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Short Communication Report

PARTITION IDENTITY

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ABSTRACT
In this short communication, we employed classical formal power series Identity due to Leonard Euler and use the idea of generating function primarily to prove that the number of partitions into parts that occurs at most twice is equal to the number of partitions into parts which are $\neq 0 \; mod \; 3$.

Keywords: Unrestricted Partition, Generating Function, Identity.

INTRODUCTION

A partition of a positive integer n is defined as a way of writing n as a summand of positive integers. Where the summand are merely differ in order in which they are written, the partitions are considered the same. We denote by $p(n)$ the number of partitions of n. Thus, for example, since 5 can be expressed by, $5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, and \; 1+1+1+1+1$, We have $p(5) = 7$, see Andrews (1984).
The function $p(n)$ is referred to as the number of unrestricted partitions of n.In this paper we are interested in partitions with some sorts of restrictions, that is, partitions in which some kind of restriction is imposed upon the parts.
In fact, we shall consider identities valid for all positive integers n of the general type.

$$p^1(n) = p^2(n)$$

Where $p^1(n)$ is the number of partition of n where the parts of n are subject to a first restriction and $p^2(n)$ is the number of partitions of n where the part of n are subject to an entirely different restrictions.
The classical identity of this type is due to Euler (1748)

Theorem (Euler)
The number of partition of n into distinct parts is equal to the number of partitions of n into odd parts.
Example: The partitions of 9 into distinct parts are
$9, 8+1, 7+2, 6+3, 6+2+1, 5+4, 5+3+1, 4+3+2$,
That is, there are 8 such partitions, and the partitions of 9 into odd parts are

$$9, 7+1+1, 5+3+1, 5+1+1+1+1, 3+3+3, 3+3+1+1+1, 3+1+\cdots+1, 1+1, \ldots+1$$

So that there are also 8 partition of 9 into odd parts.
A proof of this theorem by combinatorial methods, see Niven & Zuckerman (1980) and by means of generating function, see Alder, (1969) and Alder (1979).

In an article by Hansraj Gupta (1970), various important identities were highlighted. Euler's identity, Jacob's identity, Cauchy's identities, Ramanujan identities, Rogers – Ramanujah identities etc., just to mention a few.
In particular, Rogers - Ramanujan's identities have received considerable attention. Several proofs of these identities have been given, Andrews, (1966) besides two by Rogers himself. Alder (1954), gave a generalization of Rogers identities. A combinatorial generalization of these identities were given by Gordon (1961). Among the most striking results in the theory of partitions are the Rogers-Ramanujan's Identities Alder (1948).

Definition of Important Terms
Here we introduce some definitions related to the study following Andrews (1979), a partition of a positive integer n is a way of writing n as a sum of positive integers. The summands of the partition are known as parts.

Partition
In Singh et al (2012), a partition of a positive integer n is defined as a sequence of positive integers whose sum is n.

Partition function p(n)
In Andrews and Erikson (2004), a partition function $p(n)$ counts the number of unique partitions of the positive integer n. For example$p(5) = 7$ as seen above. For a recent reference see Ladan et al (2018).
The main purpose of this paper is to use the ideas of generating function to prove our proposition.

Proposition: The number of partitions into parts that occurs at most twice is equal to the number of partitions into parts which are ≠ 0 mod 3.
Proof **.**The proof using generating function approach.
The generating function for partitions of a number in which each part can occur at most twice is given by

$$(1+x+x^2)(1+x^2+x^4)(1+x^3+x^6)\ldots$$

The generating function for the number of partition into parts not equal to 0 mod 3 is given by

$$(1+x+x^2+\cdots)(1+x^{1.2}+x^{2.2}+\cdots)(1+x^{1.4}+x^{2.4}+\cdots)\ldots$$

We now wish to show that this two are equal,

$$(1+x+x^2+\cdots)(1+x^{1.2}+x^{2.2}+\cdots)(1+x^{1.4}+x^{2.4}+\cdots)\ldots$$

$$= \frac{1}{(1-x)(1-x^2)(1-x^4)(1-x^5)\ldots}$$

$$= \frac{1}{\prod_{i \neq o \bmod 3}(1 - x^i)}$$

$$= \frac{1}{(1-x^{3i+1})(1-x^{3i+2})}$$

$$= \frac{\prod_i^\infty(1-x^{3i})}{\prod_i^\infty(1-x^{3i})\prod_i^\infty(1-x^{3i+1})\prod_i^\infty(1-x^{3i+2})}$$

$$= \frac{\prod_i^\infty(1-x^{3i})}{\prod_i^\infty(1-x^i)}$$

$$= \left(\frac{1-x^3}{1-x}\right)\left(\frac{1-x^6}{1-x^2}\right)\left(\frac{1-x^9}{1-x^3}\right)\dots$$

$$= (1+x+x^2)(1+x^2+x^4)(1+x^3+x^6)\dots$$

which is the generating function for the partitions of a number in which each part occur at most twice.

Conclusion

By this point, we have seen a wealth of material motivated by Euler Theorem, our results employ the proof technique introduced here namely, the use of generating functions as a powerful tool used in proving partition identities.

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