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EFFECT OF BRINKMAN NUMBER AND MAGNETIC FIELD ON LAMINAR CONVECTION IN A VERTICAL PLATE CHANNEL

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ABSTRACT
The effect of Brinkman number and magnetic field on laminar convection in a vertical plate channel with uniform and asymmetric temperatures has been studied. The dimensionless form of momentum and energy balanced equations has been solved using one term perturbation series solution. The solution of the dimensionless velocity u and temperature θ were obtained. The numerical values of the skin friction and the Nusselt number were obtained and tabulated. It was found that the effect of magnetic parameter enhances the effect of flow velocity by suppressing the turbulence while Brinkman number has accelerating effect on the temperature profile.

Keywords: MHD, Laminar, Convection, vertical channel, Brinkman number

NOMENCLATURES
ν - Kinematic viscosity
α - Thermal conductivity
μ- Dynamic viscosity
T_b - Bulk temperature
$U_m -$ Mean velocity
ΔT - Temperature difference
$M\ or\ Ha - \text{magnetic parameter}$
Gr -Grashof number
Re -Reynolds number
g - Acceleration due to gravity,
Br - Brinkman number

INTRODUCTION
The study of magnetohydrodynamic (MHD) flows with viscous dissipation and buoyancy effects has many importance in industrial, technological and geothermal applications such as temperature plasmas, cooling of nuclear reactors, liquid metals etc. A comprehensive review of the study of MHD flows in relations to the applications to the above areas has been made by several authors.
Mixed convection in a vertical parallel – plate channel with uniform wall temperatures has been studied by Tao (1960). Javerci (1975) dealt with the effects of viscous dissipation and Joule heating on the fully developed MHD flow with heat transfer in a channel. The exact solution of the energy equation was derived for constant heat flux with small magnetic Reynolds number. Sparrow *et al* (1984) observed experimentally flow reversal in pure convection in a one – sided heated vertical channel. The mixed convection in a vertical parallel – plate channel with asymmetric heating, where one plate is heated and the other is adiabatic has been studied by Habchi and Acharya (1986).
Gav *et al* (1992) showed experimentally for mixed convection that the increasing buoyancy parameter will make the reversal flow wider and deeper taking into consideration the effects of viscous dissipation. Barletta (1998) demonstrated that in the case of upward flow reversal decreases with viscous dissipation. However, in the case of downward flow, the flow reversal increases with viscous dissipation. He also calculated criteria for the onset of flow reversal for both directions of flow, but without taking viscous dissipation into account.
Barletta (1999) investigated the fully developed and laminar convection in a parallel – plate channel by taking into account both viscous dissipation and buoyancy. Uniform and asymmetric temperatures are prescribed at the channel walls. The velocity field is considered as parallel. A perturbation method is employed to solve the momentum balance equation and energy balance equation. A comparison between the velocity and temperature profiles in the case of laminar forced convection with viscous dissipation is performed in order to point out the effect of buoyancy. The case of convective boundary conditions is also discussed.
Daniel *et.al.* (2010) investigated the fully developed and laminar convection in a vertical parallel plate channel by taking into account both viscous dissipation and buoyancy. Uniform and asymmetric temperatures are prescribed at the channel walls. The velocity field is considered as parallel. A perturbation method is employed to solve the momentum balance equation and the energy balance equation. A comparison with the velocity and temperature profiles in the case of laminar forced convection with various dissipation is performed in order to point out the effect the effect of buoyancy. The case of convective boundary condition is also discussed.
Joseph *et.al.* (2013) studied the MHD forced convection in a horizontal double – passage with uniform wall heat flux by taking into account the effect of magnetic parameter. They assumed the flow of the fluid to be laminar, two – dimensional, steady and fully developed. The fluid is incompressible and the physical properties are constants. Dileep and Rashin (2012) investigated the fully developed magntohydrodynamic mixed convection in a vertical channel, partially filled with clear fluid and partially, filled with a fluid – saturated porous medium is by taking into account viscous and ohmic dissipations. Alim, *et al* (2007) investigated the pressure work and viscous dissipation effects on MHD natural convection along a sphere. The laminar natural convection flow from a sphere immersed in a viscous incompressible fluid in the presence of magnetic field has been considered in this investigation.
Barik (2013) analyzed the free convection heat and mass transfer MHD flow in a vertical channel in the presence of chemical reaction. Ranuka *et al* (2009) studied the MHD effects of unsteady heat convective mass transfer flow past an infinite vertical porous plate with variable suction, where the plate's

temperature oscillates with the same frequency as that of variable suction velocity with soret effects. The governing equations are solved numerically by using implicit finite difference method.

Saleh and Hashim (2009) analysed flow reversal phenomena of the fully – developed laminar combined free and forced MHD convection in a vertical plate – channel. The effect of viscous dissipation is taken into account. Tamia and Samad (2010) studied and analyzed the radiation and viscous dissipation effects on a steady two – dimensional magneto hydrodynamics free convection flow along a stretching sheet with heat generation. The non – linear partial differential equations governing the flow field under consideration have been transformed by a similarity transformation into a system of non – linear ordinary differential equations and then solved numerically by applying Nachtsheim – Swigert shooting iteration technique together with sixth order Runge – Kutta integration scheme. Resulting in non – dimensional velocity, temperature and concentration profiles are then presented graphically for different values of the parameters of physical engineering interest.

Idowu *et.al* (2013) investigated viscous dissipation and buoyancy effects on laminar convection with transpiration. Uniform and asymmetric temperatures are prescribed at the channel walls. The velocity field is considered as parallel. A perturbation method is employed to solve the momentum balance equation and the energy balance equation. A comparison with the velocity and temperature profiles in the case of laminar forced convection with various dissipation is performed in order to point out the effect of buoyancy. The case of convective boundary condition is also discussed.

Das *et.al.* (2015) studied the fully developed mixed convection flow in a vertical channel filled with nanofluid in the presences of uniform transverse magnetic field. Anjali Devi and Ganga (2010) studied the Viscous and Joule dissipation effects on MHD non – linear flow and heat transfer past a stretching porous surface embedded in a porous medium under a transverse magnetic field. Patrulescu, *et al* (2010) investigated the steady mixed convection flow in a vertical channel for laminar and fully developed flow regime. In the modeling of heat transfer the viscous dissipation term was also considered. Temperature on the right wall is assumed constant while a mixed boundary condition (Robin boundary condition) is considered on the left wall.

The mixed convection of Newtonian fluid between vertical parallel plates channel with MHD effect and variation in Brinkman number has been studied by Alizadeh Rasul *et.al*. (2014). The effect of MHD and Brinkman number on laminar mixed convection of Newtonian fluid between vertical parallel plates channel was investigated by Salehi *et.al* (2013). The boundaries are considered to be isothermal with equal temperatures.

This work presents a solution of the problem of laminar MHD convection in a vertical channel with uniform and symmetric temperatures. The solution is found by using one term perturbation series solution.

MATERIALS AND METHODS

The problem of laminar and fully – developed flow of a Newtonian fluid in a vertical parallel – plate with effect of magnetic field and viscous dissipation and buoyancy effects were considered.

The momentum and energy balance equations (Saleh and Hashim (2009)) are;

$$g\beta(T-T_b)-\frac{1}{\rho_b}\frac{\partial P}{\partial X}+\nu\frac{d^2U}{dY^2}-\frac{\sigma_e B_0^2 U}{\rho_b}=0 \qquad (1)$$

$$\alpha\frac{d^2T}{dY^2}+\frac{\nu}{c_p}\left(\frac{dU}{dY}\right)^2+\frac{\sigma_e B_0^2 U^2}{\rho_0 c_p}=0 \qquad (2)$$

Where the acceleration due to gravity is g, ν is the kinematic viscosity which is defined by $\nu=\frac{\mu}{\rho_b}$, $P=p+\rho_b gX$ is the difference between the pressure *p* and the hydrostatic pressure *P*. α is thermal conductivity and $\mu=\rho_b\nu$ is the dynamic viscosity

Equations (2) and the boundary condition on$T$, shows that P depends on X; T depends only on Y; T_b is a constant; $\frac{\partial P}{\partial X}$ is a constant.

The bulk temperature T_b and the mean velocity U_m (Barletta, 1999) are given by.

$$T_b=\frac{1}{LU_m}\int_0^L UTdY \text{ and } U_m=\frac{1}{L}\int_0^L UdY \qquad (3)$$

The boundary conditions are:

$$U(0)=0\,,\qquad T(0)=T_0,$$
$$U(L)=0\,,\qquad T(L)=T_0 \qquad (4)$$

Due to the symmetry of the problem, $U\ and\ T$ are symmetric functions of Y, so that they must fulfill the condition

$$U'(0)=0,\quad T'(0)=0 \qquad (5)$$

Introducing the dimensionless quantities below

$$u=\frac{U}{U_m}\,,\quad \theta=\frac{T-T_b}{\Delta T}\,,\quad y=\frac{Y}{L}\,,\quad \lambda=\frac{-L^2}{\mu U_m}\frac{dP}{dX}\,,\ Gr=\frac{64L^3 g\beta\Delta T}{\nu^2}\,,$$
$$Re=\frac{4LU_m}{\nu},\ \ E=\frac{Gr}{Re},\ \ Br=\frac{\mu U_m^2}{\alpha\Delta T}$$

Where the temperature difference ΔT is given by

$$\Delta T=\frac{\mu U_m^2}{k}$$

While the Grashof number Gr is always positive, the Reynolds number Re and the parameter E can be either positive or negative. In particular, in the case of upward flow $(U_m>0)$, both Re and E are positive, while, for downward flow$(U_m<0)$, these dimensionless parameters are negative.

Therefore, as a consequence of the dimensional quantities together with the momentum balance equation (1), the energy balance equation (2) and equations (3) – (5) yields;

$$\frac{d^2u}{dy^2}-Mu=-\frac{E\theta}{16}+\lambda$$
$$\frac{d^2\theta}{dy^2}+BrMu^2=-Br\left(\frac{du}{dy}\right)^2 \qquad (6)$$

Where $M=\frac{\sigma_0\beta_0^2L^2}{\rho_0\nu}$ which is the magnetic parameter and $Br=\frac{1}{\rho_b c_p}$

$$u(1)=0,\ u'(0)=0,\theta(1)=-\varphi,\ \theta'(1)=1 \qquad (7)$$

$$\int_0^1 u\theta dy=0,\ \int_0^1 udy=1 \qquad (8)$$

From equations (6) to (8), the functions $u(y)\ and\ \theta(y)$ can be determined as well as the constants $\boldsymbol{\lambda}$ and R, if the value of dimensionless parameter E is given. In particular, it is easily verified that the choice $E=0$ correspond to the absence of buoyancy forces, i.e., to forced convection.

Perturbation Series Solution

The solutions of equations (6) to (8) are obtained by the perturbation method.

Let us expand the functions $u(y), \theta(y)$ and the constants γ and φ as a power series in the parameter E , namely

$$u(y) = u_0(y) + u_1(y)E + u_2(y)E^2 + \cdots = \sum_{n=0}^{\infty} u_n(y)E^n \quad (9)$$

$$\theta(y) = \theta_0(y) + \theta_1(y)E + \theta_2(y)E^2 + \cdots = \sum_{n=0}^{\infty} \theta_n(y)E^n \quad (10)$$

$$\gamma = \gamma_0 + \gamma_1 E + \gamma_2 E^2 + \cdots = \sum_{n=0}^{\infty} \lambda_n E^n \quad (11)$$

$$\varphi = \varphi_0 + \varphi_1 E + \varphi_2 E^2 + \cdots = \sum_{n=0}^{\infty} R_n E^n \quad (12)$$

These power series expansions are substituted in equations (6) to (8).

Thus, the original boundary value problem is mapped into a sequence of boundary value problems which can be solved in succession, in order to obtain the coefficients of the power series expansion in equations (9) to (12). A detailed analysis on the application of perturbation methods in heat transfer problems can be found

The equation which corresponds to $n = 0$ in the series is the following:

$$\frac{d^2u_0}{dy^2} - Mu_0(y) = -\lambda_0 \,, u_0'(0) = 0 \,, \quad (13)$$

$$\frac{d^2\theta_0}{dy^2} + BrM[u_0(y)]^2 = -Br\left(\frac{du_0}{dy}\right)^2 , \quad (14)$$

Subject to

$$u_0(1) = 0 \,,\ \int_0^1 u_0 dy = 1$$

$$\theta_0'(0) = 0 \,, \theta_0(1) = -\varphi_0 \,, \int_0^1 u_0\theta_0 dy = 0$$

The governing equation that corresponds to $n = 1$ is given as:

$$\frac{d^2u_1}{dy^2} - Mu_1(y) = -\lambda_1 \,, \quad (15)$$

$$\frac{d^2\theta_1}{dy^2} + 2BrM[u_0(y)u_1(y)] = -Br\frac{du_0}{dy}\frac{du_1}{dy}, \quad (16)$$

$$u_1'(0) = 0 \,, u_1(1) = 0 \,, \quad \int_0^1 u_1 dy = 0$$

$$\theta_1'(0) = 0 \,, \theta_1(1) = -\varphi_1 \,, \int_0^1 (u_0\theta_1 + u_1\theta_0) = 0$$

Equations (13) and (14) are easily solved, because the function $\theta_0(y)$ does not affect the function $u_0(y)$. The latter, together with the constantλ_0, is determined by solving equation (13), namely

$$u_0(y) = \frac{\sqrt{M}}{sinh\sqrt{M} - Mcosh\sqrt{M}}\left[cosh\sqrt{M}y + cosh\sqrt{M}\right],\ \ \lambda_0 = \frac{-M\sqrt{M}cosh\sqrt{M}}{sinh\sqrt{M} - Mcosh\sqrt{M}} \quad (17)$$

By substituting equation (17) in equation (14), one obtains the function $\theta_0(y)$ and the constant φ_0, which are given by

$$\theta_0(y) = -BrA^2\{4cosh2\sqrt{M}y + 16cosh(y+1)\sqrt{M} + 16\cosh(y-1)\sqrt{M} + 4My^2cosh2\sqrt{M} + 4My^2\} + B + [1 + BrA^3C + BD]E \ \ , \varphi_0 = [-1 - BrA^3C + BD]E \quad (18)$$

The Skin Friction

The wall shear stress i.e. the skin friction at the wall is given by

$$\tau = \left(\frac{du}{dy}\right)_{y=0} = \frac{\sqrt{M}sinh\sqrt{M}}{sinh\sqrt{M} - Mcosh\sqrt{M}} \quad (19)$$

Nusselt Number

The rate of heat Transfer i.e. the heat flux at the wall in terms of Nusselt number is given by

$$Nu = \left(\frac{d\theta}{dy}\right)_{y=0} = -32BrA^2\ sinh\sqrt{M} \quad (20)$$

Where;

$A = \frac{\sqrt{M}}{2sinh\sqrt{M} - Mcosh\sqrt{M}}$; $B = BrA^2\left[20cosh2\sqrt{M} + 4Mcosh2\sqrt{M} + 4M + 16\right]$;

$C = \frac{106}{3\sqrt{M}}sinh3\sqrt{M} + \frac{38}{\sqrt{M}}sinh\sqrt{M} + 8cosh\sqrt{M} - 8cosh3\sqrt{M} + \frac{4M}{\sqrt{M}}sinh\sqrt{M} + \frac{4M}{3}cosh3\sqrt{M} + 4cosh\sqrt{M} + \frac{2M}{\sqrt{M}}exp(3\sqrt{M}) - 4Mexp(-3\sqrt{M}) + \frac{8}{\sqrt{M}}exp(-\sqrt{M}) - \frac{8}{\sqrt{M}}$;

$D = \frac{2}{\sqrt{M}}sinh\sqrt{M} + 2cosh\sqrt{M}$; $E = \frac{1}{\frac{2}{\sqrt{M}}sinh\sqrt{M} + 2cosh\sqrt{M}}$

DISCUSSION OF RESULTS

Here, 1 – term perturbation series is employed to evaluate the dimensionless velocity profile u and temperature profile θ since the other terms resulted in a trivial solutions. The resulted equations were solved with MATLAB and the results were plotted graphically.

The effect of Brinkman number and magnetic field in a vertical channel, the velocity profile and the temperature profile are depicted graphically against y for different values of the magnetic parameter M and Brinkman number Br respectively. The skin friction τ and the Nusselt number Nu were also determined numerically for different values of magnetic and Brinkman number respectively.

Figure 1 demonstrates the variation of velocity u for different values of the magnetic parameter M. It is observed from figure 1 that the velocity decreases with increase in the magnetic parameter. This means that the magnetic parameter suppressed the turbulence of the flow.

Figure 2 depicts the effect magnetic parameter M on temperature profile θ. It is clear that the temperature rises with increase in the Brinkman number Br. The Brinkman number is a dimensionless number that show the ratio of heat produced by viscous dissipation and heat transported by moleculer conduction. The higher its value, the slower the conduction of heat produced and hence the larger the temperature rise. This is applicable to screw extruder, where the energy supplied to the polymer melt comes from the viscous heat generated by shear between elements of the flowing liquid moving at different velocities and direct heat conduction from the wall of the extruder.

Tables 1 and 2 show the variation of magnetic parameter and Brinkman number on skin friction and Nusselt number respectively. It is observed from table 1 that the skin friction decreases with increase in magnetic material. While table 2 shows that increase in Brinkman number bring rise to Nusselt number.

The results of this work is in agreement to Daniel et.al. (2010) and Idowu *et.al.* (2013). The difference is the buoyancy and transpiration term.

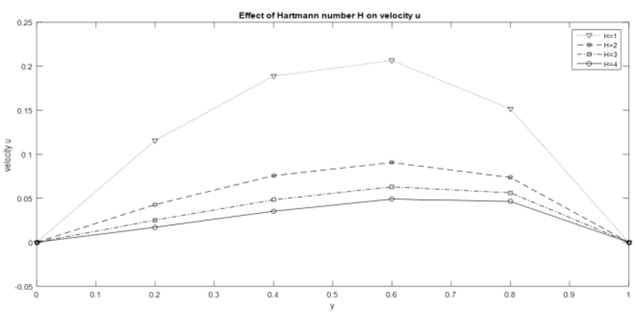

Figure 1: Effect of $M or\ Ha$ on Velocity profile

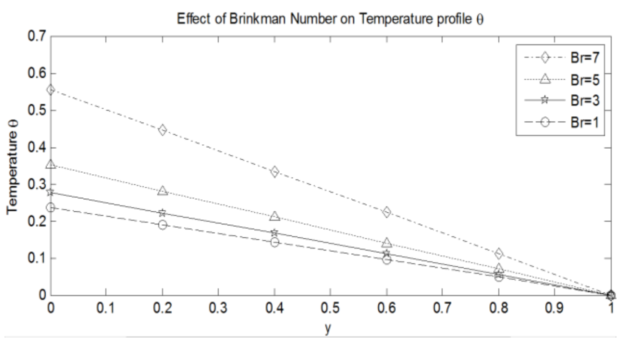

Figure 2. Effect of Br on temperature profile

Table 1: Variation of Skin Friction τ with different values of Magnetic parameter M

Magnetic Parameter	$M = 1$	$M = 3$	$M = 5$	$M = 7$
Skin Friction τ	-3.1945	-0.7895	-0.5433	-0.4358

Table 2: Variation of Nusselt number Nu with different values of Brinkman number Br

Brinkman number	$Br = 1$	$Br = 3$	$Br = 5$	$Br = 7$
Nusselt Number Nu	0.2224	0.3335	0.4447	0.5559

Summary and Conclusion

The effect of Brinkman number and Magnetic field on laminar convection in a vertical plates channel with uniform and asymmetric temperatures has been studied. The governing equations (momentum and energy balanced equations) has been written in a dimensionless form. The solution of the dimensionless velocity u and the dimensionless temperature θ has been determined.

A perturbation method has been employed to evaluate the dimensionless velocity u and temperature θ. The Skin Friction and the Nusselt number were also determined.

It can be concluded that been the effect of magnetic parameter enhances the effect of flow velocity by suppressing the turbulence while Brinkman number has accelerating effect on the temperature profile.

This study has potential application in screw extruder.

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