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EFFECTS OF HALL CURRENT ON TRANSIENT MHD NATURAL CONVECTION FLOW IN A VERTICAL MICROCHANNEL

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ABSTRACT
In this work, we studied the transient Magnetohydrodynamics natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reacting and optically thin radiating fluid past an infinite vertical plate in the presence of Hall current. The criteria for the existence of a unique solution of the equations describing the flow were established. The exact solutions for momentum and energy equations were obtained by transforming the equations and solving them using the Fourier Transform method under the relevant initial and boundary conditions. The effects of various flow parameters on the velocity, temperature and concentration were shown with the aid of graphs. Prandtl number is inversely proportional to temperature. Prandtl number has no effect on the concentration of the fluid, both thermal and solutal Grashof number are directly proportional to the velocity of the fluid, chemical reaction has a reverse effect on velocity, radiation has a reverse effect on temperature, Prandtl number has a reverse effect on the concentration of the fluid, Schmidt number has a reverse effect on the concentration of the fluid.

Keywords: Convection, MHD, Microchannel, Hall Effect, Transient.

1. INTRODUCTION
Fluid dynamics, a subdivision of fluid mechanics is an important science for solving various problems arising in aeronautical, chemical, mechanical and civil engineering field. The study of the laws governing the conversion of energy from one form to another, the direction of heat flow and the availability of energy to do work is the subject of thermodynamics. The study of biological systems is only one possible application of the knowledge of fluid dynamics. Life as we all know would not exist without fluids and without the behavior that fluids exhibit. The air we breathe and water we drink are fluids. Similarly, most of our body fluids are water based and proper motion of these fluids within our bodies even down to the cellular level is essential for good health. The study of the physical phenomenon (flow processes) occurring in oceans, atmosphere, space and down to the micro and nanoscales of biological cell activity are very important in fluid dynamics (McDonough, 2009).
The need for the study of the flow of an incompressible, viscous, chemically reacting and electrically conducting fluid through various cross sections has increased rapidly in recent years with applications in engineering problems such as Magnetohydrodynamics(MHD) generators, plasma studies, nuclear reactors, geothermal energy extraction and the boundary layer control in the field of aerodynamics. The flow and heat transfer of electrically conducting fluids in channels under the effect of transverse magnetic field occur in (MHD) pump and accelerators. The investigation of MHD phenomena in plane layers and channels with conducting fluids is quite important both for understanding the basic mechanisms and for improving the existing industrial processes and for developing new MHD devices. For instance, Chen and Weng (2005) obtained exact solution of the fully developed natural convection in an open-ended vertical parallel-plate micro-channel with asymmetric wall temperature distributions. The effects of rarefaction and fluid-wall interaction were shown to enhance the volume flow rate and to reduce the rate of heat transfer.
This result is further extended by taking into account suction/injection on the micro-channel walls by Jha *et al.* (2014). Their results showed that skin-friction as well as rate of heat transfer strongly depends on the suction/injection parameter. Numerical solutions were obtained by Buonomo and Manca (2012) for natural convection in parallel-plate vertical microchannels due to asymmetric heating by imposing constant heat flux on the boundaries. Avci and Aydin (2009) conducted a study on fully developed mixed convective flow in a vertical micro-annulus formed by two concentric microtubes. Jha *et al.* (2015) further extended the work of Avci and Aydin (2009) to the case when the vertical micro-annulus formed by two concentric micro-tubes is porous, i.e. where there is suction or injection through the annulus surfaces. Unsteady hydromagnetic Couette flow of a viscous, incompressible and electrically conducting fluid between two parallel porous plates taking Hall current into account in a rotating system was carried out by Seth *et al*. (2012). Seth and Singh (2015) investigated the unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system in the presence of an inclined magnetic field and Hall current.
In this paper, we analyze the equations governing the transient magneto-hydrodynamic natural convection flow past an infinite vertical plate with hall current effects following Seth *et al*. (2013); establish the criteria for the existence and uniqueness of solution of the transformed coupled partial differential equations; obtain an analytical solution for the coupled partial differential equations using the Fourier Transform method, and give a graphical representation of results obtained.

2. Model Formulation

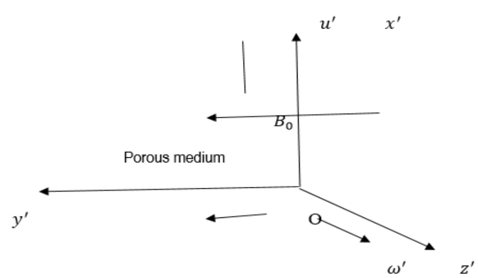

Figure 1: Geometry of the problem (Seth *et al*., 2013)

Following Seth *et al*. (2013), consider an unsteady hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reacting and optically thin radiating fluid past an impulsively moving infinite vertical plate embedded in a porous medium in the presence of thermal and mass diffusions. We choose the coordinate system in such a way that x'-axis is along the plate in upward direction, y'-axis normal to the plane of the plate and z'-axis perpendicular to the $x'y'$-plane. The fluid is permeated by uniform transverse magnetic field $\vec{B}_0$ applied in a direction parallel to the y'-axis. Both fluid and plate are in rigid body rotation with uniform angular velocity Ω about the y'-axis. Initially, i.e. at time $t' \leq 0$, both the fluid and plate are at rest and maintained at uniform temperature T'_∞. Also the level of concentration of fluid is maintained at uniform concentration C'_∞. At time $t' > 0$, the plate starts moving with uniform velocity u_0 in x'-direction against the gravitational field. At the same time the plate temperature is raised to uniform temperature T'_w and the concentration at the surface of the plate is raised to uniform concentration C'_w. The fluid considered is a gray, emitting-absorbing radiation but non-scattering medium. It is assumed that there exists a homogeneous chemical reaction of first order with constant rate K'_2 between the diffusing species and the fluid. Geometry of the problem is presented in figure 1 above.

Since the plate is of infinite extent along x' and z' directions and is electrically non-conducting, all physical quantities except pressure depend on y' and t' only. The induced magnetic field generated by fluid motion is neglected in comparison to the applied one, i.e., the magnetic field $\bar{B} \equiv (0, B_0, 0)$.

$$\frac{\partial u'}{\partial t'} + 2\Omega w' = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma {B_0}^2}{\rho(1+m^2)}(u' + m\omega') - \nu \frac{u'}{K_1'} + g\beta'(T' - T'_\infty) + g\beta^*(C' - C'_\infty) \quad (1)$$

$$\frac{\partial \omega'}{\partial t'} - 2\Omega u' = \nu \frac{\partial^2 \omega'}{\partial y'^2} + \frac{\sigma {B_0}^2}{\rho(1+m^2)}(mu' - \omega') - \nu \frac{\omega'}{K_1'} \quad (2)$$

$$\rho c_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + 16a^*\sigma^* {T'_\infty}^3 (T'_\infty - T') \quad (3)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K'_2(C' - C'_\infty) \quad (4)$$

Subject to the initial and boundary conditions:

$$\left.\begin{aligned} & t' \leq 0: u' = 0, \omega' = 0, T' = T'_\infty, C' = C'_\infty \ for\ all\ y', \\ & t' > 0: u' = u_0, \omega' = 0, T' = T'_\omega, C' = C'_\omega \ at\ y' = 0, \\ & u' \to 0, \omega' \to 0, T' \to T'_\infty, C' \to C'_\infty \ as\ y' \to \infty. \end{aligned}\right\} \quad (5)$$

Where,

u' is the fluid velocity in the x' direction, ω' is the fluid velocity in the z' direction, ν is the kinematic coefficient of viscosity, ρ is the fluid density, σ is the electrical conductivity, $m = \omega_e \tau_e$ is the Hall current parameter, ω_e is the cyclotron frequency, τ_e is the electron collision time, g is the acceleration due to gravity, β' is the volumetric coefficient of thermal expansion, T is the fluid temperature, C is the species concentration, c_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid, K'_1 is the permeability of the porous medium, q_r is the radioactive flux vector, D is the chemical molecular diffusivity.

2. METHOD OF SOLUTION

Non – dimensionalization

The governing equations (1) to (4) are non–dimensionalize using the following dimensionless variables.

$$\left.\begin{aligned} & t = \frac{t'}{t_0}, y = \frac{y'}{u_0 t_0}, u = \frac{u'}{u_0}, \omega = \frac{\omega'}{u_0}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, K^2 = \frac{\Omega \nu}{u_0^2}, M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, \\ & G_{r\theta} = \frac{\nu g \beta'(T'_w - T'_\infty)}{u_0^3}, G_{r\phi} = \frac{\nu g \beta^*(C'_w - C'_\infty)}{u_0^3}, P_r = \frac{\rho \nu c_p}{k}, S_c = \frac{\nu}{D}, R = 16a^*\sigma^*\nu^2 {T'_\infty}^3, \\ & K_1 = \frac{K_1' u_0^2}{\nu^2}, K_2 = \frac{\nu K_2'}{u_0^2}, \nu = t_0 u_0^2, c_p = \frac{1}{\rho u_0^2}. \end{aligned}\right\} \quad (6)$$

and obtain

$$\frac{\partial u}{\partial t} + 2K^2\omega = \frac{\partial^2 u}{\partial y^2} - \frac{M}{(1+m^2)}(u + m\omega) - \frac{u}{K_1} + G_{r\theta}\theta + G_{r\phi}\phi \quad (7)$$

$$\frac{\partial \omega}{\partial t} - 2K^2 u = \frac{\partial^2 \omega}{\partial y^2} + \frac{M}{(1+m^2)}(mu - \omega) - \frac{\omega}{K_1} \quad (8)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} - R\theta \quad (9)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - K_2\phi \quad (10)$$

Subject to the following initial and boundary conditions:

$$\left.\begin{aligned} & u(y,0) = 0, \omega(y,0) = 0, \theta(y,0) = 0, \phi(y,0) = 0, \\ & u(0,t) = 1, \omega(0,t) = 0, \theta(0,t) = 1, \phi(0,t) = 1, \\ & u(\infty.t) = 0, \omega(\infty,t) = 0, \theta(\infty,t) = 0, \phi(\infty,t) = 0. \end{aligned}\right\} \quad (11)$$

Further simplification gives

$$\frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial y^2} - NF + G_{r\theta}\theta + G_{r\phi}\phi \quad (12)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} - R\theta \quad (13)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} - K_2\phi \quad (14)$$

With initial and boundary conditions:

$$\left.\begin{aligned} & t \leq 0: F = 0,\ \theta = 0,\ \phi = 0\ for\ all\ y, \\ & t > 0: F = 1,\ \theta = 1,\ \phi = 1\ at\ y = 0, \\ & F \to 0, \theta \to 0, \phi \to 0\ as\ y \to \infty. \end{aligned}\right\} \quad (15)$$

Where,

$F = u + iw$, $N = \frac{M(1-im)}{(1+m^2)} + \frac{1}{K_1} - 2iK^2$, K^2 is the rotation parameter, M is the magnetic parameter, $G_{r\theta}$ is the thermal Grashof number, $G_{r\phi}$ is the solutal Grashof number, P_r is the Prandtl number, S_c is the Schmidt number, R is the radiation parameter, K_1 is the permeability parameter, K_2 is the chemical reaction parameter, m is the Hall current parameter.

Existence and Uniqueness of Solution

Theorem 3.1: Let $S_c = P_r = N = R = K_2 = G_{r\phi} = G_{r\theta} = 1$. Then the equations (12) to (14) with initial and boundary conditions (15) has a unique solution for all $t \geq 0$.

Proof: Let $S_c = P_r = N = R = K_2 = G_{r\phi} = G_{r\theta} = 1$ and $\psi(y,t) = \theta(y,t) + \phi(y,t)$. We obtain:

$$\left.\begin{aligned} & \frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial y^2} - F + \theta + \phi \\ & F(y,0) = 0, F(0,t) = 1, F \to 0\ as\ y \to \infty, \end{aligned}\right\} \quad (16)$$

$$\left.\begin{aligned} & \frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial y^2} - \psi \\ & \psi(y,0) = 0. \psi(0,t) = 2, \psi \to 0\ as\ y\ \to \infty. \end{aligned}\right\} \quad (17)$$

Using Fourier sine transform, we obtain the solution of (17) as:

$$\psi(y,t) = \frac{4}{\pi}\int_0^\infty \frac{s}{s^2+1}\left(1 - e^{-(s^2+1)t}\right)\sin sy\, ds. \quad (18)$$

And the solution of (16) as:

$$F(y,t)=\frac{2}{\pi}\int_0^\infty \frac{s}{(s^2+1)}\left(\left(1-e^{-(s^2+1)t}\right)+\frac{2}{(s^2+1)}\left(1-e^{-(s^2+1)t}\right)-2te^{-(s^2+1)t}\right)\sin sy\, ds. \quad (19)$$

Then we obtain:

$$\theta(y,t)=\frac{4}{\pi}\int_0^\infty \frac{s}{s^2+1}\left(1-e^{-(s^2+1)t}\right)\sin sy\, ds-\phi(y,t). \quad (20)$$

$$\phi(y,t)=\frac{4}{\pi}\int_0^\infty \frac{s}{s^2+1}\left(1-e^{-(s^2+1)t}\right)\sin sy\, ds-\theta(y,t). \quad (21)$$

Hence there exists a unique solution of problem (12) to (15). This completes the proof of the theorem.

Analytical Solution

We now solve the equations (12), (13) and (14) subject to the initial and boundary conditions (15) using Fourier Sine transform and we obtain,

$$\theta(y,t)=\left[1-\operatorname{erf}\left(\frac{\sqrt{y^2+R}}{2\sqrt{t(P_r)^{-1}}}\right)\right]=\operatorname{erf}c\left(\frac{\sqrt{y^2+R}}{2\sqrt{t(P_r)^{-1}}}\right) \quad (22)$$

$$\phi(y,t)=\left[1-\operatorname{erf}\left(\frac{\sqrt{y^2+K_2}}{2\sqrt{t(S_c)^{-1}}}\right)\right]=\operatorname{erf}c\left(\frac{\sqrt{y^2+K_2}}{2\sqrt{t(S_c)^{-1}}}\right) \quad (23)$$

$$F(y,t)=e^{-Nt}\left[\operatorname{erf}c\left(\frac{\sqrt{y^2+N}}{2\sqrt{t}}\right)+G_{r\phi}\left(\operatorname{erf}c\left(\frac{\sqrt{y^2+N+K_2}}{2\sqrt{t}}\right)-\operatorname{erf}c\left(\frac{\sqrt{y^2+N}}{2\sqrt{\left(1+\frac{1}{S_c}\right)t}}\right)\right)+G_{r\theta}\left(\operatorname{erf}c\left(\frac{\sqrt{y^2+N+R}}{2\sqrt{t}}\right)-\operatorname{erf}c\left(\frac{\sqrt{y^2+N}}{2\sqrt{\left(1+\frac{1}{P_r}\right)t}}\right)\right)\right] \quad (24)$$

The computation were done for solutions (22) – (24)

4. RESULTS AND DISCUSSION

The system of partial differential equations describing the heat and mass transfer flow of MHD natural convective flow past an infinite vertical plate, were solved analytically using the Fourier transform method. It was observed as the fluid is been heated and temperature increases, the thermal conductivity increases while the fluid becomes less viscous. It is quite evident that Prandtl number has no effect on the concentration of the fluid. We also noticed from the graph that an increase in Prandtl number decreases temperature therefore Prandtl number can be used for cooling of the system. If the Grashof number is increased, it increases the buoyancy of the fluid which will invariably increase the velocity of the fluid. Fig. 4.3 shows the variation of solutal Grashof number on velocity of the fluid. The solutal Grashof number has the same effect as the thermal Grashof number on the velocity of the fluid. As the solutal Grashof number increases, it increases the buoyancy force of the fluid and velocity increases as shown in Fig. 4.4 below. It is quite evident that Prandtl number has no effect on the concentration of the fluid as shown from the graph.

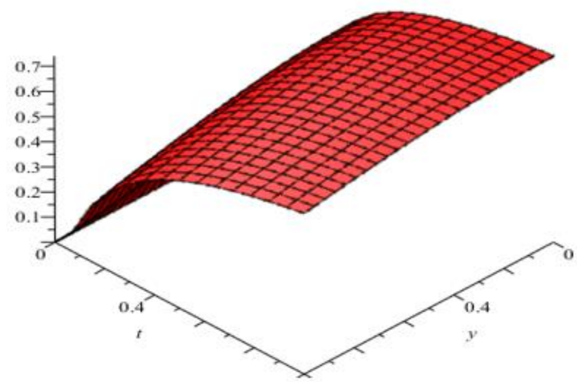

Figure 4.1: Variation of Prandtl number P_r on Concentration for present study**.**
$P_r=0.3$ (Red)

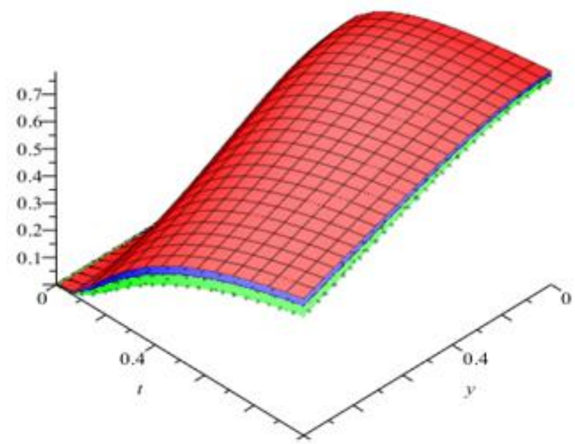

Figure 4.2: Variation of prandtl number, P_r on Temperature for present study**.**
$P_r=0.3$ (Red), $P_r=0.5$ (Blue), $P_r=0.71$ (Green).

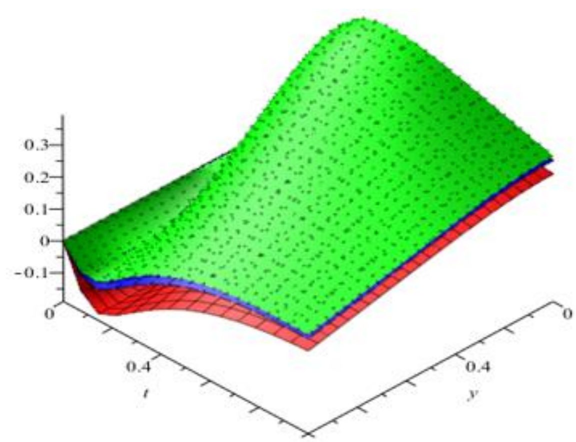

Figure 4.3: Variation of thermal Grashof number, $G_{r\theta}$ on Velocity for present study
$G_{r\theta}=2$ (Red), $G_{r\theta}=4$ (Blue), $G_{r\theta}=6$(Green).

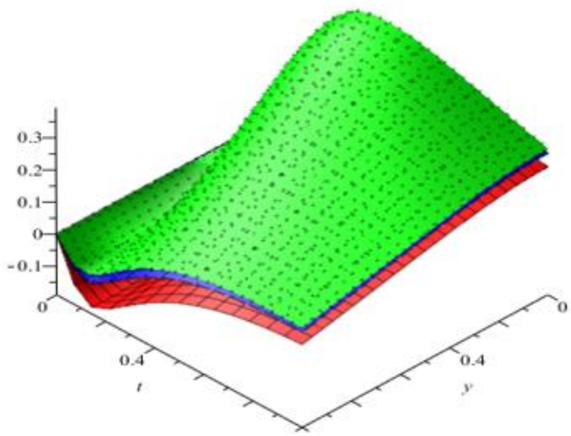

Figure 4.4: Variation of solutal Grashof number, $G_{r\phi}$ on Velocity for present study**.**
$\mathrm{G_{r\phi}} = 3$ (Red), $\mathrm{G_{r\phi}} = 5$ (Blue), $\mathrm{G_{r\phi}} = 7$ (Green)

5. Conclusion
In this research work, we solved analytically the mathematical model for studying Hall current effects on transient magnetohydrodynamic natural convection flow in a vertical microchannel. The formulated coupled partial differential equations (PDEs) were then solved analytically using the Fourier Sine Transform method and the effects of the dimensionless flow parameters were analyzed as shown from the graph.

(i) Prandtl number is inversely proportional to temperature.
(ii) Prandtl number has no effect on the concentration of the fluid.

Both thermal and solutal Grashof number is directly proportional to the velocity of the fluid.

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