Science World Journal Vol. 16(No 2) 2021
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

Full Length Research Article

MODELLING FIRE SPREAD REACTION RATE IN ATMOSPHERIC-WEATHER CONDITION

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ABSTRACT
Fire spread in any fire environment is a thing of great concern as wind is arguably the most important weather factor that influences the spread of fire. In this paper, we present equations governing the phenomenon and assume the fire depends on the space variable x. Analytical solution is obtained via perturbation method, direct integration and eigenfunction expansion technique, which depicts the influence of parameters involved in the system. The effect of change in parameters such as Peclet mass number and Equilibrium wind velocity are presented graphically and discussed. The results obtained revealed that both Peclet mass number and Equilibrium wind velocity enhanced oxygen concentration during fire spread.

Keywords: Fire spread, weather, wind, wildland fire, precipitation.

INTRODUCTION

Weather has a profound influence on wildland fire ignition potential, fire behavior, and fire severity. Local weather are affected by large-scale patterns of winds over the hemispheres (Randall, 2009). Weather is defined as the state of the atmosphere at some place and time, described in terms of such quantitative variables as temperature, humidity, cloudiness, precipitation, and wind speed and direction. Weather is dynamic and differs from the climate of a location, since observed weather over a time period constitutes climate. Fire weather, collectively, is the weather variables, especially wind, temperature, relative humidity, and precipitation that influence fire starts, fire behavior, or fire suppression (Pyne *et al.,* 1996). Kasischke *et al.* (2002) used geographic analyses to show that the most relevant weather factors affecting fire occurrence in Alaska were growing season temperature, precipitation, and lightning frequency. Short-term local weather, particularly unusual dry spells, low relative humidities, and windy weather generally associated with cold fronts, predispose wildland fuels to fire (Johnson and Miyanishi, 2001).

Wind affects fire occurrence and especially fire behavior at the synoptic, regional, and microclimate scales. Fast-moving air at high altitudes or the jet stream level has been observed to enhance wildland fire behavior by allowing drier and warmer stratospheric air to penetrate to the lower part of the troposphere (Carlson, 1980; Danielsen, 1968; Keyser and Shapiro, 1986).Randall (2009) stated that most wildland fire move in one or more directions depending on the availability of fuels, wind direction, and topography. If fuels are discontinuous (e.g., as in deserts), then a fire may not spread successfully unless wind velocity and direction are sufficient to cause a fire to ‘‘leap’’ the gap between fuels.

Mandel *et al.* (2018) studied Coupled atmosphere-wildland fire modelling with Weather Research and Forecasting (WRF) Fire. They described the coupled atmosphere-fire model WRF-Fire, as they did not support canopy fire, although canopy fire collocated with ground fire is contained in Coupled Atmospheric Wildland Fire Environment (CAWFE). In a coupled model, however, the feedback on the fire is from the wind that is influenced by the fire.
Lopes *et al.* (2017) addressed the problem of how wind should be taken into account in fire spread simulations. Their study was based on the software system Fire Station, which incorporates a surface fire spread model and a solver for the fluid flow (Navier–Stokes) equations. It was noted that, a two-way coupling method for fire behaviour analysis, where the buoyancy effects caused by the fire heat release are fully simulated. They concluded and described the underlying models for wind field and fire spread calculation. Zhiri *et al.* (2020) worked on modelling fire spread behaviour in coupled atmospheric-forest fire using direct integration and eigenfunction expansion technique to obtained analytical solution and concluded that, an increase in activation energy number causes temperature to retard but oxygen concentration is enhanced.
Wildland fire are often described by the direction the fire is spreading relative to the direction of the wind. A fire that spreads in the direction that the wind is blowing is called a heading fire, a fire that spreads in the direction the wind is blowing from is called a backing fire, and a fire that spreads perpendicular to the wind direction is called a flanking fire. The rates and completeness of combustion often differ as a function of fire spread, which in turn influences the production of smoke.
This study is aimed at establishing an analytical solutions capable of determining the nature of fire spread reaction rate in coupled atmospheric-weather condition. This will be achieved via perturbation method, direct integration and eigenfunction expansion technique.

MODEL FORMULATIONS

A wildfire model is formulated based on balance equations for energy and fuel, where the fuel loss due to burning corresponds to the fuel reaction rate. The respective equations governing forest fires propagation are:

Volume fraction of dry organic substance

$$\frac{\partial \varphi_s}{\partial t} = -k_1 \varphi_s e^{-\frac{E_1}{RT}} \qquad (1)$$

Volume fraction of moisture

$$\frac{\partial \varphi_m}{\partial t} = -k_2 \varphi_m T^{\frac{1}{2}} e^{-\frac{E_2}{RT}} \qquad (2)$$

Volume fraction of coke

$$\rho_c \frac{\partial \varphi_c}{\partial t} = \alpha_c k_1 \rho_s \varphi_s e^{-\frac{E_1}{RT}} - \frac{M_c}{M_1} k_3 S_\sigma \rho_g \varphi_c C_{ox} e^{-\frac{E_3}{RT}} \quad (3)$$

Mass concentration of oxygen

$$\left.\begin{aligned}&\rho_g\left(\frac{\partial C_{ox}}{\partial t}+v\frac{\partial C_{ox}}{\partial x}\right)=\frac{\partial}{\partial x}\left(\rho_g D_T \frac{\partial C_{ox}}{\partial x}\right)-\frac{\alpha}{C_{pg}\Delta h}\left(C_{ox}-C_{ox_\infty}\right)-\\&\left(1-\alpha_c\right)k_1\rho_s\varphi_s C_{ox}e^{-\frac{E_1}{RT}}-k_2\rho_m T^{\frac{1}{2}}\varphi_m C_{ox}e^{\frac{-E_2}{RT}}-k_3 S_\sigma \rho_g\left(1+\frac{M_c}{M_1}C_{ox}\right)\varphi_c C_{ox}e^{\frac{-E_3}{RT}}\end{aligned}\right\} \quad (4)$$

Energy balance equation

$$\left.\begin{aligned}&\left(\phi\rho_g C_{pg}+\left(1-\phi\right)\sum_{i=1}^{s+m+c}\rho_i C_{pi}\varphi_i\right)\frac{\partial T}{\partial t}+\rho_g C_{pg} v\frac{\partial T}{\partial x}=\frac{\partial}{\partial x}\left(\lambda_T\frac{\partial T}{\partial x}\right)-\frac{\alpha}{\Delta h}\left(T-T_\infty\right)\\&-4K_R\sigma T^4-k_2\rho_m q_2 T^{\frac{1}{2}}\varphi_m e^{\frac{-E_2}{RT}}+k_3 S_\sigma\rho_g q_3\varphi_c C_{ox}e^{\frac{-E_3}{RT}}\end{aligned}\right\} \quad (5)$$

With initial and boundary conditions:

$$\left.\begin{aligned}&\varphi_s(x,0)=\varphi_{so},\ \varphi_m(x,0)=\varphi_{mo},\ \varphi_c(x,0)=\varphi_{co},\ C_{ox}(x,0)=C_{ox_0},\ C_{ox}(0,t)=C_{ox_\infty}\\&C_{ox}(\mathrm{L},t)=C_{ox_\infty},\ T(x,0)=T_0,\ T(0,t)=T_\infty,\ T(\mathrm{L},t)=T_\infty\end{aligned}\right\} \quad (6)$$

Where;

ϕ_s is the volume fraction of dry organic substance, ϕ_m is the volume fraction of moisture, ϕ_c is the volume fraction of coke, C_{ox} is the concentration of oxygen, T is the temperature (in Kelvin), t is the time, x is a coordinate in the system of coordinates connected with the centre of an initial fire (distance), T_∞ is the unperturbed ambient temperature, $k_j, j=1,2,3$ are the pre- exponential factors of chemical reactions, $E_j, j=1,2,3$ are the activation energy of chemical reactions, C is the concentration, R is the universal gas constant, S_σ is the specific surface of the condensed product of pyrolysis (coke), v is the equilibrium wind velocity vector, U is the reference velocity, λ_T is the turbulent thermal conductivity, C_{ox_∞} is the unperturbed density of concentration of oxygen, $P_i, i=(s,m,c)$ is the i^{-th} phase density, that is ρ_s is the density of dry organic substance, ρ_m is the density of moisture, ρ_c is the density of coke, ρ_g is the density of gas phase (a mix of gases), Δh is the crown height, M_c is the molecular mass of carbon, M_1 is the mass of combustible forest material (CFM), C_{pg} is the thermal capacity of a gas phase, $q_j, j=2,3$ defines heat effects of processes of evaporation of burning, D_T is the diffusion coefficient, α is the coefficient of heat exchange between the atmosphere and a forest canopy, α_c is the coke number of combustible forest material (CFM), K_R is the Stefan-Boltzmann constant, $C_{p_i}, i=(s,m,c)$ is the i^{-th} phase of thermal capacity, s is the dry organic substance, m is the moisture, c is the coke, ox is the oxygen (O_2).

METHOD OF SOLUTION

Non-dimensionalisation

Here equation (1) – (6) are non-dimensionalize using the following dimensionless variables:

$$\left.\begin{aligned}&x'=\frac{x}{L},\ t'=\frac{Ut}{L},\ v'=\frac{v}{U},\ \psi_1=\frac{\varphi_s}{\varphi_{so}},\ \psi_2=\frac{\varphi_m}{\varphi_{mo}},\ \psi_3=\frac{\varphi_c}{\varphi_{co}},\ \phi=\frac{C_{ox}-C_{ox_\infty}}{C_{ox_o}-C_{ox_\infty}}\\&\in=\frac{RT_0}{E},\ \theta=\frac{E\left(T-T_o\right)}{RT_o^2},\ f=\frac{E_1}{E_3},\ r=\frac{E_2}{E_3}\end{aligned}\right\} \quad (7)$$

and we obtain;

$$\left.\begin{aligned}&\frac{\partial\psi_1}{\partial t}=-a\psi_1 e^{\frac{f\theta}{1+\in\theta}}\\&\psi_1(x,0)=1\end{aligned}\right\} \quad (8)$$

$$\left.\begin{aligned}&\frac{\partial\psi_2}{\partial t}=-b\psi_2\left(1+\in\theta\right)^{\frac{1}{2}}e^{\frac{r\theta}{1+\in\theta}}\\&\psi_2(x,0)=1\end{aligned}\right\} \quad (9)$$

$$\left.\begin{aligned}&\frac{\partial\psi_3}{\partial t}=\beta\psi_1 e^{\frac{f\theta}{1+\in\theta}}-\gamma\left(\phi+q\right)\psi_3 e^{\frac{\theta}{1+\in\theta}}\\&\psi_3(x,0)=1\end{aligned}\right\} \quad (10)$$

$$\left.\begin{aligned}&\frac{\partial\phi}{\partial t}+v\frac{\partial\phi}{\partial x}=\frac{\partial}{\partial x}\left(D_1\frac{\partial\phi}{\partial x}\right)-\beta_1\phi-\beta_2\psi_1\left(\phi+q\right)e^{\frac{f\theta}{1+\in\theta}}\\&-\beta_3\left(1+\in\theta\right)^{\frac{1}{2}}\psi_2\left(\phi+q\right)e^{\frac{r\theta}{1+\in\theta}}-\beta_4\psi_3\left(\phi+p\right)\left(\phi+q\right)e^{\frac{\theta}{1+\in\theta}}\\&\phi(x,0)=1,\ \ \phi(0,t)=0,\ \ \phi(1,t)=0\end{aligned}\right\} \quad (11)$$

$$\left.\begin{aligned}&\frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial x}=\frac{\partial}{\partial x}\left(\lambda_1\frac{\partial\theta}{\partial x}\right)-\alpha_1\left(\theta+\gamma_1\right)-R_a\left(1+4\in\theta\right)-\delta\psi_2\left(1+\in\theta\right)^{\frac{1}{2}}e^{\frac{r\theta}{1+\in\theta}}\\&+\delta_1\psi_3\left(\phi+q\right)e^{\frac{\theta}{1+\in\theta}}\\&\theta(x,0)=0,\ \ \theta(0,t)=\sigma_1,\ \ \theta(1,t)=\sigma_1\end{aligned}\right\} \quad (12)$$

Where;

$$a=\frac{k_1 L e^{\frac{-fE_1}{RT_o}}}{U},\quad b=\frac{k_2 T_o^{\frac{1}{2}} L e^{\frac{-rE_3}{RT_0}}}{U},\quad \beta=\frac{\alpha_c k_1 \rho_s \varphi_{so} L e^{\frac{-fE_3}{RT_o}}}{U\rho_c\varphi_{co}},\ \gamma=\frac{M_c k_3 S_\sigma \rho_g L}{M_1 U\rho_c}\left(C_{ox_o}-C_{ox_\infty}\right)e^{\frac{-E_3}{RT_o}},$$

$$q=\frac{C_{ox_\infty}}{C_{ox_o}-C_{ox_\infty}},\ D_1=\frac{D_T}{LU}=\frac{1}{P_{em}},\ \beta_1=\frac{\alpha L}{C_{pg}\Delta hU},\ \beta_2=\frac{(1-\alpha_c)k_1\rho_s\varphi_{so}L}{\rho_g U}e^{\frac{-fE_3}{RT_o}},$$

$$\beta_3=\frac{k_2\rho_m T_o^{\frac{1}{2}}\varphi_{mo} L e^{\frac{-rE_3}{RT_o}}}{\rho_g U},\ \beta_4=\frac{k_3 S_\sigma \rho_g \frac{M_c}{M_1}\left[C_{ox_o}-C_{ox_\infty}\right]L\varphi_{co}e^{\frac{-E_3}{RT_o}}}{\rho_g U},\ p=\frac{\frac{M_1}{M_c}+C_{ox_\infty}}{C_{ox_o}-C_{ox_\infty}},$$

$$\lambda_1=\frac{\lambda_T}{L\rho_g C_{pg}U}=\frac{1}{P_e},\ \alpha_1=\frac{\alpha L}{\rho_g C_{pg}U},\ R_a=\frac{4K_R\sigma L T_o^3}{\rho_g C_{pg}\in U},\ \delta=\frac{k_2\rho_m q_2 T_o^{\frac{1}{2}} L\varphi_{mo} e^{\frac{-rE_3}{RT_o}}}{\rho_g C_{pg}\in T_o U},$$

$$\delta_1=\frac{k_3 S_\sigma \rho_g q_3 \varphi_{co} L\left(C_{ox_o}-C_{ox_\infty}\right)}{\rho_g C_{pg}\in T_o U}e^{\frac{-E_3}{RT_o}},\ \gamma_1=\frac{T_o-T_\infty}{\in T_o},\ \sigma_1=\frac{T_\infty-T_o}{\in T_o} \qquad (13)$$

Analytical Solution

Using perturbation method, direct integration and eigenfunction expansion technique, the analytical solution of equations (8)—(12) are obtained as follows:

$$\psi_1(x,t)=1+v\left(-A_3\sum_{n=1}^{\infty}A_2\sin n\pi x-a_6\left(t+\left(\begin{array}{l}f(e-2).\\ \left(\sigma_1 t+\sum_{n=1}^{\infty}\left(A_1 t-\left(\frac{b_n-A_1}{c_2}\right)e^{-c_2 t}\right)\sin n\pi x\right)\end{array}\right)\right)\right) \qquad (14)$$

$$\psi_2(x,t)=1+v\left(\begin{array}{l}-a_7\left(\begin{array}{l}t+r(e-2)\left(\sigma_1 t+\sum_{n=1}^{\infty}\left(A_1 t-\left(\frac{b_n-A_1}{c_2}\right)e^{-c_2 t}\right)\sin n\pi x\right)+\\ \frac{1}{2}\in\left(\sigma_1 t+\sum_{n=1}^{\infty}\left(A_1 t-\left(\frac{b_n-A_1}{c_2}\right)e^{-c_2 t}\right)\sin n\pi x\right)+\\ \frac{1}{2}\in(r(e-2))\left(\begin{array}{l}\sigma^2_1 t+2\sigma_1\left(\sum_{n=1}^{\infty}\left(A_1 t-\left(\frac{b_n-A_1}{c_2}\right)e^{-c_2 t}\right)\sin n\pi x\right)\\ +\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left(\begin{array}{l}A_1^2 t-\frac{2A_1}{c_2}(b_n-A_1)e^{-c_2 t}\\ -\frac{(b_n-A_1)^2}{2c_2}e^{-2c_2 t}\end{array}\right)\sin^2 n\pi x\end{array}\right)\end{array}\right)\\ -a_7\left(A_7+\sum_{n=1}^{\infty}B_1\sin n\pi x\right)\sum_{n=1}^{\infty}A_2\sin n\pi x\end{array}\right) \qquad (15)$$

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www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
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$$\psi_3(x,t)=1+v\left(\begin{array}{l} a_9\left(A_8 t+A_9\sum_{n=1}^{\infty}\left(A_1 t-A_2 e^{-c_2 t}\right)\sin n\pi x\right) \\ +a_8\left(\begin{array}{l} A_{10}\sum_{n=1}^{\infty}\frac{A}{c_1}e^{-c_1 t}\sin n\pi x+A_{11}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left(B_2 e^{-c_1 t}+B_3 e^{-(c_1+c_2)t}\right)\sin^2 n\pi x \\ -A_{12}t-A_{13}\sum_{n=1}^{\infty}\left(A_1 t-A_2 e^{-c_2 t}\right)\sin n\pi x \end{array}\right) \\ +a_9 A_9\sum_{n=1}^{\infty}A_2\sin n\pi x-a_8\left(\begin{array}{l} A_{10}\sum_{n=1}^{\infty}\frac{A}{c_1}\sin n\pi x+A_{11}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left(B_4\right)\sin^2 n\pi x \\ +A_{13}\sum_{n=1}^{\infty}A_2\sin n\pi x \end{array}\right) \end{array}\right) \quad (16)$$

$$\phi(x,t)=\sum_{n=1}^{\infty}Ae^{-c_1 t}\sin n\pi x+v\left(\sum_{n=1}^{\infty}\left(\begin{array}{l} -2A_{49}\sum_{n=1}^{\infty}Ate^{-c_1 t}-2A_{50}\sum_{n=1}^{\infty}\left[\frac{A_1}{c_1}+A_{53}e^{-c_2 t}-A_{54}e^{-c_1 t}\right] \\ -A_{56}\left[1-e^{-c_1 t}\right]-2A_{51}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}A_{52}\left[\begin{array}{l} AA_1 te^{-c_1 t}-A_{55}e^{-(c_1+c_2)t} \\ +A_{55}e^{-c_1 t} \end{array}\right] \\ -2A_{42}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}A_{52}\left[\begin{array}{l} \frac{A_1^2}{c_1}+A_{57}e^{-c_2 t}+A_{58}e^{-2c_2 t}-\frac{A_1^2}{c_1}e^{-c_1 t} \\ -A_{57}e^{-c_1 t}-A_{58}e^{-c_1 t} \end{array}\right] \\ -2A_{43}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left[\begin{array}{l} AA_1^2 te^{-c_1 t}-A_{59}e^{-(c_1+c_2)t} \\ -A_{59}e^{-(c_1+c_2)t}-A_{60}e^{-(c_1+2c_2)t} \\ +A_{59}e^{-c_1 t}+A_{59}e^{-c_1 t}+A_{60}e^{-c_1 t} \end{array}\right] \\ +2A_{44}\sum_{n=1}^{3}\sum_{n=1}^{3}A_{61}\left[1-e^{-c_1 t}\right]-2A_{45}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}A_{52}\left[\begin{array}{l} AA_1 te^{-c_1 t} \\ -A_{55}e^{-(c_1+c_2)t} \\ +A_{55}e^{-c_1 t} \end{array}\right] \\ +2A_{46}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left[A_{62}e^{-2c_1 t}+A_{63}e^{-(2c_1+c_2)t}-A_{62}e^{-c_1 t}-A_{63}e^{-c_1 t}\right] \end{array}\right)\sin n\pi x\right) \quad (17)$$

$$\theta(x,t)=\left\{\begin{array}{l} \left(\sigma_1+\sum_{n=1}^{\infty}\left(A_1+\left(b_n-A_1\right)e^{-c_2 t}\right)\sin n\pi x\right)+ \\ v\left(\sum_{n=1}^{\infty}\left(\begin{array}{l} -2\frac{A_{72}}{c_2}\left[1-e^{-c_2 t}\right]-2A_{73}\sum_{n=1}^{\infty}\left[\frac{A_1}{c_2}+\left(b_n-A_1\right)te^{-c_2 t}-\frac{A_1}{c_2}e^{-c_2 t}\right] \\ -2A_{68}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}A_{52}\left[\begin{array}{l} \left[1-e^{-c_2 t}\right]\frac{A_1^2}{c_2}+2A_1\left(b_n-A_1\right)te^{-c_2 t} \\ +\left[e^{-c_2 t}-1\right]\frac{\left(b_n-A_1\right)^2}{c_2} \end{array}\right] \\ +2A_{69}\sum_{n=1}^{\infty}\frac{A}{\left(c_2-c_1\right)}\left[e^{-c_1 t}-e^{-c_2 t}\right]+2A_{70}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}A_{52}\left[\begin{array}{l} \frac{AA_1}{\left(c_2-c_1\right)}e^{-c_1 t}-\frac{A\left(b_n-A_1\right)}{c_1}e^{-(c_2+c_1)t} \\ -\frac{AA_1}{\left(c_2-c_1\right)}e^{-c_2 t}+\frac{A\left(b_n-A_1\right)}{c_1}e^{-c_2 t} \end{array}\right] \end{array}\right)\sin n\pi x\right) \end{array}\right\} \quad (18)$$

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www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

Where;

$$\left(\begin{array}{l}
c_1=\left(\beta_1+D_1(n\pi)^2\right),\ A=\dfrac{2\left[1-(-1)^n\right]}{n\pi},\ b_1=\left(4R_a\in+\alpha_1\right),\ b_2=\left(\sigma_1\left(4R_a\in+\alpha_1\right)+\left(R_a+\alpha_1\gamma_1\right)\right),\\
c_2=\left(b_1+\lambda_1(n\pi)^2\right),\ A_1=\dfrac{2b_2\left[(-1)^n-1\right]}{n\pi c_2},\ A_2=\left(\dfrac{b_n-A_1}{c_2}\right),\ A_3=a_6f(e-2),\ A_4=r(e-2),\\
A_5=\dfrac{1}{2}\in,\ A_6=\in\left(r(e-2)\right)\sigma_1,\ B=\dfrac{b_n-A_1}{2},\ B_1=\left(2A_1+B\right),\ A_7=\left(A_4+A_5+A_6\right),\ b_n=\dfrac{2\sigma_1}{n\pi}\left[(-1)^n-1\right]\\
A_8=\left(1+f(e-2)\sigma_1\right),\ A_9=f(e-2),\ A_{10}=\left(1+(e-2)\sigma_1\right),\ A_{11}=(e-2),\ B_2=\dfrac{AA_1}{c_1},\\
B_3=\dfrac{A\left(b_n-A_1\right)_1}{c_1+c_2},\ A_{12}=\left(1+(e-2)\sigma_1\right)q,\ A_{13}=(e-2)q,\ B_4=\left(B_2+B_3\right),\ A_{14}=\left(1+f(e-2)\sigma_1\right)q,\\
A_{15}=f(e-2)q,\ A_{16}=\left(1+r(e-2)\sigma_1\right),\ A_{17}=r(e-2),\ A_{18}=\left(1+r(e-2)\sigma_1\right)q,A_{19}=r(e-2)q,\\
A_{20}=\dfrac{1}{2}\in\left(\left(1+r(e-2)\sigma_1\right)\sigma_1\right),\ A_{21}=\dfrac{1}{2}\in r(e-2)\sigma_1,\ A_{22}=\dfrac{1}{2}\in\left(\left(1+r(e-2)\sigma_1\right)\sigma_1q\right),\\
A_{23}=\dfrac{1}{2}\in r(e-2)\sigma_1q,\ A_{24}=\dfrac{1}{2}\in\left(1+r(e-2)\sigma_1\right),\ A_{25}=\dfrac{1}{2}\in r(e-2),\ A_{26}=\dfrac{1}{2}\in\left(1+r(e-2)\sigma_1\right)q,\\
A_{27}=\dfrac{1}{2}\in r(e-2)q,\ A_{28}=(p+q)\left(1+(e-2)\sigma_1\right),\ A_{29}=(p+q)(e-2),\ A_{30}=(pq)\left(1+(e-2)\sigma_1\right),\\
A_{31}=(pq)(e-2),A_{32}=\left(a_1A_{14}+a_2A_{18}+a_2A_{22}+a_3A_{30}\right),\ A_{33}=\left(a_1A_{16}+a_2A_{20}\right),\ A_{34}=\left(a_2A_{19}+a_2A_{23}+a_2A_{26}\right),\\
A_{35}=\left(a_2A_{17}+a_2A_{21}+a_2A_{22}+a_2A_{24}\right),\ A_{36}=\dfrac{1}{2}a_1A_8,\ A_{37}=\dfrac{1}{2}a_1A_{15},\ A_{38}=\dfrac{2}{3}a_1A_9,\ A_{39}=\dfrac{1}{2}A_{33},\ A_{40}=\dfrac{1}{2}A_{34},\\
A_{41}=\dfrac{2}{3}A_{35},\ A_{42}=\dfrac{2}{3}a_2A_{27},\ A_{43}=\dfrac{3}{8}a_2A_{25},\ A_{44}=\dfrac{2}{3}a_3A_{10},\ A_{45}=\dfrac{2}{3}a_3A_{29},\ A_{46}=\dfrac{3}{8}a_3A_{11},\ A_{47}=\dfrac{1}{2}a_3A_{38},\\
A_{48}=\dfrac{1}{2}a_3A_{31},\ A_{49}=\left(A_{36}+A_{39}+A_{47}\right),\ A_{50}=\left(A_{37}+A_{40}+A_{48}\right),\ A_{51}=\left(A_{38}+A_{41}\right),\ A_{52}=\left[\dfrac{1-(-1)^n}{n\pi}\right],\\
A_{53}=\dfrac{b_n-A_1}{c_1-c_2},\ A_{54}=\dfrac{A_1\left(c_1-c_2\right)+c_1\left(b_n-A_1\right)}{c_1\left(c_1-c_2\right)},\ A_{55}=\dfrac{A\left(b_n-A_1\right)}{c_2},\ A_{56}=2\dfrac{A_{32}A_{52}}{c_1},\ A_{57}=\dfrac{2A_1\left(b_n-A_1\right)}{c_1-c_2},\\
A_{58}=\dfrac{\left(b_n-A_1\right)^2}{\left(c_1-2c_2\right)},\ A_{59}=\dfrac{AA_1\left(b_n-A_1\right)}{c_2},\ A_{60}=\dfrac{A\left(b_n-A_1\right)^2}{2c_2},\ A_{61}=\dfrac{A_{52}A^2}{c_1},\ A_{62}=\dfrac{A^2A_1}{c_1},\ A_{63}=\dfrac{A^2\left(b_n-A_1\right)}{\left(c_1+c_2\right)},\\
A_{64}=\left(a_4A_{16}+a_4A_{20}-a_5A_{12}\right),\ A_{65}=\dfrac{a_4A_{17}}{2},\ A_{66}=\dfrac{a_4A_{21}}{2},\ A_{67}=\dfrac{a_4A_{24}}{2},\ A_{68}=\dfrac{2a_4A_{25}}{3},\ A_{69}=\dfrac{a_5A_{10}}{2},\\
A_{70}=\dfrac{2a_5A_{11}}{3},\ A_{71}=\dfrac{a_5A_{13}}{2},\ A_{72}=\left(A_{64}A_{52}\right),\ A_{73}=\left(A_{65}+A_{66}+A_{67}-A_{71}\right)
\end{array}\right)$$

The computation were done using Maple 17 to generate the graphs.

Science World Journal Vol. 16(No 2) 2021
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
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RESULTS AND DISCUSSION

To conclude this analysis, we examine the effect of Peclet mass number (P_{em}) and Equilibrium wind velocity (v) on oxygen concentration $\phi(x,t)$. Analytical solution given by equation (14) to (18), is computed using computer symbolic algebraic package MAPLE 17. The numerical result obtain from the method are show in Figure 1 and 2

Figure 1 depicts the graph of oxygen concentration $\phi(x,t)$ against distance x for different values of Peclet mass number (P_{em}). It is observed that the oxygen concentration oscillates along the distance and maximum concentration increases as the Peclet mass number increases.

Figure 2 depicts the graph of oxygen concentration $\phi(x,t)$ against distance x for different values of equilibrium wind velocity (v). It is observed that the oxygen concentration oscillates along the distance and maximum concentration increases as the equilibrium wind velocity increases.

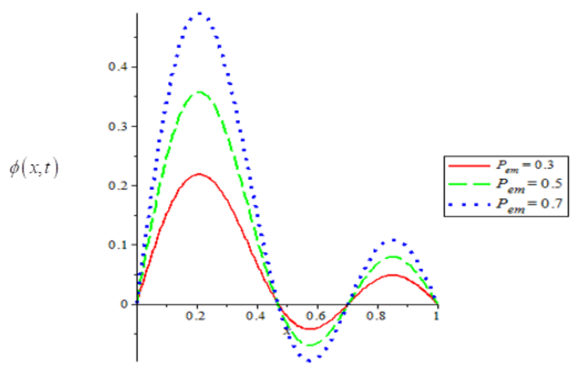

Figure 1: Graph of oxygen concentration $\phi(x,t)$ against distance x for different values of Peclet mass number (P_{em}). $(P_{em}) = 0.3$ (Red), $(P_{em}) = 0.5$ (Green) and $(P_{em}) = 0.7$ (Blue).

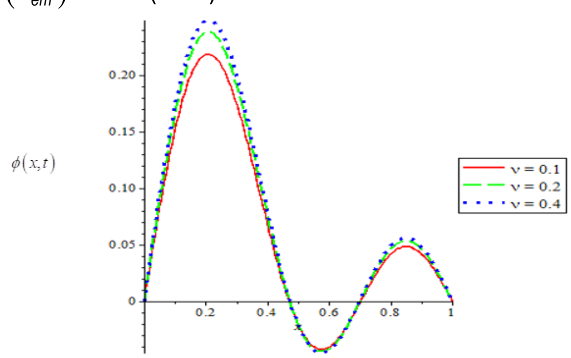

Figure 2: Graph of oxygen concentration $\phi(x,t)$ against distance x for different values of equilibrium wind velocity (v). $(v) = 0.1$ (Red), $(v) = 0.2$ (Green) and $(v) = 0.4$ (Blue).

It is worth pointing out that the effects observed in figures 1 and 2 are important for fire safety precaution as both parameters enhanced maximum oxygen concentration along the distance that escalate the rate of fire spread. Forest fire fighters could employ necessary skills and tools to retard the increase rate of these parameters discussed in this study as wind plays an important role in fire spread simulation.

Conclusion

For a high activation energy situation (i.e., as $\in \longrightarrow 0$), we have solved the equations governing the fire spread model using perturbation method, direct integration and eigenfunction expansion technique. From the result obtained, we can conclude that, Peclet mass number and Equilibrium wind velocity enhances oxygen concentration.

These results obtained are not only expected to guide fire services to manage the danger rating forest fire spread rate but to forecast the specific weather elements relating to forest fire.

REFERENCES

Carlson, T. N. (1980). Airflow through mid-latitude cyclones and the comma cloud pattern. Mon. Wea. Rev. 108, 1498–1509.

Danielsen, E. F. (1968). Stratospheric–tropospheric exchange based on radioactivity, ozone and potential vorticity. J. Atmos. Sci. 25, 502–518.

Johnson, E. A., and Miyanishi, K., eds. (2001). Forest fires: Behavior and ecological effects. *Academic Press, San Diego*, CA, 594 pp.

Kasischke, E. S., Williams, D., and Barry, D. (2002). Analysis of the patterns of large fires in the boreal forest region of Alaska. *Int. J. Wildland Fire 11(2)*, 131–144, doi:10.1071/WF02023.

Keyser, D., & Shapiro, M. A. (1986). A review of the structure and dynamics of upper-level frontal zones. Mon. Wea. Rev. 114, 452–499.

Lopes, A. M. G., Ribeiro, Viegas, D. X., & Raposo, J. R. (2017). Effect of two-way coupling on the calculation of forest fire spread: model development. *International Journal of Wildland Fire 26*, 829–843 https://doi.org/10.1071/WF16045.

Mandel, J., Jonathan, D. B., & Adam, K. K. (2018). Coupled atmosphere-wildland fire modelling with WRF-Fire version 3.3

Pyne, S. J., Andrews, P. L., and Laven, R. D. (1996). Introduction to wildland fire (second ed.) Wiley, 769 pp.

Randall, P. B., John, O. R., & David, R. W. (2009). Climatic and Weather Factors Affecting Fire Occurrence and behavior. *Developments in Environmental Science, Volume 8.* ISSN: 1474-8177/DOI:10.1016/S1474-8177(08)00002-8.

Zhiri, A. B., Olayiwola, R. O. & Odo, C. E. (2020). Modeling fire spread behavior in coupled atmospheric-forest fire. *Journal of Science, Technology, Mathematics and Education (JOSTMED), 16(4), pp 104-113*