Free vibrations analysis of sandwich beams with viscoelastic core

Analyse des vibrations libres des poutres sandwichs a cœur viscoelastique

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ABSTRACT

In this work, we treat the problem of free vibrations of sandwich beams with viscoelastic core by considering its frequency dependence. The formulation of the equation of motion is carried out by the Hamilton principle whose Euler-Bernouilli theory is applied to the faces and the Timoshenko theory to the viscoelastic core. The discretization of the bending motion equation is carried out by the finite element methods to obtain the eigenvalues problem corresponding to linear free vibrations. The difficulty of solving the eigenvalues problem due to the frequency dependence of the stiffness matrix leads us to use the asymptotic numerical methodto get the eigenmodes and the damping properties characterizing the viscoelastic sandwich beam.

RESUME

Dans ce travail, on traite le problème des vibrations libres des poutres sandwichs à âme viscoélastiques en considérant sa dépendence en fréquence. La formulation de l'équation de mouvement est réalisée par le principe de Hamilton dont la théorie d'Euler-Bernouilli est appliquée aux faces et la théorie de Timoshenko au cœur viscoélastique. La discrétisation de l'équation du mouvement de flexion est réalisée par la méthodes des éléments finis afin d'obtenir le problème aux valeurs propres correspondant aux vibrations libres linéaires des poutres composites munies des matériaux viscoélastiques. La difficulté de résoudre le problème de valeurs propres en raison de la dépendance en fréquence de la matrice de rigidité nous ramène à utiliser la méthode asymptotique numérique pour obtenir les modes propres et les propriétés amortissantes caractérisant les poutres sandwichs viscoélastiques.

***Mots*-clés**

Vibration, Elément fini, Asymptotique numérique, Matériau viscoélastique, Mode complexe, Propriétés amortissantes.

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1. INTRODUCTION

The specific mechanical properties in damping have presented the viscoelastic materials as a remarkable solution allows attenuating the vibrational amplitudes. They are widely applied in many fields because of their dissipative abilities of vibratory energies. In practice, the technique of damping of the vibratory amplitudes so-called passive technical can be realized by combining a thin layer of viscoelastic polymer between two metal or composite skins for manufacturing composite sheets with high damping capacity while maintaining its characteristics of strength and rigidity. From a mechanical point of view, the passive damping of the sandwich beams is entrained by the high shear in the viscoelastic layer due to the difference between the longitudinal displacements of the face layers and the low rigidity of the viscoelastic core. As a result, the characterization of behavior of structures with viscoelastic materials is required to define all appropriate design parameters. Several researchers have presented analytical models characterizing their damping properties [1-4]. The Hamilton principle was used to formulate the governing equation of vibration motion of sandwich beams with constant modulus of viscoelastic core and to examine their vibration amplitudes Rao [5]. However, many authors have used numerical approaches to study structures with viscoelastic materials with more complex geometries Irazu and Elejabarrieta [6]. Several approaches have been used to solve the problem of eigenvalues. Daya [7] used the asymptotic numerical methodfor the eigenvalue problem characterizing the free vibrations of viscoelastic sandwich beams taking into account the frequency dependence. Bilasse et al [8-9] used the generic approach of "Diamand" which combines both the asymptotic numerical methodand the automatic differentiation method. Arikoglu and Ozkol [10] used the differential transformation method (DTM) to solve the motion equations governing the free vibrations of the sandwich beam obtained by the Hamilton principle. The authors considered several kinematic models for their studies describing the damping of the viscoelastic layer among others Kirchhoff-Love [11], Mindlin [12], and Reddy [13]. Recently, kinematic models consist of describing the displacement field layer-by-layer, which then leads to zigzag-type models. Cai et al [14] used an analytical approach to examine the vibratory response of the beam using the Lagrange energy method. The model of Mead and Markus [15] was used by Arvin et al [16] to describe the kinematic relations between the three layers, the authors presented a higher order theory to study the free and forced vibrations of composite beams with viscoelastic core by considering asymmetrical geometries. The aim of this work is to solve the problem of free vibrations of composite sandwich beams equipped with viscoelastic materials using a numerical approach based on the finite element method. In addition, we use the asymptotic numerical methodto solve the eigenvalue problem of free and linear vibrations of viscoelastic core sandwich beams with the consideration of the frequency dependence of its properties.

2. MATHEMATICAL FORMULATION

The configuration of the viscoelastic sandwich beam presented in this work (figure1.a) is composed of a viscoelastic layer placed between two layers constituting the sandwich skins. Figure 1.b shows the geometric deformation of a viscoelastic sandwich beam. The displacement field of the viscoelastic sandwich beam is given by the zigzag model of Rao [5]. Adding that many assumptions have been considered:

- The transverse plane sections remain plane during bending.
- The transverse normal stress is very small compared to the axial normal stress.
- The shear effect is considered only for the central viscoelastic layer.

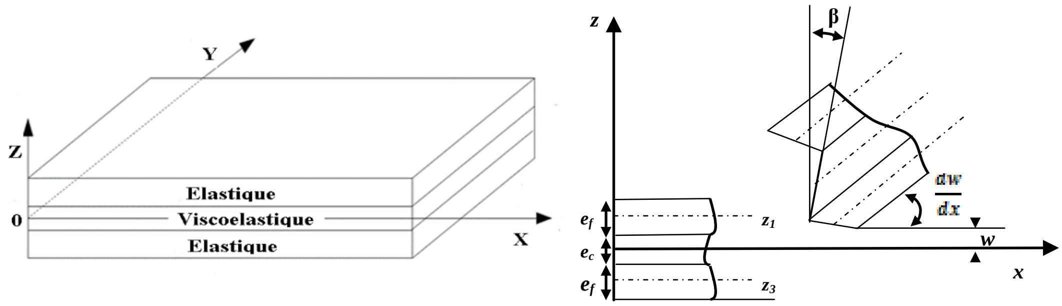

Figure 1: (a) sandwich beam with viscoelastic core; (b) deformed configuration of the sandwich beam [9]

Taking into account the hypotheses considered, the displacement and strain fields for the three composite layers are given by equations (1):

$$\begin{gathered} U_i(x,z,t) = u_i(x,t) - (z - z_i)w_{,x} \\ W_i(x,z,t) = w(x,t) \\ \varepsilon_{ni}(x,z,t) = u_{i,x} - (z - z_i)w_{,xx} \end{gathered} \tag{1}$$

where U, W et ε_n are the longitudinal displacement, the transverse displacement and the normal deformation considered in the context of small deformation and defined by the Green-Lagrange formula for each layer of the sandwich. The shear strain of the viscoelastic layer is given by:

$$\varepsilon_{c2}(x,z,t) = u_{,x} - (z)\beta(x,t) \tag{2}$$

β is the rotation of the normal to the middle plane of the central layer. The formulation of the equation of motion is described by the Hamilton principle, we consider only the potential energy and the kinetic energy. The bending motion equation is given by:

$$\begin{gathered} \delta\Pi = \delta W_p + \delta W_c = \\ \int_0^L [M_\beta \delta\beta_{,x} + M_w \delta\beta_{,xx} + T(\delta w_{,x} + \delta\beta) + (2\rho_f S_f + \rho_c S_c)\ddot{w}\delta w]dx = 0 \end{gathered} \tag{3}$$

where W_p, W_c are potential energy and kinetic energy respectively. The bending moments resulting M_β and M_w associated with the three layers are given by:

$$\begin{gathered} M_\beta = M_2 + (N_1 - N_3)\frac{h_c}{2} = \frac{E_f S_f h_f}{2}(h_c \beta_{,x} - h_f w_{,xx}) + I_c {E_c}^* * \dot{\beta}_{,x} \\ M_w = M_1 + M_3 + (N_3 - N_1)\frac{h_f}{2} = -\frac{E_f S_f h_f h_c}{2}\beta_{,x} + (\frac{E_f S_f {h_f}^2}{2} + 2E_f I_f)w_{,xx} \end{gathered} \tag{4}$$

The discretization of the equation of motion eqt.(3) by the finite element method and the expression the displacement field as a function of the nodal displacements make it possible to form the elementary matrix equation eqt.(5) which describes the free vibratory behavior of the beam.

$$[M]^e \ddot{q}_e + [K]^e q_e = 0 \tag{5}$$

where $[M]^e$ and $[K]^e$ are respectively the elementary mass and stiffness matrices given by:

$$[M]^e = (2\rho_f S_f + \rho_c S_c)\int_0^{L^e} [N_w][N_w]dx \tag{6}$$

$$\begin{aligned} [K(\omega)]^e = {} & \left(2I_c E_c(w) + \frac{E_f S_f {h_c}^2}{2}\right)\int_0^{L^e} [N_{\beta,x}]^T [N_{\beta,x}]dx - \frac{E_f S_f h_f h_c}{2}\int_0^{L^e} \left([N_{\beta,x}]^T [N_{w,xx}] + [N_{w,xx}]^T [N_{\beta,x}]\right)dx + \\ & \left(\frac{E_f S_f {h_f}^2}{2} + 2E_f I_f\right)\int_0^{L^e} [N_{w,xx}]^T [N_{w,xx}]dx + \frac{S_c}{2(1+\upsilon_c)} E_c(w)\int_0^{L^e} \left([N_{w,x}]^T [N_{w,x}] + [N_\beta]^T [N_{w,x}] + [N_{w,x}]^T [N_\beta] + \right. \\ & \left. [N_\beta]^T [N_\beta]\right)dx \end{aligned} \tag{7}$$

The global matrix system describing the free vibratory behavior of the sandwich beam after the assembly of the elementary matrices is written in the form:

$$[M]\ddot{q} + [K]q = 0 \tag{8}$$

with $[M]$ and $[K]$are respectively the global mass and stiffness matrices, q is the global displacement vector. In order to study the free vibrations of the viscoelastic core sandwich, we have to solve the problem with the following complex eigenvalues:

$$([K(\omega)] - \omega^2[M])U = 0 \tag{9}$$

The resolution of equation (9) is complex because of the frequency dependence of the stiffness matrix, the asymptotic numerical methodappears as one of the most efficient numerical methods to approximate the solution. The algorithm (figure 2) presents the main steps of this method. This method consists of expressing the variables in Taylor series and replacing them in the eigenvalues problem decomposed. The problem solving is done starting from :

- a linear initial problem $\mathbf{R(U,\lambda)=[[K_0]-\omega_0\text{^}2\ [M]]U_0}$ for $a_j=0$
- to the nonlinear problem $\mathbf{R(U,\lambda)=[[K_0]+E_n(\omega)[K_c]-\omega_n\text{^}2[M]]U_n}$ for $a_j=1$.

Therefore, a continuation procedure is applied which consists of defining a new slice of solution from a starting point (U^j, λ^j). The solution is obtained for each iteration by solving a system of linear equations. The process ends when the value of $a_{(j+1)}>1$. A Matlab code of the asymptotic numerical method has been established describing eigenvalue problem solving (9).

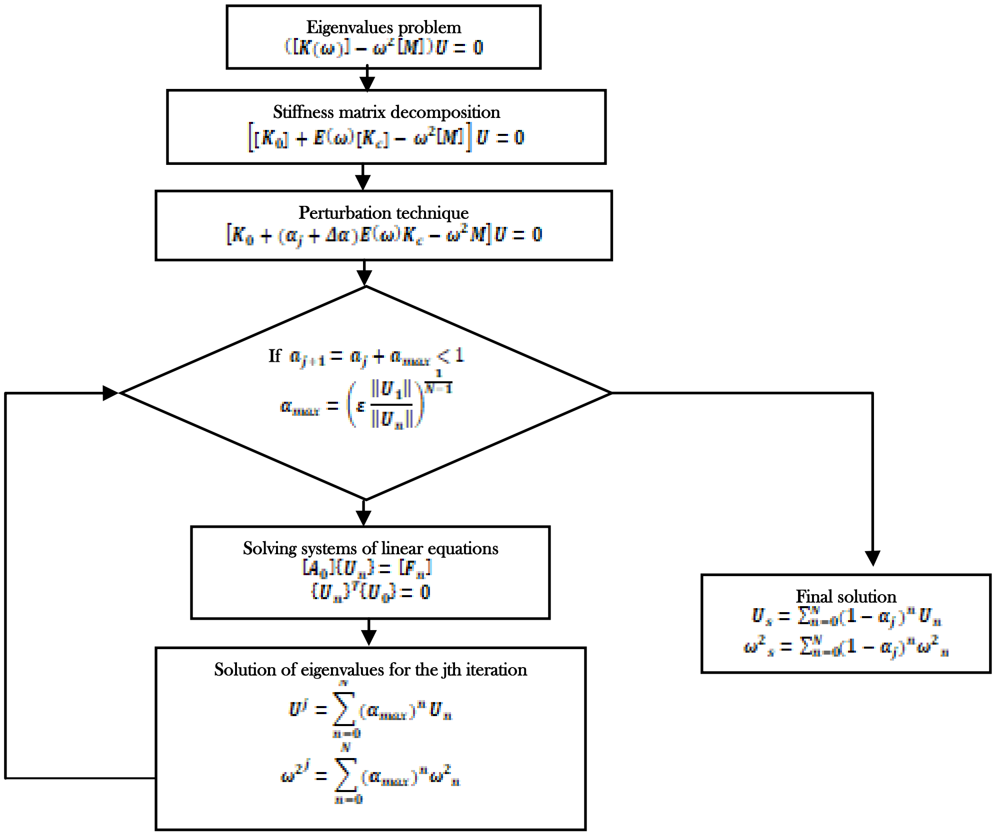

Figure 2. Algorithm of solving the eigenvalue problem by the asymptotic numerical method.

The free vibrations of the viscoelastic sandwich structures make it possible to characterize not only their mechanical properties but also their vibratory behavior under different loads, which will be the aims of future works. The damping properties of the sandwich structures that are the natural frequency f and the loss factor η can be calculated from eigenvalue problem resulting from the problem (9).

$$\omega^2 = \Omega^2(1 + i\eta) \Rightarrow \begin{cases} f = \dfrac{\Omega}{2\pi} = \dfrac{\sqrt{\omega^{2^R}}}{2\pi} \\ \eta = \dfrac{\omega^{2^I}}{\omega^{2^R}} \end{cases} \tag{10}$$

3. RÉSULTS

In order to examine and characterize the properties of sandwiches with viscoelastic materials, several law models of viscoelastic behavior have been studied. Starting from a frequency independent viscoelastic law, passing thereafter a law of viscoelastic behavior that is highly dependent on the frequency. It is Noticed that the study has already been realized by Bilasse et al [16] who used the generic approach of "Diamond" to solve the eigenvalue problem (9), this approach combines the asymptotic numerical method and automatic differentiation method. We propose our model of the asymptotic numerical methodwhose small accuracy parameter is fixed around $\varepsilon = 10^{-8}$, the truncation order of the series N = 20. Thus, we propose here a model of finite elements with 20 elements, the number of degrees of freedom nodal is three (3) which are the transverse displacement w, the rotation dw/dx, and the rotation of the central layer β.

2.1. Viscoelastic model with constant modulus

The first case studied in this work is a model of viscoelastic behavior with a frequency dependent of Young's modulus E, this model is widely used to study the viscoelastic behavior. Some research shows that this model remains only an approximation when the complex modulus of the material varies very little in frequency[8,16].

$$E = E_0(1 + i\eta_c) \tag{11}$$

where E_0 is the storage modulus, η_c is the loss factor of the viscoelastic material, the loss modulus E_d is given by:

$$E_d = E_0\eta_c \tag{12}$$

The mechanical and geometrical properties of the viscoelastic sandwich structure are presented in table 1, it consists of a viscoelastic layer sandwiched between two aluminum skins.

Table 1. Mechanical and geometrical properties of the sandwich with a frequency-independent viscoelastic core [8]

	Elastic face	Viscoelastic core
Young Modulus (Pa)	$E_f = 7.037 \times 10^{10}$	$E_c = 7.037 \times 10^5 \times 2(1 + \upsilon_c)$
Poisson Coefficient υ	$\upsilon_f = 0.3$	$\upsilon_c = 0.49$
Density (Kg/m^3)	$\rho_f = 2770$	$\rho_c = 970$
Thickness (mm)	$e_f = 1.52$	$e_c = 0.127$
Length (m)	$L = 0.1778$	
Width (m)	$l = 0.127$	

The damping properties represented by the naturel frequencies and the loss factors of the first three modes are reported in table 2 for different values of viscoelastic loss factor η_c.

Table 2. Natural frequencies and loss factors of the cantilever sandwich beam with viscoelastic core independent of frequency

η_c	N° :	f (Hz)	η	R(U,λ)
0.1	1	64.594	2.8119e-02	1.1876e-07
	2	299.26	2.4202e-02	5.0980e-08
	3	753.07	1.5283e-02	1.1492e-07
0.4	1	65.244	1.0583e-01	1.1868e-07
	2	300.38	9.4985e-02	5.0965e-08
	3	753.85	6.0888e-02	1.1492e-07
0.8	1	66.945	1.7920e-01	2.0183e-07
	2	303.58	1.8003e-01	5.8352e-08
	3	756.26	1.2028e-01	1.1500e-07
1.2	1	68.971	2.1699e-01	2.0188e-07
	2	308.03	2.5113e-01	5.8328e-08
	3	760.12	1.7696e-01	1.1496e-07

The results show that as the damping factor of the viscoelastic core increases, the naturel frequencies also increases, thus implying the improvement of the overall damping of the structure. It is noticed that low frequencies can generate large amplitudes when they are close to the loading frequencies inducing what is called resonance. We can see the efficiency of our Matlab program of the asymptotic numerical method , wich residue was generally less than 2.5×10-7 that validates the precision of the results obtained. Figures 3.a, 3.b, 3.c and 3.d show the variation in amplitudes of the deflection of the three first eigenmodes normalized with respect to the maximum deflection $W_f = W(x_0)$ for different loss factor values $\eta c = \{0.1, 0.4, 0.8, 1.2\}$.

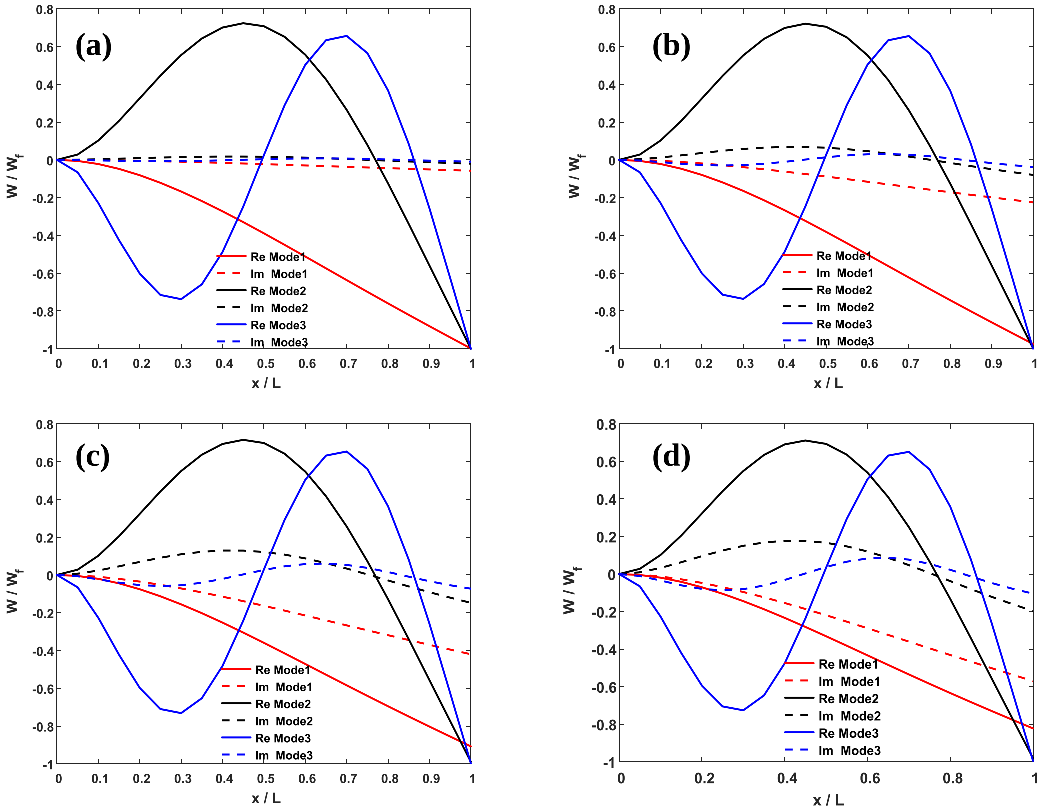

Figure 3. Eigen modes of the cantilever sandwich beam with viscoelastic core for different values of viscoelastic loss factor η_c ((a) -0.1; (b) -0.4; (c) -0.8; (b) -1.2).

The position x_0 depends on the mode order and the boundary conditions of the beam. The figures show the importance of the imaginary part of the eigenmodes particularly for the large values of damping factor of viscoelastic core. Several researchers such as Daya [12], Boutyour [17] considered that the real part of the stiffness matrix, which implies the negligence of the loss modulus, this approach generally, leads to less accurate results in vibration analysis. The consideration of the imaginary part of the eigenmodes and the use of complex modes is very necessary especially with a high viscoelastic damping. This means that the magnitudes relative to the imaginary parts of the complex modes are indices representing the damping capacity induced in the structure. Table.3 reports the natural frequencies and the loss factor of the simply supported sandwich beam corresponding to the first three modes for η_c={0.1,0.4,0.8,1.2}.

Table 3. Natural frequencies and loss factors of the simply supported sandwich beam with viscoelastic core independent of frequency

η_c	N°	*f* (Hz)	η	R(U,λ)
0.1	1	149.59	3.5003e-02	3.7593e-09
	2	492.79	1.9470e-02	2.3415e-09
	3	1047.5	1.0526e-02	2.0584e-09
0.4	1	150.55	1.3695e-01	3.7590e-09
	2	493.34	7.7637e-02	2.3604e-09
	3	1047.8	4.2074e-02	2.0701e-09
0.8	1	153.49	2.5598e-01	3.6937e-09
	2	495.09	1.5375e-01	2.3180e-09
	3	1048.8	8.3948e-02	2.0499e-09
1.2	1	157.92	3.4621e-01	3.7600e-09
	2	497.97	2.2691e-01	2.3332e-09
	3	1050.3	1.2542e-01	2.0265e-09

The same findings have been made, where the natural frequency increases with the increase in the damping factor of the viscoelastic core. Obviously, the eigenfrequecies obtained for simply supported beam are higher than those obtained for the clamped-free beam, the latter condition typically allows show the lowest frequency that can reach larger magnitudes. Thus, we can see again the effectiveness of our Matlab program of the asymptotic numerical method that residue is less than 1.7×10-5, which validates the precision of the results obtained. Figures 4.a, 4.b, 4.c, 4.d show the normalized amplitude variations of three first vibration modes for different values of η_c.

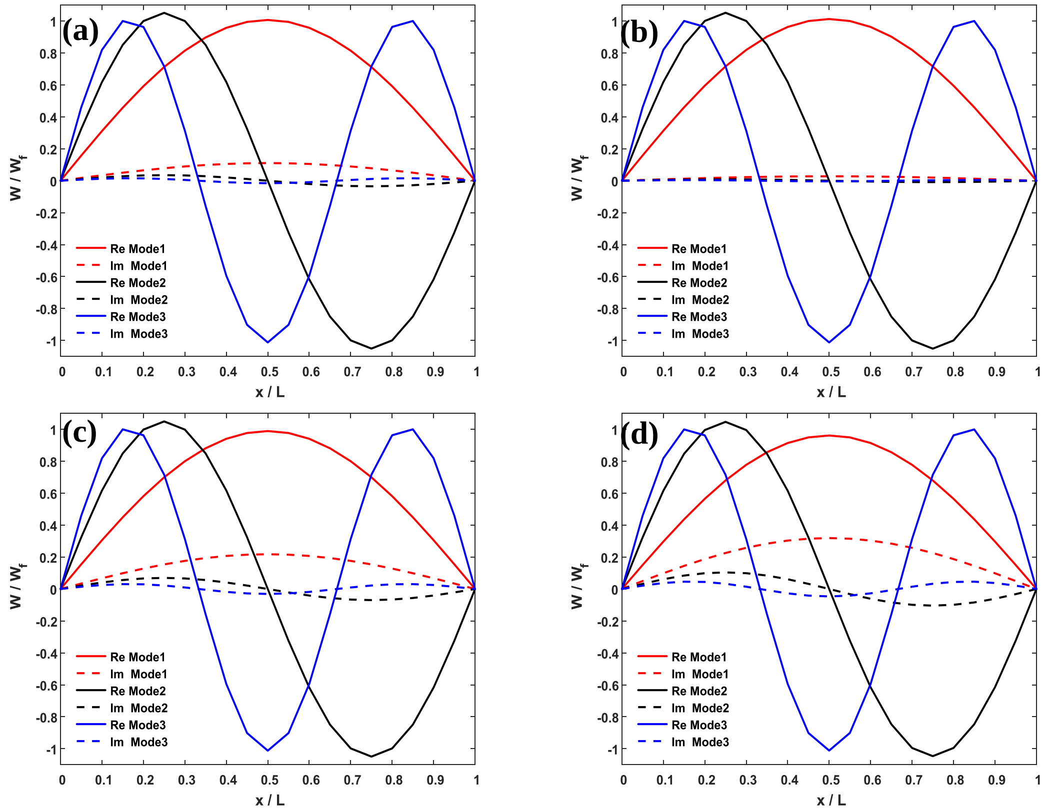

Figure 4. Eigen modes of the simply supported sandwich beam with viscoelastic core for different values of viscoelastic loss factor ηc ((a) 0.1; (b) 0.4; (c) 0.8; (b) 1.2)

In order to study the effect of loss factor of the viscoelastic layer on the overall damping of the sandwich, a study of variation of loss factor of the structure as a function of viscoelastic loss factor is presented in figures 5.a and 5.b for the clamped-free and simply supported boundary conditions respectively. We can see the proportional relationship between the two loss factors, which shows that dispersion becomes significant when the viscoelastic loss factor is greater. This property can be used in systems working under dynamic load in order to reduce or attenuate the structural vibrations generated by these types of stresses in the context of the passive damping system by integrating the viscoelastic materials.

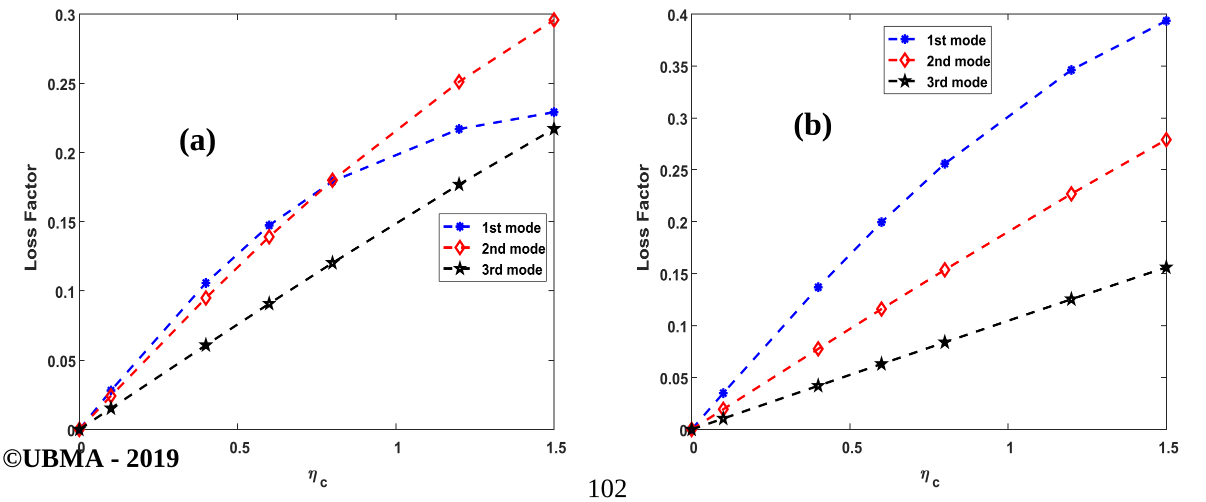

Figure 5. Variation of viscoelastic loss factor of the sandwich beam with frequency independent viscoelastic core for different values of viscoelastic loss factor ηc ((a) clamped-free; (b) simply supported)

2.2. Viscoelastic model with frequency dependent module

In the previous section, we studied the free vibrations of a viscoelastic sandwich beam between two elastic skins, which the modulus of the viscoelastic core was independent of the frequency. In this section, we present another model of viscoelastic behavior strongly dependent on frequency. The viscoelastic core consists of polymeric material ISD112 , which viscoelastic modulus is described by the generalized Maxwell model considered for two different temperatures 20 ° and 27 ° (Tab. 4). The mechanical and geometrical properties of sandwiches are presented in table 5.

Table 4. Shear module of ISD112 viscoelastic core

$$G_c^*(\omega) = G_0\left(1 + \sum_{j=1}^{3} \frac{\Delta_j}{\omega - i\Omega_j}\right) \quad (13)$$

20°C				27°C		
j	G_0	Δ_j	Ω_j	G_0	Δ_j	Ω_j
1	5.11×10^4	2.8164	31.1176	5×10^5	0.746	468.7
2		13.1162	446.4542		3.265	4742.4
3		45.4655	5502.5318		43.248	71532.5

Table 5. Mechanical and geometrical properties of the sandwich with a frequency dependent ISD112 viscoelastic core

	Elastic face	viscoelastic core
Young's modulus (Pa)	$E_f=7.037\times10^{10}$	(14)
Poisson Coefficient	$\upsilon_f=0.3$	$\upsilon_c = 0.5$
Density (Kg/m^3)	$\rho_f = 2770$	$\rho_c = 1600$
Thickness mm	$e_f = 1.52$	$e_c = 0.127$
Length mm	$L = 177.8$	
Width mm	$l = 12.7$	

The natural frequencies and the loss factors corresponding to the first three eigenmodes with the two boundary conditions (Clamped-Free; Supported-Supported) are reported in table 6 and table 7 for the two viscoelastic modulus considered at temperatures 20° and 27° respectively.

Table 6. Natural frequencies and loss factors of the sandwich beam with dependent frequency viscoelastic core ISD112 at 20 ° for different values of viscoelastic loss factor η_c.

	Encastrée-Libre			Appuyée-Appuyée		
N°	*f* (Hz)	η	R	*f* (Hz)	η	R
1	64.93	2.45e-01	2.78e-08	157.67	2.77e-01	.63e-08
2	321.29	2.21e-01	8.87e-08	544.22	2.36e-01	4.84e-09
3	843.96	1.68e-01	4.55e-08	1160.50	1.26e-01	5.65e-09

Table 7. Natural frequencies and loss factors of the sandwich beam with dependent frequency viscoelastic core ISD112 at 27 ° for different values of viscoelastic loss factor

	Encastrée-Libre			Appuyée-Appuyée		
N° :	f (Hz)	*η*	R(U,λ)	f (Hz)	η	R(U,λ)
1	65.54	1.72e-01	3.92e-08	159.47	3.07e-01	5.85e-09
2	325.26	3.03e-01	3.71e-08	548.41	3.88e-01	5.16e-09
3	853.82	3.29e-01	6.39e-08	1160.9	3.29e-01	6.35e-09

The loss factors obtained for the two sandwiches are very high which are very close to those obtained with a constant viscoelastic model and viscoelastic loss factor η_c = 1.2. This illustrates the highly damping that can be brought by this model of the viscoelastic core. On the other hand, it is noticed that the loss factors obtained for the ISD112 core at 27 ° are generally higher compared to those obtained with the ISD112 core at 20 °, implying that the mechanical properties of the materials depend not only on the frequency but also on the temperature. However, it is possible to note the precision of the results obtained, the calculated residual of which is generally less than 7×10^{-8}. The real and imaginary modes are presented in figure 6 and figure 7 for the first three eigenmodes of the sandwich beam with two viscoelastic core at temperatures 20° and 27° respectively. The results show the importance of the imaginary part of the eigenmodes which plays an important role in improving the accuracy of different simulations based on this modal base that can be used to examine the forced vibratory behavior of various configurations of sandwich beams with viscoelastic core.

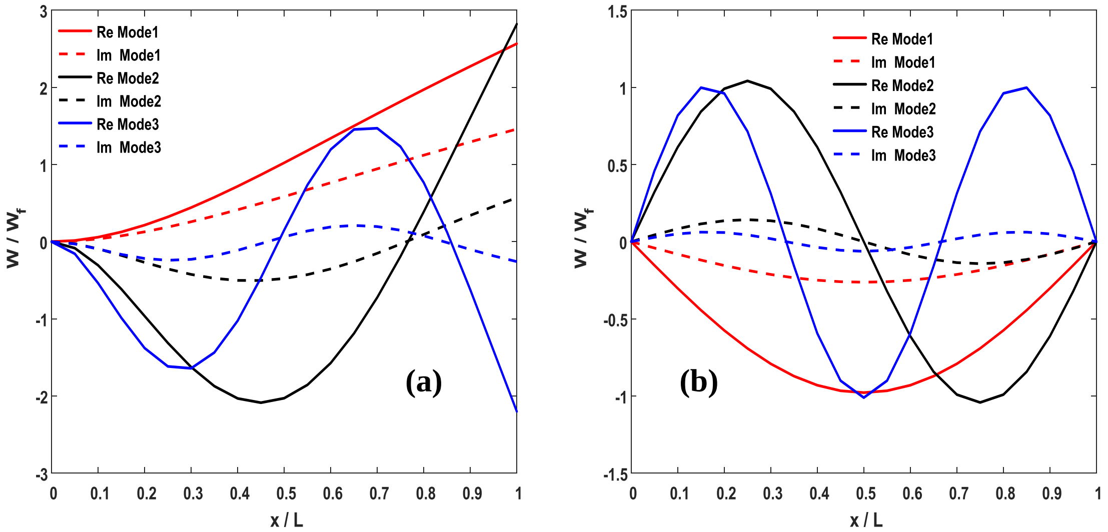

Figure 6. Eigenmodes of sandwich beam with viscoelastic core ISD112 at 20° ((a) clamped-free; (b) simply supported)

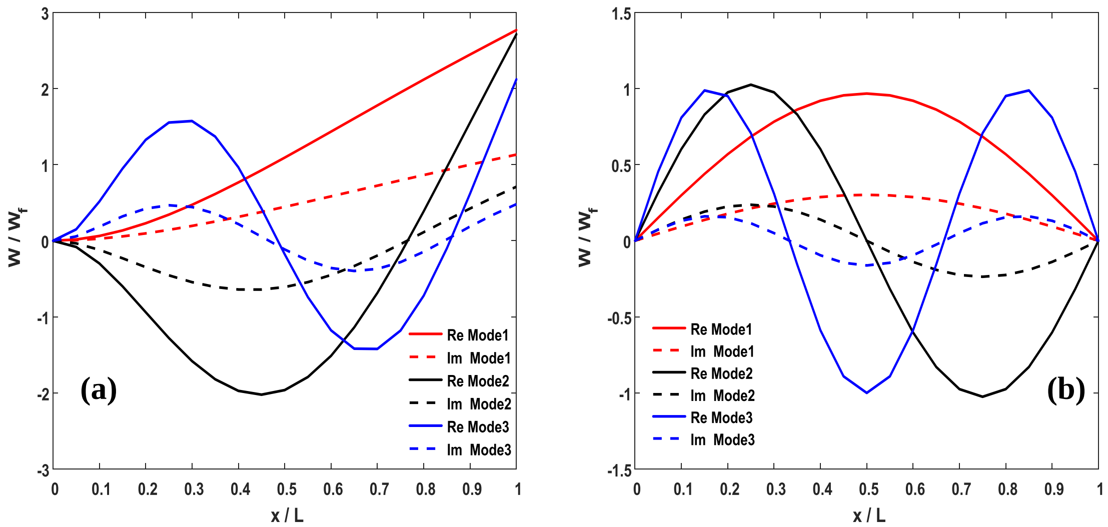

Figure 7. Eigenmodes of sandwich beam with viscoelastic core ISD112 at 27° ((a) clamped-free; (b) simply supported)

3. CONCLUSION

In this work, we studied the free vibrations of viscoelastic sandwich beams with different models of viscoelastic behavior, the first of which is frequency independent model, and the second of which is frequency dependent model. Different boundary conditions were examined to evaluate the accuracy of our numerical algorithm based on finite element discretization and the asymptotic-numerical method, and to construct a modal basis to study linear and non-linear vibrations behavior of sandwich beams under various dynamic load. The following conclusions can be drawn from this work:

- The efficiency of the present numerical algorithm based on the finite element method and the asymptotic numerical method to solve the eigenvalue problem related to the free vibrations of viscoelastic sandwich beams. The obtained results are very acceptable for approximating the damping properties of different sandwich configurations.
- The effect of the increase in damping factor of the viscoelastic core with frequency-independent modulus on the damping properties of the sandwich, involving the improvement of the overall damping of sandwich beams.
- The importance of the imaginary parts of the eigenmodes which make it possible to improve the precision of the results particularly when this modal basis is used to study the forced vibrations.

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