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MODELING AND FORECASTING DAILY STOCK RETURNS OF GUARANTY TRUST BANK NIGERIA PLC USING ARMA-GARCH MODELS, PERSISTENCE, HALF-LIFE VOLATILITY AND BACKTESTING

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ABSTRACT
This study investigated the forecasting ability of GARCH family models, and to achieve superior and more reliable models for volatility persistence, half-life volatility and backtesting, the study combined the ARMA and GARCH models. The study modeled and forecasted the Guaranty Trust Bank (GTB) daily stock returns using data from January 2, 2001 to May 8, 2017 obtained from a secondary source. The ARMA-GARCH models, persistence, half-life and backtesting were used to analyse the data using student t and skewed student t distributions, and the analyses were carried out in R environment using rugarch and performanceAnaytics Packages. The study revealed that using the lowest information criteria values alone could be misleading so backtesing was also carried out. The ARMA(1,1)-GARCH(1,1) models fitted exhibited high persistency in the daily stock returns while it took about 6 days for mean-reverting of the models, but failed backtesting. However, backtesting showed that ARMA(1,1)-eGARCH(2,2) model with student t distribution passed the test and was suitable for evaluating the GTB stock returns, and required about 16 days for the persistence volatility to return to its average value of the stock returns. The study recommended addition of backtesting approach in evaluating the performance of GARCH model in order to avoid misleading results. Also, the GTB stocks can be predicted since most of the estimated models were stable.

Keywords: Stock returns, Guaranty Trust (GT) Bank, Generalized Autoregressive Conditional Heteroskedasticity (GARCH), Persistence, Volatility, Backtesting

INTRODUCTION

Autoregressive Moving Average (ARMA) and Autoregressive Integrated Moving Average (ARIMA) models are popular and excellent for modeling and forecasting univariate time series data as proposed by Box & Jenkins (1970), and its extension with exogenous variables as Autoregressive Integrated Moving Average with Explanatory Variable (ARIMAX) (Kongcharoen & Kruangpradit, 2013). These models are applied in almost all fields of endeavours such as engineering, geophysics, business, economics, finance, agriculture, medical sciences, social sciences, meteorology, quality control etc. (Kirchgassner & Wolters, 2007; Adenomon, 2017a; Adenomon, 2017b; Cooray, 2008; Dobre & Alexandru, 2008; Gujarati, 2003; Adekeye & Aiyelabegan, 2006). The ARMA and ARIMA models are used to model conditional expectation of a process but in ARMA model, the conditional variance is constant. This means that ARMA model cannot capture process with time-varying conditional variance (volatility) which is mostly common with economic and financial time series data.

Actually, with economic and financial time series data, time-varying is more common than constant volatility, and accurate modeling of time volatility is of great importance in financial time series analysis (Ruppert, 2011). Financial time series contains uncertainty, volatility, excess kurtosis, high standard deviation, high skewness and sometimes non normality (Pedroni, 2001; Grigoletto & Lisi, 2009; Emenogu & Adenomon, 2018; Emenogu *et al.*, 2018). To model and capture properly the characteristics of financial time series models such as Auto-Regressive Conditional Heteroscedastic (ARCH), Generalized Auto-Regressive Conditional Heteroscedastic (GARCH), multivariate GARCH, Stochastic volatitlity (SV) and various variants of the models have been proposed to handle these characteristics of financial time series (Lawrance, 2013).

From the foregoing, considering the flexibility and simplicity of the ARMA model and the capability of the GARCH model to capture volatility in financial time series, combining the ARMA model with the GARCH model for the innovations, yielding the so-called ARMA-GARCH model, provides the econometricians and financial analyst with a more flexible and yet tractable model that allows the model to capture the mean and variance components that is common with financial time series volatility (Lange, 2011; Panait & Slavescu, 2012) meaning that the ARMA-GARCH model will produced more reliable estimates for financial analyst to take a better decision. Most financial time series analyses in Nigeria scarcely incorporate backtesting approach in selecting GARCH models.

This paper therefore investigates the persistence, half-life volatility and forecasting (Backtesting that is providing real life model) of daily stock returns of Guaranty Trust Bank, Nigeria plc using ARMA-GARCH Models. The remaining sections are as follows: Empirical review, Materials and Methods, Results, Discussion of Results, Conclusion and Recommendations.

Empirical literature on the persistence, half-life Volatility and Backtesting of Stocks Returns
The volatility of asset returns is a measure of how much the returns fluctuates around its means (Marra, 2015). In addition, volatility is the purest measure of risk in financial markets and by this, it has becomes the expected price of uncertainty. A good volatility model and forecast help impact the public confidence significantly and by extension on the broader global economy. What comes to mind again is the persistence and half-life volatility of any given stock.

The persistence of financial stock is the extent to which events today have an efficient influence on the whole future history of a stochastic process, and as such is a central issue in financial time series, macroeconomic theory and policy (Caporale & Pittis, 2001). In a stationary GARCH process, the persistence volatility returns back to its means at the long term horizon and it is a rate calculated by the sum of GARCH and ARCH coefficients. And in many financial time series it is usually close to 1 (Ahmed *et al*., 2018; Engle & Patton, 2001; Vosvrda, 2006). While on the other hand, the half-life of the volatility shocks measure the average time period for the volatility to return back to it mean value in the long run horizon (Ahmed *et al*., 2018; Sahai, 2016).

Engle & Patton (2001) examined the Dow Jones Industrial index from 23 August 1988 to 22 August 2000. Their result indicated that the volatility returns are quite persistent.

Magnus & Fosu (2006) modeled and forecasted the volatility of returns on Ghana Stock exchange using GARCH models. They found that presence of high level of persistence in the returns in the stock market.

Vosvrda (2006) compared empirical analysis of persistence and dependence patterns among capital market using univariate and multivariate measures. The results revealed that the univariate measure shows a low level of persistence while multivariate measure shows that the persistence change depended on structure in different period of lags.

Panait & Slavescus (2012) investigated the volatility and persistence of seven Romanian companies traded on Bucharest Stock Exchange and three market indices from 1997-2012 using GARCH-in-Mean Models. They found out that persistency is more in the daily returns as compared to weekly and monthly series.

Emenike & Ani (2014) examined the nature of volatility of stock returns in the Nigerian banking sector using ARMA-GARCH models using data covering 3rd January to December 2012. Their results revealed volatility persistence was high for the sample period they considered.

Usman *et al*. (2017) examined the performance of eleven competing GARCH models for fitting the rate of returns of monthly observations on the index returns series of the market over a period of January 1996 to December 2015. The overall results revealed increased volatility of the market returns.

Chu *et al*. (2017) provided the first GARCH modeling of the seven most popular cryptocurrencies using twelve GARCH models fitted for each cryptocurrencies. Their work concluded IGARCH and GJR-GARCH models provided the best fits in terms of modeling of the volatility in the most popular and largest cryptocurrencies.

Kuhe (2018) examined the volatility persistence and asymmetry with exogenous break in Nigerian stock market using data from 3rd July 1999 to 12th June 2017 using standard symmetric GARCH (1,1), asymmetric EGARCH (1,1) and GJR-GARCH (1,1) models. The study revealed among other results a high persistence of shocks in the return series for the estimated models.

Ahmed *et al*. (2018) examined and compared the mean reversion phenomenon in developed and emerging stock markets, it employed data from 1st January to 30th June 2016 using GARCH (1,1) model. There results revealed that South Korean market has the slowest mean reversion and thus has the highest half-life period while Pakistan stock exhibited fastest reverting process.

Backtesting approach is very useful in GARCH model selection, but not often applied in the Nigerian context. Summinga-Sonagadu and Narsoo (2019) employed three backtesting procudures namely Kupiec's test, a duration-based test and an asymmetric VaR loss function on Intraday of 1-min EUR/USD exchange rate returns. Their results revealed that VaR prediction of the MC-GARCH model performed better using the asymmetric loss function.

Tay *et al.* (2019) investigated the efficiency of the Value-at-Risk (VaR) backtesting in model selection from different types of GARCH models with skewed and non-skewed innovation distributions. The study implemented both simulation and real life data application (NASDAQ Index). The study revealed that AIC and VaR backtesting approach were able to select the correct model with their corresponding innovation distributions.

MATERIALS AND METHODS

Model Specification

ARMA-GARCH Models
This study focuses on the ARMA-GARCH models that are robust for forecasting the volatility of financial time series data; so ARMA-GARCH model and some of its extensions are presented in this section.

ARMA-GARCH specification is employed to model the conditional mean and conditional variance (volatility) of any financial time series because of its superiority in modelling such series. GARCH models model conditional variances much as the conditional expectation by an ARMA model (Ruppert 2011). Therefore ARMA model can be combined to any form of GARCH model.

The ARMA (p,q)-GARCH (1,1) model can be specified as follows:
ARMA(p,q)-GARCH(1,1) model can be specified as follows:

$$\left.\begin{array}{l} r_t = \sum_{i=1}^{p} \theta_i r_{t-i} + \sum_{j=1}^{q} \phi_j\, \epsilon_{t-j} + \epsilon_t \\ \epsilon_t \sqrt{\sigma_t^2 Z_t}, \qquad Z_t \sim D(0, \sigma_t^2) \\ \sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{array}\right\} \quad (1)$$

Where, r_t is the daily rate of return, θ is the AR(p) term in the mean equation in order to account for time dependence in returns, ϕ is the MA(q) term in the mean equation, ϵ_t is the residual term in the mean equation, Z_t is the standardized residual sequence of iid random variable with mean zero and variance σ_t^2, while D represents distribution of the shock returns.

TGARCH(p, q) Model

The Threshold GARCH model is another model used to handle leverage effects, and a TGARCH (p, q) model is given by the following:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}(\alpha_i + \gamma_i N_{t-i})^2 a_{t-i}^2 + \sum_{j=1}^{q}\beta_j \sigma_{t-j}^2 \quad (2)$$

where N_{t-i} is an indicator for negative a_{t-i}, that is,

$$N_{t-i} = \begin{cases} 1 & if\ \epsilon_{t-i} < 0, \\ 0 & if\ \epsilon_{t-i} \geq 0, \end{cases}$$

and α_i, γ_i and β_j are nonnegative parameters satisfying conditions similar to those of GARCH models, (Tsay, 2005). When $p = 1$ and $q = 1$, the TGARCH model becomes

$$\sigma_t^2 = \omega + (\alpha + \gamma N_{t-1})a_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (3)$$

EGARCH Model

The Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) Model was proposed by Nelson (1991) to overcome some weaknesses of the GARCH model in handling financial time series as pointed out by Enocksson & Skoog (2012). In particular, to allow for asymmetric effects between positive and negative asset returns, he considered the weighted innovation:

$$g(\epsilon_t) = \theta\epsilon_t + \gamma[|\epsilon_t| - E(|\epsilon_t|)], \quad (4)$$

where θ and γ are real constants. Both ϵ_t and $|\epsilon_t| - E(|\epsilon_t|)$ are zero mean iid sequences with continuous distributions. Therefore, $E[g(\epsilon_t)] = 0$. The asymmetry of $g(\epsilon_t)$ can easily be seen by rewriting it as:

$$g(\epsilon_t) = \begin{cases} (\theta + \gamma)\epsilon_t - \gamma E(|\epsilon_t|)\ if\ \epsilon_t \geq 0, \\ (\theta - \gamma)\epsilon_t - \gamma E(|\epsilon_t|)\ if \epsilon_t < 0. \end{cases} \quad (5)$$

An EGARCH(*m, s*) model, according to (Tsay 2005; Dhamija & Bhalla 2010; Jiang 2012; Ali 2013; Grek 2014), can be written as

$$a_t = \sigma_t\epsilon_t,$$
$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{s}\alpha_i \frac{|a_{t-i}| + \theta_i a_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{m}\beta_j \ln(\sigma_{t-i}^2), \quad (6)$$

which specifically results in EGARCH (1, 1) being written as

$$a_t = \sigma_t\epsilon_t,$$
$$\ln(\sigma_t^2) = \omega + \alpha([|a_{t-1}| - E(|a_{t-1}|)]) + \theta a_{t-1} + \beta\ln(\sigma_{t-1}^2) \quad (7)$$

where $|a_{t-1}| - E(|a_{t-1}|)$ are iid and have mean zero. When the EGARCH model has a Gaussian distribution of error term, then $(|\epsilon_t|) = \sqrt{2/\pi}$, which gives:

$$\ln(\sigma_t^2) = \omega + \alpha\left(\left[|a_{t-1}| - \sqrt{2/\pi}\,\right]\right) + \theta a_{t-1} + \beta\ln(\sigma_{t-1}^2) \quad (8)$$

The Absolute Value GARCH (AVGARCH):

The absolute value generalized autoregressive conditional heteroskedasticity (AVGARCH) is an extension of an Asymmetric GARCH (AGARCH) model which is specified as:

$$a_t = \sigma_t\epsilon_t,$$
$$\sigma_t = \omega + \sum_{i=1}^{p}\alpha_i\big(|\epsilon_{t-i} + b| - c(\epsilon_{t-i} + b)\big)^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2, \quad (9)$$

Nonlinear (Asymmetric) GARCH, or N(A)GARCH or NAGARCH

NAGARCH plays key role in option pricing with stochastic volatility because, as we shall see later on, NAGARCH allows for closed-form expressions of European option prices in spite of the rich volatility dynamics. A NAGARCH may be written as

$$\sigma_{t+1}^2 = \omega + \alpha\sigma_t^2(z_t - \delta)^2 + \beta\sigma_t^2 \quad (10)$$

And if $z_t \sim iidN(0,1)$, z_t is independent of σ_t^2 as σ_t^2 is only a function of an infinite number of past squared returns, it is possible to easily derive the long run, unconditional variance under NGARCH and the assumption of stationarity:

$$E[\sigma_{t+1}^2] = \bar{\sigma}^2 = \omega + \alpha E[\sigma_t^2(z_t - \delta)^2] + \beta E[\sigma_t^2]$$
$$= \omega + \alpha E[\sigma_t^2]E(z_t^2 + \delta^2 - 2\delta z_t) + \beta E[\sigma_t^2]$$
$$= \omega + \alpha\bar{\sigma}^2(1 + \delta^2) + \beta\bar{\sigma}^2, \quad (11)$$

where $\bar{\sigma}^2 = E[\sigma_t^2]$, and $E[\sigma_t^2] = E[\sigma_{t+1}^2]$ because of stationary. Therefore

$$\bar{\sigma}^2[1 - \alpha(1 + \delta^2) + \beta] = \omega \Longrightarrow \bar{\sigma}^2 = \frac{\omega}{1 - \alpha(1 + \delta^2) + \beta} \quad (12)$$

Which exists and is positive if and only if $\alpha(1 + \delta^2) + \beta < 1$. This has two implications:

(i) The persistence index of a NAGARCH(1,1) is $\alpha(1 + \delta^2) + \beta < 1$ and not simply $\alpha + \beta$

(ii) a NAGARCH(1,1) model is stationary if and only if $\alpha(1 + \delta^2) + \beta < 1$.

See details in (Nelson 1991; Hall & Yao 2003; Enders 2004; Christoff ersen, *et al.* 2008; Engle & Rangel 2008).

Persistence

The low or high persistency in volatility exhibited by financial time series can be determined by the GARCH coefficients of a stationary GARCH model. The persistence of a GARCH model can be calculated as the sum of GARCH (β_1) and ARCH (α_1) coefficients that is $\alpha + \beta_1$. In most financial time series, it is very close to one (1) (Banerjee & Sarkar, 2006; Ahmed *et al*., 2018). Persistence could take the following conditions:

If $\alpha + \beta_1 < 1$: The model ensures positive conditional variance as well as stationary.

If $\alpha + \beta_1 = 1$: we have an exponential decay model, then the half-life becomes infinite. Meaning the model is strictly stationary.

If $\alpha + \beta_1 > 1$: The GARCH model is said to be non-stationary, meaning that the volatility ultimately detonates toward the infinitude (Ahmed *et al*., 2018). In addition, the model shows that the conditional variance is unstable, unpredicted and the process is non-stationary (Kuhe, 2018).

Half-Life Volatility

Half-life volatility measures the mean reverting speed (average time) of a stock price or returns. The mathematical expression of half-life volatility is given as

$$Half-Life = \frac{\ln(0.5)}{\ln(\alpha_1+\beta_1)}$$

It can be noted that the value of $\alpha+\beta_1$ influences the mean reverting speed (Ahmed *et al*. 2018), which means that if the value of $\alpha+\beta_1$ is closer to one (1), then the volatility shocks of the half-life will be longer.

The unconditional (Kupiec) test also refer to as POF-test (Proportion of failure) with its null hypothesis given as

$$H_0: p = \hat{p} = \frac{y}{T}$$

Here y is the number of exceptions and T is the number of observations.

The test is given as

$$LR_{POF} = 2\ln\left(\frac{(1-p)^{T-y}p^{y}}{\left[1-(\frac{y}{p})^{T-y}(\frac{y}{T})^{y}\right]}\right) \quad (13)$$

Under the null hypothesis that the model is correct and LR_{POF} is asymptotically chi-squared (χ^2) distributed with degree of freedom as one (1). If the value of the LR_{POF} statistic is greater than the critical value (or $p\ value < 0.01$ for 1% level of significant or $p\ value < 0.05$ for 5% level of significant) the null hypothesis is rejected and the model then is inaccurate.

The Christoffersen's Interval Forecast Test combined the independence statistic with the Kupiec's POF test to obtained the joint test (Christoffersen, 1998; Nieppola, 2009). This test examined the properties of a good VaR model, the correct failure rate and independence of exceptions, that is condition coverage (cc). the conditional coverage (cc) is given as

$$LR_{cc} = LR_{POF} + LR_{ind}$$

Where

$$LR_{ind} = \sum_{i=2}^{n}\left[-2\ln\left(\frac{p(1-p)^{u_i-1}}{(\frac{1}{u_i})(1-\frac{1}{u_i})^{u_i}}\right)\right] - 2\ln\left(\frac{p(1-p)^{u-1}}{(\frac{1}{u})(1-\frac{1}{u})^{u-1}}\right) \quad (14)$$

Where u_i is the time between exceptions I and $i=1$ while u is the sum of u_i.

If the value of the LR_{cc} statistic is greater than the critical value (or $p\ value < 0.01$ for 1% level of significant or $p\ value < 0.05$ for 5% level of significant) the null hypothesis is rejected and that leads to the rejection of the model.

Distributions of GARCH models

In this study we employed two innovations namely student t and skewed student t distributions they can account for excess kurtosis and non-normality in financial returns (Heracleous, 2003; Wilhelmsson, 2016; Kuhe, 2018).

The student t distribution is given as

$$f(y) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1+\frac{y^2}{v}\right)^{-\frac{(v+1)}{2}}; \qquad -\infty < y < \infty \quad (15)$$

The Skewed student t distribution is given as

$$f(y;\ \mu,\sigma,v,\lambda) = \begin{cases} bc\left(1+\frac{1}{v-2}\left(\frac{b\left(\frac{y-\mu}{\sigma}\right)+a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}}, & if\ y < -\frac{a}{b} \\ bc\left(1+\frac{1}{v-2}\left(\frac{b\left(\frac{y-\mu}{\sigma}\right)+a}{1+\lambda}\right)^2\right)^{-\frac{v+1}{2}}, & if\ y \geq -\frac{a}{b} \end{cases} \quad (16)$$

Where v is the shape parameter with $2 < v < \infty$ and λ and is the skewness parameter with $-1 < \lambda < 1$. The constants $a, b\ and\ c$ are given as

$$a = 4\lambda c\left(\frac{v-2}{v-1}\right);\ b = 1+3(\lambda)^2 - a^2;\ c = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{\pi(v-2)}\Gamma(\frac{v}{2})}.$$

Where μ and σ are the mean and standard deviation of the skewed student *t* distribution respectively.

Calculation of Stock Returns

The returns was calculated using the formula below

$$R_t = \ln P_t - \ln P_{t-1}, \quad (17)$$

where

R_t is rate of returns of Guaranty Trust Bank (GTB) stock, P_t is the price of the stocks at time t, while P_{t-1} is the price of the stocks at time t-1, which is the previous day price of the stocks.

RESULTS

Data Source

The data used in this study was collected from www.cashcraft.com under stock trend and analysis. Daily stock price for Guaranty Trust Bank Nigeria plc from January 2nd 2001 to May 8th 2017 (a total of 4017 observations) was collected from the website. A total observation becomes 4016.

Preliminary Analysis/Descriptive Statistics

The analyses in this study were carried in R environment using rugarch package by Ghalanos (2018) and PerformanceAnalytics package by Peterson *et al*. (2018). The section begins with the descriptive statistics of the daily stock price of GT Bank Nigeria, plc. Figures 1, 2, 3 and 4 presents the plot of the daily actual price of GT bank stock, the plot of the log Transform of the actual price of GT bank stock, the plot of log transformed of stock returns of GT Bank daily stock price and the plot of cleansed log transform of stock returns of GT Bank respectively.

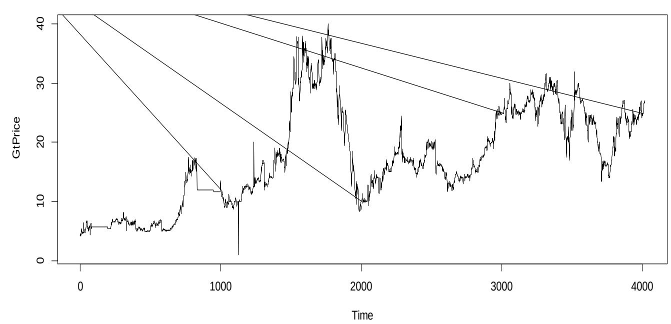

Figure 1: Plot of the Actual price of GT Bank Plc stock

Figure 1 above presents the Actual price of the Guaranty Bank Plc stock from January 2nd 2001 to May 8th 2017. The figure exhibited some trend.

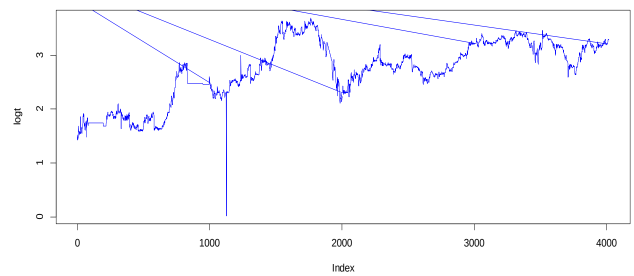

Figure 2: Plot of the log Transform of the Actual price of GT Bank Plc stock

Figure 2 above presents the log transform of the Actual price of the Guaranty Bank Plc stock from January 2nd 2001 to May 8th 2017. The figure exhibited some pattern and achieved stability through transformation.

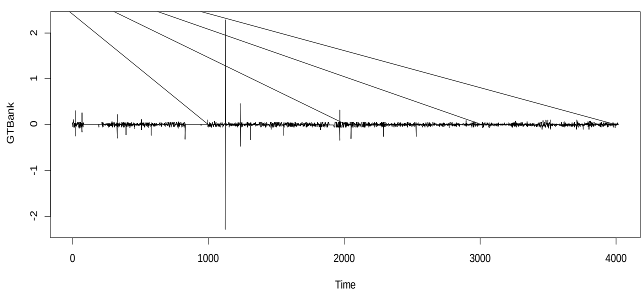

Figure 3: Plot of log transform of stock returns of GT Bank Plc

Figure 3 above presents the log transform of the stock returns of the Guaranty Bank stock plc from January 2nd 2001 to May 8th 2017. The figure actually exhibited the pattern of a typical financial time series; that is volatility.

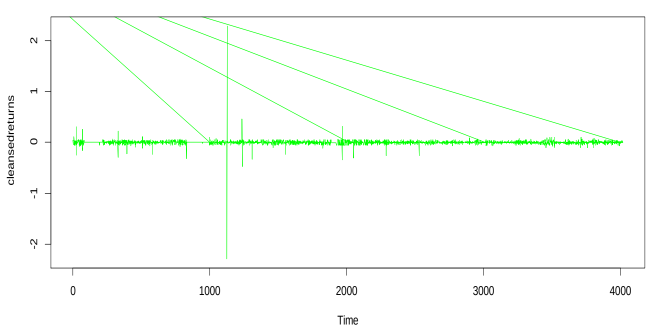

Figure 4: Plot of cleansed log transform of stock returns of GT Bank Plc

Figure 4 above presents the cleansed log transform of the stock returns of the Guaranty Bank Plc from January 2nd 2001 to May 8th 2017. This is done to remove the effects of possible outliers if any in the financial time series. The analysis of the financial time series in this study will be based on this cleansed series.

Table 1: Summary Statistics of Daily stock Returns of Guaranty Trust Bank Nigeria Plc

Statistics	Log of returns of Daily Stock price	Actual Daily Stock Price	Log transform of Daily Actual Stock price
Min	-2.28279	1.02	0.01980263
Max	2.28279	39.98	3.688379
Median	0	16.13	2.780681
Mean	0.0004655802	17.32804	2.71587
Estimated sd	0.059 99086	8.334726	0.5533098
Estimated skewness	-0.2103046	0.3325078	-0.5420392
Estimated kurtosis	1049.868	2.242218	2.418776
Jarque-Bera Normality Test	X-squared: 182929273.4134 p Value: < 2.2e-16	X-squared: 170.2176 p Value: < 2.2e-16	X-squared: 253.2493 p Value: < 2.2e-16
Number of Observations	4016	4017	4017
ARCH Test	Chi-squared = 830.2 p-value < 2.2e-16	Chi-squared =3984.9 p-value < 2.2e-16	Chi-squared = 3978.3 p-value < 2.2e-16
ADF-first difference test	test-statistic is: -91.9653 p-value: < 2.2e-16	test-statistic is: -45.1483 p-value: < 2.2e-16	test-statistic is: -61.079 p-value: < 2.2e-16

Table 1 above examined the characteristics of the financial time series used in this study. The actual stock price, the log transform of the stock price and the log transform of the stock returns exhibited the characteristics of a typical financial time series (i.e evidence of volatility) (Abdulkareem & Abdulkareem, 2016). The series exhibited large standard deviation, skewness and kurtosis. The series further exhibited non-normality using Jarque-Bera Statistic (p-values < 0.05) and shows the presence of ARCH effects (p-values < 0.05), and all the type of series exhibited stationarity at first difference. In addition the averages of the stock series revealed positive values; this implies that the stock price is gaining. With these characteristics revealed above, GARCH and ARMA-GARCH models are appropriate in studying the volatility of the Guaranty Trust Bank stock returns.

ARMA-GARCH Model Performances

Table 2: The Performance of the ARMA (1,1)-GARCH(1,1) Models using Information Criteria with respect to the distributions

Models	Information Criteria	Student t distribution	Skewed student t distribution
ARMA(1,1)-eGARCH(1,1)	Akaike	-4.8457	-4.8480
	Bayes	-4.8347	-4.8354
	Shibata	-4.8457	-4.8480
	Hannan-Quinn	-4.8418	-4.8435
ARMA(1,1)-TGARCH(1,1)	Akaike	**-6.0027**	**-6.0064**
	Bayes	**-5.9917**	**-5.9939**
	Shibata	**-6.0027**	**-6.0064**
	Hannan-Quinn	**-5.9988**	**-6.0020**
ARMA(1,1)-NAGARCH(1,1)	Akaike	-5.0575	-5.0519
	Bayes	-5.0466	-5.0393
	Shibata	-5.0575	-5.0519
	Hannan-Quinn	-5.0536	-5.0474
ARMA(1,1)-AVGARCH(1,1)	Akaike	-5.9990	-5.9566
	Bayes	-5.9864	-5.9425
	Shibata	-5.9990	-5.9566
	Hannan-Quinn	-5.9945	-5.9516

In table 2 above, four competing models are compared using student t distribution and skewed student t distribution. The following information criteria such as Akaike, Bayes, Shibata and Hannan-Quinn were used in selecting the preferred model. The results revealed ARMA(1,1)-TGARCH(1,1) as preferred model with the least values of the information criteria using student t and skewed student t distributions.

Table 3: The Performance of the ARMA(1,1)-GARCH(2,2) Models using Information Criteria with respect to the distributions

Models	Information Criteria	Student t distribution	Skewed student t distribution
ARMA(1,1)-eGARCH(2,2)	Akaike Bayes Shibata Hannan-Quinn	-5.1904 -5.1748 -5.1904 -5.1849	-5.1334 -5.1162 -5.1335 -5.1273
ARMA(1,1)-TGARCH(2,2)	Akaike Bayes Shibata Hannan-Quinn	-5.9878 -5.9721 -5.9878 -5.9822	**-5.9916** **-5.9743** **-5.9916** **-5.9855**
ARMA(1,1)-NAGARCH(2,2)	Akaike Bayes Shibata Hannan-Quinn	-5.0607 -5.0450 -5.0607 -5.0551	-5.0621 -5.0449 -5.0621 -5.0560
ARMA(1,1)-AVGARCH(2,2)	Akaike Bayes Shibata Hannan-Quinn	**-6.0110** **-5.9922** **-6.0110** **-6.0043**	-5.9200 -5.8996 -5.9200 -5.9127

In table 3 above, four competing models are compared with respect to student t distribution and skewed student t distribution. The following information criteria such as Akaike, Bayes, Shibata and Hannan-Quinn were used in selecting the preferred model. The results revealed ARMA (1,1)-AVGARCH(2,2) is preferred for student t distribution and ARMA(1,1)-TGARCH(2,2)model is preferred for skewed student t distribution.

Persistence and Half-life Volatility of ARMA-GARCH Models

Table 4: The persistence and half-life volatility of the ARMA (1,1)-GARCH(1,1) models with respect to the distributions

Models	Distributions	Persistence	Half-life (Days)
ARMA(1,1)-eGARCH(1,1)	Student t distribution	0.8822875	5.534669
	Skewed student t distribution	0.8853582	5.692593
ARMA(1,1)-TGARCH(1,1)	Student t distribution	0.9515151	13.94671
	Skewed student t distribution	0.9503758	13.6184
ARMA(1,1)-NAGARCH(1,1)	Student t distribution	**0.992533**	**92.48072**
	Skewed student t distribution	**0.9855705**	**47.68934**
ARMA(1,1)-AVGARCH(1,1)	Student t distribution	0.9481713	13.02416
	Skewed student t distribution	0.939799	11.16372

Evidence from persistence and half-life volatility in table 4 above shows that the Guaranty Trust Bank stock returns can be modeled and predicted since all the persistence values are all less than 1 (one). ARMA (1,1)-NAGARCH(1,1) exhibited the highest persistence and half-life volatility values while ARMA(1,1)-eGARCH(1,1) exhibited the lowest persistence and half-life volatility values. For all the models, the days of mean-reverting ranges from 5 days to 95 days

Table 5: The persistence and half-life volatility of the ARMA (1,1)-GARCH(2,2) models with respect to the distributions

Models	Distributions	Persistence	Half-life (Days)
ARMA(1,1)-eGARCH(2,2)	Student t distribution	0.9745043	26.83874
	Skewed student t distribution	0.9576603	16.02202
ARMA(1,1)-TGARCH(2,2)	Student t distribution	0.9425471	11.71463
	Skewed student t distribution	0.9399941	11.20117
ARMA(1,1)-NAGARCH(2,2)	Student t distribution	0.9810724	36.27328
	Skewed student t distribution	**0.986655**	51.59337
ARMA(1,1)-AVGARCH(2,2)	Student t distribution	**0.9876131**	55.61095
	Skewed student t distribution	0.9537416	14.63495

Evidence from persistence and half-life volatility in table 5 above shows that the Guaranty trust stock returns can be modeled and predicted since all the persistence values are all less than 1 (one). ARMA (1,1)-AVGARCH(2,2) exhibited the highest persistence and half-life volatility values with respect to student t distribution while ARMA(1,1)-NAGARCH(2,2) exhibited the highest persistence and half-life volatility values with respect to skew student t distribution. The ARMA (1,1)-TGARCH(2,2) exhibited the lowest persistence and half-life volatility values for both distributions under consideration. For all the models, the days of mean-reverting ranges from 10 days to 60 days.

Backtesting Evaluation of the Estimated ARMA-GARCH Models

Table 6: Backtesting of the ARMA (1,1)-GARCH(1,1): GARCH Roll Forecast (Backtest Length: 1016)

Model	Distributions	Alpha	Expected Exceed	Actual VaR Exceed	Unconditional Coverage (Kupiec) H_0: Correct Exceedances	Conditional Coverage (Christoffersen) H_0: Correct Exceedances and independence of Failure
ARMA(1,1)-eGARCH(1,1)	Student t	1%	10.2	4	accept	Reject
		5%	50.8	60	Accept	Accept
	Skewed student t	1%	10.2	6	Accept	Accept
		5%	50.8	55	Accept	Accept
ARMA(1,1)-TGARCH(1,1)	Student t	1%	10.2	38	Reject	Reject
		5%	50.8	96	Reject	Reject
	Skewed student t	1%	10.2	38	Reject	Reject
		5%	50.8	96	Reject	Reject
ARMA(1,1)-NAGARCH(1,1)	Student t	1%	10.2	28	Reject	Reject
		5%	50.8	90	Reject	Reject
	Skewed student t	1%	10.2	30	Reject	Reject
		5%	50.8	90	Reject	Reject
ARMA(1,1)-AVGARCH(1,1)	Student t	1%	10.2	38	Reject	Reject
		5%	50.8	96	Reject	Reject
	Skewed student t	1%	10.2	37	Reject	Reject
		5%	50.8	97	Reject	Reject

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Backtesting approach is a means to select and use financial GARCH models for real life application. This approach revealed ARMA(1,1)-eGARCH(1,1) as good model irrespective of the distribution but only failed at 1% alpha level in student t distribution, while other models failed the Backtesting Furthermore, coefficients of the ARMA(1,1)-eGARCH(1,1) model for
both distributions (see Tables 8 and 9 at the appendix) are more significant when compared to the other models (that is, ARMA(1,1)-TGARCH(1,1); ARMA(1,1)-NAGARCH(1,1) and ARMA(1,1)-AVGARCH(1,1)) (see Tables 10 to 15 at the appendix). These results led to the consideration of higher order GARCH model as ARMA (1,1)-GARCH(2,2) models.

Table 7: Backtesting of the ARMA(1,1)-GARCH(2,2): GARCH Roll Forecast (Backtest Length: 1016)

Model	Distributions	Alpha	Expected Exceed	Actual VaR Exceed	Unconditional Coverage (Kupiec) H_0: Correct Exceedances	Conditional Coverage (Christoffersen) H_0: Correct Exceedances and independence of Failure
ARMA(1,1)-eGARCH(2,2)	Student t	1%	10.2	9	Accept	Accept
		5%	50.8	58	Accept	Accept
	Skewed student t	1%	10.2	7	Accept	Accept
		5%	50.8	66	Reject	Reject
ARMA(1,1)-TGARCH(2,2)	Student t	1%	10.2	31	Reject	Reject
		5%	50.8	89	Reject	Reject
	Skewed student t	1%	10.2	34	Reject	Reject
		5%	50.8	90	Reject	Reject
ARMA(1,1)-NAGARCH(2,2)	Student t	1%	10.2	41	Reject	Reject
		5%	50.8	109	Reject	Reject
	Skewed student t	1%	10.2	30	Reject	Reject
		5%	50.8	92	Reject	Reject
ARMA(1,1)-AVGARCH(2,2)	Student t	1%	10.2	34	Reject	Reject
		5%	50.8	86	Reject	Reject
	Skewed student t	1%	10.2	36	Reject	Reject
		5%	50.8	89	Reject	Reject

Backtesting approach revealed ARMA(1,1)-eGARCH(2,2) as good model irrespective of the distribution at 1% and 5% alpha levels, while other models failed the Backtesting. Furthermore, coefficients of the ARMA(1,1)-eGARCH(2,2) model for both distributions are more significant (see Tables 16 and 17 at appendix) when compared to the other models (that is, ARMA(1,1)-TGARCH(2,2); ARMA(1,1)-NAGARCH(2,2) and ARMA(1,1)-AVGARCH(2,2)) see Tables 18 to 23 in the Appendix.

DISCUSSION

The log transform of the Guaranty Trust Bank stock returns exhibited the characteristics of a typical financial time series that is evidence of volatility (Abdulkareem & Abdulkareem, 2016) as shown in Table 1. The series exhibited large standard deviation, skewness and kurtosis. The series further exhibited non-normality using Jarque-Bera Statistic (p-values<0.05), shows the presence of ARCH effects (p-values<0.05) and the series exhibited stationarity at first difference. In addition the average value of the returns revealed a positive value which implies that the stock price is gaining (Kuhe, 2018). With these characteristics of the stock returns, the GARCH and ARMA-GARCH models are appropriate in studying the volatility of the Guaranty Trust Bank stock returns (Emenike & Ani, 2014; Ahmed *et al*., 2018).

In table 2, the four competing models were compared using student t distribution and skewed student t distribution. The following information criteria: Akaike, Bayes, Shibata and Hannan-Quinn were used to select the preferred model. The results revealed ARMA(1,1)-TGARCH(1,1) as preferred model with the least values of the information criteria for both student t and skew student t distributions.

In table 3, the four competing models of higher order were compared with respect to student t distribution and skewed student t distribution. The following information criteria: Akaike, Bayes, Shibata and Hannan-Quinn were employed to select the preferred model. The results revealed ARMA(1,1)-AVGARCH(2,2) is preferred for student t distribution and ARMA(1,1)-TGARCH(2,2)model is preferred for skew student t distribution.

Evidence from persistence and half-life volatility in table 4 shows that the Guaranty Trust Bank stock returns can be modeled and predicted since all the persistence values are all less than 1. This also means that the models ensure positive conditional variance as well as stationarity (Banerjee & Sarkar, 2006; Ahmed *et al*., 2018). The ARMA(1,1)-NAGARCH(1,1) exhibited the highest persistence and half-life volatility values while ARMA(1,1)-eGARCH(1,1) exhibited the lowest persistence and half-life volatility values for both distributions. For all the models, the days of mean-reverting ranges from 5 days to 95 days (that is within three (3) months).

Evidence from persistence and half-life volatility in table 5 shows that the Guaranty Trust Bank stock returns can be modeled and predicted since all the persistence values are all less than 1. This also means that the models ensured positive conditional variance as well as stationary (Banerjee & Sarkar, 2006; Ahmed *et al*., 2018). ARMA(1,1)-AVGARCH(2,2) exhibited the highest persistence and half-life volatility values with respect to student t distribution while ARMA(1,1)-NAGARCH(2,2) exhibited the highest persistence and half-life volatility values with respect to skewed student t distribution. The ARMA(1,1)-TGARCH(2,2) exhibited the lowest persistence and half-life volatility values for both distributions under consideration. For all the models, the days of mean-reverting ranges from 10 days to 60 days.

Backtesting approach is a means to select and use financial GARCH models for real life application. This approach revealed ARMA(1,1)-eGARCH(1,1) as good model for both distributions but only failed the Conditional Coverage (Christoffersen), this is Correct Exceedances and independence of Failure at 1% alpha level in student t distribution. This contradicts the results from the information criteria that selected ARMA(1,1)-TGARCH(1,1) as the preferred model. This suggests that models should not be selected by information criteria alone but should be selected in addition by how significant the coefficients of the model are, and possibly by backtesting approach (Christoffersen 1998; Christoffersen & Pelletier 2004; Nieppola 2009). The other models under considerations failed the Backtesting. Furthermore, coefficients of the ARMA(1,1)-eGARCH(1,1) model for both distributions (see Tables 8 and 9 in Appendix) are more significant when compared to the other models (that is, ARMA(1,1)-TGARCH(1,1); ARMA(1,1)-NAGARCH(1,1) and ARMA(1,1)-AVGARCH(1,1)) (see Tables 10 to 15 in Appendix). These results led the study to consider higher order GARCH

model as ARMA(1,1)-GARCH(2,2) models which is in line with Starica (2003), and Hansen & Lunde (2005) that opined that the GARCH(1,1) was clearly inferiors to models that can accommodate a leverage effect. But our results contradicts the work of Namugaya *et al*. (2014) that GARCH(1.1) outperformed the higher order of GARCH models, this could be because their work did not consider how good is their model.
Backtesting approach revealed ARMA(1,1)-eGARCH(2,2) in Table 7 as good model in respective of the distribution at 1% and 5% alpha levels, while other models failed the Backtesting. Furthermore, coefficients of the ARMA(1,1)-eGARCH(2,2) model for both distributions (see Tables 16 and 17 in Appendix) are more significant than those of the other models (that is, ARMA(1,1)-TGARCH(2,2); ARMA(1,1)-NAGARCH(2,2) and ARMA(1,1)-AVGARCH(2,2)) (see Tables 18 to 23 in Appendix). As mention earlier, ARMA(1,1)-eGARCH(2,2) was selected because it completely passed the backtesting though ARMA(1,1)-AVGARCH(2,2) was selected by information criteria. This suggests model should not be selected by information criteria lone but should be selected in addition, by how significant the coefficients of the model are, and possibly by backtesting approach (Christoffersen, 1998; Nieppola, 2009). Lastly, in all the models considered, there were no ARCH effects in the residuals of the estimated models.

Conclusion and Recommendations
This study revealed that the models considered ensured positive conditional variance as well as stationary (Banerjee & Sarkar, 2006; Ahmed *et al*., 2018). The study further revealed that using the lowest information criteria values only could not be enough to select preferred GARCH model rather we should add the use of backtesing. The models fitted exhibited high persistency in the daily stock returns and the results further revealed ARMA(1,1)-eGARCH (2,2) model with student t distribution provides a suitable model for evaluating the GT bank stock returns among the competing models. This study recommended that researchers should adopt backtesting approach while fitting GARCH models while the GT bank stock returns has the ability to return to its mean price returns.

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Appendix

Table 8: Estimates of ARMA(1,1)-eGARCH(1,1) with std

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*
Conditional Variance Dynamics
-----------------------------------
GARCH Model      : eGARCH(1,1)
Mean Model       : ARFIMA(1,0,1)
Distribution     : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value
Pr(>|t|)
ar1     0.090426    0.135239   0.66864
0.503724
ma1     0.015825    0.133828   0.11825
0.905871
omega  -0.638445    0.020334 -31.39719
0.000000
alpha1  0.192669    0.052765   3.65144
0.000261
beta1   0.882287    0.001346 655.31371
0.000000
gamma1  1.831176    0.019753  92.70358
0.000000
shape   2.100000    0.009205 228.12482
0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value
Pr(>|t|)
ar1     0.090426    0.143390   0.63063
0.52828
ma1     0.015825    0.149513   0.10584
0.91571
omega  -0.638445    0.081165  -7.86604
0.00000
alpha1  0.192669    0.171535   1.12321
0.26135
beta1   0.882287    0.002625 336.12545
0.00000
gamma1  1.831176    0.357599   5.12075
0.00000
shape   2.100000    0.049998  42.00152
0.00000
```

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```
LogLikelihood : 9737.114

Information Criteria
------------------------------------
Akaike       -4.8457
Bayes        -4.8347
Shibata      -4.8457
Hannan-Quinn -4.8418


Weighted Ljung-Box Test on Standardized
Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.06633  0.7968
Lag[2*(p+q)+(p+q)-1][5]   0.08466  1.0000
Lag[4*(p+q)+(p+q)-1][9]   0.23001  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized
Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                  0.0004513  0.9831
Lag[2*(p+q)+(p+q)-1][5] 0.0013750  1.0000
Lag[4*(p+q)+(p+q)-1][9] 0.0022635  1.0000
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3] 0.0004649 0.500 2.000  0.9828
ARCH Lag[5] 0.0011178 1.440 1.667  1.0000
ARCH Lag[7] 0.0016103 2.315 1.543  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  10.9421
Individual Statistics:
ar1    1.2313
ma1    1.2554
omega  2.7533
alpha1 0.8187
beta1  2.1565
gamma1 0.6688
shape  0.2326


Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.69 1.9 2.35
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          0.98272 0.3258
Negative Sign Bias 0.33742 0.7358
Positive Sign Bias 0.02935 0.9766
Joint Effect       1.04044 0.7915

Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20       852   1.903e-168
2    30      1131   1.637e-219
3    40      1420   7.300e-273
4    50      1694   7.905e-323
```

Table 9: Estimates of ARMA(1,1)-eGARCH(1,1) with sstd

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(1,1)
Mean Model      : ARFIMA(1,0,1)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
ar1     0.090252    0.059767    1.51006 0.131029
ma1     0.013462    0.058206    0.23129 0.817091
omega  -0.355651    0.021578  -16.48217 0.000000
alpha1  0.617628    0.165270    3.73708 0.000186
beta1   0.885358    0.004641  190.76635 0.000000
gamma1  5.501824    0.189529   29.02895 0.000000
skew    1.000633    0.009679  103.38505 0.000000
shape   2.010000    0.000613 3278.52434 0.000000

Robust Standard Errors:
        Estimate  Std. Error    t value Pr(>|t|)
ar1     0.090252    0.038433    2.34831 0.018859
ma1     0.013462    0.025677    0.52431 0.600066
omega  -0.355651    0.091076   -3.90499 0.000094
alpha1  0.617628    0.518456    1.19128 0.233542
beta1   0.885358    0.026923   32.88446 0.000000
gamma1  5.501824    0.598899    9.18656 0.000000
skew    1.000633    0.007893  126.77568 0.000000
shape   2.010000    0.001837 1094.18647 0.000000

LogLikelihood : 9742.685

Information Criteria
------------------------------------
Akaike       -4.8480
Bayes        -4.8354
Shibata      -4.8480
Hannan-Quinn -4.8435

Weighted Ljung-Box Test on Standardized
Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.07050  0.7906
Lag[2*(p+q)+(p+q)-1][5]   0.08858  1.0000
Lag[4*(p+q)+(p+q)-1][9]   0.23150  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                        statistic p-value
Lag[1]                  0.0004545   0.983
Lag[2*(p+q)+(p+q)-1][5] 0.0013845   1.000
Lag[4*(p+q)+(p+q)-1][9] 0.0022795   1.000
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3] 0.0004681 0.500 2.000  0.9827
ARCH Lag[5] 0.0011254 1.440 1.667  1.0000
ARCH Lag[7] 0.0016218 2.315 1.543  1.0000
```

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```
Nyblom stability test
------------------------------------
Joint Statistic:  12.5781
Individual Statistics:
ar1    1.2117
ma1    1.2345
omega  2.6897
alpha1 0.9021
beta1  1.7730
gamma1 0.6292
skew   0.1230
shape  0.2369

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.89 2.11 2.59
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          0.98086 0.3267
Negative Sign Bias 0.33726 0.7359
Positive Sign Bias 0.03006 0.9760
Joint Effect       1.03727 0.7922


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     961.4   9.289e-192
2    30    1308.9   2.714e-257
3    40    1647.1   7.026e-321
4    50    1970.2    0.000e+00
```

Table 10: Estimates of ARMA (1,1)-TGARCH(1,1) with std

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(1,1)
fGARCH Sub-Model        : TGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.260521    0.018296 14.23931  0.00000
ma1    -0.111401    0.018403 -6.05358  0.00000
omega   0.000000    0.000000  0.18102  0.85635
alpha1  0.695885    0.015610 44.58006  0.00000
beta1   0.499255    0.008670 57.58337  0.00000
eta11  -0.005608    0.021908 -0.25598  0.79797
shape   3.117434    0.057788 53.94586  0.00000


Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
ar1     0.260521    0.327748  0.794880 0.426683
ma1    -0.111401    0.423033 -0.263339 0.792289
omega   0.000000    0.000065  0.000551 0.999560
alpha1  0.695885    2.953122  0.235644 0.813709
beta1   0.499255    2.233770  0.223503 0.823144
eta11  -0.005608    0.246792 -0.022723 0.981871
shape   3.117434    1.074083  2.902415 0.003703

LogLikelihood : 12060.42

Information Criteria
------------------------------------

Akaike       -6.0027
Bayes        -5.9917
Shibata      -6.0027
Hannan-Quinn -5.9988

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                  2.709e-10       1
Lag[2*(p+q)+(p+q)-1][5] 2.575e-08       1
Lag[4*(p+q)+(p+q)-1][9] 5.878e-08       1
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.002273   0.962
Lag[2*(p+q)+(p+q)-1][5]  0.006825   1.000
Lag[4*(p+q)+(p+q)-1][9]  0.011387   1.000
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]  0.002273 0.500 2.000  0.9620
ARCH Lag[5]  0.005458 1.440 1.667  0.9998
ARCH Lag[7]  0.008126 2.315 1.543  1.0000


Nyblom stability test
------------------------------------
Joint Statistic:  278.5786
Individual Statistics:
ar1      0.2703
ma1      0.1850
omega  127.6935
alpha1  65.3278
beta1    8.8435
eta11    1.1637
shape    3.4515

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.69 1.9 2.35
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.1211 0.9036
Negative Sign Bias  0.3729 0.7093
Positive Sign Bias  0.4161 0.6774
Joint Effect        0.3281 0.9547


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     855.3   3.746e-169
2    30    1208.4   6.256e-236
3    40    1450.8   2.785e-279
4    50    1752.6    0.000e+00
```

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Table 11: Estimates of ARMA(1,1)-TGARCH(1,1) with sstd

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(1,1)
fGARCH Sub-Model        : TGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.217562    0.036887  5.89804  0.00000
ma1    -0.054003    0.036536 -1.47808  0.13939
omega   0.000000    0.000000  0.17964  0.85743
alpha1  0.732986    0.016282 45.01939  0.00000
beta1   0.474753    0.008953 53.02883  0.00000
eta11   0.007986    0.021498  0.37149  0.71027
skew    1.003418    0.011781 85.17358  0.00000
shape   3.107679    0.056652 54.85523  0.00000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
ar1     0.217562    0.483360  0.450103 0.652636
ma1    -0.054003    0.516754 -0.104505 0.916769
omega   0.000000    0.000067  0.000538 0.999571
alpha1  0.732986    2.841856  0.257925 0.796465
beta1   0.474753    2.133175  0.222557 0.823880
eta11   0.007986    0.182467  0.043768 0.965089
skew    1.003418    0.011739 85.479756 0.000000
shape   3.107679    1.053970  2.948546 0.003193

LogLikelihood : 12068.86

Information Criteria
------------------------------------

Akaike       -6.0064
Bayes        -5.9939
Shibata      -6.0064
Hannan-Quinn -6.0020

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                  3.349e-08  0.9999
Lag[2*(p+q)+(p+q)-1][5] 1.764e-07  1.0000
Lag[4*(p+q)+(p+q)-1][9] 3.366e-07  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.002415  0.9608
Lag[2*(p+q)+(p+q)-1][5]  0.007253  1.0000
Lag[4*(p+q)+(p+q)-1][9]  0.012101  1.0000
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]  0.002416 0.500 2.000  0.9608
ARCH Lag[5]  0.005801 1.440 1.667  0.9998
ARCH Lag[7]  0.008635 2.315 1.543  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  281.753
Individual Statistics:
ar1      0.19640
ma1      0.16059
omega  131.29433
alpha1  68.81920
beta1   10.30766
eta11    1.58122
skew     0.05679
shape    3.51394

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.89 2.11 2.59
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.1977 0.8433
Negative Sign Bias  0.3610 0.7181
Positive Sign Bias  0.4341 0.6643
Joint Effect        0.3590 0.9486


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20       960   1.798e-191
2    30      1290   3.095e-253
3    40      1638   5.095e-319
4    50      1936    0.000e+00
```

Table 12: Estimates of ARMA(1,1)-NAGARCH(1,1) with std

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(1,1)
fGARCH Sub-Model        : NAGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.202254    0.161208  1.25462  0.20962
ma1    -0.136256    0.166396 -0.81887  0.41286
omega   0.000000    0.000000  0.11911  0.90519
alpha1  0.348048    0.012211 28.50355  0.00000
beta1   0.643824    0.009687 66.46287  0.00000
eta21   0.043594    0.089369  0.48779  0.62570
shape   3.715735    0.102281 36.32859  0.00000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.202254    0.414763  0.48764 0.625807
ma1    -0.136256    0.515821 -0.26415 0.791661
omega   0.000000    0.000073  0.00049 0.999609
alpha1  0.348048    1.863233  0.18680 0.851819
beta1   0.643824    1.833290  0.35119 0.725450
eta21   0.043594    0.332024  0.13130 0.895540
shape   3.715735    1.523823  2.43843 0.014751
```

```
LogLikelihood : 10162.54

Information Criteria
------------------------------------

Akaike       -5.0575
Bayes        -5.0466
Shibata      -5.0575
Hannan-Quinn -5.0536


Weighted Ljung-Box Test on Standardized
Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.03067   0.861
Lag[2*(p+q)+(p+q)-1][5]   0.03263   1.000
Lag[4*(p+q)+(p+q)-1][9]   0.05507   1.000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.001840  0.9658
Lag[2*(p+q)+(p+q)-1][5]  0.006133  1.0000
Lag[4*(p+q)+(p+q)-1][9]  0.010398  1.0000
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]  0.002144 0.500 2.000  0.9631
ARCH Lag[5]  0.005173 1.440 1.667  0.9998
ARCH Lag[7]  0.007636 2.315 1.543  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  230.4385
Individual Statistics:
ar1      0.3625
ma1      0.3992
omega  101.8067
alpha1  51.9765
beta1    7.8753
eta21    1.3598
shape    4.1795

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.69 1.9 2.35
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.9102 0.3628
Negative Sign Bias  0.4771 0.6333
Positive Sign Bias  0.1879 0.8510
Joint Effect        1.0520 0.7887


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20      1161   2.433e-234
2    30      1491   4.173e-296
3    40      1744    0.000e+00
4    50      1987    0.000e+00
```

Table 13: Estimates of ARMA(1,1)-NAGARCH(1,1) with sstd

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(1,1)
fGARCH Sub-Model        : NAGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
ar1      0.21159    0.166371  1.27179  0.20345
ma1     -0.14934    0.172436 -0.86605  0.38646
omega    0.00000    0.000000  0.11943  0.90494
alpha1   0.34296    0.011964 28.66595  0.00000
beta1    0.64255    0.009524 67.46838  0.00000
eta21    0.01319    0.112685  0.11705  0.90682
skew     1.00799    0.012574 80.16590  0.00000
shape    3.73914    0.102750 36.39075  0.00000
Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
ar1      0.21159    0.251290  0.842011 0.399782
ma1     -0.14934    0.358117 -0.417011 0.676671
omega    0.00000    0.000073  0.000492 0.999607
alpha1   0.34296    1.829657  0.187445 0.851312
beta1    0.64255    1.838481  0.349501 0.726713
eta21    0.01319    1.457099  0.009052 0.992778
skew     1.00799    0.041364 24.368965 0.000000
shape    3.73914    1.625545  2.300239 0.021435

LogLikelihood : 10152.18

Information Criteria
------------------------------------

Akaike       -5.0519
Bayes        -5.0393
Shibata      -5.0519
Hannan-Quinn -5.0474

Weighted Ljung-Box Test on Standardized
Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.03295   0.856
Lag[2*(p+q)+(p+q)-1][5]   0.03493   1.000
Lag[4*(p+q)+(p+q)-1][9]   0.05748   1.000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.001834  0.9658
Lag[2*(p+q)+(p+q)-1][5]  0.006111  1.0000
Lag[4*(p+q)+(p+q)-1][9]  0.010360  1.0000
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]  0.002136 0.500 2.000  0.9631
ARCH Lag[5]  0.005153 1.440 1.667  0.9998
```

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```
ARCH Lag[7]  0.007607 2.315 1.543  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  232.8581
Individual Statistics:
ar1      0.37484
ma1      0.41837
omega  101.51981
alpha1  55.11488
beta1    8.72212
eta21    0.65460
skew     0.07016
shape    4.15469

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.89 2.11 2.59
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.9112 0.3622
Negative Sign Bias  0.4772 0.6333
Positive Sign Bias  0.1882 0.8508
Joint Effect        1.0544 0.7881


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20      1255   1.619e-254
2    30      1659    0.000e+00
3    40      1917    0.000e+00
4    50      2194    0.000e+00
```

Table 14: Estimates of ARMA(1,1)-AVGARCH(1,1) with std

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(1,1)
fGARCH Sub-Model        : AVGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.216232    0.020440 10.57901 0.000000
ma1    -0.091422    0.022696 -4.02803 0.000056
omega   0.000000    0.000000  0.17852 0.858313
alpha1  0.722956    0.015914 45.42756 0.000000
beta1   0.476527    0.008804 54.12823 0.000000
eta11  -0.026953    0.021307 -1.26499 0.205876
eta21   0.000193    0.001045  0.18441 0.853692
shape   3.141671    0.058494 53.70923 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
ar1     0.216232    0.228626  0.945789 0.344256
ma1    -0.091422    0.543642 -0.168165 0.866453
omega   0.000000    0.000068  0.000531 0.999576
alpha1  0.722956    2.763034  0.261653 0.793589
beta1   0.476527    2.080736  0.229018 0.818855
eta11  -0.026953    0.287347 -0.093800 0.925268
eta21   0.000193    0.002573  0.074930 0.940270
shape   3.141671    1.430933  2.195539 0.028125

LogLikelihood : 12053.98

Information Criteria
------------------------------------

Akaike       -5.9990
Bayes        -5.9864
Shibata      -5.9990
Hannan-Quinn -5.9945

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                  3.257e-08  0.9999
Lag[2*(p+q)+(p+q)-1][5] 1.757e-07  1.0000
Lag[4*(p+q)+(p+q)-1][9] 3.368e-07  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.002416  0.9608
Lag[2*(p+q)+(p+q)-1][5]  0.007254  1.0000
Lag[4*(p+q)+(p+q)-1][9]  0.012102  1.0000
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]  0.002416 0.500 2.000  0.9608
ARCH Lag[5]  0.005801 1.440 1.667  0.9998
ARCH Lag[7]  0.008636 2.315 1.543  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  281.1214
Individual Statistics:
ar1      0.2856
ma1      0.4489
omega  130.6555
alpha1  69.7212
beta1   11.1180
eta11    0.5819
eta21    0.3538
shape    3.9707

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.89 2.11 2.59
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.1974 0.8435
Negative Sign Bias  0.3617 0.7176
Positive Sign Bias  0.4444 0.6568
Joint Effect        0.3668 0.9470


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20       961   1.130e-191
2    30      1315   1.142e-258
3    40      1572   5.360e-305
```

```
4    50      1898    0.000e+00
*---------------------------------*
```

Table 15: Estimates of ARMA(1,1)-AVGARCH(1,1) with sstd

```
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(1,1)
fGARCH Sub-Model        : AVGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.186919    0.052008  3.59404 0.000326
ma1    -0.044629    0.054695 -0.81596 0.414520
omega   0.000000    0.000000  0.18133 0.856108
alpha1  0.603595    0.013305 45.36510 0.000000
beta1   0.535856    0.008274 64.76511 0.000000
eta11  -0.023806    0.021919 -1.08608 0.277441
eta21   0.000161    0.001017  0.15863 0.873957
skew    1.008536    0.012038 83.78155 0.000000
shape   3.330057    0.069111 48.18388 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
ar1     0.186919    0.754665  0.247684  0.80438
ma1    -0.044629    0.299470 -0.149028  0.88153
omega   0.000000    0.000064  0.000561  0.99955
alpha1  0.603595    2.862902  0.210833  0.83302
beta1   0.535856    2.346030  0.228410  0.81933
eta11  -0.023806    0.347020 -0.068602  0.94531
eta21   0.000161    0.000145  1.112047  0.26612
skew    1.008536    0.025408 39.693201  0.00000
shape   3.330057    2.369263  1.405524  0.15987

LogLikelihood : 11969.9

Information Criteria
------------------------------------

Akaike       -5.9566
Bayes        -5.9425
Shibata      -5.9566
Hannan-Quinn -5.9516


Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                  6.605e-09  0.9999
Lag[2*(p+q)+(p+q)-1][5] 1.011e-08  1.0000
Lag[4*(p+q)+(p+q)-1][9] 1.728e-08  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.001817   0.966
Lag[2*(p+q)+(p+q)-1][5]  0.005456   1.000
Lag[4*(p+q)+(p+q)-1][9]  0.009103   1.000
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]  0.001817 0.500 2.000  0.9660
ARCH Lag[5]  0.004364 1.440 1.667  0.9999
ARCH Lag[7]  0.006496 2.315 1.543  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  284.2378
Individual Statistics:
ar1      0.32177
ma1      0.17936
omega  119.81614
alpha1  71.63048
beta1    9.27011
eta11    0.99530
eta21    0.63851
skew     0.05967
shape    4.18341

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:          2.1 2.32 2.82
Individual Statistic:     0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          0.04364 0.9652
Negative Sign Bias 0.38816 0.6979
Positive Sign Bias 0.36206 0.7173
Joint Effect       0.28283 0.9632

Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20      1035   1.779e-207
2    30      1371   2.107e-270
3    40      1672    0.000e+00
4    50      2011    0.000e+00
```

Table 16: Estimates of ARMA(1,1)-eGARCH(2,2) with std

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(2,2)
Mean Model      : ARFIMA(1,0,1)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
ar1      0.68786    0.014697   46.8036        0
ma1     -0.53668    0.015623  -34.3512        0
omega   -0.25233    0.001059 -238.2264        0
alpha1   0.39354    0.053218    7.3948        0
alpha2  -0.52520    0.002786 -188.5413        0
beta1    0.73683    0.000159 4631.2482        0
beta2    0.23768    0.000247  963.5852        0
gamma1   3.69469    0.006275  588.8134        0
gamma2   0.44676    0.002619  170.5717        0
shape    2.10000    0.000751 2797.4873        0

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
ar1      0.68786    0.048823   14.089  0.00000
```

```
ma1     -0.53668    0.040005  -13.415  0.00000
omega   -0.25233    0.004415  -57.147  0.00000
alpha1   0.39354    0.241286    1.631  0.10289
alpha2  -0.52520    0.003159 -166.235  0.00000
beta1    0.73683    0.002479  297.207  0.00000
beta2    0.23768    0.002884   82.418  0.00000
gamma1   3.69469    0.073306   50.401  0.00000
gamma2   0.44676    0.004863   91.866  0.00000
shape    2.10000    0.003104  676.648  0.00000

LogLikelihood : 10432.39

Information Criteria
------------------------------------

Akaike       -5.1904
Bayes        -5.1748
Shibata      -5.1904
Hannan-Quinn -5.1849

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                  0.0006631  0.9795
Lag[2*(p+q)+(p+q)-1][5] 0.0019914  1.0000
Lag[4*(p+q)+(p+q)-1][9] 0.0033223  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                         statistic p-value
Lag[1]                   0.0003884  0.9843
Lag[2*(p+q)+(p+q)-1][11] 0.0023365  1.0000
Lag[4*(p+q)+(p+q)-1][19] 0.0039019  1.0000
d.o.f=4


Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[5] 0.0003889 0.500 2.000  0.9843
ARCH Lag[7] 0.0010005 1.473 1.746  1.0000
ARCH Lag[9] 0.0015138 2.402 1.619  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  192.3745
Individual Statistics:
ar1     1.17129
ma1     1.21514
omega  13.94563
alpha1  1.66821
alpha2  0.05803
beta1   8.09970
beta2   6.96569
gamma1 13.50861
gamma2  3.15886
shape   9.79736

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.29 2.54 3.05
Individual Statistic:    0.35 0.47 0.75


Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          0.95871 0.3378
Negative Sign Bias 0.48252 0.6295
Positive Sign Bias 0.05428 0.9567
Joint Effect       1.05095 0.7889


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     870.7   1.918e-172
2    30    1210.9   1.861e-236
3    40    1417.3   3.413e-272
4    50    1678.0   1.790e-319
```

Table 17: Estimates of ARMA(1,1)-eGARCH(2,2) with sstd

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(2,2)
Mean Model      : ARFIMA(1,0,1)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
ar1     0.311104    0.018626    16.7028 0.000000
ma1     0.031624    0.013750     2.2999 0.021455
omega  -0.428565    0.023080   -18.5689 0.000000
alpha1  0.859577    0.148896     5.7730 0.000000
alpha2  1.059502    0.134331     7.8872 0.000000
beta1   0.031654    0.001147    27.5924 0.000000
beta2   0.926006    0.001006   920.5848 0.000000
gamma1 10.000000    0.022874   437.1748 0.000000
gamma2  9.399735    0.072643   129.3967 0.000000
skew    1.005109    0.009240   108.7818 0.000000
shape   2.010257    0.000173 11597.2175 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.311104    0.049329   6.3068  0.00000
ma1     0.031624    0.020087   1.5743  0.11542
omega  -0.428565    0.150406  -2.8494  0.00438
alpha1  0.859577    0.774569   1.1097  0.26711
alpha2  1.059502    0.747112   1.4181  0.15615
beta1   0.031654    0.004750   6.6637  0.00000
beta2   0.926006    0.002727 339.5184  0.00000
gamma1 10.000000    1.405909   7.1128  0.00000
gamma2  9.399735    1.282506   7.3292  0.00000
skew    1.005109    0.006673 150.6302  0.00000
shape   2.010257    0.000824 2438.2258  0.00000

LogLikelihood : 10318.96

Information Criteria
------------------------------------

Akaike       -5.1334
Bayes        -5.1162
Shibata      -5.1335
Hannan-Quinn -5.1273

Weighted Ljung-Box Test on Standardized
Residuals
------------------------------------
```

```
                        statistic p-value
Lag[1]                  0.0006631  0.9795
Lag[2*(p+q)+(p+q)-1][5] 0.0019914  1.0000
Lag[4*(p+q)+(p+q)-1][9] 0.0033223  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                         statistic p-value
Lag[1]                   0.0003884  0.9843
Lag[2*(p+q)+(p+q)-1][11] 0.0023365  1.0000
Lag[4*(p+q)+(p+q)-1][19] 0.0039019  1.0000
d.o.f=4

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[5] 0.0003889 0.500 2.000  0.9843
ARCH Lag[7] 0.0010005 1.473 1.746  1.0000
ARCH Lag[9] 0.0015138 2.402 1.619  1.0000


Nyblom stability test
------------------------------------
Joint Statistic:  87.344
Individual Statistics:
ar1     0.6185
ma1     0.8204
omega  31.2178
alpha1  2.0947
alpha2  2.4273
beta1   8.3024
beta2   7.4301
gamma1 15.1495
gamma2  8.0161
skew    0.1114
shape  26.7576

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.49 2.75 3.27
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                  t-value   prob sig
Sign Bias          0.8896 0.3738
Negative Sign Bias 0.0580 0.9538
Positive Sign Bias 0.3984 0.6904
Joint Effect       0.9086 0.8234

Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20      1139   1.296e-229
2    30      1730    0.000e+00
3    40      2211    0.000e+00
4    50      2707    0.000e+00
```

Table 18: Estimates of ARMA(1,1)-TGARCH(2,2) with std

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(2,2)
fGARCH Sub-Model        : TGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
ar1     0.188006    0.019754   9.51725 0.000000
ma1    -0.044100    0.016926  -2.60537 0.009178
omega   0.000000    0.000000   0.56731 0.570503
alpha1  0.737207    0.022620  32.59082 0.000000
alpha2  0.007113    0.000221  32.23497 0.000000
beta1   0.391203    0.048539   8.05958 0.000000
beta2   0.067454    0.031730   2.12587 0.033514
eta11  -0.021401    0.022010  -0.97233 0.330887
eta12  -0.571818    0.013305 -42.97626 0.000000
shape   3.119393    0.058240  53.56058 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
ar1     0.188006    0.361081  0.520676 0.602593
ma1    -0.044100    0.019349 -2.279187 0.022656
omega   0.000000    0.000007  0.005365 0.995720
alpha1  0.737207    2.342134  0.314759 0.752945
alpha2  0.007113    0.006765  1.051482 0.293037
beta1   0.391203   14.595151  0.026804 0.978616
beta2   0.067454   11.374815  0.005930 0.995268
eta11  -0.021401    0.712753 -0.030025 0.976047
eta12  -0.571818    3.397238 -0.168319 0.866333
shape   3.119393    5.171723  0.603163 0.546400

LogLikelihood : 12033.5

Information Criteria
------------------------------------

Akaike       -5.9878
Bayes        -5.9721
Shibata      -5.9878
Hannan-Quinn -5.9822

Weighted Ljung-Box Test on Standardized
Residuals
------------------------------------
                        statistic p-value
Lag[1]                  6.448e-09  0.9999
Lag[2*(p+q)+(p+q)-1][5] 6.551e-09  1.0000
Lag[4*(p+q)+(p+q)-1][9] 8.576e-09  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                         statistic p-value
Lag[1]                    0.001951  0.9648
Lag[2*(p+q)+(p+q)-1][11]  0.011736  1.0000
Lag[4*(p+q)+(p+q)-1][19]  0.019599  1.0000
d.o.f=4


Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[5]  0.001954 0.500 2.000  0.9647
ARCH Lag[7]  0.005026 1.473 1.746  0.9999
ARCH Lag[9]  0.007604 2.402 1.619  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  287.706
```

```
Individual Statistics:
ar1      0.2268
ma1      0.1519
omega  124.3815
alpha1  69.1607
alpha2   7.1574
beta1   11.0448
beta2    6.4155
eta11    0.8532
eta12    6.4544
shape    3.2976

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.29 2.54 3.05
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                    t-value   prob sig
Sign Bias          0.002147 0.9983
Negative Sign Bias 0.384569 0.7006
Positive Sign Bias 0.377895 0.7055
Joint Effect       0.290821 0.9617
Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     926.9   2.054e-184
2    30    1253.4   1.733e-245
3    40    1511.5   3.960e-292
4    50    1814.4    0.000e+00
```

Table 19: Estimates of ARMA(1,1)-TGARCH(2,2) with sstd

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
------------------------------------
GARCH Model     : fGARCH(2,2)
fGARCH Sub-Model        : TGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.180819    0.027211  6.64515 0.000000
ma1    -0.040789    0.027805 -1.46695 0.142391
omega   0.000000    0.000000  0.25422 0.799323
alpha1  0.751149    0.019581 38.36156 0.000000
alpha2  0.021083    0.000863 24.42460 0.000000
beta1   0.338048    0.034700  9.74188 0.000000
beta2   0.099881    0.023224  4.30084 0.000017
eta11  -0.032566    0.022312 -1.45958 0.144405
eta12  -0.009352    0.036362 -0.25720 0.797028
skew    1.008252    0.011963 84.28353 0.000000
shape   3.119786    0.059674 52.28027 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
ar1     0.180819    0.400435  0.451555  0.65159
ma1    -0.040789    0.065709 -0.620747  0.53477
omega   0.000000    0.000032  0.001112  0.99911
alpha1  0.751149    0.749862  1.001716  0.31648
alpha2  0.021083    0.006765  3.116443  0.00183
beta1   0.338048    6.615345  0.051101  0.95925
beta2   0.099881    6.442248  0.015504  0.98763
eta11  -0.032566    0.527177 -0.061775  0.95074
eta12  -0.009352    1.470815 -0.006359  0.99493
skew    1.008252    0.063847 15.791770  0.00000
shape   3.119786    3.512696  0.888146  0.37446

LogLikelihood : 12042.07

Information Criteria
------------------------------------

Akaike       -5.9916
Bayes        -5.9743
Shibata      -5.9916
Hannan-Quinn -5.9855

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                  1.325e-07  0.9997
Lag[2*(p+q)+(p+q)-1][5] 5.397e-07  1.0000
Lag[4*(p+q)+(p+q)-1][9] 9.579e-07  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                         statistic p-value
Lag[1]                    0.001474  0.9694
Lag[2*(p+q)+(p+q)-1][11]  0.008864  1.0000
Lag[4*(p+q)+(p+q)-1][19]  0.014803  1.0000
d.o.f=4
Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[5]  0.001475 0.500 2.000  0.9694
ARCH Lag[7]  0.003796 1.473 1.746  0.9999
ARCH Lag[9]  0.005743 2.402 1.619  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  275.0765
Individual Statistics:
ar1      0.2426
ma1      0.1458
omega  120.3569
alpha1  59.1178
alpha2   5.9049
beta1    8.7332
beta2    5.8779
eta11    1.0777
eta12    6.0758
skew     0.1025
shape    3.6370

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.49 2.75 3.27
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.1533 0.8782
Negative Sign Bias  0.3937 0.6938
Positive Sign Bias  0.3207 0.7484
Joint Effect        0.2768 0.9643

Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     957.9   5.097e-191
2    30    1265.1   5.619e-248
```

```
3    40    1599.4   9.439e-311
4    50    1889.7    0.000e+00
```

Table 20: Estimates of ARMA(1,1)-NAGARCH(1,1) with std

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(2,2)
fGARCH Sub-Model        : NAGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.267628    0.170384  1.57074 0.116244
ma1    -0.204195    0.180250 -1.13284 0.257281
omega   0.000000    0.000000  0.12858 0.897688
alpha1  0.361676    0.018067 20.01888 0.000000
alpha2  0.027698    0.009210  3.00738 0.002635
beta1   0.370490    0.081133  4.56642 0.000005
beta2   0.216297    0.058472  3.69914 0.000216
eta21   0.056881    0.058199  0.97736 0.328392
eta22   0.367374    0.021812 16.84251 0.000000
shape   3.725628    0.107154 34.76898 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
ar1     0.267628    0.652530  0.410138  0.68170
ma1    -0.204195    0.564150 -0.361951  0.71739
omega   0.000000    0.000060  0.000595  0.99952
alpha1  0.361676    0.726879  0.497574  0.61878
alpha2  0.027698    0.068116  0.406631  0.68428
beta1   0.370490    7.019245  0.052782  0.95791
beta2   0.216297    6.860580  0.031528  0.97485
eta21   0.056881    0.198935  0.285929  0.77493
eta22   0.367374    1.733692  0.211903  0.83218
shape   3.725628    3.399272  1.096008  0.27308

LogLikelihood : 10171.79

Information Criteria
------------------------------------

Akaike       -5.0607
Bayes        -5.0450
Shibata      -5.0607
Hannan-Quinn -5.0551

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.03455  0.8525
Lag[2*(p+q)+(p+q)-1][5]   0.04001  1.0000
Lag[4*(p+q)+(p+q)-1][9]   0.06476  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                         statistic p-value
Lag[1]                    0.002135  0.9631
Lag[2*(p+q)+(p+q)-1][11]  0.014612  1.0000
Lag[4*(p+q)+(p+q)-1][19]  0.024386  1.0000
d.o.f=4

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[5]  0.002531 0.500 2.000  0.9599
ARCH Lag[7]  0.006381 1.473 1.746  0.9998
ARCH Lag[9]  0.009663 2.402 1.619  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  228.1006
Individual Statistics:
ar1     0.3208
ma1     0.3712
omega  93.0616
alpha1 48.2432
alpha2 20.2380
beta1   6.3303
beta2   5.2483
eta21   2.1330
eta22   4.9865
shape   4.1481

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.29 2.54 3.05
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.9311 0.3519
Negative Sign Bias  0.5031 0.6149
Positive Sign Bias  0.1984 0.8427
Joint Effect        1.1151 0.7734


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20      1136   5.322e-229
2    30      1467   6.164e-291
3    40      1702    0.000e+00
4    50      1873    0.000e+00
```

Table 21: Estimates of ARMA(1,1)-NAGARCH(2,2) with sstd

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(2,2)
fGARCH Sub-Model        : NAGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
ar1      0.27314    0.164993   1.65545 0.097834
ma1     -0.21735    0.177319  -1.22578 0.220283
omega    0.00000    0.000000   0.12625 0.899532
alpha1   0.39866    0.019042  20.93536 0.000000
alpha2   0.07167    0.009134   7.84656 0.000000
beta1    0.27848    0.077779   3.58034 0.000343
beta2    0.19095    0.051386   3.71606 0.000202
eta21   -0.11927    0.064471  -1.84992 0.064325
eta22   -0.75844    0.008760 -86.58119 0.000000
```

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```
skew     1.00208    0.012026  83.32973 0.000000
shape    3.45518    0.083425  41.41662 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
ar1      0.27314    0.277317  0.984928 0.324660
ma1     -0.21735    0.312482 -0.695573 0.486696
omega    0.00000    0.000064  0.000558 0.999554
alpha1   0.39866    1.439615  0.276921 0.781841
alpha2   0.07167    0.085600  0.837260 0.402447
beta1    0.27848    6.335548  0.043955 0.964941
beta2    0.19095    6.104333  0.031281 0.975045
eta21   -0.11927    5.516330 -0.021621 0.982751
eta22   -0.75844    5.670235 -0.133759 0.893593
skew     1.00208    0.113344  8.841085 0.000000
shape    3.45518    1.131894  3.052565 0.002269

LogLikelihood : 10175.73

Information Criteria
------------------------------------

Akaike       -5.0621
Bayes        -5.0449
Shibata      -5.0621
Hannan-Quinn -5.0560

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.05674  0.8117
Lag[2*(p+q)+(p+q)-1][5]   0.06030  1.0000
Lag[4*(p+q)+(p+q)-1][9]   0.08085  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                         statistic p-value
Lag[1]                    0.003088  0.9557
Lag[2*(p+q)+(p+q)-1][11]  0.020713  1.0000
Lag[4*(p+q)+(p+q)-1][19]  0.034493  1.0000
d.o.f=4

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[5]  0.003576 0.500 2.000  0.9523
ARCH Lag[7]  0.009064 1.473 1.746  0.9997
ARCH Lag[9]  0.013686 2.402 1.619  1.0000


Nyblom stability test
------------------------------------
Joint Statistic:  245.1376
Individual Statistics:
ar1      0.2838
ma1      0.3408
omega  104.3987
alpha1  54.9018
alpha2   8.7199
beta1    8.7570
beta2    6.9370
eta21    0.7783
eta22    3.9323
skew     0.1141
shape    3.4639
```

```
Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.49 2.75 3.27
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.7966 0.4257
Negative Sign Bias  0.5196 0.6034
Positive Sign Bias  0.2699 0.7873
Joint Effect        0.9490 0.8136

Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20      1244   4.108e-252
2    30      1611   1.047e-321
3    40      1900    0.000e+00
4    50      2164    0.000e+00
```

Table 22: Estimates of ARMA(1,1)-AVGARCH(2,2) with std

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(2,2)
fGARCH Sub-Model        : AVGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.158970    0.015930  9.97959 0.000000
ma1    -0.137234    0.019347 -7.09344 0.000000
omega   0.000000    0.000000  0.32907 0.742103
alpha1  0.736912    0.022551 32.67807 0.000000
alpha2  0.005087    0.000510  9.97569 0.000000
beta1   0.374727    0.041856  8.95270 0.000000
beta2   0.063603    0.026411  2.40821 0.016031
eta11  -0.045385    0.024043 -1.88765 0.059073
eta12   0.709412    0.171149  4.14499 0.000034
eta21   0.000162    0.000961  0.16819 0.866435
eta22   8.347500    0.822924 10.14371 0.000000
shape   3.088752    0.070739 43.66432 0.000000

Robust Standard Errors:
        Estimate  Std. Error    t value Pr(>|t|)
ar1     0.158970    0.099143   1.603451 0.108835
ma1    -0.137234    0.077725  -1.765629 0.077458
omega   0.000000    0.000019   0.001899 0.998485
alpha1  0.736912    0.239248   3.080123 0.002069
alpha2  0.005087    0.000033 153.551376 0.000000
beta1   0.374727   10.078103   0.037182 0.970340
beta2   0.063603    8.950449   0.007106 0.994330
eta11  -0.045385    0.877183  -0.051739 0.958736
eta12   0.709412    0.067197  10.557231 0.000000
eta21   0.000162    0.001038   0.155686 0.876280
eta22   8.347500    0.913383   9.139099 0.000000
shape   3.088752    5.353848   0.576922 0.563992

LogLikelihood : 12082.03


Information Criteria
------------------------------------
```

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```
Akaike       -6.0110
Bayes        -5.9922
Shibata      -6.0110
Hannan-Quinn -6.0043

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                  2.408e-06  0.9988
Lag[2*(p+q)+(p+q)-1][5] 7.945e-06  1.0000
Lag[4*(p+q)+(p+q)-1][9] 1.349e-05  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                         statistic p-value
Lag[1]                    0.001238  0.9719
Lag[2*(p+q)+(p+q)-1][11]  0.007444  1.0000
Lag[4*(p+q)+(p+q)-1][19]  0.012432  1.0000
d.o.f=4

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[5]  0.001239 0.500 2.000  0.9719
ARCH Lag[7]  0.003188 1.473 1.746  0.9999
ARCH Lag[9]  0.004823 2.402 1.619  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  -1051.369
Individual Statistics:
ar1      1.4224
ma1      1.5537
omega  113.9610
alpha1  28.8623
alpha2   7.2632
beta1    5.1249
beta2    7.1273
eta11    1.9999
eta12    7.2619
eta21    0.5528
eta22    7.1282
shape    6.1527

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.69 2.96 3.51
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.2294 0.8186
Negative Sign Bias  0.3973 0.6911
Positive Sign Bias  0.3141 0.7535
Joint Effect        0.3056 0.9590


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20      1040   1.372e-208
2    30      1559   1.494e-310
3    40      2088    0.000e+00
4    50      2603    0.000e+00
```

Table 23: Estimates of ARMA(1,1)-AVGARCH(2,2) with sstd

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(2,2)
fGARCH Sub-Model        : AVGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
ar1     0.101731    0.024077  4.225306 0.000024
ma1    -0.001814    0.032550 -0.055728 0.955558
omega   0.000000    0.000000  0.213259 0.831125
alpha1  0.799645    0.018756 42.635197 0.000000
alpha2  0.008552    0.000802 10.663941 0.000000
beta1   0.273720    0.028337  9.659585 0.000000
beta2   0.046177    0.007158  6.450902 0.000000
eta11  -0.052875    0.024251 -2.180361 0.029231
eta12   0.710317    0.160919  4.414136 0.000010
eta21   0.158922    0.007346 21.634544 0.000000
eta22   9.417915    0.800430 11.766066 0.000000
skew    1.003885    0.011379 88.224025 0.000000
shape   2.822039    0.041927 67.308817 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
ar1     0.101731    1.067858  0.095266 0.924103
ma1    -0.001814    1.465060 -0.001238 0.999012
omega   0.000000    0.000042  0.000854 0.999319
alpha1  0.799645    1.406233  0.568643 0.569598
alpha2  0.008552    0.000516 16.584686 0.000000
beta1   0.273720    4.509151  0.060703 0.951596
beta2   0.046177    4.998391  0.009238 0.992629
eta11  -0.052875    0.604364 -0.087489 0.930283
eta12   0.710317    1.239174  0.573218 0.566497
eta21   0.158922    0.556172  0.285742 0.775076
eta22   9.417915    2.795129  3.369403 0.000753
skew    1.003885    0.044910 22.353023 0.000000
shape   2.822039    1.009785  2.794693 0.005195

LogLikelihood : 11900.28

Information Criteria
------------------------------------

Akaike       -5.9200
Bayes        -5.8996
Shibata      -5.9200
Hannan-Quinn -5.9127

Weighted Ljung-Box Test on Standardized
Residuals
------------------------------------
                        statistic p-value
Lag[1]                  5.637e-06  0.9981
Lag[2*(p+q)+(p+q)-1][5] 1.794e-05  1.0000
Lag[4*(p+q)+(p+q)-1][9] 3.022e-05  1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
------------------------------------
                         statistic p-value
```

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```
Lag[1]                      0.001205  0.9723
Lag[2*(p+q)+(p+q)-1][11]  0.007248  1.0000
Lag[4*(p+q)+(p+q)-1][19]  0.012105  1.0000
d.o.f=4


Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[5]  0.001207 0.500 2.000  0.9723
ARCH Lag[7]  0.003104 1.473 1.746  0.9999
ARCH Lag[9]  0.004696 2.402 1.619  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  -1109.483
Individual Statistics:
ar1      1.5266
ma1      1.3519
omega  103.5352
alpha1  16.0413
alpha2  11.4688
beta1    7.7769
beta2   11.1323
eta11   23.9089
eta12   11.4688
eta21   56.5608
eta22   11.1304
skew     0.1852
shape    3.4749

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         2.89 3.15 3.69
Individual Statistic:    0.35 0.47 0.75
```

```
Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.2267 0.8207
Negative Sign Bias  0.3927 0.6945
Positive Sign Bias  0.2951 0.7679
Joint Effect        0.2865 0.9625


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20      1142   2.338e-230
2    30      1506   3.365e-299
3    40      1826    0.000e+00
4    50      2153    0.000e+00
```