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Linear Programming Approach to Sustainable Management of *Gmelina Arborea* Plantations in a Nigerian Lowland Rainforest

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Abstract

A linear programming (LP) model was used to prescribe timber harvest in the management of even-aged Gmelina arborea plantations in Omo Forest Reserve, Southwestern, Nigeria. The plantations now being managed for timber production are to be exploited within fifteen years based on a 5-year harvesting period. Data was generated from eight compartments which were established between 1991 and 2000, and covering an area of 9489 hectares with a total volume of 9,716,726.0m^3*. Sample plots (20m X 20m) were randomly laid out in each compartment. In each sample plot, tree diameters and heights were measured and tree volumes estimated. The maximization* *problem of wood volume on compartmental level was formulated and solved. The solution of the LP showed that about 12.5m^3 million of wood was maximized. The optimal solution prescribed 3500ha (4,703,426.0m^3) to be cut in period I (2008 –2012); 3000ha (3,400,946.0m^3) is to be cut in period II (2013 – 2017) while 2989ha (4,402,982.0m^3) will be cut in period 2018 – 2022). Sensitivity analysis and opportunity costs were used to highlight the effects of changing some of the model parameters on the prescribed optimal solution.*

Key words: Linear programming, timber harvest, *Gmelina arborea*, Omo forest reserve, optimal solution, sensitivity analysis

Introduction

The tropical rainforest is a mechanism of incredible complexity whose parts work together with humming precision. It is valued for the tremendous varieties of living things (biodiversity), its significance in the ecosystem and the immense potentials in economics, industry, medicine and agriculture (Awake, 2003). The tropical rainforest has been referred to as 'man's pride and heritage'. In spite of the extensive potential, the tropical forests of the world are being destroyed at the rate of 11.3 million hectares a year with Nigeria having its full share of the destruction (Iyamabo, 1990). Osho (1995) and Awake (2003) identified the causal factors to include slash and burn agriculture, population explosion, over dependence of developing countries on wood for cooking and heating fuel, illegal and unsustainable timber extraction by logging industries, widespread poverty, greed and inappropriate government policies.

To meet the challenge of providing high quality wood for the increasing population and rapid industrialization at sustained yield production level while preserving our natural heritage for future generation, studies were carried out and extensive plantation of indigenous species such as *Terminalia* , *Triplochiton* and *Nauclea* have been established. The pace of the study was however comparatively slow and what was produced was limited compared to

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what was needed and time was running out (Iyamabo, 1990). On the other hand, exotic tree species were introduced into Nigeria as indeed other parts of the world. Some of these species proved very successful; examples are *Pinus caribea*, P. *occarpa*, Tectona *grandis* and *Gmelina arborea* in several tropical countries. Vast plantations of these introduced species were to be almost everywhere. Within the last four decades, the federal and state governments of Nigeria in conjunction with the World Bank have heavily invested in plantation establishment of those exotic species in most parts of the country (predominantly *G. arborea* and *T. grandis*) to provide raw materials in the form of poles, timber, veneer, wood particles, pit props, pulp and fuel wood. Omoluabi *et al, (1990)* estimated the areas of established plantations in Nigeria by 1990 to be 213,730 hectares. Of these areas, *G. arborea* plantations cover 83,377 hectares (41.8%) of the total forest plantation.

According to International Tropical Timber Organisation – ITTO (2001), these plantations have answers for more than a few global problems. They reduce deforestation, restore degraded land, ameliorate climate change, improve local livelihood, return good profits, create employment and bolster national economies. However, not all the plantations are in good shape. These plantations have outgrown their planned maximum volume production rotation of 8 years (Akachukwu 1981; Evans, 1992). Furthermore, silvicultural management has been limited; leaving stands untended and the established stocking has been reduced on several occasions. Natural and man-made agencies such as diseases, wind and most noticeable fire have superimposed their effects on the variations caused by sites, climate and management. Global concern for the sustainable management of these plantations has been expressed so as to achieve benefits people expect of them (Evans, 1999; ITTO, 2003)

In order to enhance the successful management of these plantations and guarantee the steady supply of timber from the plantations, viable exploitation rates have to be developed.

Linear programming is a method of allocating limited resources to competing activities in an optimal manner (Buonogiono and Gilles, 1987). According to Leuschener (1984), LP is a common analysis tool in the management and planning of renewable resources, which has gained wide acceptance in forestry, and other fields like warehousing, product mix determination, production scheduling, budget allocation, farm planning and military problems in combat and logistics. As an analytical technique, Lp will help in evaluating existing forest conditions and serve as a tool for short term and long term planning for future sequence of harvests from the forest whether industrial or public.

The main objective of this study therefore is to prescribe an optimal harvesting schedule by compartments for *G. arborea* plantations in Omo Forest reserve, Southwestern Nigeria by the use of Linear Programming (Bell, 1977; Osho, 1995). The main interest lies in the maximization of wood from the established plantation through rational exploitation that will generate constant flow of woods on sustainable basis.

Methods

Omo Forest Reserve (6^0 35' N, 4^0 40' E) lies within the high forest zone in south-western Nigeria. The climate of the area is moist monsoon with the annual rainfall varying from about 1,500mm to 1,800mm, most of it from March to October. Rainfall distribution is bimodal with a marked decline in August. This reserve has a long history of uncontrolled exploitation (Hall, 1977) and is described in the plantation project document as low-value logged over high forest.

Model development

The compartments are to be exploited over a fifteen-year period – a rotation age of *G. arborea* for timber. This means that each compartment is to receive a single cutting over the exploitation period. For simplicity, the planning is to be done on 5-year basis, which means that the schedule will be done for 3 cutting periods of five years each. However, a compartment that is not completely cleared within a cutting period can be extended to another period.

Let C_{ij} represent the amount of wood as a result of cutting a hectare of land from compartment i during year j. Let X_{ij} represent the fraction of compartment i in the year j. Then $C_{ij} X_{ij}$ represents the amount of wood taken from the ith compartment in the jth year. Also $\sum C_{ij} X_{ij}$ represent the total wood taken from all compartments in all the years. The intention is maximize this total wood. Let the objective function be represented by Z

$$Z = \sum \sum C_{ij} X_{ij} \qquad \text{..... (1)}$$

The variable X_{ij} is constrained in a variety of ways:

(a) The hectares to be cut in each compartment can neither be negative nor can it exceed the total number of hectares in each compartment. If a_i represents the number of hectares in compartment i, it follows that

$$\sum a_i X_{ij} \leq \alpha_i \qquad \text{.....(2)}$$

where α_i represent the total hectares in compartment i and such that $X_{ij} \geq 0$ for all i and j

(b) There is need for restrictions on how much can be cut each period so that wood volume can be adequately distributed over cutting period. It follows that

$$\sum a_i X_{ij} \leq \beta_j \qquad \text{.....(3)}$$

where β_j represents the maximum number of hectares which can be cut in the j period.

The whole exploitation process can now be formulated as a linear programming problem thus:

Maximize: $$Z = \sum \sum C_{ij} X_{ij} \qquad \text{.....(4)}$$

Subject to:

i. $\sum a_i X_{ij} \leq \alpha_i$, (hectares cut from each compartment cannot be more than the area of that compartment) ………(5)

ii. $\sum a_i X_{ij} \leq \beta_j$, (hectares cut for each period cannot be more than the area available for that cutting period) ………(6)

iii. $X_{ij} \geq 0$ for all *i* and *j* (negative hectares cannot be cut) …..(7)

where for emphasis and clarity,

Z = total wood volume (m^3)

C_{ij} = wood volume per hectare accruing as a result of cutting compartment *i* in year *j* (m^3).

X_{ij} = plantable area to be cut from compartment *i* in year *j* (ha)

α_i = plantable area in compartment *i* (ha)

β_j = plantable area required to be cut in year *j* (ha)

i = compartments 1, 2 ………8

j = cutting periods I, II and III (2013 - 2017, 2018 – 2022, 2023 – 2027 respectively)

Estimation of parameters of the model (Cij, ai, αi, βj)

The eight plantations, which were established from the year 1991 through 2000, were treated as eight compartments. In each compartment, four sample plots of size 20m X 20m were randomly laid and complete enumeration of all trees in each plot was carried out. The height of each standing tree was measured using Suunto altimeter and diameter at breast height (dbh) was measured using the conventional girth tape. The projection of wood volume per compartment and period was carried out by the use of the multiple linear regression model:

$V = \beta_0 + \beta_1 A + \beta_2 G + \beta_3 S$ (*Ajayi et al*, 2007) …………(8)

where,

A = age of trees in the compartment (years)

G = basal area /ha (m^2 ha^{-1})

S = site index (m)

β_0, β_1, β_2 , β_3 = regression coefficients

The projected wood volumes (rounded off as whole numbers) per compartment and by period are presented in table I.

Maximize $Z = 1424X_{1,1} + 1574X_{1,2} + 1670X_{1,3} + 1386X_{2,1} + 1558X_{2,2} + 1642X_{2,3} + 1285X_{3,1} + 1445X_{3,2} + 1541X_{3,3} + 1259X_{4,1} + 1419X_{4,2} + 1515X_{4,3} + 1238X_{4,1} + 1398X_{4,2} + 1494X_{4,3} + 1096X_{5,1} + 1256X_{5,2} + 1352X_{5,3} + 698X_{7,1} + 769X_{7,2} + 1029X_{7,3} + 517X_{8,1} + 677X_{8,2} + 773X_{8,3}$

subject to:

(i) $1000X_{1,1} + 1000X_{1,2} + 1000X_{1,3} \leq 1000$

$2338X_{2,1} + 2338X_{2,2} + 2338X_{2,3} \leq 2338$

$1230X_{3,1} + 1230X_{3,2} + 1230X_{3,3} \leq 1230$

$1570X_{4,1} + 1570X_{4,2} + 1570X_{4,3} \leq 1570$

$1002X_{5,1} + 1002X_{5,2} + 1002X_{5,3} \leq 1002$

$927X_{6,1} + 927X_{6,2} + 927X_{6,3} \leq 927$

$230X_{7,1} + 230X_{7,2} + 230X_{7,3} \leq 230$

$1192X_{8,1} + 1192X_{8,2} + 1192X_{8,3} \leq 1192$

(ii) $1000X_{1,1} + 2338X_{2,1} + 1230X_{3,2} + 1570X_{4,1} + 1002X_{5,1}$

$+ 927X_{6,1} + 230X_{7,1} + 1192X_{8,1} \leq 3500.$

$1000X_{1,2} + 2338X_{2,2} + 1230X_{3,2} + 1570X_{4,2} + 1002X_{5,2}$

$+ 927X_{6,2} + 230X_{7,2} + 1192X_{8,2} \leq 3000.$

$1000X_{1,3} + 2338X_{2,3} + 1230X_{3,3} + 1570X_{4,3} + 1002X_{5,3}$

$+\ 927X_{6,3} + 230X_{7,3} + 1192X_{8,3} \leq 2989.$

(iii) $Xij \geq 0$ for all i and j

The Linear programming problem was solved on a personal computer using a management science decision package (Quantitative systems for business plus - QSB+).

Results

Summaries of the prescribed solutions are presented in tables II and III

The prescribed optimal harvesting regimes were 3500ha, representing about 4,703, 426 m^3 of wood for period I (2008 – 2012). For period II (2013 – 2017), 3000ha with wood content of 3,400,946 m^3 was prescribed. The harvesting for period III (2018 – 2022) was put at 2989ha with wood volume of 4,402,982 m^3. Harvestings for period I are to be carried out in compartments 2 and 4. The harvesting for period II are to be conducted in compartments 3, 4, 5 and 8 while the final cutting for period III will take place in compartments 5, 6 and 7.

Sensitivity Analysis

Tables IV and V show the sensitivity analysis performed on the Coefficients of the objective function (variation of wood volume) the Right hand side (RHS) values of the constraints respectively.

The minimum wood volume for each compartment is bounded below, while on the other hand, maximum wood volume for each compartment unbounded above except for compartments 4 and 8. However, the maximum value for the amount of land in each compartment is bounded above, which in practical terms put an upper limit on the wood volume.

Opportunity cost

Table VI contains wood volume coefficients that can be used to compute the opportunity cost for deviating from the prescribed optimal schedule.

The coefficient is multiplied by the corresponding area (ha.) to determine the loss in wood volume.

The coefficient is multiplied by the corresponding area (ha.) to determine the loss in wood volume.

Discussion

There is a reduction in wood volume during the second period. This may help to check any drop in wood price during the first period as may be caused by factors that affect the demand for *G. aborea*. This will however depend on the period of such fluctuation since a harvesting period last for five years.

Sensitivity analysis is a useful tool to management in helping to prepare for any eventuality and in presenting alternative courses of action to accommodate fluctuations – a case of being forewarned and being forearmed. It is not uncommon for wanting to change the development decision model. Conditions often change, particularly in harvesting a plantation over years. Some of the causes for change include resources availability as occasioned by compartment size and the quality of wood available on each compartment. Change is inevitable in a dynamic system like forest plantations. Hence, sensitivity analysis is tailored to demonstrate how the system or model is affected when a change occurs in one or more of the parameters. The change might be so significant that the model would have to be reviewed. The range of values for wood volume and hectares in this study would have to be maintained in the optimal solution. These range of values are surely of interest to the plantation management (Osho, 1995).

There is a cost of delay (opportunity cost) in postponing the harvesting of trees much beyond optimal schedule. Similarly, if harvesting is carried out much earlier than scheduled, a loss can be incurred. According to Price (1989), a long time delay entails greater costs than short time delay. For example, if the management decides to delay the harvesting of say 500 hectares from compartment 2 until the second cutting period, the loss in wood volume will be (66.27 X

500) m^3 = 33,135.0 m^3. If for some reasons, the delay extends till the third cutting period, the loss in wood volume will be (206.28 X 500) m^3 = 103,140.0 m^3. The opportunity cost in this study is measured in wood volume. It is valued by the amount of wood in the period it is not used. Definitely, this type of information is very useful in management.

Conclusion

It can be concluded that linear programming, though very costly and time consuming during data collection and preparation, it is nevertheless a good technique for generating information both for short-term and long-term planning and decision making in logging management. In Nigeria, there is limited or negligible forest management, particularly plantation. (Okojie, 1981). The conversion of many Nigeria's moist rainforests to exotic species like *Pines, Eucalyptus, Teak* and *Gmelina* poses new problem. For resource-poor countries of Sub–Saharan Africa, fertilization and thinning are yet to be accepted as management practices of merit. This situation puts a serious doubt as to the productivity of these plantations in the long run.

However, the existence of thousands of hectares of forest plantations in a country like Nigeria, which has suffered from severe over exploitation of its existing indigenous trees, has some socio-economic benefits. These benefits include:

i. Accessibility of previously inaccessible villages due to link roads, which were built during plantation establishment. This situation improved the marketing of farm produce derived under departmental taungya (a practice of growing arable crops with trees) with positive effects on the their standard of living.

ii. Improvement of health services in the villages due to the establishment of clinics, which enhanced regular, visits of medical doctors to the forestry workers.

The environmental impact of clear felling of forest compartments of plantation is another issue of great concern. Clear felling will definitely lead to soil erosion and loss of soil fertility with severe consequences on the future productivity of the forest plantations. The present state of knowledge of plantation forestry in Nigeria cannot guarantee that *Gmelina arborea* can be grown perpetually on the same site. A controlled exploitation to plantation would appear more sensible (Osho, 1995).

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Table I: Tree volume projection per compartment by period (m^3 ha^{-1})

Compartment	Period I 2013 - 2017	Period II 2018 - 2022	Period III 2023 - 2027	Area per compartment (Ha.)
1	1424	1574	1670	1000
2	1386	1558	1642	2338
3	1285	1445	1541	1230
4	1259	1419	1515	1570
5	1238	1398	1494	1002
6	1096	1256	1352	927
7	698	769	1029	230
8	517	677	773	1192
Total				**9489**

Table II: Harvesting schedule for maximum wood volume – Area data (ha.)

Compartment	Period I 2013 - 2017	Period II 2018 - 2022	Period III 2023 - 2027	Area per compartment (Ha.)
1	0	0	1000	1000
2	2338	0	0	2338
3	0	1230	0	1230
4	1162	408	0	1570
5	0	170	823	1002
6	0	0	927	927
7	0	0	230	230
8	0	1192	0	1192
Maximum ha. to cut	**3500**	**3000**	**2989**	**9489**

Table III: Harvesting schedule for maximum wood volume – Volume data (m^3)

Compartment	Period I 2013 - 2017	Period II 2018 - 2022	Period III 2023 - 2027	Volume per compartment (m^3)
1	0	0	1,670,000	1,670,000
2	3,240,468	0	0	3,240,468
3	0	1,777,350	0	1,777,350
4	1,462,958	578,952	0	2,041,910
5	0	237,660	1,243,008	1,480,668
6	0	0	1,253,304	1,253,304
7	0	0	236,670	236,670
8	0	806,984	0	806,984
Total	**4,703,426**	**3,400,946**	**4,402,982**	**12,507,354**

Table IV. Sensitivity analysis for the objective coefficients – wood volume (m^3/ha)

Compartment	Minimum	Original	Maximum
1	1669.81	1670	Infinity
2	1319.73	1386	Infinity
3	1423.16	1445	Infinity
4	1214.77	1259	1303.5
5	1478.70	1494	Infinity
6	1344.80	1352	Infinity
7	799.04	1029	Infinity
8	658.80	677	821.0

Table V. Sensitivity analysis for the Constraints – Hectarage (ha)

Compartment	Minimum	Original	Maximum
1	1000	1000	1832
2	2338	2338	3500
3	1230	1230	2422
4	1570	1570	2762
5	1002	1002	2194
6	927	927	1759
7	230	230	1062
8	1192	1192	Infinity

Table VI: Coefficients for calculating opportunity cost (m^3 ha^{-1})

Compartment	Area (Ha.)	Period I 2013 - 2017	Period II 2018 - 2022	Period III 2023 - 2027
1	1000	48.28	0.19	0
2	2338	0	66.27	206.28
3	1230	34.65	0	21.84
4	1570	0	0	54.42
5	1002	57.88	0	0
6	927	72.71	7.18	0
7	230	285.52	237.96	0
8	1192	38.52	0	18.2
Total	**4,703,426**	**3,400,946**	**4,402,982**	**12,507,354**