Energetic evaluation of a perfect damped elastoplastic oscillator

Bilan énergétique d'un oscillateur elastoplastique parfait amorti

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ملخص:

هذا المقال يعالج السلوك الديناميكي لمتذبذب مخمد مرن بشكل مثالي ومعرض لجهد خارجي ذو نوع زلزالي. النموذج الانسيابي مقدم كنظام ذو درجة تغير واحدة متمثل في حالتين :إثارة متناظرة (بلاستيكي بشكل مثالي) ، إثارة غيرمتماثلة في مجال الضغط. تم تأسيس حصيلة الطاقة أساسًا بالنظر إلى الطاقة المتشتتة أثناء التذبذبات ، وفقاللمتغيرات الهيكلية للنموذج (التردد الزاوي المكافئ ، التخميد ، عاملاللاتماثل في قوة الإثارة ...). تشتت في طاقة الدورة الحدودية المتواجدة في منطقة التوافق السلوكية والتي يسيطر عليها مخطط التشعب.

الكلمات المفتاحية: متذبذب مخمد مرن بشكل مثالي، النموذج الانسيابي، الطاقة المتشتتة، الدورة الحدودية، التوافق، مخطط التشعب.

Résumé :

L'article traite le comportement dynamique d'un oscillateur élastoplastique parfait amorti, sollicité par une charge extérieure, de type sismique. Le modèlerhéologiqueest présenté comme un système à un seul degré de liberté (1DDL), pour les deux cas : une excitation symétrique (élastoplasticité parfaite), et une Asymétrie dans le domaine de la compression. Le bilan énergétique est essentiellement établi à partir de l'énergie dissipée au cours des oscillations, en fonction des paramètres structuraux du modèle (fréquence angulaire équivalente, l'amortissement, paramètre d'asymétrie de la force d'excitation…).Une dissipation de l'énergie du cycle limite dans la zone comportementale de l'accommodation contrôlée par la frontière de bifurcation.

Mots Clés: Oscillateur élastoplastique parfait amorti, modèle rhéologique, énergie dissipée, cycle limite, accommodation, frontière de bifurcation.

Abstract

This paper deals with the dynamic behaviour of a perfect damped elastoplastic oscillator, subjected to an external seismic load. The rheological model is presented as a single-degree-of-freedom system, for the two cases: a symmetrical excitation (perfect elastoplasticity); and an asymmetry in the compression field. The energizing assessment is essentially established from the dissipated energy map during the oscillations, according to the structural parameters of the model (equivalent angular frequency, damping, parameter of the asymmetric excitation force…). A dissipation of the limit cycle energy observed for the alternating plasticity behavioural area, which is controlled by the bifurcation boundary.

Keywords: Perfect Damped Elastoplastic Oscillator, Rheological model, Dissipated energy, Limit cycle, Alternating plasticity, Bifurcation boundary.

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1. Introduction

Knowing that the cyclical behaviour of dynamical systems is strongly related to the hysteretic character which depends on the history of the material, a difficult subject, in which the analysis aims to present the response of dynamic models according to the external excitation, these models represented by rheological oscillators, which may reflect the behaviour of civil engineering structures subjected to cyclic vibrations of seismic kind. The study of the response -often periodically- of these oscillators will help in the control of the dynamic stability of structures, based on dynamic parameters, and thus determine and draw the energy map. However, the simplest model for dynamic analysis is that a single degree of freedom (s-d-o-f) as in Fig. 1, and the behavior that has been the subject of many research studies [1], [2], [3], is the elastoplastic that being more representative in term of its cyclical (dynamical) character. Thus, the chosen model -from the literature- for the study and the energetic evaluation of elastoplastic structures is that, where the elastoplastic behavior is perfect for both cases, asymmetric: the force amplitudes are equal in both domains (tension and compression) and a perfect elastoplasticity with asymmetry in the compression field.

2. Presentation of the rheological model

Let us consider the single degree of freedom model (Fig. 1), a rheological system composed of a mass (M), which is attached to an elastoplastic spring (K_0), and damping coefficient noted (C), necessarily positive. The inelastic system is submitted to a harmonic external force $F(t)$ defined by its amplitude (F_0) and its pulsation (Ω).

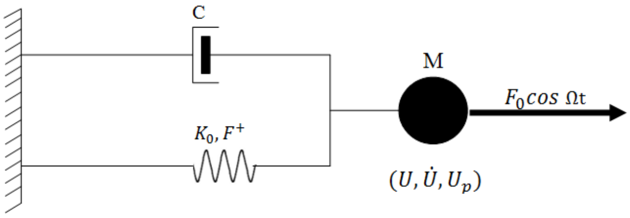

Figure.1. Dynamic s-d-o-f model (Rheological).

This oscillator (Fig. 1), who is characterized by its position (U), displacement rate $\left(\dot{U}\right)$ and an internal plastic variable noted$\left(U_p\right)$named the plastic displacement, can be approached by an inelastic incremental behaviour, (elastic perfectly plastic, see Fig. 2 (a and b).

(a) Symmetric case ($F^+ = F^-$) (b) Asymmetric Case ($F^+ \neq F^-$)

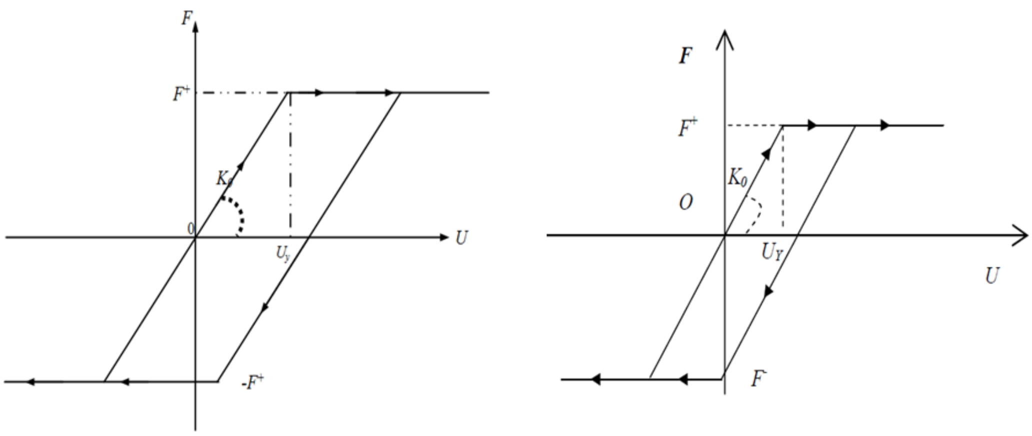

Figure. 2**:** Incremental perfect elastoplastic behaviour

For this inelastic system, two types of dynamic states can be distinguished [2], [3], [4], [5], [6], [7], [8]:

- ***Elastic State*** $\hat{\boldsymbol{E}}$: (reversible state)
- ***Plastic State*** $\hat{\boldsymbol{P}}$: (irreversible state) associated to the evolution of the plastic displacement.

This plastic state can be divided into two plastic states $\widehat{\boldsymbol{P}^{+}}$ and $\widehat{\boldsymbol{P}^{-}}$ in function of the sign of the elastic displacement $(U - U_p)$. [6], [7], [8]

3. Formulation of the motion equations

The equations of motion for this perfect elastoplastic oscillator damped have been formulated, using dimensionless variables, for a system with one degree of freedom (s-d-o-f), and for the three states [7], [8] as follows:

3.1. Symmetric System

$$\begin{cases} \text{State}\hat{E}: & \ddot{u} + 2\zeta\dot{u} + v = f_0 cos(\omega\tau)\ ;\ \dot{u}_p = 0 \\ \text{State}\hat{P}^{+}: & \ddot{u} + 2\zeta\dot{u} + 1 = f_0 cos(\omega\tau)\ ;\ \dot{u}_p = \dot{u} \\ \text{State}\hat{P}^{-}: & \ddot{u} + 2\zeta\dot{u} - 1 = f_0 cos(\omega\tau)\ ;\ \dot{u}_p = \dot{u} \end{cases} \text{with :} \left| \begin{array}{l} f_0 = \frac{F_0}{F^{+}} \\ \omega = \Omega\sqrt{\frac{M}{K_0}} \\ \zeta = \frac{C}{2\sqrt{MK_0}} \end{array} \right. \quad (1)$$

These equations of motion in the reduced domain phases $(v, \dot{u})$, are controlled by:

$$\begin{cases} \text{State}\hat{E}: & (|v| < 1)\text{or}[v = 1\ \text{ and}(\dot{u}\ \leq 0)]\text{or}[v = -1\ \text{ and } (\dot{u}\ \geq 0)] \\ \text{State}\hat{P}^{+}: & (v = 1)\text{and}\dot{u} \geq 0 \\ \text{State}\hat{P}^{-}: & (v = -1)\text{and}\dot{u} \leq 0 \end{cases} \quad (2)$$

The dynamic system (Eq. 1), represent the perfect damped elastoplastic model. However, the response of this system of differential equations, depends to the kind of elastoplasticity (symmetric or asymmetric), defined by the condition of equality of forces (F^{+}) and (F^{-}). Therefore, from the works of Challamel [7], and Hammouda [8], the perfect damped elastoplastic model could be seen as "Symmetric" (Fig. 2.a) if:$(F^{+} = F^{-})$, in otherwise, the model is "Asymmetric" (Fig. 2.b).

3.2. Asymmetric system

The asymmetry of the perfect damped elastoplastic model will be introduced through:

$$\frac{F^{-}}{F^{+}} = -1 - \varepsilon \ \text{ with}(\varepsilon > 0) \quad (3)$$

$$\begin{cases} \text{State}\hat{E}: & \ddot{u} + 2\zeta\dot{u} + v = f_0 cos(\omega\tau)\ ;\ \dot{v} = \dot{u} \\ \text{State}\hat{P}^{+}: & \ddot{u} + 2\zeta\dot{u} + 1 = f_0 cos(\omega\tau)\ ;\ \dot{v} = 0 \\ \text{State}\hat{P}^{-}: & \ddot{u} + 2\zeta\dot{u} - (1+\varepsilon) = f_0 cos(\omega\tau)\ ;\ \dot{v} = 0 \end{cases} \text{with :} \left| \begin{array}{l} f_0 = \frac{F_0}{F^{+}} \\ \omega = \Omega\sqrt{\frac{M}{K_0}} \\ \zeta = \frac{C}{2\sqrt{MK_0}} \end{array} \right. \quad (4)$$

The equations of motion of the asymmetric model are also controlled by:

$$\begin{cases} \text{State}\hat{E}: & (-1 - \varepsilon < v < 1) \\ & \text{or}[v = 1\ \ and\ (\dot{u}v\ \leq 0)]\text{or}[v = -1 - \varepsilon\ \ \text{ and}(\dot{u}v\ \geq 0)] \\ \text{State}\hat{P}^{+}: & (v = 1)\text{and}\dot{u} \geq 0 \\ \text{State}\hat{P}^{-}: & (v = -1 - \varepsilon)\text{and}\dot{u} \leq 0 \end{cases} \quad (5)$$

4. Dynamic response of the perfect damped elastoplastic model

4.1 Response of the Symmetric system

The response of the symmetric dynamic model (Fig. 2.a), expressed by the equations of motion (Eq.1), in the case of forced vibrations of the elastic state ($\hat{E}$) can be written:

$$\hat{E}:\begin{cases} v(\tau) = \begin{bmatrix} A\cos\left[\sqrt{1-\zeta^2}(\tau-\tau_i)\right] + \\ B\sin\left[\sqrt{1-\zeta^2}(\tau-\tau_i)\right] \end{bmatrix} e^{-\zeta(\tau-\tau_i)} \\ \quad + \frac{f_0}{(1-\omega^2)^2+4\omega^2\zeta^2}\left[(1-\omega^2)\cos(\omega\tau) + 2\omega\zeta\sin(\omega\tau)\right] \\ \dot{u}(\tau) = \begin{bmatrix} \left(-\zeta A + B\sqrt{1-\zeta^2}\right)\cos\left[\sqrt{1-\zeta^2}(\tau-\tau_i)\right] + \\ \left(-\zeta B - A\sqrt{1-\zeta^2}\right)\sin\left[\sqrt{1-\zeta^2}(\tau-\tau_i)\right] \end{bmatrix} e^{-\zeta(\tau-\tau_i)} \\ \quad + \frac{\omega f_0}{(1-\omega^2)^2+4\omega^2\zeta^2}\left[2\omega\zeta\cos(\omega\tau) - (1-\omega^2)\sin(\omega\tau)\right] \\ \text{with: } \begin{array}{l} A = v_i - f_0\frac{(1-\omega^2)\cos(\omega\tau_i)+2\omega\zeta\sin(\omega\tau_i)}{(1-\omega^2)^2+4\omega^2\zeta^2} \\ B = \frac{\dot{u}_i+\zeta A}{\sqrt{1-\zeta^2}} - \frac{\omega f_0}{\sqrt{1-\zeta^2}}\frac{2\omega\zeta\cos(\omega\tau_i)-(1-\omega^2)\sin(\omega\tau_i)}{(1-\omega^2)^2+4\omega^2\zeta^2} \end{array} \end{cases} \tag{6}$$

In the case of two plastic states $\hat{P}^+$, $\hat{P}^-$:

$$\hat{P}:\begin{cases} v(\tau) = \pm 1 \\ \dot{u}(\tau) = \left[\dot{u}_i + \frac{v_i}{2\zeta} - f_0\frac{2\zeta\cos(\omega\tau_i)+\omega\sin(\omega\tau_i)}{4\zeta^2+\omega^2}\right]e^{-2\zeta(\tau-\tau_i)} \\ \quad -\frac{v_i}{2\zeta} + f_0\frac{2\zeta\cos(\omega\tau)+\omega\sin(\omega\tau)}{4\zeta^2+\omega^2} \end{cases} \tag{7}$$

4.2 Response of the Asymmetric system

Once the equivalence between asymmetric loading and asymmetric resistance is formulated analytically [8], the periodic solution of the asymmetric model (Fig. 2b) is given for the elastic vibrations, by:

$$\hat{E}:\begin{cases} v(\tau) = \begin{bmatrix} \cos\left[\sqrt{1-\zeta^2}(\tau-\tau_i)\right]\left(v_i - f_0\frac{(1-\omega^2)\cos(\omega\tau_i)+2\omega\zeta\sin(\omega\tau_i)}{(1-\omega^2)^2+4\omega^2\zeta^2}\right) + \\ \sin\left[\sqrt{1-\zeta^2}(\tau-\tau_i)\right]\left(\frac{\dot{u}_i+\zeta v_i}{\sqrt{1-\zeta^2}} + f_0\frac{-\zeta(1+\omega^2)\cos(\omega\tau_i)+\omega(1-\omega^2-2\zeta^2)\sin(\omega\tau_i)}{\sqrt{1-\zeta^2}((1-\omega^2)^2+4\omega^2\zeta^2)}\right) \end{bmatrix} e^{-\zeta(\tau-\tau_i)} \\ \quad + f_0\frac{\left[(1-\omega^2)\cos(\omega\tau)+2\omega\zeta\sin(\omega\tau)\right]}{(1-\omega^2)^2+4\omega^2\zeta^2} \\ \dot{u}(\tau) = \begin{bmatrix} \left(\dot{u}_i + f_0\frac{-2\omega^2\zeta\cos(\omega\tau_i)+\omega(1-\omega^2)\sin(\omega\tau_i)}{(1-\omega^2)^2+4\omega^2\zeta^2}\right)\cos\left[\sqrt{1-\zeta^2}(\tau-\tau_i)\right] + \\ \left(-\frac{v_i+\zeta\dot{u}_i}{\sqrt{1-\zeta^2}} + f_0\frac{(2\omega^2\zeta^2+1-\omega^2)\cos(\omega\tau_i)+\left(\omega\zeta(1+\omega^2)\right)\sin(\omega\tau_i)}{\sqrt{1-\zeta^2}((1-\omega^2)^2+4\omega^2\zeta^2)}\right)\sin\left[\sqrt{1-\zeta^2}(\tau-\tau_i)\right] \end{bmatrix} e^{-\zeta(\tau-\tau_i)} \\ \quad + \omega f_0\frac{\left[2\omega\zeta\cos(\omega\tau)-(1-\omega^2)\sin(\omega\tau)\right]}{(1-\omega^2)^2+4\omega^2\zeta^2} \end{cases}$$

(8)

For the two plastic states $\hat{P}^{+}$, $\hat{P}^{-}$, the response is written as:

$$\hat{P}: \begin{cases} v(\tau) = 1 \quad \text{or} v(\tau) = -1 - \varepsilon \\ \dot{u}(\tau) = \left[\dot{u}_i + \frac{v_i}{2\zeta} - f_0 \frac{2\zeta\cos(\omega\tau_i) + \omega \sin(\omega\tau_i)}{4\zeta^2 + \omega^2}\right] e^{-2\zeta(\tau - \tau_i)} \\ \qquad\qquad - \frac{v_i}{2\zeta} + f_0 \frac{2\zeta\cos(\omega\tau) + \omega\sin(\omega\tau)}{4\zeta^2 + \omega^2} \end{cases} \tag{9}$$

5. Evolution of the Dynamical Systems

By following the evolution of the dynamic systems presented by the equations of motion (Eq.1; 4), which are controlled by the conditions of space phases (Eq.2, 5), expressed by the response (Eq.6;7; 8 and 9), it can clearly distinguished two types of movement called behavioural areas: “Shakedown” which represents an elastic stationary phase, and the “Alternating plasticity” which is an alternation of two plastic phases through an elastic state (Fig. 3) [7], [8].

The limit cycles controlled by the alternating plasticity area presents a central symmetry [6], [8], and thus, these cycles depend of the structural parameters (f_0,ζ,ω). So, according to the works of Challamel [6], [7], and Hammouda [8], [9], there is a bifurcation boundary (Fig. 3) between the two behavioural areas: Shakedown and Alternating plasticity, in which one can consider an asymptote phase to $|v|$= 1, this boundary [2], [3] is expressed by

$$f_0 = \sqrt{(1-\omega^2)^2 + 4\omega^2\zeta^2} \tag{10}$$

This bifurcation boundary (Eq.10) is the same as in the works of Liu and Huang [10]:

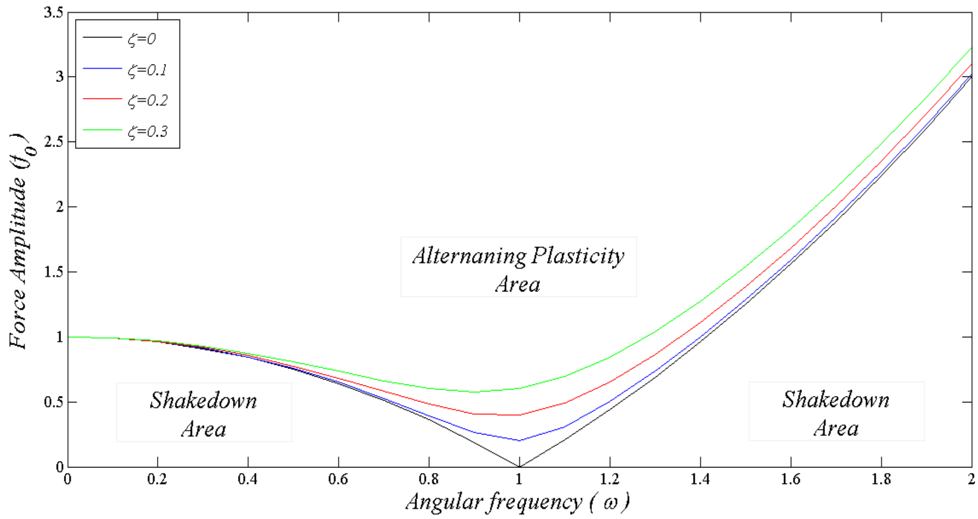

Figure.3: Bifurcation boundary between the behavioural areas

In the reduced phase space, from the works of Capecchi [11], the displacement (v) can represent the force associated to the dimensionless dynamic system [7],[8],[9],[12]. The representation of the behavioural relationship can be expressed by the elastic displacement versus total displacement, which is equivalent to the relationship Force-Displacement illustrated in (Fig. 2).

For the elastoplastic oscillations, the chosen relationship: Force -Displacement, is asymptotic to ($|\, v\,|$ = 1), as shown from a hysteresis case of the relation (v,u), which can be called “Adopted Behaviour” (Fig.4). Similar results are given by Ahn [13].

6. Dissipated energy of the elastoplastic oscillator

6.1 Energetic assessment of the symmetric system

The energy aspect of this perfect, damped and symmetrical elastoplastic oscillator, assumes that the dissipated energy by the oscillator is indeed, the air of the hysteresis loop coming from the limit cycle of the (v, u) behaviour [9], [14].

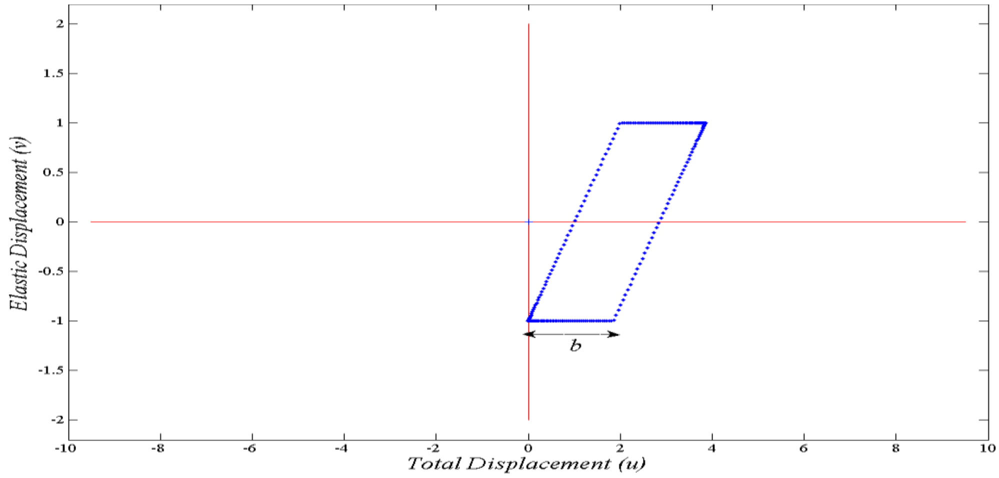

Figure. 4: Adopted Behaviour (hysteresis loop),
For:$(\boldsymbol{v_0}, \boldsymbol{\dot{u}_0}) = (\mathbf{0}, \mathbf{1}); \boldsymbol{\zeta} = \mathbf{0.1}; \boldsymbol{\omega} = \mathbf{0.5}\ ; \boldsymbol{f_0} = \mathbf{1}$

The calculus of the dissipated energy during oscillations, for a set of appropriate structural parameters (which represents the behavioural area of alternating plasticity in Fig. 3, where there is an alternation between the elastic and plastic phases), may be approximately equal to :[9]

$$E(\text{Limit Cycle}) = \int_0^{\tau} f(\zeta)u(\zeta)d(\zeta) \tag{11}$$

$$\overline{E} = 2 * b \tag{12}$$

The energetic assessment can be illustrated through a curve often called “energy map” (Fig.5), independent of the initial conditions, because it concerns the limit cycle (after convergence), which shows the agreement with the boundary bifurcation (Fig.3) [8].

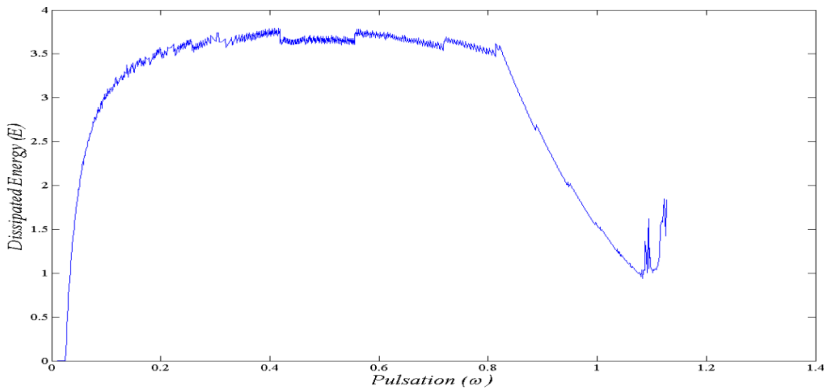

Figure. 5: Energy Assessment for the perfect damped and symmetric elastoplastic oscillator for:$(\boldsymbol{v_0}, \boldsymbol{\dot{u}_0}) = (\mathbf{0}, \mathbf{0}); \boldsymbol{\zeta} = \mathbf{0.1}; \boldsymbol{f_0} = \mathbf{1}$

This assessment of the dissipated energy (Fig. 5) during the oscillations of perfect damped elastoplastic model in equal amplitude values of excitation force (Symmetric) , highlights an energy variation with very close values of the equivalent pulse , and valid for the same occasion the next one (Fig. 6), presented by the recent works of Hammouda et al , [9] , in the energetic and rheological study of this type of hysteretic oscillations, and we can affirm that for small values of pulse , energy is greatly dissipated , following the super-harmonic motions due to the fact that the system acts as a rigid plastic oscillator. This phenomenon is previously mentioned in the works of Capecchi [15] for an undamped system, and those of Liu and Huang [10] for the same damped system, as well as Challamel and Gilles [6] . We can say that this work discusses the most important perspective of the previous ones [8-9] by studying the variation of the energy according to the structural parameters, such as damping, which represents the energy dissipation device widely discussed in this topic.

The energy -theoretically- dissipated by the studied oscillator, can be expressed, according to the damping parameter(ζ), which should not -in any case- be ignored, for their connection, because of the damping term is always linked to the reverse conversion side of the energy to return the dynamic system to its initial position [14], the following curve(Fig.6) exhibit this relationship:

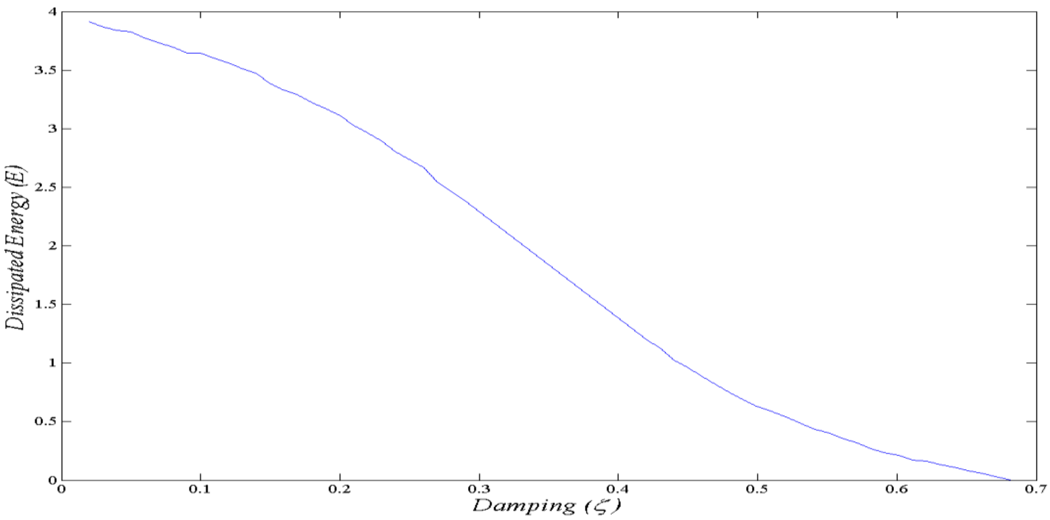

Figure.6: Energy Assessment for the perfect damped and symmetric elastoplastic oscillator for:$(\boldsymbol{v_0}, \dot{\boldsymbol{u}}_{\boldsymbol{0}}) = (\boldsymbol{0}, \boldsymbol{0}); \boldsymbol{\omega} = \boldsymbol{0.5}; \boldsymbol{f_0} = \boldsymbol{1}$

The energy assessment represented by this curve (Fig. 6) expresses the variation of the energy according to different damping rates, and criticizes the idea to always increase the damping of an oscillator to obtain better energy dissipation, something to deal with delicacy, because we must not deny the existence of the dissipated energy by the periodic motions of the oscillator, which causes the increase of the stored and absorbed energy by its stiffness. This result validates the third conclusion of the works of Zhang and Iwan [16], which assert the existence of the phenomenon of small oscillations ride on the component (damping), causing long-period excitations, therefore, this undesirable phenomenon makes the dissipation of reduced energy. That feeds also, on choosing a good set of parameters taken from the bifurcation boundary (Fig.3), to stay in the Alternating plasticity behavioural area.

For an undamped model (Fig. 7), it can clearly realize that, although the model is not damped, but we can notice a decrease in the dissipated energy during the cyclical movements, which confirms the discussion of the curve above (Fig. 6):

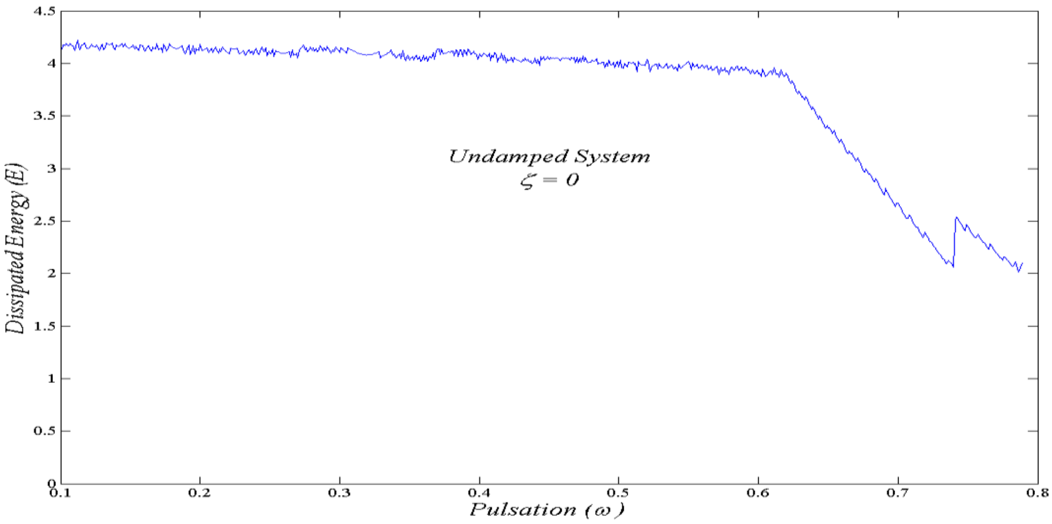

Figure. 7 **:** Energy Assessment for the perfect damped and symmetric elastoplastic oscillator for:$(\boldsymbol{v_0}, \boldsymbol{\dot{u}_0}) = (\mathbf{0}, \mathbf{0}); \boldsymbol{\zeta} = \mathbf{0}; \boldsymbol{f_0} = \mathbf{1}$

The variation of the energy, dissipated by the elastoplastic damped model, can be also expressed (Fig. 8) in function of the parameter of the outside excitation force (f_0), for a fixed damping and pulsation coefficients:

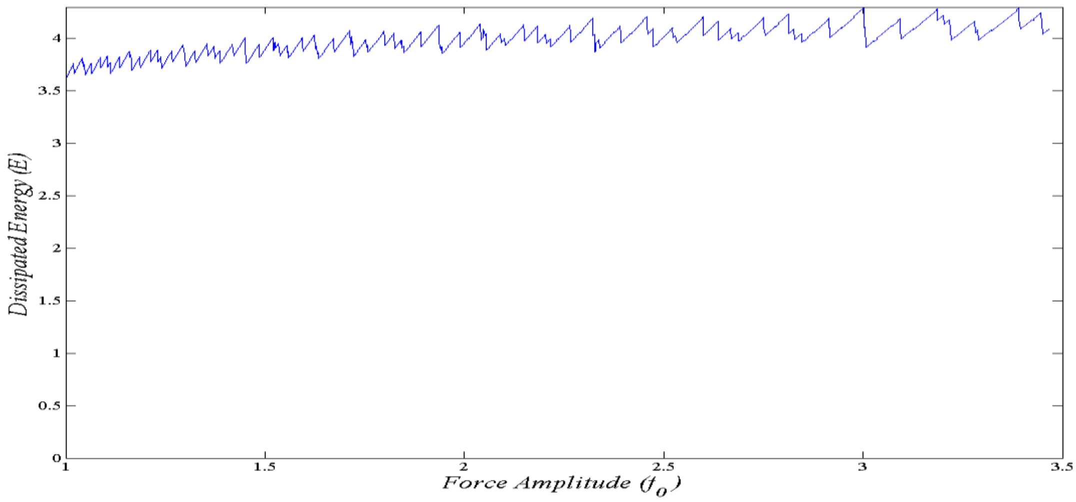

Figure. 8: Energy Assessment for the perfect damped and symmetric elastoplastic oscillator for:$(\boldsymbol{v_0}, \boldsymbol{\dot{u}_0}) = (\mathbf{0}, \mathbf{0}); \boldsymbol{\zeta} = \mathbf{0.1}; \boldsymbol{\omega} = \mathbf{0.5}$

This curve shows the high variability in the dissipated energy by increasing the amplitude of the external force, but moderately, the relationship is not stronger than that of previous curves, and it is known that a dynamic load with high amplitude and low pulsation, does not necessarily give the same effect as another load of a higher pulse, and through a lower amplitude [14].
In this section of a perfect damped, and symmetric elastoplastic oscillator (Fig. 2.a), we have treated the variation of the dissipated energy, according to the different structural parameters and establish an energy assessment, thus to carefully study and analyse this type of non linear oscillator, and especially to give an idea about the choice of parameters playing a role in the response of this model. The authors perspective in previous study [9], was to study the oscillator when it presents an asymmetry ($F^+ \neq F^-$) in its behaviour (Fig.2.b), whether in traction or compression, a prospect discussed in what follows.

6.2 Energetic assessment of the asymmetric system

In this section, we will enter directly to the energy assessment of the asymmetric system without going through the evolution of this system (see [8]).This system brings up a new structural parameter denoted (ε), which represents the difference between (F^{+})and(F^{-}) in compression or tension field, but without affecting the choice of the other parameters, that it was previously mentioned [8],[9],and it was concluded that the bifurcation boundary does not depend on the asymmetry parameter (ε).

The following curve (Fig.9) presents the Adopted behaviour ($v - u$) during the oscillations (hysteresis loops):

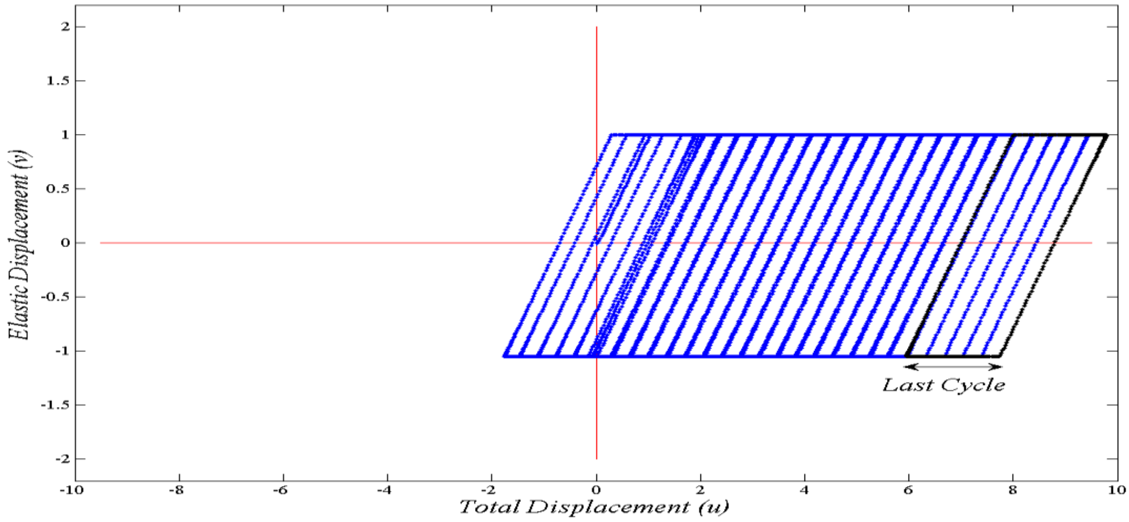

Figure. 9. Behaviour of the perfect damped and asymmetric in compression elastoplastic oscillator ($F^{+}< F^{-}$), "Dynamic ratcheting" for:$(\boldsymbol{v_0}, \boldsymbol{\dot{u}_0}) = (\mathbf{0}, \mathbf{0}); \boldsymbol{\zeta} = \mathbf{0.1}; \boldsymbol{\omega} = \mathbf{0.75}, \boldsymbol{f_0} = \mathbf{0.8}, \boldsymbol{\varepsilon} = \mathbf{0.05}$

This curve clearly shows the effect of "rochet" observed for this type of oscillator, which caused by the asymmetry of compression amplitude, this effect occurs when increasing a field strength compared to the other (traction or compression), and therefore leads to a divergence of the limit cycle [8].

We can also study the energy assessment of such oscillators, starting with the variation of the dissipated energy according to the pulsation (Fig. 10), and this for a fixed asymmetry rate (ε):

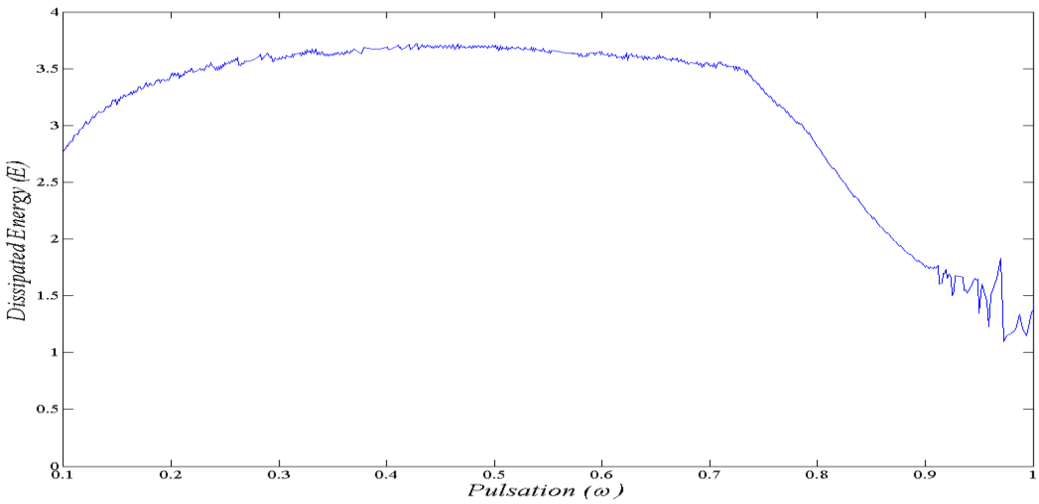

Figure.10: Energy Assessment of the perfect damped and a symmetric elastoplastic oscillator for :$\boldsymbol{\zeta} = \mathbf{0.1};\ \boldsymbol{f_0} = \mathbf{0.8};\ \boldsymbol{\varepsilon} = \mathbf{0.05}$

The variation of the dissipated energy which is equal to the air of the last hysteresis loop, depending on the equivalent angular frequency of the system for a fixed asymmetry, is not different from that of the symmetric oscillator. This result leads us to express the energy dissipation according to the progressive variation of the asymmetry rate (ε), to investigate its influence (Fig. 11).

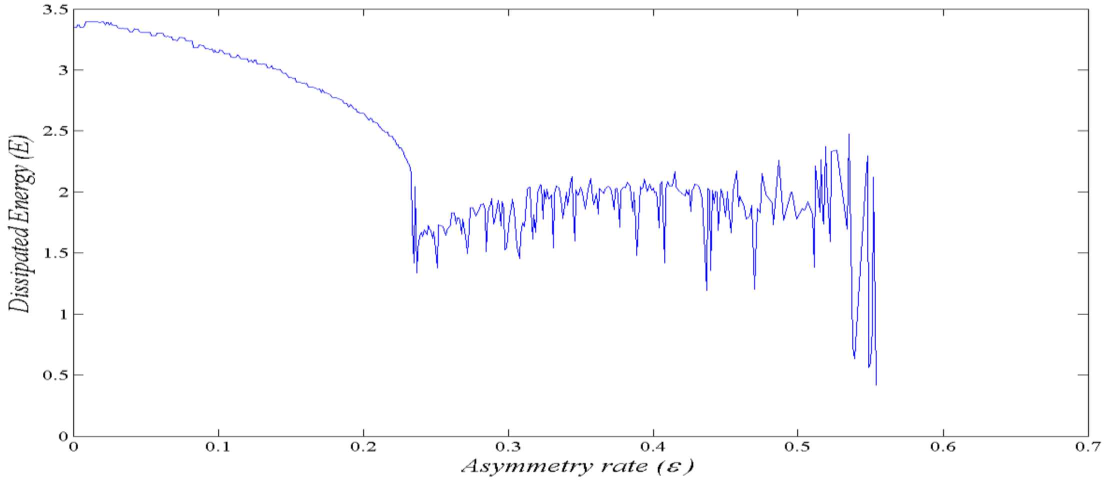

Figure. 11. Energy Assessment of the perfect damped and asymmetric elastoplastic oscillator for: $\boldsymbol{\zeta = 0.1;\ f_0 = 0.8;\ \omega = 0.75}$

This curve (Fig. 11) shows the "Dynamic ratcheting", for an asymmetry between the two domains leading to a significant decrease of the dissipated energy for higher asymmetry, something to be avoided and must be taken into consideration on seismic design [8].

Besides the parameter that we have just to distinguish, who plays a very important role in the energy assessment of the asymmetric model, the dissipated energy also depends on other structural parameters (Fig. 12).

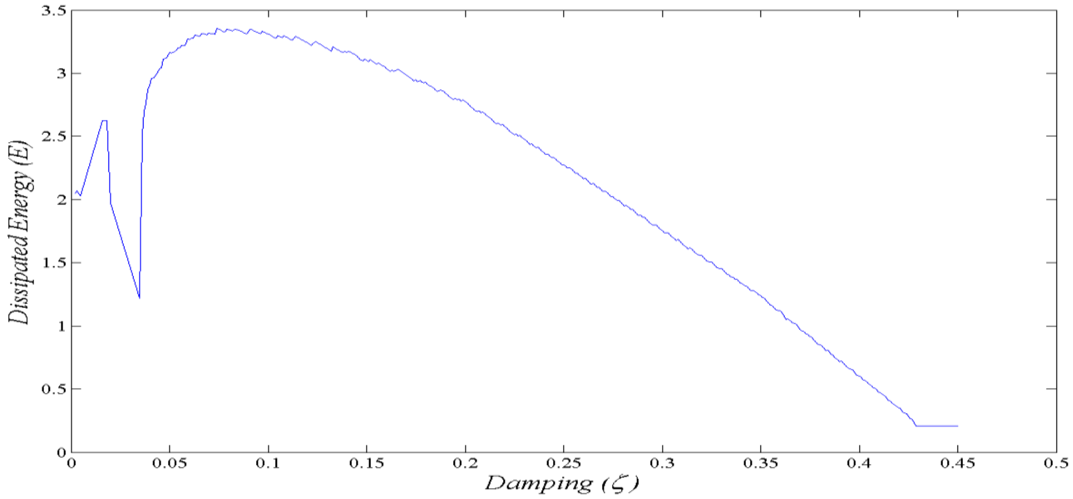

Figure. 12. Energy Assessment of the perfect damped and asymmetric elastoplastic oscillator for : $\boldsymbol{\varepsilon = 0.05;\ f_0 = 0.8;\ \omega = 0.75}$

In terms of damping and energy dissipation, the above figure (Fig. 12) supports the assumption discussed earlier (Fig.6), concerning the choice of damping parameter, which must not tolerate the excess of oscillations energy absorption, and consequently, reducing dissipation [14].

So, we can at the last point of the energy assessment analysis, varying the force amplitude, to study the variation of the dissipated energy while remaining -naturally- in the alternating plasticity area (Eq.10, Fig.3).

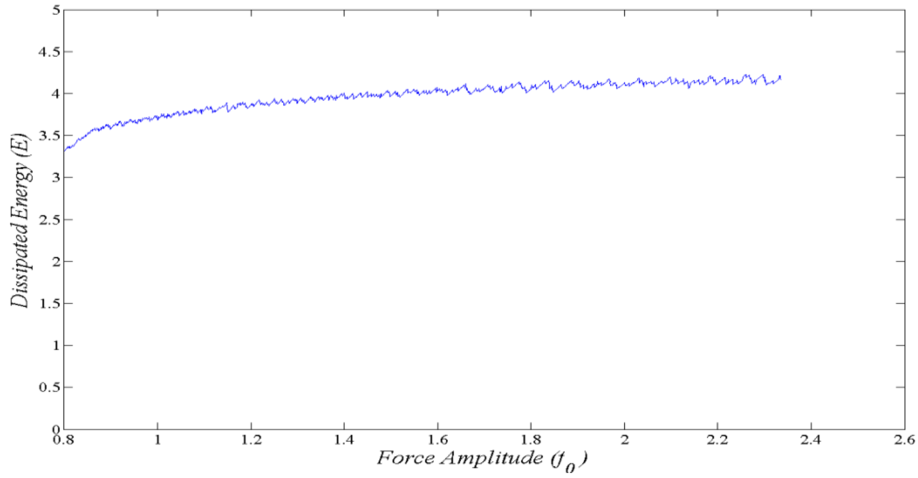

Figure. 13. Energy Assessment of the perfect damped and asymmetric elastoplastic oscillator for :$\boldsymbol{\varepsilon} = \mathbf{0.05}$; $\boldsymbol{\zeta} = \mathbf{0.1}$; $\boldsymbol{\omega} = \mathbf{0.75}$

The influence of the amplitude of the external excitation of the asymmetric in compression model, cannot give us a clear vision on the choice of this parameter, because of the increase in the dissipated energy is caused -of a part- by the increase of the kinematic energy from the cyclic movements.

7. Conclusion

This work represents a part of a wide program studying the energy assessment by exposing the variation of the dissipation energy of a perfect damped elastoplastic oscillator, whose can reflect the behaviour of structures subjected to dynamic loads of seismic type in both cases: symmetric and asymmetric, depending on structural parameters that characterize the behavioural areas of this oscillator. It was found that the dissipated energy in the first case where the behaviour is symmetrical, essentially depends on the equivalent angular frequency between the external force and the own pulsation of the model.

Regarding the damping term, often associated with the dissipated energy, we have demonstrated that the dissipation of energy is not exclusively due to the devices and damping elements of the model, but it can also caused by the cyclic movements in the hysteresis loops.
The second case brings up another structural parameter, which plays a very important role in the energy dissipation, which is manifested by the “Dynamic ratcheting” in alternating plasticity that implies the divergence of phase trajectories, and therefore the domination of one domain to the other, without contradicting the choice of the other parameters.

We can, at the end of this study indicate that the main prospects is to consider another behaviour law, more complex such as in the presence of kinematic hardening elastoplasticity [17], which will also appear the hardening rate as a new structural parameter and this will probably could enrich the topic.

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