Science World Journal Vol 13(No 4) 2018
www.scienceworldjournal.org
ISSN 1597-6343
Published by Faculty of Science, Kaduna State University

Full Length Research Article

SOME PROPERTIES OF SOFT MULTISET

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ABSTRACT
The concepts of relative null, relative semi-null, relative absolute, relative semi-absolute and absolute soft multisets were presented. It is shown that the relative absolute soft multiset is equivalent to the complement of relative null soft multiset, the complement of relative absolute soft multiset is equivalent to the relative null soft multiset. It is further established that a relative semi-absolute soft multiset is equivalent to the complement of a relative semi-null soft multiset and the complement of a relative semi-absolute soft multiset is equivalent to a relative semi-null soft multiset. Related results are also established.

Keywords: Multiset, Soft Set, Soft Multiset, absolute soft multiset, relative absolute soft multiset

INTRODUCTION
The concept of a set formulated by George Cantor requires that an object must be definite and distinct. Thus, the issue of repetition and vagueness becomes a problem for philosophers, logicians, mathematicians and computer scientists etc. To handle these, tools such as rough sets (Pawlak, 1982), soft sets (Molodtsov, 1999), fuzzy sets (Zadeh, 1965), multisets (Yager, 1981), etc., emerged.

Soft set which is an approximate description of an object consisting of predicate and approximate value set comes up as a result of the requirements for exact solutions in classical sets, as some problems are so complicated in such a way that, only an approximate solution is possible. This theory attracts applications in various fields such as decision making, medical diagnosis, algebra, data analysis, forecasting, game theory etc. (Maji & Roy, 2002; Majumdar & Samanta, 2008; Zou & Xiao, 2008; Xiao *et al.*, 2009; Qin & Hong, 2010).

Multiset which is an unordered collection of objects where duplicates of objects are considered significant was developed with the aim of addressing repetitions. The theory has applications in many fields such as mathematics, computer science and social sciences to mention a few (Blizard, 1991; Singh *et al.*, 2007; Isah Tella, 2015; Singh & Isah, 2015, 2016].

Soft multisets was introduced in (Alkhazaleh, 2011) using the idea of universes. Since then, various scholars contributed to the development of the theory using different approaches (Neog & Sut, 2012; Majumdar, 2012; Babitha & John, 2013). The theory attracts applications in decision making, mathematics, computer science etc.

In this paper some properties such as relative null, relative semi-null, absolute, relative absolute and relative semi-absolute soft multisets were presented and related results were established.

Preliminaries

Soft set

Definition 2.1.1 (Molodtsov, 1999)
Let U be an initial universe set and E a set of parameters with respect to U. Let $P(U)$ denote the power set of U and $A \subseteq E$. A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F: A \longrightarrow P(U)$. Thus, (F, A) is defined as

$$(F, A) = \{F(e) \in P(U): e \in E, F(e) = \emptyset \text{ if } e \notin A\}.$$

Definition 2.1.2 (Pei & Miao, 2005) Let (*F*, *A*) and (*G*, *B*) be two soft sets over a common universe *U*, we say that
(a) (*F*,*A*) is a soft subset of (*G*,*B*), denoted by $(F,A) \tilde{\subseteq} (G,B)$, if
(i) $A \subseteq B$, and
(ii) $\forall \ e \in A, F(e) \tilde{\subseteq} G(e)$.
(*F*, *A*) is soft equal to (*G*, *B*), denoted (*F*, *A*) = (*G*, *B*), if $(F,A) \tilde{\subseteq} (G,B)$ and $(G,B) \tilde{\subseteq} (F,A)$

Multisets (mset, for short) (Jena *et al.*, 2001; Girish & John, 2012)

Definition 2.2.1: An mset *M* drawn from the set *X* is represented by a function *Count* M or C_M defined as $C_M: X \longrightarrow \mathbb{N}$, where $\mathbb{N}$ is a set of natural numbers including zero.
Let M be a multiset from X, then M can be represented as $M = \{k_1/x_1, k_2/x_2, \dots, k_n/x_n\}$ with x_1 appearing k_1 times, x_2 appearing k_2 times and so on.

Definition 2.2.2: Let M and N be two msets drawn from a set X. Then
(i) $M \subseteq N$ if $C_M(x) \leq C_N(x), \forall x \in X$.
(ii) $M = N$ if $C_M(x) = C_N(x), \forall x \in X$.
(iii) $M \cup N = max\{C_M(x), C_N(x)\}, \forall x \in X$.
(iv) $M \cap N = min\{C_M(x), C_N(x)\}, \forall x \in X$.
(v) $M - N = max\{C_M(x) - C_N(x), 0\}, \forall x \in X$.

Definition 2.2.3: Let M be a multiset drawn from a set X. The support set or root set of M denoted by M^* is a subset of X given by $M^* = \{x \in X: C_M(x) > 0\}$.
The power multiset of a given mset M, denoted by $P(M)$ is the multiset of all submultisets of M, and the power set of a multiset M is the root set of $P(M)$, denoted by $P^*(M)$.

Soft Multiset (Soft mset, for short) (Osmanoglu & Tokat, 2014; Tokat *et al.*, 2015)

Definition 2.3.1: Let U be a universal multiset, E be a set of parameters and $A \subseteq E$. Then a pair (F, A) is called a soft multiset where F is a mapping given by $F: A \longrightarrow P^*(U)$. For all $e \in A$, the mset $F(e)$ is represented by a count function $C_{F(e)}: U^* \longrightarrow \mathbb{N}$.

Definition 2.3.2: Let (F, A) and (G, B) be two soft multisets over U. Then

(i) (F, A) is a soft submultiset of (G, B) written $(F, A) \sqsubseteq (G, B)$ if
  (a) $A \subseteq B$
  (b) $C_{F(e)}(x) \leq C_{G(e)}(x), \forall x \in U^*, \forall e \in A$.

$(F, A) = (G, B)$ iff $(F, A) \sqsubseteq (G, B)$ and $(G, B) \sqsubseteq (F, A)$.
Also, if $(F, A) \sqsubset (G, B)$ and $(F, A) \neq (G, B)$ then (F, A) is called a proper soft submset of (G, B), and (F, A) is a whole soft submset of (G, B) if $C_{F(e)}(x) = C_{G(e)}(x), \forall x \in F(e)$.

(ii) **Union:**
$(F, A) \sqcup (G, B) = (H, C)$ where $C = A \cup B$ and $C_{H(e)}(x) = max\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in C, \forall x \in U^*$.

(iii) **Intersection:**
$(F, A) \sqcap (G, B) = (H, C)$ where $C = A \cap B$ and $C_{H(e)}(x) = min\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in C, \forall x \in U^*$.

(iv) **Difference:**
$(F, E) \backslash (G, E) = (H, E)$ where $C_{H(e)}(x) = max\{C_{F(e)}(x) - C_{G(e)}(x), 0\}, \forall x \in U^*$.

(v) **Null:**
A soft multiset (F, A) is called a Null soft multiset denoted by Φ if $\forall e \in A, F(e) = \emptyset$.

(vi) **Complement:**
The complement of a soft multiset (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$ where $F^c: A \longrightarrow P^*(U)$ is a mapping given by $F^c(e) = U \backslash F(e), \forall e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x),\ \forall x \in U^*$.

Some properties of soft multisets

Definition 3.1 Let U be a universal multiset, E be a set of parameters and $A \subseteq E$. Then
(i) (F, A) is called a *relative null* soft multiset with respect to A denoted $(F, A)_\emptyset$ if $F(e) = \emptyset, \forall e \in A$.
(ii) (F, A) is called a *relative semi-null* soft multiset with respect to A, denoted $(F, A)_{\emptyset 1}$ if $\exists\ e \in A$ such that $F(e) = \emptyset$.
(iii) (F, A) is called a *relative absolute* soft multiset with respect to A denoted $(F, A)_U$ if $F(e) = U, \forall e \in A$.
(iv) (F, A) is called a *relative semi-absolute* soft multiset with respect to A, denoted $(F, A)_{U1}$ if $\exists\ e \in A$ such that $F(e) = U$.
(v) (F, E) is said to be the absolute soft multiset denoted $(F, E)_U$ if $F(e) = U, \forall e \in E$.

Example 3.2
Let $= \{5/w, 2/x, 3/y\}, E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ and $A = \{e_1, e_2, e_7\}$. Then
(i) If $F(e_1) = \emptyset,\ F(e_2) = \emptyset, F(e_7) = \emptyset$, then (F, A) is a relative null.
(ii) If $F(e_1) = \{2/w, 1/y\},\ F(e_2) = \emptyset, F(e_7) = \{4/w, 1/y, 1/x\}$ then (F, A) is a relative semi-null.
(iii) If $F(e_1) = \{5/w, 2/x, 3/y\}, F(e_2) = \{5/w, 2/x, 3/y\}, F(e_7) = \{5/w, 2/x, 3/y\}$, then (F, A) is a relative absolute.
(iv) If $F(e_1) = \{5/w, 2/x, 3/y\}, F(e_2) = \{1/x, 3/y\}, F(e_7) = \{4/w, 2/x, 1/y\}$, then (F, A) is a relative semi-absolute.
(v) If $F(e_1) = \{5/w, 2/x, 3/y\}, F(e_2) = \{5/w, 2/x, 3/y\}, F(e_3) = \{5/w, 2/x, 3/y\}, F(4) = \{5/w, 2/x, 3/y\}, F(e_5) = \{5/w, 2/x, 3/y\}, F(e_6) = \{5/w, 2/x, 3/y\}, F(e_7) = \{5/w, 2/x, 3/y\}$, then (F, E) is the absolute soft multiset.

Theorem 3.3
(i) $(F, A)_U = ((F, A)_\emptyset)^c$
(ii) $((F, A)_U)^c = (F, A)_\emptyset$
(iii) $(F, A)_{U1} = ((F, A)_{\emptyset 1})^c$
(iv) $((F, A)_{U1})^c = (F, A)_{\emptyset 1}$

Proof
(i) Let $e \in A$, then each element of $((F, A)_\emptyset)^c$ will be $F^c(e) = U \backslash F(e)$
but by definition of $(F, A)_\emptyset,\ F(e) = \emptyset$
thus, $F^c(e) = U \backslash F(e) = U \backslash \emptyset = U$
In other words, each element of $((F, A)_\emptyset)^c$ will be equal to U.
That is, $((F, A)_\emptyset)^c \sqsubseteq (F, A)_U$ ……………….(1)
Conversely,
Consider $(F, A)_U$, by definition each element of $(F, A)_U$ will be equal to U.
i. e., $(F, A)_U \sqsubseteq ((F, A)_\emptyset)^c$ ……………(2)
From (1) and (2) $(F, A)_U = ((F, A)_\emptyset)^c$.

(ii) Let $e \in A$, then each element of $((F, A)_U)^c$ will be $F^c(e) = U \backslash F(e)$
but by definition of $(F, A)_U,\ F(e) = U$,
thus, $F^c(e) = U \backslash U = \emptyset$
i.e., each element of $((F, A)_U)^c$ will be $\emptyset$.
i.e., $((F, A)_U)^c \sqsubseteq (F, A)_\emptyset$ ……………….(1)
Conversely,
Let consider $(F, A)_\emptyset$. By definiton of $(F, A)_\emptyset$, each of its element $F(e) = \emptyset$.
i. e., $(F, A)_\emptyset \sqsubseteq ((F, A)_U)^c$ ……………(2)
From (1) and (2) we have $((F, A)_U)^c = (F, A)_\emptyset$.

(iii) Let an element $F(e) \in (F, A)_{U1}$, this imply $F(e) = U$ for atleast one $e \in A$.
i.e., $(F, A)_{U1} \sqsubseteq ((F, A)_{\emptyset 1})^c$ …………….. (1)
Conversely,
Consider $((F, A)_{\emptyset 1})^c$, then an element $F^c(e)$ of $((F, A)_{\emptyset 1})^c$ will be of the form $F^c(e) = U \backslash F(e)$, but $F(e) = \emptyset$ for atleast one $e \in A$ by the definition of $(F, A)_{\emptyset 1}$.
i.e., $F^c(e) = U$ for atleast one $e \in A$.
i.e., $((F, A)_{\emptyset 1})^c \sqsubseteq (F, A)_{U1}$ …………………(2)
From (1) and (2), $(F, A)_{U1} = ((F, A)_{\emptyset 1})^c$.

(iv) Let $e \in A$, then each element of $((F, A)_{U1})^c$ will be $F^c(e) = U \backslash F(e)$,

but by definition of $(F,A)_{U1}$, $F(e) = U$ for atleast one $e \in A$.
i.e., an element of $((F,A)_{U1})^c$ will be $U \setminus U = \emptyset$ for atleast one $e \in A$.
i.e., $((F,A)_{U1})^c \sqsubseteq (F,A)_{\emptyset 1}$ ....(1)
Conversely,
Let consider $(F,A)_{\emptyset 1}$, by definition, we have $F(e) = \emptyset$ for atleast one $e \in A$.
i. e., $(F,A)_{\emptyset 1} \sqsubseteq ((F,A)_{U1})^c$ .... (2)
From (1) and (2) we have $((F,A)_{U1})^c = (F,A)_{\emptyset 1}$.

Corollary 3.4
(i) Every relative null soft multiset is a semi null soft multiset, but the converse is not always true.
(ii) Every relative absolute soft multiset is a relative semi absolute soft multiset, however, the converse is not always true.

Conclusion
The paper presents an important aspect of soft multisets that are useful in many areas of mathematics, computer science, decision making and such other areas that deals with uncertainty.

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