from a given initial condition such that it can be interpreted as a description of the initial state of the system. Hence the forms of equations listed above, if they possess an initial condition upon which the initial system can be interpreted from will be referred to as an Evolution equation. Hunter(1996) gave the general form of an evolution equation as

$$u' = f(u) \qquad (1)$$

where the prime (') which denotes a time derivative. The state of the system at time t is given by y(t)∈X, f is a given vector field on the space X which is the state space of the system. As given by, the evolution of a system depending on a continuous time variable t is thus given by (1) and the Initial Value Problem. Following the conditions of an evolution equation is given as

$$u' = f(u), \;\; u(0) = u_0 \qquad (2)$$

where y(0)=y_0 is the initial condition of (1). In [3], it states that the class of Evolution equation includes first of all, ordinary differential equations and systems of the form

$$u' = f(t,u), \;\; u'' = f(t,u,u') \qquad (3)$$

where u(t) is the solution of the Cauchy problem. It should be noted that real life processes that can be reduced to mathematical equations having an origin (initial value) and whose numerical solution can be facilitated can be termed to form an evolution equation. Schnaubelt (2019) expressed his opinion in the following way: let A be a closed operator on X and x∈D(A). Then the Cauchy problem or evolution equation is given as

$$u' = Au(t), \quad u(0) = x, \quad t \geq 0 \qquad (4)$$

where a (classical) solution of (4) is a function of $u \in C^1(\mathbb{R}_{\geq 0}, X)$ taking values in D(A) and satisfies (4). It should be noted that $Au \in C(\mathbb{R}_{\geq 0}, X)$ and $u \in C(\mathbb{R}_{\geq 0}, D(A))$.

If we have that the states u(t)∈X is a physical system having its properties encoded in A and its domain, it will be possible to predict the future of the system by (4).Hence a lot of initial values of x will be required, as well as determining also the initial value approximately to avoid a blow up of the solutions under little variation or changes of the data.

According to Nicklos (2016), a control problem refers to a significant loss of control where control once existed. Hence the optimal solution of such will be the best technique amongst techniques in resolving such a problem. Becerra (2008) defines Optimal control as the process of determining control and state trajectories for a dynamic system over a period of time to minimize a performance index. Given a fixed time and no terminal or path constraints on the states or control variables, we define a general continuous time optimal control problem as

$$\text{Min } J = \varphi(X(t_f)) + \int_0^{t_f} L(X(t), u(t), t)\,dt, \qquad (5)$$

$$\text{Subject to} \quad X'(t) = f(X(t), u(t), t), \;\; X(t_0) = X_0, \qquad (6)$$

where $[0, t_f]$ is the time interval of interest, $X: [t_0, t_f] \longrightarrow W^n$ is the state vector, , $\varphi: W^n \times W \longrightarrow W$ is a terminal cost function $L: W^n \times W \longrightarrow W^n$ is a vector field. Equation (5) is known as the Bolza problem, Apanapudor, et al (2016). Halicka et al (2015) defined an optimal solution problem for an optimal allocation of resources as

$$\text{Maximize} \quad \sum_{i=0}^{k-1} g u_i x_i + h(1 - ut)x_i \qquad (7)$$

Subject to

$$x_{i+1} = b u_i x_i + h(1 - ut)x_i, \;\; i = 0, \cdots, k-1, \qquad (8)$$

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$x_0 = a,$ (9)

$u_i \in [0,1], \quad i = 0, \cdots, k-1,$ (10)

where a,b,c,g,h are given constants and maximum is to be found with respect to the variables control problem as

$$\text{Min}_{u=U} J_u = \text{Min}_{u=U} \int_0^1 \left(z_1(\tau) - \frac{u(\tau)}{2}\right) dt, \quad (11)$$

$$\text{Subject to} \quad u - z_1 + z_2 e^{-\frac{2}{10}z_2^2}, \quad (12)$$

$$\frac{dz_2}{d\tau} = u - \pi^{\frac{5}{2}} z_1 cos\left(\frac{5}{2}u\right), \quad (13)$$

$$z_1(0) = 0, z_2(0) = 1, |u(\tau)| \le 1, \tau \in [0,1], \quad (14)$$

where u∈U of all piecewise continuous functions on [0,1].
Various methods for solving control problems of evolution equations exist in literature, but researchers often seek faster converging methods. This research work is poised to study and apply the Steepest Descent Method (SDM) and the Extended Conjugate Gradient Method (ECGM) to a control problem of evolution equations. The aim is to obtain optimal solution methods to given control problems of evolution equations and to achieve this, we shall identify control problems of evolution equations, apply some optimal control techniques on them to determine their optimal solution and compare results emanating from the techniques to determine which is better, considering their convergence and some other factors.

METHODOLOGY

Nonlinear optimal control problem of an Evolution Equation

As stated by Connor(2019), evolution equations are a set of differential equations used to mathematically model isotopic changes. We shall consider the equation of the form

$$\text{Min } J(X,U) = \text{Min} \int_0^\sigma [x^T(t)Qx(t) + u^T(t)Ru(t)]dt, \quad (15)$$

$$\text{Subject to:} \quad x\prime(t)=Cx(t)+Du(t), \ 0\le t\le\sigma, \quad x(0)=x_0, \quad (16)$$

where x(t) is an n×1 vector matrix for the state variables of the system with x^T (t) being its transpose, u(t) is an n×1 vector matrix for the control variables applied to the system, 0≤t≤σ, σ is a specified final time, C and D represent square constant matrices of order n, while Q and R are symmetric, positive definite, constant square matrices of order r. Our aim here is that of employing the ECGM and SDM on (15) subject to (16) to determine which method will proffer an optimal solution for the evolution equation.

Formulation of the CGM

Zuchlke et al (2015) presented the formulation of the CGM as follows: Let

Ax=b, (17)

be the set of linear equations where A is a known n×n symmetric, real and positive definite matrix, b is a known vector and x is the unknown. The solution of the problem is given by x^*. Let P be a set of n conjugate vectors such that

$P = \{P_k : \forall\, i \neq k, k \in [1,n], \langle P_i, P_k\rangle_A = 0\}.$ (18)

If $x^* \in P$, then

$x^* = \sum_{i=1}^{n} \alpha_i P_i,$ (19)

and

$b = Ax^* = \sum_{i=1}^{n} \alpha_i A P_i.$ (20)

Substituting (19) and (20) into (17) gives

$P_k^T = \alpha_k P_k^T A P_k.$ (21)

Hence the coefficients α_k can be computed as

$\alpha_k = \frac{\langle P_k, b\rangle}{\|P_k\|_A^2},$ (22)

from a linear system by finding n conjugate vectors Connor(2019), Encyclopedia of Mathematics(2020) . Rao (2009) gave the Fletcher Reeves algorithm for the non-linear system as follows: For

$\beta_i = \frac{\nabla f_i^T \nabla f_i}{\nabla f_{i-1}^T \nabla f_{i-1}},$ (23)

a) Start with an arbitrary initial point X_1
b) Set the first search direction S=-∇〖f(X〗_1)=-∇f_1
c) Find the point X_2 according to the relation

$X_2 = X_1 + \lambda_1^* S_1,$ (24)

where λ_1^* is the optimal length in the direction S_1. Set i=2 and go to the next step:

d) $\nabla f_1 = \nabla f(X_1)$ and set

$S_1 = -\nabla f_1 + \frac{|\nabla f_1|^2}{|\nabla f_{i-1}|^2} S_{i-1}.$ (25)

e) Compute the optimal length λ_1^* in the direction of S_1 and find the next new point

$X_{i+1} = X_i + \lambda_1^* S_i.$ (26)

f) Test for optimality of the point X_(i+1) and stop the process if X_(i+1) is optimum, otherwise set the value of i=i+1 and revert to step d).

In constructing the CGM for (15), we have

$$\text{Min J(X, U)} = \text{Min} \int_0^\sigma [\phi^*(t)]dt$$

Let $x(0) = x_0$ be the initial point and- the first search direction be given as

$$S_1 = \begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}$$

Then using the relation $X_{n+1} = X_n + \lambda_1^* S_i$, we have

$$X_n = X_1 + \lambda_1^* S_1 \implies X_1 = \begin{pmatrix}x_0^*\\ x_0^*\end{pmatrix} + \begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}\lambda_1^* =$$

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$$\begin{pmatrix} x_{0\_}^* + \lambda_1^*\alpha_1 \\ x_0^* + \lambda_1^*\alpha_2 \end{pmatrix} \quad (27)$$

Substituting $x_1 = x_0^* + \lambda_1^*\alpha_1$ and $u_1 = x_0^* + \lambda_1^*\alpha_2$ into (3.13), we obtain

$$\mathrm{J(x)} = \mathrm{J}\begin{pmatrix} \lambda_1^*\alpha_1 \\ \lambda_1^*\alpha_2 \end{pmatrix} = \int_0^\sigma [(\lambda_1^*\alpha_1)^\mathrm{T}\mathrm{Q}(\lambda_1^*\alpha_1)^\mathrm{T} + (\lambda_1^*\alpha_2)^\mathrm{T}\mathrm{R}(\lambda_1^*\alpha_2)^\mathrm{T}]\mathrm{dt}. \quad (28)$$

Formulation of the ECGM

It is expected to find an optimal control function $\boldsymbol{x}^*(\cdot)$defined on the closed interval [0,σ] along with a corresponding trajectory $\boldsymbol{x}^*(\cdot)$ given by (16) which minimizes (15). Otunta (1998) quoting Ibiejugba and Buraimoh-Igbo (1985) however proposed the use of the ECGM by connecting (15) with a control operator A given as

$$\mathbf{Min}\, \boldsymbol{J(X,U,\mu)} = \boldsymbol{Min}\langle z,\bar{A}z\rangle_k = \mathbf{Min} \int_0^\sigma [\boldsymbol{x^T(t)Qx(t)} + \boldsymbol{u^T(t)Ru(t)}]\boldsymbol{dt} + \boldsymbol{\mu}\int_0^\sigma \|\boldsymbol{x\prime(t)} - \boldsymbol{Cx(t)} - \boldsymbol{Du(t)}\|^2 \boldsymbol{dt} \quad (29)$$

where k is the Cartesian product space of $\boldsymbol{H[0,\sigma]}$ and called the Sobolev space of absolutely continuous functions $\boldsymbol{x}(\cdot)$ and $\boldsymbol{L_2^n[0,\sigma]}$ is the Hilbert space consisting of classes of square integrable functions from $[\mathbf{0},\boldsymbol{\sigma}]$ into $\mathbb{R}^{\boldsymbol{n}}$.. Ibiejugba and Buraimoh-Igbo (1985) further opined that the space k is endowed with the norm and inner product defined respectively as follows:

$$\|z\|_k^2 = \|x\|_{L_2}^2 + \|\dot{x}\|_{L_2}^2 + \|u\|_{L_2}^2 \quad (30)$$

where $z^T = (x,u), x \in L_2[0,\sigma],\ x \in L_2[0,\sigma]$ and $\langle\cdot,\cdot\rangle_k = \langle\cdot,\cdot\rangle_{H^1} + \langle\cdot,\cdot\rangle_{L_2}$ with the norm and inner product defined by

$$\|u\| = \left\{\int_0^\sigma <u,u>_E^2 dt\right\}^{\frac{1}{2}} \ and \ \langle u_1,u_2\rangle = \int_0^\sigma \langle u_1(t),u_2(t)\rangle_E dt,$$

respectively, where $\|\cdot\|_E$ and $\langle\cdot,\cdot\rangle_E$ denote the norm and scalar product in the Euclidean n-dimensional space.

Problem Formulation

Following the algorithm prepared by Rao (2009), we present the methodology for equation (15) via the CGM as follows: Let the initial point be given as

$$\mathrm{X}(0) = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$$

and the initial search direction be given as

$$\mathrm{S}_1 = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}.$$

Using the relation $X_{n+1} = X_n + \lambda^* S_1$, we have for $n = 0$

$$\mathrm{X}_1 = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix} + \lambda^*\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \Rightarrow \mathrm{X}_1 = \begin{pmatrix} x_0 + \lambda^*\alpha_1 \\ x_1 + \lambda^*\alpha_2 \end{pmatrix}.$$

Substituting $x(t) = x_0 + \lambda^*\alpha_1$ and $u(t) = x_1 + \lambda^*\alpha_2$ into (31) gives

$$\mathrm{Min\ J(X,U)} = \mathrm{Min}\int_0^\sigma [(x_0 + \lambda^*\alpha_1)^\mathrm{T}\mathrm{Q}(x_0 + \lambda^*\alpha_1)^\mathrm{T} + (x_1 + \lambda^*\alpha_2)^\mathrm{T}\mathrm{R}(x_1 + \lambda^*\alpha_2)^\mathrm{T}]\mathrm{dt}. \quad (31)$$

With the aid of Mathematical Software, such as Liberty Basic Programming Language (LBPL), Equation (31) is solved subject to the constraint

$$X(t) = Cx(t) + Du(t), \quad 0 \le t \le \sigma. \quad (32)$$

Formulating the ECGM for (15), we follow from (28) on (29) to obtain

$$\mathrm{Min\ J(X,U,\mu)} = \mathrm{Min}\int_0^\sigma [(x_0+\lambda_1^*\alpha_1)^\mathrm{T}\mathrm{Q}(x_0+\lambda_1^*\alpha_1)^\mathrm{T} + (x_1+\lambda^*\alpha_2)^\mathrm{T}\mathrm{R}(x_1+\lambda^*\alpha_2)^\mathrm{T}]\mathrm{dt} + \mu\int_0^\sigma \|(x_0+\lambda_1^*\alpha_1)\prime - C(x_0+\lambda_1^*\alpha_1) - D(x_1+\lambda_1^*\alpha_2)\|^2 dt. \quad (33)$$

The Steepest Descent Method (SDM) Algorithm

The SDM comes into play when we want to find the minimum of a function iteratively based on our inability to use analytic methods. The Newton's method though effective, can also be unreliable as opined by Lamberts (2011). Hence, if we have a function $f: W^n \longrightarrow W^n$that is differentiable at x_0, the vector − *vg*(U_0) becomes the direction of the steepest descent. In [9] it was presented that given the function

$$H(t) = g(U_0,\varphi t), \quad (34)$$

where φ is a unit vector. By the chain rule,

$$H'(t) = \frac{\partial g}{\partial U_1}\frac{\partial U_1}{\partial t} + \cdots + \frac{\partial g}{\partial U_n}\frac{\partial U_n}{\partial t} = \frac{\partial g}{\partial U_1}\varphi_1 \cdots + \frac{\partial g}{\partial U_n}\varphi_n = \nabla g(U_0,\varphi t). \quad (35)$$

Therefore,

$$H'(t) = \nabla g(U_0)\cdot\varphi = \|\nabla g(U_0)\|\cos\theta, \quad (36)$$

where θ is the angle between $\nabla g(U_0)$ and φ. We then see that $H'(0)$ is minimized when $\theta = \pi$ which

$$\varphi = \frac{\nabla g(U_0)}{\nabla g(U_0)},\quad H'(0) = \nabla g(U_0). \quad (37)$$

Hence, by finding the minimum of $H(t)$ for the choice of φ, it becomes possible to reduce the problem of minimizing a function of several variables to a single variable minimization problem. This is achieved when we find the value of φt, for $t > 0$ that minimizes

$$H_0(t) = g(U_0 - t\nabla g(U_0)). \quad (38)$$

We find the minimum at t_0 and set

$$U_1 = U_0 - t_0\nabla g(U_0), \quad (39)$$

and we continue the process such that U_1 is searched in the direction of $-\nabla g(U_0)$ to obtain U_2 by minimizing $H_0(t) = g(U_0 - t\nabla g(U_0))$ and so on. With this process, we obtain the method of steepest descent that is, given an initial guess U_0, the method computes a sequence of iterates U_k, where

$$U_{k+1} = U_k - t_k\nabla g(U_k),\quad k = 0,1,2,\cdots \quad (40)$$

for $y_k = t_k$ where $y_k > 0$ minimizes the function

$$H_k(t) = g(U_k - t_k\nabla g(U_k)). \quad (41)$$

From the above, we generate a simple Steepest Descent Algorithm given by Freund (2004) as follows:

Step0: Given U_n, set $n = 0$;
Step1: $\varphi_n = -\nabla g(U_n)$, if $\varphi_n = 0$, then stop.
Step 2: Solve $Min\ g(U_k + x\varphi_n)$ for the step size x_n. Perhaps chosen by an exact or inexact line search.
Step3: $U_{n+1} \longrightarrow U_n + x_n\varphi_n, n \longrightarrow n+1$.Go to **Step1**.

It should be noted from **Step 2** that since $\varphi_n = -\nabla g(U_n)$ is a descent direction, it follows that $g(U_{k+1}) < g(U_k)$. The above algorithm is iterated using Liberty Basic Programming Language (LBPL) to obtain the optimal solution of an evolution equation.

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NUMERICAL ILLUSTRATION
Test Problem1: Given

Test Problem1: Given

$$\frac{dx}{dt} = 0.5x(t) + \mu(t), \qquad x(0) = 1$$

Objective Function = $\int_0^1 (x^2(t) + \mu^2(t))dt$
Minimize f(x) subject to the constraint.

Test Problem2

$$J[z(x,t), u(x,t)] = Min \iint_{[0.1]} \{z^2(x,t) + u^2(x,t)\}\, dt,$$

subject to a wave propagation equation

$$\frac{\partial^2 z(x,t)}{\partial t^2} = c^2 \frac{\partial^2 z(x,t)}{\partial x^2}, \quad 0 < x < 1, \quad t > 0,$$

$$\begin{cases} z(0,t) = z(1,t) = 0, & t > 0, \\ z(0,t) = \sin(5\pi) + 2\sin(7\pi x), & 0 < x < 1 \\ z_t(x,0) = 0, & 0 < x < 1. \end{cases}$$

The computational results of the above two problems are obtained by the LBPL and presented the tables below. Graphical representations and discussion of the obtain results are also presented.

DISCUSSION OF RESULTS
Comparison of Function Values For ECGM and SDM for Test Problem 1
Table 1 and Figure 1 above show the convergence comparison of ECGM and SDM behavior of function value for problem1. Graphically, it can be seen that the problem converge date 6th iteration. Thereafter, the two methods increased asymptotically which shows that both methods are employable for solving evolution problem. On the table, it is discovered that the function value experienced unstable divergence at μ =0.2.
Table 2 and Figure 2 show the convergence comparison of ECGM and SDM behavior of function value for problem1.Graphically, it can be seen that the problem converges in parallel format 2 dimension iteration. On the table, it can be observed that the function value converges simultaneously but it is at al sow paste in the case of SDM while unstable in the case of ECGM at μ = 0.4.
Table 3 and Figure 3 show the convergence comparison of ECGM and SDM behavior of function value for problem1. Graphically, it can be seen that the problem converges towards the 8th iteration. While on the table of result, it can be observed that the function value, though diverges at some points but later converges simultaneously at μ=0.5 in the case of SDM while the case is difference in the case of ECGM.
Table 4 and Figure 4 show the convergence comparison of ECGM and SDM behavior of function value for problem1. Graphically, it can be seen that the function values experienced a parallel decreasing profile. While on the table of result, both the ECGM and SDM experienced asymptotic decreasing values at μ = 0.6.
Table 5 shows the convergence comparison of ECGM and SDM behavior of function valueforproblem2. It is observed that, the function values converges asymptotically. On the case of SDM, the values are more reduced and closer compare with ECGM results which has a greater values to SDM but also converged as observed at μ =0.02.

Table 1: Function Values of ECGM and SDM results at μ=0.2

Iteration	ECGM Function Values	SDM Function Values
0	1.5	1.5
1	0.23915442	0.23915442
2	0.91358266e-1	0.91493495e-1
3	0.70091878e-1	0.87189614e-1
4	0.71226375e-1	0.13287558
5	0.88367666e-1	0.13526607
6	0.13545319	0.11760518
7	0.17706986	0.11681635
8	0.20793221	0.12321445
9	0.23102709	0.12351908
10	0.24850100	0.12114356

Table 2: Function Values of ECGM and SDM results at μ=0.4

Eration	ECGM Function Values	SDM Function Values
0	1.2	1.2
1	1.08770177	1.08770177
2	2.35224398	1.65246825
3	2.15587783	1.79608904
4	2.14478995	1.76501471
5	2.28875145	1.80049614
6	2.06998756	1.79272017
7	2.15096097	1.80167243
8	2.16538518	1.79970396
9	2.16519281	1.80197467
10	2.16735393	1.80147495

Table 3: Function Values of ECGM and SDM RESULTS at μ=0.5

Iteration	ECGM Function Values	SDM Function Values
0	1.6	1.6
1	0.33086284	0.33086284
2	0.4766892	0.47646478
3	0.49333688	0.47728958
4	0.49053783	0.45968538
5	0.47566322	0.45960216
6	0.45284677	0.46100512
7	0.43932224	0.46101202
8	0.43434972	0.46089379
9	0.43311611	0.46089321
10	0.43373679	0.46090313

Table 4: Function Values of ECGM and SDM RESULTS at μ=0.6

Iteration	ECGM Function Values	SDM Function Values
0	1.4	1.4
1	0.22893705	0.22893705
2	0.60971878e-1	0.40714412e-1
3	0.19924843e-1	0.22100576e-1
4	0.17856642e-1	0.24078513e-1
5	0.25888878e-1	0.17548265e-1
6	0.13109322e-1	0.18797065e-1
7	0.24520282e-1	0.16758186e-1
8	0.19111099e-1	0.01719310

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9	0.19859658e-1	0.16573076e-1
10	0.20112453e-1	0.16709357e-1

Table 5: Function Values of ECG M and SDM RESULTS at μ=0.2

Iteration	ECGM Function Values	SDM Function Values
0	1.02	1.02
1	0.36673749	0.36673749
2	0.9388051	0.36670793
3	0.94751808	0.36667836
4	0.94974126	0.36664878
5	0.94806304	0.3666192
6	0.9460535	0.3665896
7	0.9451472	0.36655999
8	0.9452299	0.36653037
9	0.94563364	0.36650074
10	0.9459049	0.3664711

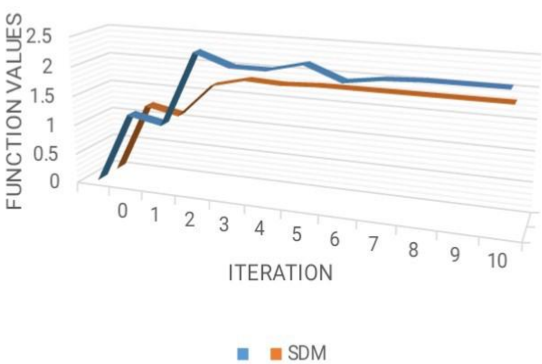

Figure1: Graphical Presentation of Function Values at μ=0.2

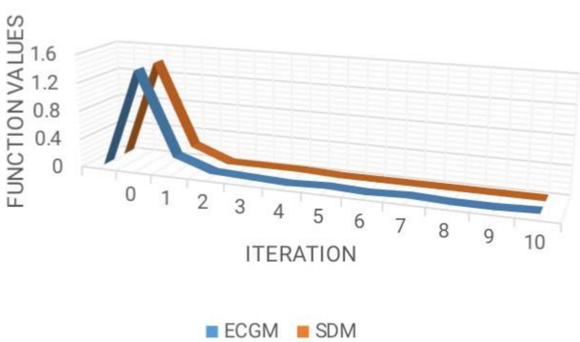

Figure 2: Graphical Presentation of Function Values at μ=0.4

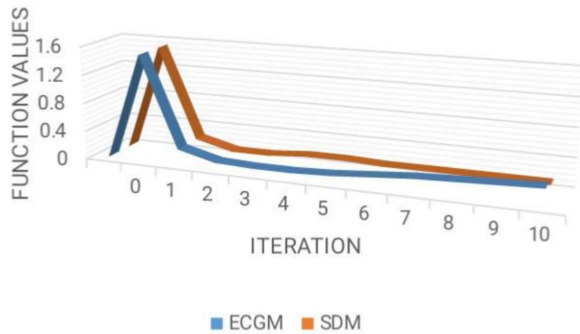

Figure 3: Graphical Presentation of Function Values at μ=0.5

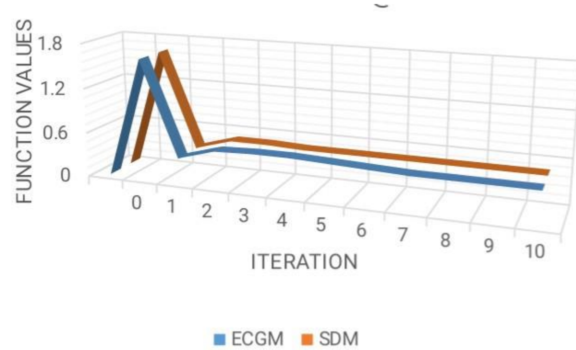

Figure 4: Graphical Presentation of Function Values at μ=0.6

CONCLUSION

Various optimal solution techniques exists in literature to find the solutions of control problems of evolution equations. These evolution equations, though analytic in nature are difficult to arrive at using some analytic methods, and researchers have resulted to using numerical methods to find approximate solutions to these equations. The use of algorithms has been of immense value to the numerical methods which are iterative in nature to help solve these equations. Also, the aid of computer software has also played a major role in arriving at the optimal solutions of the equations using identified optimal solution techniques. It is noted that the use of iterative methods via numerical means have the ability to give approximate solutions of equations having analytical forms and solutions. Hence control problems of evolution equations which are analytic in nature can be resolved using numerical means and iterative methods.

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