International Journal of Science and Technology
(STECH)
Bahir Dar- Ethiopia
Vol. 6 (1), S/No13, February, 2017: 74-87
ISSN: 2225-8590 (Print) ISSN 2227-5452 (Online)
DOI: http://dx.doi.org/10.4314/stech.v6i1.6

Analytic Determination of Values on the Truth Table: Possible Implications

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Abstract

The paper refers to the traditional approach used in the specification of values on the statement constant columns of the truth table as pictorial. It differentiates it from the α_y^x coordinate model, which it introduces as an analytic approach for the achievement of the same objective. The coordinate is stated as: $\alpha_y^x = \frac{x}{2\left(\frac{2^n}{2^y}\right)}$. The α_y^x is shown by analysis to derive from the coordinate of x and y as $\alpha_y^x = \frac{x}{y}$. y is shown to derive from the coefficient of C_y and $C_y = \frac{2^n}{2^y}(T,F)^{2^y/2}$. C_y is an algebraic function introduced in the paper for the computation of truth table columns. The paper shows by implication that the delineation of the value of any truth table unit no longer stands in need of pictorial aids due to the introduction of the coordinate. Results from analysis shows that the specific value of a truth table unit could be determined set theoretically, without an actual truth table provided the α_y^x coordinate is applied.

Key Words: Algebraic Function, Coordinate, Columns, Rows, Truth Table, Truth Value

Introduction

Irving Copi defines logic as the science of reasoning. In doing so, Copi distinguishes between logic and psychology. Reasoning is an aspect of thinking, which is the domain of psychology. But logic is not a branch of psychology, because in studying reasoning, logic is not concerned with the latter as thinking (Copi xiii). Logic studies the relation of propositions in order to ascertain the relation of entailment, which ultimately leads to the relation of truth.

The above understanding of logic is consistent with Bertrand Russell's definition of pure mathematics as a class of propositions of the form: "$p \supset q$" (Russell 3). Considering that the *Principles of Mathematics* (1992) was written by Russell to demonstrate that pure mathematics is identical with logic, it could be submitted that logic is a science that studies entailment among propositions. Russell's conception of logic agrees with Aquinas' idea of paradigmatic truth as scientia, which is syllogistic (McDonald 19).

Russell's definition is also a correct representation of Aristotle's structure for logic as syllogistic. Although logic for Aristotle was syllogism, further developments in the discipline have shown that syllogistic logic presupposes the ultimate meaning of the notion of proposition. A proposition is any sentence that is capable of taking a truth value (Efemini 92). Although entailment is a relation among propositions, the essence of studying entailment is not to discover propositions that generally relate with one another, but to determine the relation of truth among them. Propositions that are found to be entailed by others are dependent on such subalterns for their truth values (Hausman 334).

The above argument is equally valid in syllogistic logic. The square of opposition, which is the culminating aspect of that logic, is a diagrammed demonstration of the relation of truth in syllogistic logic. Notice that:

1. The Universal Affirmative and the Particular Negative propositions are contraries if they contain the same subject and predicate terms.
2. The Universal Affirmative and Particular Negative propositions are contradictories if they contain the same subject and predicate terms.
3. The Universal Negative and the Particular Affirmative propositions are contradictories if they contain the same subject and predicate terms (Moore 256).

The concepts of contraries and contradictories derive their meaning from the notion of contradiction. A contradiction occurs where the same proposition is as a necessity assigned two incompatible truth values. Hence, "A and not – A" is a contradiction and it is not true (Layman 313). Truth functional logic is not the ultimate logic, but it is central to all logical analysis. The logic of truth function is the propositional logic.

The understanding of logic as shown above implies its definition as the science that studies the truth functional relations of propositions. This definition was championed by Jan Lukasiewicz, who understood truth extensions as the subject matter of logic. Lukasiewicz argued in his modal logic that the proposition $\varphi \supset \psi$ means extensionally $\Box\varphi \supset \Box\psi$ and $\Diamond\varphi \supset \Diamond\psi$, where $\Box$ and $\Diamond$ mean 'necessity and possibility', respectively. The concern of logic in this sense is to study actual or possible truth values of propositions (Font 2).

A system of logic that is dedicated to the analysis of truth functions of compound propositions is the propositional logic. A key component of propositional logic is the relation of compound propositions. Compound propositions are in turn defined in terms of the truth value of simple propositions. The truth functional implication of the definition of compound propositions by simple or simpler propositions is expressed on the truth table.

The Truth Table

The truth table is a logical tool for the analysis of truth functions (Goodstein 2). The concept of the truth table as an instrument for propositional analysis is due to Charles Sanders Peirce and was perfected by Ludwig Wittgenstein and Bertrand Russell (Anellis 1). Given that the truth table is an instrument for the analysis of compound statement, it follows that there is no truth table for simple statements. The compound statement 'John is married but Joseph is celibate' has two components (i.e., two simple statements), namely: 1. John is married. 2. Joseph is celibate. The two statements form a conjunction by virtue of the connective 'and', in this case the connective is represented by 'but'. As a conjunction, the compound statement is defined in terms of its truth function. Hence, the statement is true if both of its component conjuncts are true, otherwise it is false (Langer 304). Consequently, a conjunction will fail to be true if one or both of the conjuncts are false (Barwise 71). These possibilities could be expressed on the truth table.

Without any prejudice against the validity of many valued logic, the relation of truth value has shown itself to be bivalent in all modes of analysis, including fuzzy systems (Mamadu 96). Even the concept of 'Tertium' has not assumed any definite truth value and therefore could not be termed a third definite value. Those who repudiate the principle of excluded middle confuse the concept of 'being true in general' with that of 'actual particular truths'. A deeper analysis of the distinctions between these concepts is beyond the scope of the paper. Meanwhile, it is noteworthy that the following gradation of truth values did not provide an alternative to bivalence:

1. Very True

2. True
3. Not Very True
4. Undetermined
5. Not Very False
6. False
7. Very False

The above so called many valued truth value expressions or fuzzy values actually show a gradation of only two truth values (i.e., true and false). So, it would not be out of place to refer to the truth table as a matrix expression of all possible combinations of True and False values under any compound proposition. The possibilities under the conjunction above has traditionally been shown to be determinable by the formula 2^n(where *n* is the total number of dissimilar simple statements in the proposition and *2* is the abstract coefficient of all possible truth values) (Carnap 11). The abstract coefficient of all possible truth values is therefore bivalent or two.

After 2^n has been determined, the actual combination of truth values stands in need of a diagram called the truth table. Let the conjunction of 'John is married' and 'Joseph is celibate' be symbolized as: M • C. The combination of truth values under the compound statement could follow as expressed on either Table 1 or Table 2 below.

Table 1: Showing all Possible Combinations of Truth Values in Statement Constant Columns

	COLUMN ONE (1)	COLUMN TWO (2)
1.	T	T
2.	T	F
3.	F	T
4.	F	F

Table 2: Showing all Possible Combinations of Truth Value in Statement Constant Columns

	COLUMN ONE (1)	COLUMN TWO (2)
1.	F	F
2.	F	T
3.	T	F
4.	T	T

Without the aid of the above diagram, it remains absolutely cumbersome to determine the arrangement of values on the truth table. Let us imagine a set of propositions S that forms a compound proposition P. Let the total number of dissimilar simple statements in S be n. If it is such that: $n = 2000$, what would be the structure of values' combinations on *P's* truth table? This problem could only be adequately resolved by constructing a truth table like the ones on Tables 1 and 2 above.

There is a pattern that emerges on Tables 1 and 2 that is worthy of consideration. Let the first type of value in order of arrangement in both columns 1 and 2 of either of the two tables be referred to as v_1 and the second type of value in the same columns be v_2. These interpretations would follow: On Table 1: $v_1 = T$ but on Table 2: $v_1 = F$. On Table 1: $v_2 = F$ but on Table 2: $v_2 = T$. But whether it is on Table 1 or on Table 2, what emerges is that v_1 is followed by v_2 and both form the same pattern of truth values on the two tables. This pattern is expressed on Table 3 as follows:

Table 3: Showing Abstract Pattern of Values on a Two Statement Constant Truth Table Columns

	COLUMN ONE (1)	COLUMN TWO (2)
1.	v_1	v_1
2.	v_1	v_2
3.	v_2	v_1
4.	v_2	v_2

If we return to our question above, it could be asked what the v_1 and v_2 pattern would be for 2^n $(n = 2000)$. The diagram for this could be very cumbersome. Even more herculean is the answer to the following question: Assuming the above pattern as a given, how can we without an actual truth table determine what the truth value of row *1000* and column *1780* would be in a *2000* components compound proposition? The problem associated with this kind of question is that, if one were to construct a truth table for a *2000* statement constant proposition, it would take a whole day to achieve it. Many electronic systems place a limit on the total number of values admissible for construction. But logic places no limit on how long a problematic proposition could be.

Another very difficult question that faces a logic teacher in a logic class is how to justify to students the $v_1$and v_2 pattern for all truth tables as on Table 3 above. The second question would be answered first. The first question would be answered on the basis of the answer to the second question.

Analytic Determination of the Nature of Values on the Truth Table

Introducing the Algebraic Function for Truth Values Permutation

We could evolve two formulas that could aid in the combination of truth values on the statement constant columns of the truth table. The first formula is in response to the second problem, which is the justification to students, the grounds for the $v_1$and v_2

pattern assumed by truth values on truth tables. The solution therefore has a didactic value. Let $C_y = \frac{2^n}{2^y}(T,F)^{2^y/2}$ be a formula for the permutation of values on the statement constant columns of the truth table. With the aid of the above formula it could be shown that all the statement constant columns of any number of components of any compound statement whatever is automatically computed. But there is a requirement prior to the application of the formula to any proposition (i.e., the user of the formula must first determine the specific number of dissimilar simple statement constants in the proposition). Once that is done, the interpretation of the formula is as follows:

i. y means the unique column number or a unique number assigned to a particular component statement in the compound proposition, as a way of numbering. If a column number is used, then $C_y = C_1 \;\; or\, C_2\, or\, C_3\, etc.$ Note that the statement numbers and the column numbers are identical. Once a column has been assigned a unique number that number must be used consistently throughout the analysis. The assignment of column numbers must be serial from the natural number *1* upward. If $C_y = C_3$, then $C_3 = \frac{2^n}{2^3}(T,F)^{2^3/2}$. All other expressions of the equation must follow the pattern shown here.
ii. n is a symbol that represents the total number of dissimilar statement constants in the compound proposition. In the conjunction above, the total number of dissimilar statement constant is *2*. Hence, $n = 2$. Let $[(P \supset Q) \bullet (Q \supset R)] \supset [P \supset (Q \bullet R)]$ be a compound statement. Even though the argument places occupied by the components of the proposition are seven, $n = 3$, because there are only three dissimilar statement constants (P, Q and R) in the proposition.
iii. T, F stands for the truth values, True and False, respectively.

We would allow all the laws of algebra to apply to our equation. Hence, given a compound proposition, $(P \supset Q) \supset R$, the following analysis applies.1. $n = 3$; 2. $C_y = C_1$, C_2 and C_3.

1. $C_y = \frac{2^n}{2^y}(T,F)^{2^y/2}$.
2. $C_1 = \frac{2^3}{2^1}(T,F)^{2^1/2}$
3. $C_1 = \frac{8}{2}(T,F)^{2/2}$
4. $C_1 = 4(T,F)$
5. $C_1 = 4T'^s$ and $4F'^s$ ……………………………………………… (1)

Hence, column one would contain four T's and four F's spread vertically. Note that by the arithmetic principle of commutation, the equation could be expressed conversely as follows:

1. $C_1 = \frac{2^3}{2^1}(F,T)^{2^1/2}$
2. $C_1 = \frac{8}{2}(F,T)^{2/2}$
3. $C_1 = 4(F,T)$
4. $C_1 \ = 4F'^s$ and $4T'^s$ ……………………………...………… (2)

Hence, C_1 would contain four F's and four T's in just that order. But we would prefer the previous order of *(T, F)* as in equation (1) above for all subsequent analysis in the paper. Consequently, we would adopt C_1 = *4(T, F)* instead of $C_1 = 4(F,T)$.

1. $C_2 = \frac{2^3}{2^2}(T,F)^{2^2/2}$
2. $C_2 = \frac{8}{4}(T,F)^{4/2}$
3. $C_2 = 2(T,F)^2$
4. $C_2 \ = 2T'^s$ and $2F'^s$ into two places …………………………… (3)

Hence, C_1 would contain two T's and two F's expressed twice just in that order within the column.

1. $C_3 = \frac{2^3}{2^3}(T,F)^{2^3/2}$
2. $C_3 = \frac{8}{8}(T,F)^{8/2}$
3. $C_3 = (T,F)^4$
4. $C_3 \ = T$ and F expressed together in that order, four times within the column ………………………………………………………………(4)

Now, if the results of C_1 *to* C_3 were translated into separate columns, the following truth table diagram would emerge.

Table 4: Showing the Arrangements of Values as Expressed in Equation (1) Above

	C_1	
1.	T	
2.	T	
3.	T	*4T's*
4.	T	
5.	F	
6.	F	*4F's*
7.	F	
8.	F	

Table 5: Showing Arrangements of Values as Expressed in Equation (3) Above

	C_2	
1.	T	*2T's*
2.	T	
3.	F	*2F's*
4.	F	
5.	T	*2T's*
6.	T	
7.	F	*2F's*
8.	F	

Table 6: Showing Arrangements of Values as Expressed in Equation (4) Above

	C_3	
1.	T	*One T One F*
2.	F	
3.	T	*One T One F*
4.	F	
5.	T	*One T One F*
6.	F	
7.	T	*One T One F*
8.	F	

If Tables 4, 5 and 6 were brought together as columns 1, 2 and 3 of a truth table with eight rows and three columns, then the conditions for the statement constant columns for our proposition $(P \supset Q) \supset R$ would have been satisfied as shown below on Table 7.

Table 7: Showing the Statement Constants Columns of $(P \supset Q) \supset R$

	C_1	C_2	C_3
1.	T	T	T
2.	T	T	F
3.	T	F	T
4.	T	F	F
5.	F	T	T
6.	F	T	F
7.	F	F	T
8.	F	F	F

Note that any of P, Q and R could be assigned to any of the columns. But once that assignment is made, the decision must be maintained throughout the process.

Introduction of a Coordinate Model for Truth Value Determination on Truth Table Columns

It could also be demonstrated how all the values on Table 3 above, could be computed on the basis of an algebraic function, which with the aid of five rules coordinates the serial numbers of rows and columns to independently determine the value of each unit of the statement constants columns of the truth table.

Let (M • C) be a compound statement (conjunction). Let a virtual truth table of (M • C) contain at least one row and one column. The definite truth value of row one on column one of (M • C) is determined by the algebraic function α_y^x and a set of five rules, (where, $\alpha_y^x = \frac{x}{y}$). The value of y has already been given in $C_y = \frac{2^n}{2^y}(T,F)^{2^y/2}$. If the coefficient of C_y is abstracted and substituted for y in $\alpha_y^x = \frac{x}{y}$, then $y = 2(\frac{2^n}{2^y})$, where *2* is the sum of the coefficient of $T = 1$ and $F = 1$ and $\frac{2^n}{2^y}$ is the coefficient of the entire function. Hence, if $2(\frac{2^n}{2^y})$ is substituted for y in $\alpha_y^x = \frac{x}{y}$ then the coordinate could be stated as follows: $\alpha_y^x = \frac{x}{2(\frac{2^n}{2^y})}$. With the aid of the α_y^x coordinate one could determine the unique value of any statement constant column of any compound proposition without a truth table diagram.

As earlier stated, the coordinate only operates under certain conditions called rules to determine the value of a truth table statement constant unit. The following five rules are the conditions with which the coordinate functions:

1. **RULE 1:** *IF* $\alpha_y^x = \frac{x}{y}$ *AND* $x <= (\frac{y}{2})$ *THEN*, $\alpha_y^x = v_1$ (where v_1 is any type of truth value (i.e., either *T* or *F*) that is first entered on the first column of the truth table).

   INTERPRETATION: If the result of the α_y^x coordinate is a proper fraction (i.e., the numerator is less than the denominator), and the fraction is such that when the denominator is divided by *2*, then the numerator is consequently either less than or equal to the quotient, then the value of the unit concerned is v_1; v_1 is already defined.

2. **RULE 2:** *IF* $\alpha_y^x = \varphi\frac{x}{y}$ *AND* $x <= (\frac{y}{2})$ *THEN*, $\alpha_y^x = v_1$(where v_1 is any type of truth value (i.e., either *T* or *F*) that is first entered on the first column of the truth table).

   INTERPRETATION: If the result of the α_y^x coordinate is a mixed number (i.e., a whole number and a proper fraction) and the fractional part of it is such that when the denominator is divided by *2*, then the numerator is either less than or equal to the quotient, then the value of the unit concerned is v_1; v_1 is already defined.

3. **RULE 3:** *IF* $\alpha_y^x = \frac{x}{y}$ *AND* $x > (\frac{y}{2})$ but $x < y$ *THEN*, $\alpha_y^x = v_2$ (where v_2 is any type of truth value (i.e., either *T* or *F*), other than the first type of truth value that is entered on the first column of the truth table).

   INTERPREATION: If the result of the α_y^x coordinate is a proper fraction (i.e., the numerator is less than the denominator), and the fraction is such that when the denominator is divided by 2, the numerator is consequently greater than the quotient, then the value of the unit concerned is v_2; v_2 is already defined.

4. **RULE 4:** *IF* $\alpha_y^x = \varphi\frac{x}{y}$ *AND* $x > (\frac{y}{2})$ but $x < y$ *THEN*, $\alpha_y^x = v_2$ (where v_2 is any type of truth value (i.e., either *T* or *F*), other than the first type of truth value that is entered on the first column of the truth table).

   INTERPRETATION: If the result of the α_y^x coordinate is a mixed number (i.e., a whole number and a proper fraction) and the fractional part of it is such that when the denominator is divided by *2*, then the numerator is greater than the quotient, then the value of the unit concerned is v_2; v_2 is already defined.

5. **RULE 5:** *IF* $\alpha_y^x = n$ (where n is any natural number) *THEN*, $\alpha_y^x = v_2$(where v_2 is any type of truth value (i.e., either *T* or *F*), other than the first type of truth value that is entered on the first column of the truth table).

   INTERPREATION: If the result of the α_y^x coordinate is a natural number, then the value of the unit concerned is v_2; v_2 is already defined.

Let the conjunction in Section 2.0 above be written as *M* • *C*. It could be shown by the α_y^x coordinate that the result of Row one on Column one or Row-1: Column-1 automatically determines the total number of rows contained on the truth table concerned or the total number of x to be computed for each y. Hence:

$$\alpha_1^1 = \frac{1}{the\ total\ number\ of\ rows\ on\ the\ truth\ table\ or\ the\ total\ number\ of\ x\ for\ each\ y}.$$

This result has to be determined prior to the application of Rules one to five to delineate the values on the table. It is the α_1^1 result that will determine the total number of rows on the truth table. The application of the rules will then show that α_1^1 is necessarily v_1. The conjunction *M* • *C* contains two statement constants *M* and *C*. Hence, *n = 2*; $\alpha_y^x = \frac{x}{2(\frac{2^n}{2^y})}$; $v_1 = T$ and $v_2 = F$; the total number of rows is the denominator of α_1^1. The computation of columns one and two is as follows: $\alpha_1^1 = \frac{1}{2(\frac{2^2}{2^1})} = \frac{1}{4}$. α_1^1 is therefore just one out of the four rows on the truth table.

i. COLUMN ONE (C_1)

1. $\alpha_1^1 = \frac{1}{2(\frac{2^2}{2^1})} = \frac{1}{4}\left(x = 1, y = 4; \frac{y}{2} = \frac{4}{2} = 2; x <= \frac{y}{2}, R1\right) = v_1 \dots\dots\dots\dots (T)$.
2. $\alpha_1^2 = \frac{2}{2(\frac{2^2}{2^1})} = \frac{2}{4} = \frac{1}{2}\left(x = 1, y = 2; \frac{y}{2} = \frac{2}{2} = 1; x <= \frac{y}{2}, R1\right) = v_1 \dots (T)$.
3. $\alpha_1^3 = \frac{3}{2(\frac{2^2}{2^1})} = \frac{3}{4}\left(x = 3, y = 4; \frac{y}{2} = \frac{4}{2} = 2; x > \frac{y}{2}, R3\right) = v_2 \dots\dots\dots\dots (F)$.
4. $\alpha_1^4 = \frac{4}{2(\frac{2^2}{2^1})} = \frac{4}{4} = 1\left(\alpha_1^4 = 1; \alpha_y^x = n, R5\right) = v_2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (F)$.

ii. COLUMN TWO (C_2)

1. $\alpha_2^1 = \frac{1}{2(\frac{2^2}{2^2})} = \frac{1}{2}\left(x = 1, y = 2; \frac{y}{2} = \frac{2}{2} = 1; x <= \frac{y}{2}, R1\right) = v_1 \dots\dots\dots (T)$.
2. $\alpha_2^2 = \frac{2}{2(\frac{2^2}{2^2})} = \frac{2}{2} = 1\left(\alpha_2^2 = 1; \alpha_y^x = n, R5\right) = v_2 \dots\dots\dots\dots\dots\dots\dots (F)$.
3. $\alpha_2^3 = \frac{3}{2(\frac{2^2}{2^2})} = \frac{3}{2} = 1\frac{1}{2}\left(x = 1, y = 2; \frac{y}{2} = \frac{2}{2} = 1; x <= \frac{y}{2}, R2\right) = v_1 \dots (T)$.

4. $\alpha_2^4 = \frac{4}{2(\frac{2^2}{2^2})} = \frac{4}{2} = 2\left(\alpha_2^4 = 2; \alpha_y^x = n, R5\right) = v_2$ ... ... (F).

The order of the truth values in the analysis is identical with that of the truth table on Table 6 below.

Table 6: Showing the Diagram of a Two Statement Constants Truth Table Columns

	$\boldsymbol{C_1}$	$\boldsymbol{C_2}$
1.	T	T
2.	T	F
3.	F	T
4.	F	F

Analytic Determination of Values on the Truth Table: Possible Implications

The truth table diagram has a value that renders it necessary. As a pictorial representation of truth values, it facilitates the determination of truth value possibilities available to a compound statement. Without the truth table, it would be traditionally difficult to ascertain the order of truth values for the correct analysis of a compound statement. But that necessity could be eliminated by the development of an approach that assumes all the processes of truth value analysis without a table. Hence, the truth table could be understood as a virtual table. The α_y^x coordinate may as well have rendered diagrams of truth tables irrelevant because it is now possible to determine the unique relation of one unit of a column to another without reference to the truth table diagram. This may be one of the possible implications of the coordinate. This implication is indicated for all Boolean type algebra. But it is noteworthy that elegant as the algebra appears, the foundation of the binary numbers used in its system is questionable.

Let us consider a set theoretic approach to truth value analysis for any proposition whatsoever. Let *P* be a compound proposition with two dissimilar statement constants *M* and *C*. Let the compound proposition *P* be a conjunction. The relation between *M* and *C* is a binary relation expressed by the sign *'•'* as follows: *M* • *C*. *P* is a set with *M* and *C* as subsets. Columns one and two above could be shown to be the elements of *M* and C as follows: *M* = $(\alpha_1^1, \alpha_1^2, \alpha_1^3, \alpha_1^4)$ and *C* = $(\alpha_2^1, \alpha_2^2, \alpha_3^3, \alpha_4^4)$. *M* and *C* are properly ordered sets with a definite ordering by their superscripts. The subscripts are reference or domain numbers for *M* and *C* respectively. But if the subscripts were replaced by m and c, respectively, then the sets *M* and *C* would remain ordered as follows: *M* =

$(\alpha_m^1, \alpha_m^2, \alpha_m^3, \alpha_m^4)$ and $C = (\alpha_c^1, \alpha_c^2, \alpha_c^3, \alpha_c^4)$. Each of α_y^x has already been shown to be properly determined or defined by means of the coordinate of its subscript and superscript numerals. Hence, *M = (T, T, F, F),* the truth values follow from α_1^1 ... α_1^4 as earlier computed and *C = (T, F, T, F),* the truth values are those of α_2^1 ... α_2^4 as above. Let each element of M and C determined by means of the coordinate of its subscript and superscript numerals be properly specified by subscripts so that each subscript of a defined value corresponds to the superscript of its corresponding coordinate as follows: *M* = $(\alpha_1^1, \alpha_1^2, \alpha_1^3, \alpha_1^4)$, defined as (T_1, T_2, F_3, F_4) and *C* = $(\alpha_2^1, \alpha_2^2, \alpha_3^3, \alpha_4^4)$ defined as (T_1, F_2, T_3, F_4). Despite the obvious similarity of the values each value is distinguished from another by means of the subscripts. Hence, although T_1 and T_2 are identical truth values, as members of a well-ordered set, they are distinct on the basis of their position in the series as specified by their subscripts. Based on the above definitions, the set *M* and *C* shall henceforth be stated as the set of definitions of α_y^x for both *M* and *C* and every operation on *M* and *C* shall be an operation on these definitions.

Consequently, $M = (T_1, T_2, F_3, F_4)$ and $C = (T_1, F_2, T_3, F_4)$. Let β be a set of binary operators and τ a set of unary operators. "An operator on a set is binary if it combines or operates on two elements of a set to produce another element of the set"(Anderson 67). Anderson equally defines a unary operator as follows: "An operator on a set is unary if it operates on one element of a set to produce another element of the set" (Anderson 67). The unique set which is referred to by Anderson above is the set: *S = (T, F)* or *(0, 1)* by Boolean algebra or (v_1, v_2) as in this paper or all such binary sets. $\beta =$ (the set of all logical connectives except the negation). τ = (the set of all connectives that operate like the negation). The members of β set relate two and only two elements of *P* in each operation by relating one element of one subset of *P* say *M* to just one element of another subset of *P* say *C*, such that the subscript of the two elements related correspond to one another. The result of the operation of the β relation • on *P* through an operation on its subsets *M* and *C* is the set *R*. *R* = (T, F, F, F). If the unique subscripts of the relation were referenced then *R* = (T_1, F_2, F_3, F_4). The summation of *R = (T, 3F).* All sets that give rise to the type of set *R* are determinable by a Karnaugh map (Anderson 35). *M* • *C* is that type of relation. Hence, $(M \bullet C) = (T, 3F)$.

Conclusion

The above results have shown that the analytic approach to the determination of values on the truth table could be extended to analysis of values by logical connectives. Such analysis had been shown have implication for the necessity of the truth table because from the α_y^x coordinate to the results, it is possible to generate truth value combinations without any mention of the concept of table. By implication therefore, the truth table has become a virtual instrument for truth value analysis.

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