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Full Length Research Article

THE EFFECT OF HYDROGEN ON TRANSIENT FLOW OF HYDROGEN NATURAL GAS MIXTURE

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ABSTRACT
Hydrogen is a high pressure gas, hence hydrogen natural gas mixture require an accurate prediction of transient flow parameter. The reduce order modelling is used in analysis of transient flow, where viscosity change is neglected thereby reducing the governing equations to Euler equation based on the assumption. The hydrogen natural gas mixture is homogenous, with pressure and velocity considered as principal dependent variable where polytrophic process is admitted. For improvement on the accurate prediction of flow parameters on the analysis of transient flow of hydrogen natural gas mixture in pipeline implicit Steger-Warming flux vector splitting method (FSM) used in the construction of efficient reduce order model. The result shows significant improvement on efficiency, accuracy and uniqueness when compared to normal conventional numerical techniques. The effect of heat transfer is observed on heat flux and internal energy of transient flow. The methods of reduced order with and without static correction show significant agreement for different gas ratio. The prediction of flow parameter along the pipeline can improve gas delivery and the analysis of transient flow behaviour at any point during the flow.

Keywords: Heat flux, Energy, Hydrogen natural gas, Transient flow, Ratio

INTRODUCTION

Hydrogen is the lightest, most abundant gas in the universe and attention as globally receiving alternative energy source, been the most cleanness fuel gas hence seen as an important gas to sustain energy generation (Elaoud & Hadj-Taı¨eb, 2008). Many researches have been conducted on its production, transportation and storage. Hydrogen is known to be mostly transported through the existing natural gas pipelines to reduce transportation cost. With its low volumetric energy content high pressure is required for effective transportation (Elaoud, Hafsi, & Hadj Taieb, 2017)

Hydrogen has a societal benefit including the reduction of greenhouse gas effect at zero level. Coal dominate World energy generation in the 19th century, petroleum oil, natural gas and nuclear energy in the 20th and 21st centuries with a clear indication of the possibility hydrogen will takes over before the end of 21st century(Lakoba & Yang, 2007; Winter, 2009). With this hydrogen is expected to contribute positively to world economy mostly on transportation sector going by amount of increase of hydrogen gas car on roads but shall still takes decades before a significant fraction is made compared to cars on roads (Wang, 2011). Although base on optimization model the hydrogen cars could also have negative impact in the economic growth due to it storage and source of production (Baba Galadima Agaie, 2014).

In a step toward realizing hydrogen economy there is need to transport hydrogen from its produce place to where it can be consume. To construct a separate pipeline for hydrogen transportation will be more costly for industries therefore there is need of using exiting natural gas pipeline (Baba G. Agaie, Khan, Alshomrani, & Alqahtani, 2017).

The transportation cost of hydrogen depend on distance and quantity, during which pressure is lost. The smoothness of pipeline walls cut operational cost during transportation up to 10% hence the skin fraction and pressure drag for configuration with reliable computational fluid technique is required in calculation of turbulent flow (Baufume, Hake, Linssen, & Markewitz, 2011; Mohammadi, Shojaei, & Arabi, 2012). The (Peet, Sagaut, & Charron, 2009)

The Reduce Order Modelling (ROM) technique can be use in analysis of computational fluid dynamic CFD. This can be achieved by transforming large set of primitive variables into smaller decoupled equations (Romanowski & Dowell, 1996) For accuracy and efficient analysis of transient flow in pipeline ROM technique has demonstrated high significant impact (Behbahani-Nejad & Shekari, 2010) . The advantage of ROM is the construction of Eigen analysis in unsteady flows where only few of the original nodes are retained.

Computational efficiency is highly demanding in CFD analysis which results to high computational cost. ROM is often use in the analysis of aerodynamic problem and design of wing fuselage system and exhibited a low cost in computation (Jun, Park, Kang, Lee, & Cho, 2010)

ROM has been a well-known numerical technique but not much used in the transient flow analysis. In the research by (Behbahani-Nejad & Shekari, 2010) were ROM technique is used remarkable accuracy was achieved in natural gas transient flow analysis with low computational cost. ROM can also be applied in subsonic unsteady flow.

This research is to study the effect of hydrogen presents on transient flow perimeters and improve on accurate prediction with reduction on computational time.

MATERIALS AND METHODOLOGY

One dimensional equation is sufficient in the description of a compressible gas in pipeline as reported by (Daneshyar, 1976). Therefore, this research our problem shall be defined by sets of partial differential motion equations. The general gas flow dynamics through a pipeline can fully be analyzed using the usual conservation equations of mass, momentum and energy equation.

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}=0 \quad (1)$$

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$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial\left(\rho u^2+\rho c^2\right)}{\partial x}+\frac{f\rho u|u|}{2D}+\rho g\sin\theta=0 \qquad (2)$$

$$\frac{\partial}{\partial t}\left[\left(e+\frac{u^2}{2}\right)\rho\right]+\frac{\partial}{\partial x}\left[\left(h+\frac{u^2}{2}\right)\rho u\right]=\rho q-\rho ug\sin\theta \qquad (3)$$

where ρ is the density of fluid mixture, f is coefficient of pipe friction, P the pressure is equal to ρc^2, u is the velocity of fluid mixture, D is the pipeline diameter, g is the gravitational acceleration, θ is the pipe inclination and c is a celerity wave. The internal energy per unit mass is e and *h* the specific enthalpy. If the kinetic energy and wave effect on temperature change are neglected equation (3) reduces into

$$\frac{\partial}{\partial t}[\rho e]+\frac{\partial}{\partial x}[\rho ue]=\rho q-\rho ug\sin\theta \qquad (4)$$

Since the two mixed components are in gaseous form, them homogeneous mixture is assumed. For a polytrophic flow the density of the resultant mixture will depend on mass ratio of each gas. To develop density of the mixture ρ of hydrogen-natural gas mixture (HCNG), hydrogen fluid mass ratio will be used as the determining factor of the density. Therefore the mass ratio of the mixture is defined by

$$\phi=m_h/(m_h+m_g) \qquad (5)$$

where ϕ, m_i are the mass ratio of HCNG, mass of hydrogen and natural gas for *i=h or g* respectively.
From the definition of density of gas

$$\rho=\frac{m_m}{V_m},\ \rho_h=\frac{m_h}{V_h} \text{ and } \rho_g=\frac{m_g}{V_g}$$

where V_m, V_h, V_g, ρ_h and ρ_g are volumes of HNG, hydrogen, natural gas and the densities of hydrogen and natural gas respectively. The gas mixture density taking the reciprocal will become

$$\frac{1}{\rho}=\frac{V_m}{M_m}$$

Since $V_m=V_h+V_g$ the density of gas definition on we get

$$\frac{1}{\rho}=\frac{1}{\rho_h}\left(\frac{m_h}{M_m}\right)+\frac{1}{\rho_g}\left(\frac{m_g}{M_m}\right)$$

Hence, simplifying using density definition equation (5) becomes

$$\frac{1}{\rho}=\frac{\phi}{\rho_h}+\frac{(1-\phi)}{\rho_g} \text{ which is } \rho=\left[\frac{\phi}{\rho_h}+\frac{(1-\phi)}{\rho_g}\right]^{-1} \qquad (6)$$

For a polytrophic assumption the flow inlet is constant under these process polytrophic index of the densities of hydrogen and natural gas are expressed as

$$\frac{p}{p_0}=\left(\frac{\rho_h}{\rho_{h_0}}\right)^{n_h} \text{ and } \frac{p}{p_0}=\left(\frac{\rho_g}{\rho_{g_0}}\right)^{n_g}$$ substituting into equation (6)

$$\rho=\left[\frac{\varphi}{\rho_h}+\frac{1-\varphi}{\rho_g}\right]^{-1}=\left[\frac{\varphi}{\rho_{h0}}\left(\frac{p_0}{p}\right)^{-\frac{1}{n'}}+\frac{(1-\varphi)}{\rho_{g0}}\left(\frac{p_0}{p}\right)^{-\frac{1}{n''}}\right]^{-1} \qquad (7)$$

Equation (7) can be written as

$$\rho(e^p)=\left[\frac{\phi}{\rho_{h_0}}\left(\frac{p_0}{e^p}\right)^{-\frac{1}{n_1}}+\frac{1-\phi}{\rho_{g_0}}\left(\frac{p_0}{e^p}\right)^{-\frac{1}{n_2}}\right]^{-1} \qquad (8)$$

This removed the singularity in (8)
From the definition of celerity pressure wave and taken the derivative of (7) with respect to p and simplifying we have

The Celerity of pressure wave is defined as:

$$c=\left(\frac{d\rho}{dp}\right)^{-1/2}$$

Integrating and taken into account (8) we have

$$c=\left[\frac{\varphi}{\rho_{h0}}\left(\frac{p_0}{p}\right)^{-\frac{1}{n'}}+\frac{(1-\varphi)}{\rho_{g0}}\left(\frac{p_0}{p}\right)^{-\frac{1}{n''}}\right]\times\left[\frac{1}{p}\left[\frac{\phi}{n'\rho_{h0}}\left(\frac{p_0}{p}\right)^{-\frac{1}{n'}}+\frac{(1-\phi)}{n''\rho_{g0}}\left(\frac{p_0}{p}\right)^{-\frac{1}{n''}}\right]\right]^{-\frac{1}{2}}$$

To remove singularity

$$c(e^p)=\left[\frac{\varphi}{\rho_{h0}}\left(\frac{p_0}{e^p}\right)^{-\frac{1}{n'}}+\frac{(1-\varphi)}{\rho_{g0}}\left(\frac{p_0}{e^p}\right)^{-\frac{1}{n''}}\right]\times\left[\frac{1}{e^p}\left[\frac{\phi}{n'\rho_{h0}}\left(\frac{p_0}{e^p}\right)^{-\frac{1}{n'}}+\frac{(1-\phi)}{n''\rho_{g0}}\left(\frac{p_0}{e^p}\right)^{-\frac{1}{n''}}\right]\right]^{-\frac{1}{2}}$$

Initial and Boundary Conditions

The flow initially assumed to be at steady state condition, therefore the initial condition will be given as

$$\frac{\partial\rho(x,0)}{\partial x}=0 \qquad (9)$$

$$\frac{\partial\rho u(x,0)}{\partial x}=-c^2\frac{\partial\rho}{\partial x}-f\frac{\rho u^2}{2D} \qquad (10)$$

$$\frac{\partial}{\partial x}[\rho ue]=\rho q-\rho ug\sin\theta \qquad (11)$$

Boundary condition depends on valves operational time and locations coupled with compressor supply and pressure regulator. This work involved valves which are placed in upstream and downstream of pipe with different valves and operational times.

The Automatic Closure Valve (ACV) is placed at the upstream and Rapid Closure Valve (RCV) at the downstream, boundary condition are based on reaction and actuation time effect on the inlet and outlet fluid.

At $x=0$

$$\rho(x,t)=\rho_0(t),\ \frac{\partial u}{\partial x}=u_0(t)\quad T(x,t)=T_0 \tag{12}$$

At *x=L*

$$\rho u(L,t)=\rho u_L(t),\ \frac{\partial u}{\partial x}=u_L(t),\ \rho eu=h_L(t) \tag{13}$$

where ρ, $\rho u=m_{0,1}$ and $\rho eu=h_{0,1}$ are the density and the inlet and outlet gas mass flux respectively.

Solution procedure

To construct ROM, the set of partial differential equation are written in vector flux form and discretized using forward and backward difference on time and space. The scheme referred to as implicit Steger-Warming Splitting Scheme. Is then linearized using steady state solution and a small perturbation over the solution. The result is referred to as linear form can be written in the form of eigen value problem. Then perform eigenmode analysis and subsequent construction of ROM through by setting the flow analysis with Vortex Lattices Method (VLM) Behbahani-Nejad *et al.* (2004).

There Writing equations (1), (2) and (4) in matrix form we have

$$\frac{\partial Q}{\partial t}=-\frac{\partial E(Q)}{\partial x}+H(Q) \tag{14}$$

where

$$E=\begin{bmatrix}\rho u\\ \rho u^2+c^2\rho\\ \rho ue+pu\end{bmatrix} \tag{15}$$

$$H=\begin{bmatrix}0\\ \frac{f\rho u|u|}{2d}-\rho g\sin(\theta)\\ q\rho-\rho gu\sin(\theta)\end{bmatrix} \tag{16}$$

Differentiating with respect to Q equations (15) and (16) we get

$$\frac{E(Q)}{\partial Q}=\begin{bmatrix}0&1&0\\ -u^2&2u&r-1\\ -ue&e&u\end{bmatrix} \tag{17}$$

$$\frac{\partial H(Q)}{\partial Q}=\begin{bmatrix}0&0&0\\ \frac{fu|u|}{2d}-g\sin(\theta)&\frac{f|u|}{2d}&0\\ q-gu\sin(\theta)&-g\sin(\theta)&0\end{bmatrix} \tag{18}$$

Let $A=\frac{\partial E(Q)}{\partial Q}$ and $B=\frac{\partial H(Q)}{\partial Q}$ where A and B are Jacobian matrix of *E(Q)* and *H(Q)* respectively.

To obtain implicit Steger-Warming flux vector splitting scheme similarly procedure as: Since the flux vector is *E=AQ* and *E* homogenous, hence there exit two sub vectors associated with positive and negative eigenvalues of its Jacobian matrix, such that their sum is equal to E. Similarly A can be split in the same manner as did on flux vector

Such that

$$A=A^+ + A^-$$
$$E=E^+ + E^-$$

where A^+, is transpose of a matrix whose rows are right and left eigenvector of matrix of *A* and E^+ is the produce of with E

$$A^+=\frac{u}{2c}\begin{bmatrix}-u+c&\frac{u+c}{u}&\frac{(r-1)(c-u)}{cu}\\ -(u^2+c^2)&\frac{(u+c)^2}{u}&\frac{(r-1)(-u^2+2uc+c^2)}{cu}\\ \frac{-c^2(u+c)}{r-1}&\frac{c^2(u+c)}{u(r-1)}&\frac{c(u+c)}{u}\end{bmatrix} \tag{19}$$

$$A^-=\frac{u}{2c}\begin{bmatrix}u-c&\frac{-(u-c)}{u}&\frac{(r-1)(u-c)}{cu}\\ (u-c)^2&\frac{-(u-c)^2}{u}&\frac{(r-1)(u-c)^2}{cu}\\ \frac{c^2(u-c)}{r-1}&\frac{-c^2(u-c)}{u(r-1)}&\frac{c(u-c)}{u}\end{bmatrix} \tag{20}$$

$$E^+=\begin{bmatrix}\frac{\rho(u+c)}{2}\\ \frac{\rho(u+c)^2}{2}\\ \frac{\rho c^2(u+c)}{2(r-1)}\end{bmatrix},\ E^-=\begin{bmatrix}\frac{\rho(u-c)}{2}\\ \frac{\rho(u-c)^2}{2}\\ \frac{\rho c^2(u-c)}{2(r-1)}\end{bmatrix} \tag{21}$$

To obtain implicit Steger-Warming flux vector splitting scheme using Taylor series expansion

By Taylor series expansion the left hand side of (14) in order 2

$$Q^{n+1}=Q^n+\frac{1}{2}\left[\left(\frac{\partial Q}{\partial t}\right)^n+\left(\frac{\partial Q}{\partial t}\right)^{n+1}\right]\Delta t+\mathrm{O}(\Delta t)^3 \tag{22}$$

Substituting (13) into (21)

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$$Q^{n+1}-Q^{n}=-\frac{1}{2}\left[\left(\frac{\partial E}{\partial x}-H\right)^{n}+\left(\frac{\partial E}{\partial x}-H\right)^{n+1}\right]\Delta t+\mathrm{O}(\Delta t)^{3} \quad (23)$$

Linearize the nonlinear term (22) by Taylor series expansion,

$$\left(\frac{\partial E}{\partial x}-H\right)^{n+1}$$

we have

$$E^{n+1}=E^{n}+\Delta t\left(\frac{\partial E}{\partial x}\right)^{n}+O(\Delta t)^{2}$$

$$E^{n+1}=E^{n}+A^{n}(Q^{n+1}-Q^{n}) \quad (24)$$

Similarly

$$H^{n+1}=H^{n}+B^{n}(Q^{n+1}-Q^{n}) \quad (25)$$

$$\left(\frac{\partial E}{\partial x}-H\right)^{n+1}=\left(\frac{\partial E}{\partial x}-H\right)^{n}+\left(\frac{\partial A}{\partial x}-B\right)^{n}(Q^{n+1}-Q^{n}) \quad (26)$$

where A and B are Jacobian matrices of E(Q) and H(Q) respectively.

Substituting (24),(25) and (26) into (23) and simplifying

$$Q^{n+1}-Q^{n}+\frac{1}{2}\Delta t\left(\frac{\partial A}{\partial x}-B\right)^{n}\left(Q^{n+1}-Q^{n}\right)=-\Delta t\left(\frac{\partial E}{\partial x}-H\right)^{n} \quad (27)$$

Substituting the split matrices into (27)

$$Q^{n+1}-Q^{n}+\frac{1}{2}\Delta t\left(\frac{\partial(A^{+}+A^{-})}{\partial x}-B\right)^{n}$$
$$*\left(Q^{n+1}-Q^{n}\right)=-\Delta t\left(\frac{\partial(E^{+}+E^{-})}{\partial x}-H\right)^{n} \quad (28)$$

Taking backward and forward difference space step on positive and negative part of split matrices (28) and simplifying (Hanif Chaudhry, 2008)

$$\left[1+\tfrac{\Delta t}{\Delta x}(A_i^{n(+)}-A_i^{n(-)})-\Delta tB_i^{n}\right]\left(Q_i^{n+1}-Q_i^{n}\right)$$
$$-\left(\tfrac{\Delta t}{\Delta x}A_{i-1}^{n(+)}\right)(Q_{i-1}^{n+1}-Q_{i-1}^{n})+\left(\tfrac{\Delta t}{\Delta x}A_{i+1}^{n(-)}\right)(Q_{i+1}^{n+1}-Q_{i+1}^{n})$$
$$=-\tfrac{\Delta t}{\Delta x}\left[E_i^{n(+)}-E_{i-1}^{n(+)}+E_{i+1}^{n(-)}-E_i^{n(-)}\right]+\Delta tH_i^{n}$$

$$\left[1+\tfrac{\Delta t}{\Delta x}(A_i^{n(+)}-A_i^{n(-)})-\Delta tB_i^{n}\right]\Delta Q_i$$
$$-\left(\tfrac{\Delta t}{\Delta x}A_{i-1}^{n(+)}\right)\Delta Q_{i-1}+\left(\tfrac{\Delta t}{\Delta x}A_{i+1}^{n(-)}\right)\Delta Q_{i+1} \quad (29)$$
$$=-\tfrac{\Delta t}{\Delta x}\left[E_i^{n(+)}-E_{i-1}^{n(+)}+E_{i+1}^{n(-)}-E_i^{n(-)}\right]+\Delta tH_i^{n}$$

(28) is referred to as the implicit Steger-warming flux vector splitting method (FSM) using backward time difference scheme deformation (Hoffman and Chaiang, 2000)

The LFSM is similar as in (28) and becomes (30)

$$-\left(\frac{\Delta t}{\Delta x}A_{j+1}^{0}\right)\widehat{Q}_{j-1}^{n+1}+\left(1+\frac{\Delta t}{\Delta x}\left(A_j^{0+}-A_j^{0-}\right)-\Delta tB_j^{0}\right)\widehat{Q}_j^{n+1}$$
$$+\left(\frac{\Delta t}{\Delta x}A_{j+1}^{0-}\right)Q_{j+1}^{n+1}=\widehat{Q}_j^{n} \quad (30)$$

and can be written as

$$W^{o}\widehat{Q}^{n+1}=I\widehat{Q}^{n}+V^{n+1} \quad (31)$$

where V^{n+1} and W^{0} are imposed value from the boundary condition and the coefficient matrix of the unknown term $\hat{Q}_j^{n+1}$ respectively.

Construction of ROM using VLM

Vortex lattice method is use to construct ROM that requires static correction and ROM that does not requires static correction.

ROM with static correct requirement

The homogeneous part of (30) $W^{o}\widehat{Q}^{n+1}=I\widehat{Q}^{n}$ is satisfied by setting

($$Q_j=x_j\exp(-i\omega_j t)\alpha_j\exp(iz_j x) \quad (32)$$

Then (31) is the general eigenmode (Giles, 1983)

Considering the VLM for small perturbation from (31), then (30) reduces to

$$\widehat{Q}_j=x_j\exp(\lambda_j t),\ z_i=\exp(\lambda_i\Delta t) \quad (33)$$

where λ_i, x_i eigenvalue and the corresponding eigenvector.

From the eigenvalue problem the following generalization is made

$$z_j W^{0}x_j=Ix_j \quad (34)$$

$$ZW^{n}X=IX \quad (35)$$

(35) is the general right eigenvalue.

where Z and X are the diagonal matrix of eigenvalue and a matrix whose columns are the corresponding eigenvector.

Similarly considering the left eigenvalue problem of the homogenous part of (34) $W^{o}\widehat{Q}^{n+1}=I\widehat{Q}^{n}$ we have

$$\left(W^{0}\right)^{T}YZ=IY \quad (36)$$

For normalized eigenvalue the orthogonal conditions are satisfied

$$Y^{T}W^{0}X=I,\ Y^{T}IX=Z \quad (37)$$

where *Y* is the matrix whose columns are the left eigenvectors of W^{0}

From eigenmode analysis base on time step, (36) described the fluid flow behaviour at individual nodes

Therefore

$$\widehat{Q}=X\widehat{c} \quad (38)$$

where $\widehat{c}$ is the vector of the normal node coordinate (Behbahani-Nejad and Shekari, 2010)

From (38)

$\hat{Q} = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix}$, and $\hat{c} = \begin{bmatrix} \hat{c}_1 \\ \hat{c}_2 \\ \vdots \\ \hat{c}_n \end{bmatrix}$ normal node coordinates.

Since $\hat{Q}$ is known at the initial point then using orthogonal conditions (37) and multiplying (38) by $Y^T W^0$ and evaluation to obtain

$$\hat{c} = Y^T W^0 \hat{Q} \tag{39}$$

Going back to (34) and multiplying through by Y^T considering the orthogonal conditions we get.

$$\hat{c}^{n+1} - Z\hat{c}^n = Y^T V^{n+1} \tag{40}$$

The left-hand side of (40) is now the diagonal of each mode that marched forward in time independently and less computational cost. The results are then reassembled using (39) to obtain the fluid flow bahaviours. The advantage of this modal is to satisfactory ROM can construct with few numbers of the original modes. In the present work *m* modes are retain with the largest eigenvalues z_i for $m < N$ where *N* is the total number of original modes. With this the order of X, *Y* are reduced to $N \times m$ matrices, and Z is reduced to $m \times m$ matrix. Unfortunately, satisfactory results cannot give a satisfactory result unless a large number of *m* is used. This is as a result of neglected modes which are not orthogonal to the downwash and however does not participate in the response Hall (1994).

For a satisfactory ROM to can be constructed from (39) is decomposed into two system parts, known as quasi-steady and systems dynamic part and can be represented as

$$Q^n = Q_s^n + Q_d^n \tag{41}$$

where Q_s^n, and Q_d^n are quasi-steady and dynamic parts.

Such that

$$Q_d^n = XC_d^n$$

Substituting (39) into (38) and multiply through by Y^T and simplifying we have

$$C_d^{n+1} = ZC_d^n + Y^T V^{n+1} - Y^T (W^0 Q_s^{n+1} + IQ_s^n) \tag{42}$$

Then (41) is replaced by (42) in the construction of ROM. Since static correction is carried out on (41) it will give satisfactory result where the system is dominated by few eigen mode (Behbahani-Nejad et al., 2004)

ROM without Static Correction

The construction of ROM is unsatisfactory and requires static correction due to existence of zero eigenvalue in the eigensystem (Esfahaian and Behbahani-Nejad, 2002). However, the zero eigenvalues can be removed by defining a new eigenvalue problem whose eigenvalues are non-zero of the previous eigenvalue problem (Behbahani-Nejad, *et al.*, 2004). In high pressure HCNG transient flow analysis we let Q_b and Q_s to be the vectors of fluid motion due to disturbance and steady state fluid respectively.

Equation (34) can now be written as

$$\begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{bmatrix}^{n+1} + \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{bmatrix}^{n} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}^{n+1} \tag{43}$$

where $W_{i,j}$, $I_{i,j}$ are sub square matrices from W^o and I with $1 \le i, j \le 3$

Expanding (43) we have

$$W_{11}\Gamma_1^{n+1} + W_{12}\Gamma_2^{n+1} + W_{13}\Gamma_3^{n+1} + I_{11}\Gamma_1^n + I_{12}\Gamma_2^n + I_{13}\Gamma_3^n = V_1^{n+1} \tag{44}$$

$$W_{21}\Gamma_1^{n+1} + W_{22}\Gamma_2^{n+1} + W_{23}\Gamma_3^{n+1} + I_{21}\Gamma_1^n + I_{22}\Gamma_2^n + I_{23}\Gamma_3^n = V_2^{n+1} \tag{45}$$

$$W_{31}\Gamma_1^{n+1} + W_{32}\Gamma_2^{n+1} + W_{33}\Gamma_3^{n+1} + I_{31}\Gamma_1^n + I_{32}\Gamma_2^n + I_{33}\Gamma_3^n = V_3^{n+1} \tag{46}$$

Since the matrices W^o and *I* are tridiagonal and identity matrices respectively then following sub matrices are zero matrix $W_{1,3}$ and $W_{3,1}$ while $I_{i,j} = 0$ if $i \ne j$, therefore equations (44) and (46) reduces and hence are simplified to get

$$\Gamma_1^{n+1} = W_{11}^{-1}\left[V_1^{n+1} - \left(W_{12}\Gamma_2^{n+1} + I_{11}\Gamma_1^n\right)\right] \tag{47}$$

$$\Gamma_3^{n+1} = W_{33}^{-1}\left[V_3^{n+1} - \left(W_{32}\Gamma_2^{n+1} + I_{33}\Gamma_3^n\right)\right] \tag{48}$$

Substituting (48) and (47) into (46) and evaluating we get

$$\left[W_{22} - W_{21}W_{11}^{-1}W_{12} - W_{23}W_{33}^{-1}W_{32}\right]\Gamma_2^{n+1} - W_{21}W_{11}^{-1}\Gamma_1^n - W_{23}W_{33}^{-1}\Gamma_3^n = V_2^{n+1} - W_{31}W_{11}^{-1}V_1^{n+1} - W_{23}W_{33}^{-1}V_3^{n+1} \tag{49}$$

Assuming $\Gamma_1^n = \Gamma_3^n = -\Gamma_2^n$ then (36) become

$$W_{new}^o \Gamma_s^{n+1} + B_{new}\Gamma_s^n = V_{new}^{n+1} \tag{50}$$

where

$W_{new}^o = \left[W_{22} - W_{21}W_{11}^{-1}W_{12} - W_{23}W_{33}^{-1}W_{32}\right]$,

$B_{new} = (-W_{21}W_{11}^{-1} - W_{23}W_{33}^{-1})$ and

$V_{new}^{n+1} = V_2^{n+1} - W_{31}W_{11}^{-1}V_1^{n+1} - W_{23}W_{33}^{-1}V_3^{n+1}$

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However (50) is eigenvalue problem that fully define the previous problem and its eigenvalues are nonzero on which ROM can now be constructed without static correction. Then ROM can now be constructed without static correction.

To analysed dynamic behaviour of HCNG, for change on surrounding temperature to be achieved (42) and (48) are used for when static correction is required and without static correction requirement.

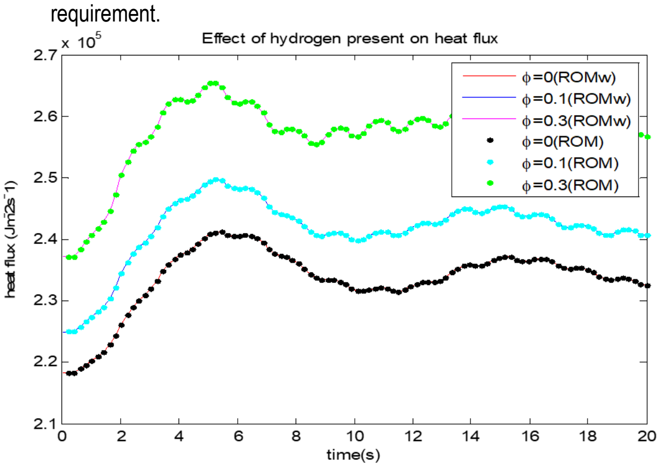

Figure 1: Heat flux with respect to ratio of gas presents

The presents of hydrogen in the gas mixture shows a significant effect on heat flux of the fluid flow analysis. From figure 1 there is steady is the graph with increase on initial heat flux due to percentage of hydrogen presents.

The heat flux continuously varies during simulation due to the ratio of hydrogen gas presence the system. The effect of pipeline surrounding on heat transfer was observed from figure 1. For different mass ratio the heat flux have significant difference and never converged throughout the simulation period. There is significant agreement on the two methods of computation as can be seen in heat flux result presented in Figure1. Change in heat flux is noted from different presence of hydrogen which is observed through the simulation.

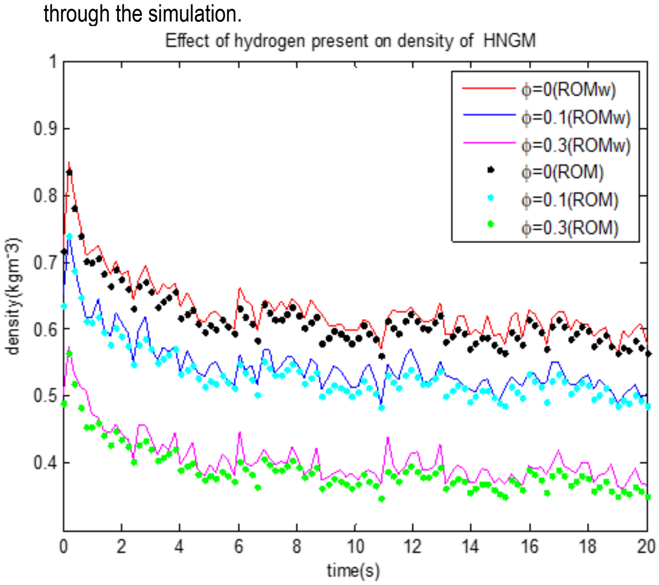

Figure 2. The density as hydrogen natural gas mixture for different ratio

In figure 2 the simulation of HNG density of mixture is determined by mass ratio with similarity in the graph. The methods of simulation has strong agreement for various percentage of hydrogen present. This also is in agreement with results on method comparison as presented on the paper A Novel Technique of Reduce Order Modelling without Static Correction for Transient Flow of Non-isothermal Hydrogen-Natural Gas Mixture (B. G. Agaie, Khan, Yacoob, & Tlili, 2018).

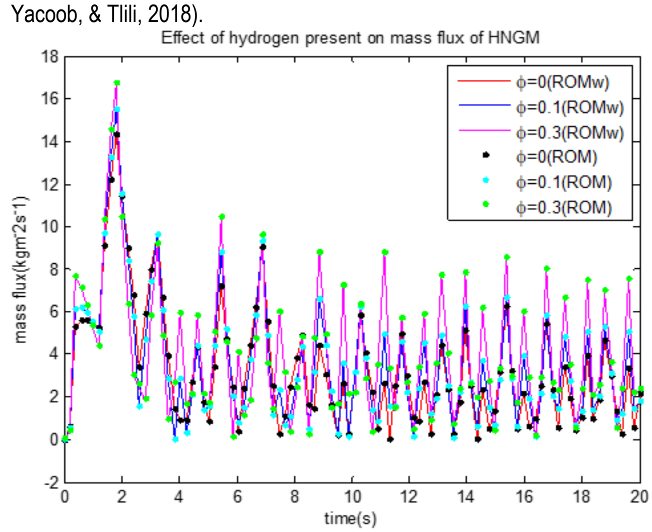

Figure 3: Effect of Mass Flux on Hydrogen natural gas mixture

There are difficulty in the obtaining velocity from industrial data of pipeline flow, but these can easily be conducted using analysis of mass flux solution results (Yanpeng and Yuxing 2010). Mass flux is the product of density and velocity therefore; the result gives us a clear inside of what is happening on velocity profile of HCNG.

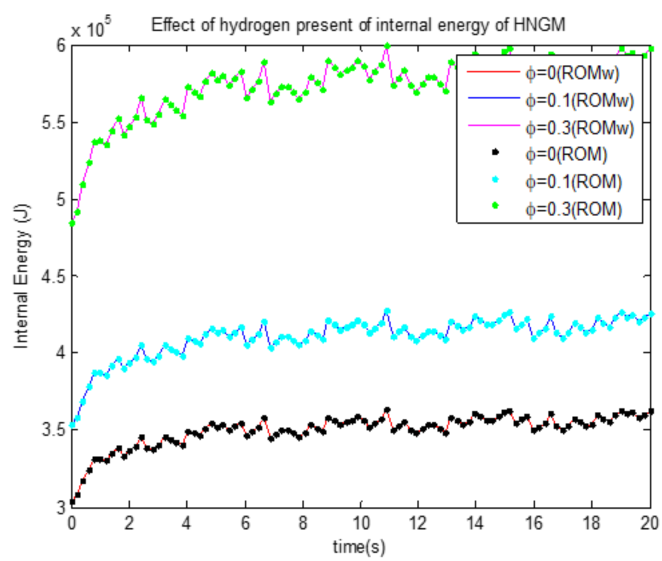

Figure 4: Effect of Internal energy for different hydrogen present

Internal energy of the mixture is observed to be highly affect when the present of hydrogen is more. From the graph the sinusoidal remain but energy per joule increases for increase in percentage. The result from both static correction ROM and without static correction shows a clear agreement.

Conclusion

ROM without static correction requirement was also compared with the conventional ROM and generated a satisfactory result for different parameters during simulation. It has advantage on non-existence of zero eigenvalue and therefore reduces the computational time. These as in line with results early published on the paper novel technique of reduce order modelling without static correction for transient flow of non-isothermal hydrogen-natural gas mixture (B Agaie 2018) which indicates computational efficiency in the approach and accuracy. The technique was also use on Eigen analysis of unsteady flows about airfoils, cascades, and wings. The results are stable and satisfactory when compare existing (Behbahani-Nejad, Haddadpour, & Esfahanian, 2004). Since the eigenvalues involved in eigensystem are non-zero and the real part of the eigenvalues are negative the computational time during simulation. As presented in the results the presence of hydrogen leads to decrease on heat flux, this also agrees with early reported of (Subani, Amin, & Agaie, 2015; Uilhoorn, 2009).

Heat flux, Internal energy and density of the mixture have similar behavior in the simulation as presented for all cases of hydrogen present. These was also observed when body force is considered in the computation and has similar to what is obtain in micro channel as presented by (Dang, Teng, & Chu). For good storage and transportation of HCNG to be stable the involve substantial challenges on heat and mass transfer is required (Wenqi, Mingyao, Baosheng, & Zhulin, 2006). It is observed that the mass of hydrogen present must always be taken into account for any HCNG flow analysis, which also agrees with the experimental and development of model on active cooling system for hydrogen storage (Pourpoint, Velagapudi, Mudawar, Zheng, & Fisher, 2010).

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