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Full Length Research Article

DEVELOPMENT OF A 2-DIMENSIONAL ANGULATION ALGORITHM TARGET LOCATING ERROR ESTIMATION TECHNIQUE

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ABSTRACT
A multiangulation (MANG) system determines an emitting target location using the angle of arrival (AOA) measurement estimated from its emission with an angulation algorithm. Prior to deployment of the system, it is important to know if the horizontal coordinate (HC) root mean square error (RMSE)s obtained by the system at certain target locations given an AOA error are within approved standards set by the international regulatory bodies. For this reason, a MANG system target locating error estimation technique based on Euclidean geometrical analysis and linear regression is proposed in this paper. This is to assist in the systematic determination and prediction of the HC RMSE obtained by the angulation algorithm of the MANG system. The proposed technique is validated by comparison with the Monte Carlo (MC) simulation at some randomly selected target locations using a square receiving station (RS) configuration. Result comparison shows that the proposed technique predicts the target HC RMSE obtained by the angulation algorithm within a system coverage of 10 km by 10 km with a prediction accuracy of about ±5 m.

Keywords: Multiangulation system; Angle of arrival: Angulation Algorithm; Error prediction; Linear regression

INTRODUCTION
Wireless positioning system (WPS)s either in an active or passive mode determines the location of an emitting target using it emission detected at spatially deployed receiving station (RS) (Zekavat and Buehrer, 2012). The Target locating process of any WPS is usually in two stages (Zekavat and Buehrer, 2012; Yaro and Sha'ameri, 2018). The first stage involves the estimation of a position dependent signal parameter (PDSP) such as angle of arrival (AOA), time difference of arrival (TDOA) and frequency difference of arrival from the target emission (Wang and Ho, 2015; Zhu and Feng, 2015; Yaro and Sha'ameri, 2018). In the second stage, the PDSP obtained in the first stage is used as an input to a localization algorithm such as a fingerprinting algorithm, angulation algorithm and lateration algorithm to obtain the target location (Zekavat and Buehrer, 2012). The multiangulation (MANG) system is an example of a WPS which has an AOA measurement as its PDSP and an angulation algorithm as it localization algorithm for target locating (Yaro and Sha'meri, 2016; Sha'ameri *et al.*, 2017). The AOA estimation is not the scope of this paper but assumed to be obtained using any of the available techniques mentioned in literature (ITU-R, 2010; Wielandt *et al.*, 2014) however, it is obtained with an error.

The angulation algorithm is the scope of this paper and is used by the MANG system to obtain the location of the target with the AOA measurement vector and coordinates of the RSs at its input (Sha'ameri *et al.*, 2017; Yaro *et al.*, 2017). Error in the AOA measurement vector results in error in the location of the target obtained by the angulation algorithm of the MANG system. The target locating error of the MANG system is a function of the target location relative to the RS configuration (Sha'ameri *et al.*, 2017). However, documents from international bodies such as International Civil Aviation Organization (ICAO) and MANG system manufacturers only provide the target horizontal coordinate (HC) root mean square error (RMSE) and AOA RMSE of the system with no link to the emitting target's location (ITU-R, 2010). Prior to the deployment of the MANG system for target locating, it is important to know if the target HC RMSE obtained by the system at certain target locations within the system coverage is within the approved set standard. According to Federal Aviation Administration (FAA), the maximum allowed HC RMSE for any WPS used in civil aviation is 1 km (ICAO, 2016). Thus, a MANG system target locating error estimation technique is proposed in this paper. Using the geometry of the RS configuration, a mathematical function is obtained that relates the target location, AOA error and the coordinates of the RS. Monte Carlo (MC) simulation is performed to obtain a numerical value for the target location error obtained by the angulation algorithm with respect to the AOA error. The results are then equated to the geometrical mathematical function using linear regression and a mathematical expression is produced for proposed technique.

Angulation Algorithm Development
Let $\mathbf{x}_{[m]} = (x, y)$ in meter indicated by the subscript "[*m*]" be the instantaneous location of an emitting target in 2-dimensional Euclidean space while $\mathbf{s}^{i}_{[m]} = (x_i, y_i)$ is the coordinate of the *i*-th RS. The AOA of the emitting target at the *i*-th RS is related to the coordinates of the emitting source and that of the *i*-th RS by a line of bearing (LOB) as shown in Eq. (1) (Yaro and Sha'meri, 2016):

$$y = x\tan\left(\theta_{i,[rad]}\right) + y_i - x_i\tan\left(\theta_{i,[rad]}\right) \qquad (1)$$

where: $\theta_{i,[rad]}$ in radian indicated by the subscript "[rad]" is the AOA of the emitting target obtained at the *i*-th RS.

In practical application, the AOAs at each RS are obtained with error. Modeling the AOA error as a zero mean Gaussian random

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variable, the measured AOA of the emitting target at the i-th RS is mathematically expressed as shown in Eq. (2) (Griffin *et al.*, 2015):

$$\hat{\theta}_{i,[rad]} = \theta_{i,[rad]} + N(0, \sigma^{i}_{[rad]}) \quad (2)$$

where: $\sigma^{i}_{[rad]}$ is the AOA error standard deviation (SD) at the i-th RS and ranges from 0 to $\pi/36$ (5^0).

With a total of four RSs that is for $i = 1$ to 4, four LOB equations are obtained after substituting Eq. (2) into Eq. (1) shown in Eq. (3) to Eq. (6).

$$y = x\tan\left(\hat{\theta}_{1,[rad]}\right) + y_1 - x_1\tan\left(\hat{\theta}_{1,[rad]}\right) \quad (3)$$

$$y = x\tan\left(\hat{\theta}_{2,[rad]}\right) + y_2 - x_2\tan\left(\hat{\theta}_{2,[rad]}\right) \quad (4)$$

$$y = x\tan\left(\hat{\theta}_{3,[rad]}\right) + y_3 - x_3\tan\left(\hat{\theta}_{3,[rad]}\right) \quad (5)$$

$$y = x\tan\left(\hat{\theta}_{4,[rad]}\right) + y_4 - x_4\tan\left(\hat{\theta}_{4,[rad]}\right) \quad (6)$$

Eq. (3) to Eq. (6) can be presented in matrix form as shown in Eq. (7).

$$\mathbf{Ax} = \mathbf{b} \quad (7)$$

where:

$$\mathbf{A} = \begin{bmatrix} -\tan\left(\hat{\theta}_{1,[rad]}\right) & 1 \\ -\tan\left(\hat{\theta}_{2,[rad]}\right) & 1 \\ -\tan\left(\hat{\theta}_{3,[rad]}\right) & 1 \\ -\tan\left(\hat{\theta}_{4,[rad]}\right) & 1 \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} y_1 - x_1\tan\left(\hat{\theta}_{1,[rad]}\right) \\ y_2 - x_2\tan\left(\hat{\theta}_{2,[rad]}\right) \\ y_3 - x_3\tan\left(\hat{\theta}_{3,[rad]}\right) \\ y_4 - x_4\tan\left(\hat{\theta}_{4,[rad]}\right) \end{bmatrix}$$

and $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

The unknown from the matrix Eq. (7) is the target location **x**. The least square (LS) problem in Eq. (7) is an overdetermined total least square (TLS) problem which can be solved using Singular Value Decomposition (SVD) TLS. Let matrix **C** be a concatenation of matrix **A** and **b.** Taking the SVD of matrix **C** as indicated in Eq. (8) (Markovsky, Sima and Van Huffel, 2010),

$$\mathbf{C} = [\mathbf{A}, \mathbf{b}] = \mathbf{U\Sigma V^T} = \sum_{i=0}^{n+1} u_i \sigma_i v_i^T \quad (8)$$

The solution to Eq. (7) using the SVD TLS technique is presented in Eq. (9).

$$\hat{\mathbf{x}} = -\frac{1}{v_{(n+1,n+1)}}[v_{(1,n+1)}, v_{(2,n+1)}, \cdots, v_{(n,n+1)}]^T \quad (9)$$

The Eq. (9) is the estimated target location obtained by the angulation algorithm given the measured AOAs at each RSs.

Proposed Mang System Target Locating Error Estimation Technique

In this section of the paper, the detailed description of the proposed target locating error estimation technique is presented. For simplicity, a pair of RS is used for illustration as shown in Figure 1.

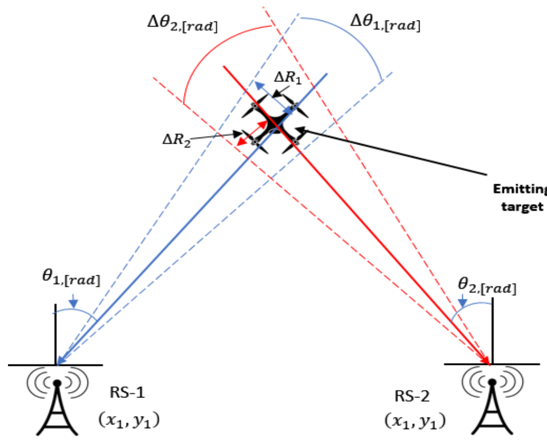

Figure 1: AOA range error ambiguity. [$\Delta\theta_i$ is the AOA estimation error]

The AOA range error ambiguity ΔR_i due to $\Delta\theta_i$ at the i-th RS is mathematically expressed as shown in Eq. (10) (Sivagnanam *et al.*, 2012).

$$\Delta R_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \times \tan\left(\frac{\Delta\theta_{i,[rad]}}{2}\right)$$

$$1 \le i \le 4 \quad (10)$$

For small values of $\Delta\theta_{i,[rad]}$, Eq. (11) is valid:

$$\tan\left(\frac{\Delta\theta_{i,[rad]}}{2}\right) \approx \frac{\Delta\theta_{i[rad]}}{2} \quad (11)$$

After substituting Eq. (11) into Eq. (10), the resulting expression is shown in Eq. (12).

$$\Delta R_i \approx \frac{\Delta\theta_{i,[rad]}}{2}\sqrt{(x - x_i)^2 + (y - y_i)^2} \quad (12)$$

Eq. (13) shows the average range error ambiguity for the 4 RSs based on Eq. (12).

$$\Delta R_{avg} = \frac{1}{4}\sum_{i=1}^{4}\Delta R_i = \frac{1}{4}\left(\Delta R_1 + \Delta R_2 + \Delta R_3 + \Delta R_4\right) \quad (13)$$

Let HC_{rmse_mc} be the HC RMSE in locating the emitting target obtained using the MC simulation. Eq. (14) presents the

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mathematical relationship between the average range ambiguity ΔR_{avg} in Eq. (13) and the L_{rmse_mc}.

$$HC_{rmse\_mc}\left(x,y\right)=\alpha\times\Delta R_{avg} \tag{14}$$

where: α is the correlation coefficient.

Using linear regression, the correlation coefficient in Eq. (14) is determined by minimizing the error that is expressed in Eq. (15).

$$E=\arg\_\min_{\alpha}\left\|HC_{rmse\_mc}\left(x,y\right)-\Delta R_{avg}\right\|_2 \tag{15}$$

The mathematical expression for the HC RMSE obtained using the proposed technique is presented in Eq. (16).

$$HC_{rsmse\_proposed}\left(x,y\right)=\alpha\times\Delta R_{avg} \tag{16}$$

The HC RMSE obtained by the proposed technique using Equation (13) is validation in the next section.

Using HC RMSE as a performance measure, the performance of the proposed target locating error estimation technique is validated by comparing with MC simulation results. The mathematical expression of the HC RMSE obtained using the MC simulation presented in Eq. (17).

$$HC_{rmse\_mc}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\left(x-x_i\right)^2+\left(y-y_i\right)^2\right]} \tag{17}$$

where: (x,y) are the known coordinates of the emitting target and (x_i,y_i) are estimated coordinates of the emitting target at the *i*-th MC simulation realization. For the Monte Carlo simulation results are obtained after $N=500$ realizations.

The configuration in which the RSs are deployed affects the locating capability of the MANG system. As report in (Sha'ameri *et al.*, 2017), the RS configuration that results in the best accuracy in target locating for a WPS with a total of four RSs is square. Thus, the square RS configuration with a separation of 250 m between RS pair is adopted in this paper. Figure 2 shows the distribution of the RSs labelled 1, 2, 3 and 4.

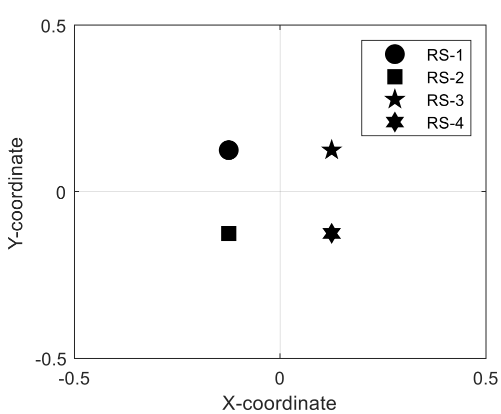
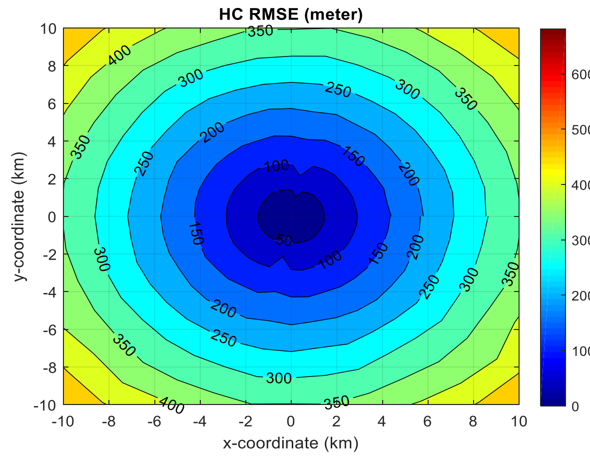
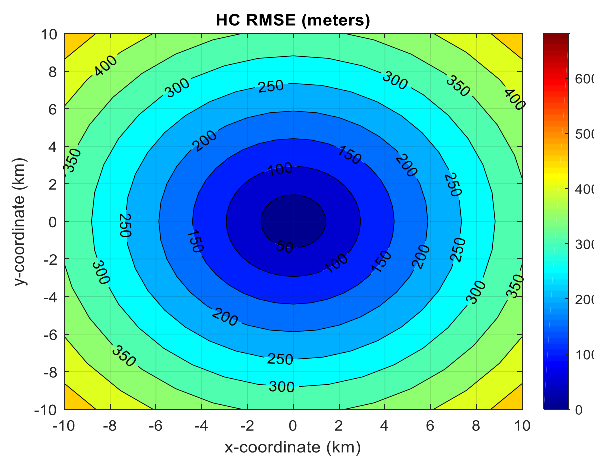


Figure 2: Distribution of the RSs in square configuration

With an AOA error SD of π/90 (ITU-R, 2010), the HC RMSE of the proposed technique on Eq. (16) and that of the MC simulation based on Eq. (17) are obtained for a MANG system coverage of 10 km by 10 km and is compared. This can be seen in Figure 2. For both techniques, the HC RMSE increases with the target horizontal range but invariant with the horizontal bearing.



(a) HC RMSE obtained using MC simulation



(b) HC RMSE obtained using the proposed technique

Figure 2: HC RMSE comparison at AOA error SD of π/90

Table 1 shows the HC RMSE comparison between the proposed technique and the MC simulation result at eight randomly selected target locations. At target location A, the HC RMSEs obtained using the proposed technique and the MC simulation are 136.20 m and 139.00 m respectively with an absolute HC RMSE difference of about 2.80 m. This means that proposed target locating error estimation technique predicts the HC RMSE obtained by the angulation algorithm of the multiangulation system based on an AOA error SD of π/90 at target location A with a prediction error of 2.80 m compared to the MC simulation.

Table 1. HC RMSE comparison

Target location	Coordinates (m)		HC RMSE (m)		Absolute HC RMSE Difference (m)
	x	y	MC simulation	Proposed technique	
A	2	3	139.00	136.20	2.8
B	9	3	345.90	340.40	5.5
C	3	-5	208.50	204.30	4.2
D	8	-9	421.30	426.60	5.3
E	-4	-7	284.20	277.30	6.9
F	-8	-5	345.00	340.40	4.6
G	-6	1	209.40	204.30	5.1
H	-3	9	355.50	348.40	7.1

Extending the analysis to target at locations B, C, D, E, F, G, and H, the absolute HC RMSE differences between the proposed technique and the MC simulation are 5.50 m, 4.20 m, 5.30 m. 6.90 m, 4.60 m, 5.10 m and 7.10 m respectively. Within the 10 km by 10 km MANG system coverage, the average HC RMSE difference between the proposed technique and the MC simulation is about 5 m. This means that based on an AOA error SD of $\pi/90$ and within a system coverage of 10 km by 10 km with the RS in a square configuration, the proposed target locating error estimation technique has a prediction accuracy of about ±5m.

Conclusion

In this paper, a technique to assist in the systematic determination and predication of the HC RMSE obtained by the angulation algorithm of the MANG system is proposed. This is to ensure that the HC RMSE at certain target locations given an AOA measurement error is within the approved standard set by the regulatory bodies. The proposed technique is validated through comparison with the MC simulation. Simulation results shows that within a MANG system coverage of 10 km by 10 km and based on an AOA error SD of $\pi/90$, the proposed technique has a HC RMSE prediction accuracy of about ± 5 m with the RS in a square configuration. This paper focuses on using the AOA measurement error to determine the target locating error of the angulation algorithm of the MANG system. However, the value of the AOA error depends on the AOA estimation technique used.

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