Science World Journal Vol 14(No 1) 2019
www.scienceworldjournal.org
ISSN 1597-6343
Published by Faculty of Science, Kaduna State University

Full Length Research Article

A DISCRETE-TIME MATHEMATICAL MODEL FOR THE CONTROL OF WEEDS POPULATION DENSITY TOWARDS IMPROVING CROP YIELDS

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ABSTRACT

In this paper, a mathematical model for the control of single weed species population density is proposed. The model's steady-state solutions were obtained and analysed for local and global stabilities. The analysis reveals that the model is locally asymptotically stable and as well globally stable. Graphical simulations were carried out to support the analytic analysis of the model for the global stability and concludes that, weed proliferation may be controlled if the control strategy is target at the recruitment factors. Base on this finding, it is recommended that for effective control, weeds management tactics should be targeted at the recruitment stage rather than the usual practice of controlling mature weed through the application of herbicides. Hence, application of the results of this work may reduce or eradicate the weeds density and improve crop yield at its optimum capacity for sustainable food production.

Keywords: Steady-state, local stability, global stability, weeds density control, recruitment stage.

INTRODUCTION

Weed is a term generally applied to any uncultivated plant species that grows and proliferate naturally in a place it is not wanted. Weeds exist only in natural environments that have been disturbed by human's activities such as agricultural lands, recreational parks, and irrigation dams (Akobundu, 1987 in Nasir, *et al*, 2016). Weeds have had a detrimental effect on crop yield since the beginning of agriculture. Out of all the harmful pests (weeds, insects, pathogens etc) that impair crop performance, weeds have the biggest potential to reduce crop yield, and are the most common and difficult to control (Hartzler, 2000).

Farmers and weed scientist who involved in managing weeds have long recognized how weeds can harm crop growth and productivity by competing for light, nutrients and space as well as hampering harvesting operations, reducing quality of the harvested crop and human activities (Nasir, 2014). Weeds however form an important part of the land ecosystem, contributing food and cover for animals and birds which are an important indicator of biodiversity health (Parsons *et al,* 2009).

Managing crop pests such as weeds to limit both crop production loss and environmental impacts is a major challenge of agriculture (Nathanie, 2010) and one of the key elements of most agricultural system.

The weed management strategies attempt to limit the deleterious effects of weeds growing with crop plants and the development of such weeds management system strategies requires thorough qualitative in-sight in the behaviour of weeds in agro-ecosystems and their effects. These effects can be quite variable and involves understanding the dynamic of weeds population (Nasir *et al*, 2016).

Population dynamics involve the study of population growth, composition and spatial dispersion. The objectives are to identify the causes of numerical change in population and to explain how this cause act and interact to produce the observed pattern. The most common method currently employed to manage weeds is the use of herbicides. In many developing countries, control of weed which was mainly based on manual weeding has been shifted to frequent and systematic applications of herbicides. The use and application of herbicides was one of the main factors enabling intensification of agriculture in the past decades (Kropff 1993). Besides, nonchemical methods, such as hand weeding, sanding, flooding, and proper fertilization, remain integral for managing weed populations; new tactics such as flame cultivation, priority ratings have been developed to aid in weed management planning. Despite many efforts, biological control of weeds remains elusive on the commercial scale. However, evaluation of new herbicides, precision agriculture technology, investigation of other management practices for weeds and their natural enemies among others are research areas whose results will translate into new use recommendations for the weed control (Sander, 2018).

As a consequence, there is an increasing need for improved strategies in weed control. An important element of such management strategies is the development of population models that are capable of predicting the results of control measures on weed densities.

The aim of this research work is to introduce a control parameter into the discrete deterministic homogeneous model proposed by Nasir *et al* (2015) that studied the weed population dynamics, then obtain its steady states densities and analyse them for local and global stabilities for the purpose of controlling weeds proliferation to improve crop yield.

MATERIALS AND METHODS

Baseline for studies of population dynamics of weeds as with most plant species are usually on analysis of single species in defined habitats and often experimentally manipulated.

In this paper, the researchers considered the work of Nasir et al (2015) in which they proposed a Discrete-Time mathematical model to described the proliferation of homogeneous population density dynamics of single weed species as given in (1).

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$$N_{t+1} = \frac{\beta N_t}{1 + aN_t} + \gamma N_t \quad (1)$$

This is a non-linear difference equation for the density of mature weeds. Where

N_t = density of established weeds in year t

N_{t+1} = density of established weeds in year *t*+1

β = weed recruitment factor (i.e fraction of seeds that germinate, become mature and produce seeds,) $\beta > 0$

a = Crowding coefficient (equivalent to the intra-specific competitions)

γ = the density independent fraction of N_t surviving in the seed bank to the next season.

$\frac{\beta}{1 + aN_t}$ represents the density-dependent net recruitment rate from generation to generation. Therefore, Equation (1) gives homogenous model for a single weed proliferation with no control (Nasir *et al,* 2015).

In this work, a control practice to reduce the population density of established weeds was considered by introducing a control parameter δ into equation (1). Hence, it then becomes;

$$N_{t+1} = \frac{(\beta - \delta) N_t}{1 + aN_t} + \gamma N_t \quad (2)$$

Where $(\beta - \delta)$ is a control strategy, such that the control measure δ acts on the recruitment factor β of the density-dependent portion (i.e post emergency application) and applied in the year t. It is assumed that, the established weed densities in the year of control are reduced by a factor of δ prior to the seed production. So, $0 \le \delta \le 1$. Hence, $\delta = 0$ means no control, while $\delta = 1$ implies complete eradication of the weeds. Therefore, equation (2) gives the homogeneous model of a single weed proliferation where a control is applied every year.

Analysis of the Model

In this section, analysis of the homogeneous weed proliferation and control models (2) for single species was carried out. The existence and stability of the associated steady- states (fixed-point) solutions are determined and interpreted biologically.

The first step in understanding the dynamics of model population is to determine the steady-State solutions and their stabilities/equilibrium (Cushing & Yicang, 1994). That is, usually the first step to take in order to study the dynamics of any system is to find its steady- state solutions. So, the steady-state solutions of the models are obtained as follows

Steady-state solutions of the control model

A point is assumed to be a solution of the steady-state of the model equations only if all of its components are non-negative for biological and ecological significance

To solve for the steady-states of (2), it is assumed that

$N_{t+1} = N_t$, implies that $\Delta N = N_{t+1} - N_t = 0$

So, we let $N_{t+1} = N_t = \bar{N}$ (3)

There are two nonnegative solutions of the steady-states for the single species weed model (2). So applying assumption (3), the steady-state of (2) satisfies the equation

$$\bar{N} = \frac{(\beta - \delta)\bar{N}}{1 + a\bar{N}} + \gamma \bar{N} \quad (4)$$

From equation (4) we have

$$\bar{N}\left(1 - \frac{(\beta - \delta)}{1 + a\bar{N}} - \gamma\right) = 0 \quad (5)$$

The zero steady state solution is $\bar{N} = 0$. That is when $E_1 = 0$. Then, non-zero steady-state is obtained from (5) as

$$\bar{N} = \frac{(\beta - \delta) - (1 - \gamma)}{a(1 - \gamma)} \quad (6)$$

So, the second fixed point E_2 as given in (6) exists and positive provided $\gamma < 1$ and $\beta > \delta + (1 - \gamma)$. Hence, the two non-negative steady-states are $E_1(0)$ and $E_2\left(\frac{(\beta - \delta) - (1 - \gamma)}{a(1 - \gamma)}\right)$

Local Stability Analysis of the steady-state solutions

We obtained the derivative of RHS of equation (4)

$$f'(\bar{N}) = \frac{\beta - \delta}{(1 + a\bar{N})^2} + \gamma \quad (7)$$

Evaluating (7) at $\bar{N} = 0$ gives

$f'(0) = \beta - \delta + \gamma$

E₁(0) is stable if $\beta - \delta + \gamma < 1$, So if this happens, the weed density would approach zero and it goes extinct. Otherwise, it is unstable, and then there exists a unique positive value.

Evaluating (7) at $\bar{N} = \frac{(\beta - \delta) - (1 - \gamma)}{a(1 - \gamma)}$ after simplification gives

$$f'(\bar{N}) = \frac{(1 - \gamma)^2}{\beta - \delta} + \gamma \quad (8)$$

E_2 is stable if $\frac{(1 - \gamma)^2}{\beta - \delta} + \gamma < 1$

It implies $\frac{1 - \gamma}{\beta - \delta} < 1$ (9)

Proposition 1

If $\beta + \gamma - \delta > 1$, then the non-zero steady-state E_2 is locally stable, otherwise it is not stable.

Proof

Stability theorem for discrete one-dimensional population models is adopted (Luis and Rodrigues, 2016).

Theorem 1: If $f(x)$ is differentiable at $\bar{x}$ then, a population model is locally stable if $|f^{/}(\bar{x})| \le 1$. It is asymptotically stable if $|f^{/}(\bar{x})| < 1$. Here $\bar{x}$ is the unique equilibrium point of function $X_{t+1} = f(x_t)$. (Paul, 2007).
Now suppose $E_1(0)$ is stable. It implies that $\beta + \gamma - \delta < 1$.
For E_2 to be stable, equation (9) must hold using T**heorem 2.1,** that is

$$\left|\frac{1-\gamma}{\beta-\delta}\right| < 1$$

Implies, $1-(\beta-\delta) < \gamma < 1+(\beta-\delta)$
then $1 < \beta+\gamma-\delta < 1+2(\beta-\delta)$
This completes the proof. Hence, the non-zero steady-state E_2 is locally stable. While $E_1(0)$ is unstable.

Global Stability of the steady-state $E_2(N)$
It is important to know whether or not a model is globally stable. Models having this property are predictable, while those that do not can exhibit unexpected behaviour (Heinschel, 1994). One of the tools used to prove global stability in difference equations is the Schwarzian derivative, which was first introduced into the study of one-dimensional dynamical system by David Singer (Heinschel, 1994; Eduarodo, 2007).
The Schwarzian derivative (**S**) of ***f*** at a point ***x*** is given by

$$\boldsymbol{S}(f,x) = \frac{f^{///}(x)}{f^{/}(x)} - \frac{3}{2}\left(\frac{f^{//}(x)}{f^{/}(x)}\right)^2$$

For any real valued function f with at least three continuous derivatives, wherever $f^{/}(x) \ne 0$. The following proposition can be deduced from Singer's results (Heinschel, 1994, Eduardo 2007);

Theorem 2:
Suppose f is C^3 and has at most one critical point $\bar{x}$(maximum). If $|f^{/}(\bar{x})| \le 1$ and $\boldsymbol{S}(f,x) < 0$ for all $x \ne \bar{x}$ then $\bar{x}$ is globally stable.

Calculation of the Schwarzian for (4)

$$f'(\bar{N}) = \frac{\beta-\delta}{(1+a\bar{N})^2} + \gamma\ ,$$

$$f''(\bar{N}) = \frac{-2a(\beta-\delta)}{(1+a\bar{N})^3} \qquad \text{and}$$

$$f'''(\bar{N}) = \frac{6a^2(\beta-\delta)}{(1+a\bar{N})^4}$$

$$\mathbf{S}(f,\bar{N}) = \frac{6a^2(\beta-\delta)}{(1+a\bar{N})^2[(\beta-\delta)+\gamma(1+a\bar{N})^2]} - \frac{3}{2}\left[\frac{-2a(\beta-\delta)}{(1+a\bar{N})[(\beta-\delta)+\gamma(1+a\bar{N})^2]}\right]^2$$

$$= \frac{6a^2(\beta-\delta)[(\beta-\delta)+\gamma(1+a\bar{N})^2] - 6a^2(\beta-\delta)^2}{\left\{(1+a\bar{N})[(\beta-\delta)+\gamma(1+a\bar{N})^2]\right\}^2}$$

$$= \frac{6a^2(\beta-\delta)\gamma(1+a\bar{N})^2}{\left\{(1+a\bar{N})[(\beta-\delta)+\gamma(1+a\bar{N})^2]\right\}^2}$$

This gives

$$\mathbf{S}(f,\bar{N}) = \frac{6a^2\gamma(\beta-\delta)}{[(\beta-\delta)+\gamma(1+a\bar{N})^2]^2} \qquad (10)$$

We notice that, if $\beta < \delta$ then $\mathbf{S}(f,\bar{N}) < 0$ everywhere. Hence, non-zero steady-state E_2 is globally stable and the model may exhibit some predictable behaviours. So, the weed proliferation may be controlled or eradicated.

Graphical Profile of the Global Stability for Weed Density Dynamics
In this section, we give the graphical profile to support our theoretical analysis of the global stability of our model equation via the software package Mathematica 5.2. In all the figures, we fixed all parameters $\beta = 0.6$, $b = 20$, and $\gamma = 0.5$ except δ and N so that we can investigate the effect of control δ on the population of weed and be able to plot the Schwarzian in 3-dimensions.

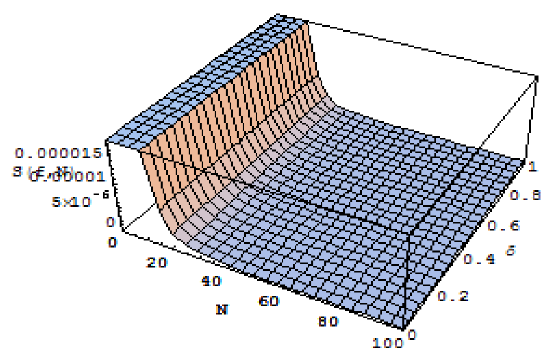

Figure. 1: Schwarzian of model (1) with no control

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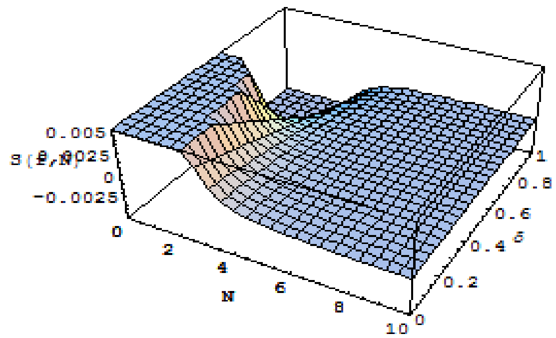
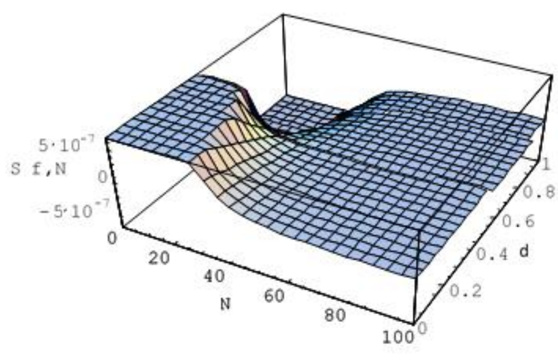


(a)



(b)

Figure. 2: Schwarzian of model (2) with control

Interpretation

The trough-like structures of each of the Schwarzian graphs (Figure 2a and b) shows the region where the Schwarzian is negative for N = 10, 100 and $\delta = 1$, which indicate $S(f, N) < 0$ everywhere, provided $\beta < \delta$. Figure. 1 shows that Schwarzian is positive everywhere since its graph has no trough. So, we conclude that the weed proliferation may be controlled or eradicated if the control is targeted at the recruitment factor.

Conclusion

The steady-state solutions of the control model equation (2) for single weed species density was meticulously analysed for local and global stabilities. The analyses show that the model is locally asymptotically stable and is globally stable, which is as a result of the introduction of a control parameter. Also, based on the results of the analysis in this paper, weed proliferation may be controlled if the control strategy is targeted at the recruitment factors.

Base on the outcome of the analyses, it is therefore recommended that weeds scientists' and farmers' efforts on weed management should be targeted at the recruitment stage rather than the usual practice of controlling mature weed through the application of herbicides. On the other hand, application of effective control measure to eradicate the weeds species would improve crop yield at its optimum capacity for sustainable food production.

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