Confining pressure effects on stress intensity factors: A 3D finite element analysis

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ملخص

في الطبقات الأرضية العميقة، يحدث انكسار الصخور الكتلية تحت تأثير ضغط الاحتباس. لكن جل الدراسات السابقة الخاصة بانكسار الصخور اهتمت بالظروف العادية وتجاهلت تأثير الاجهادات الناتجة عن ضغط الاحتباس. في هذه الدراسة، تم بحث تأثير هذا الضغط على الانكسار المختلط (كيفية I وكيفية II) باستعمال عينة اختبار. عدد معتبر من الأنماط المنتهية للعنصر 3D تم تحليلها بدلالة الضغط وانكسار العينة البرازيلية على شكل قرص. الضغط المتساوي التوزيع الممتد من 0 إلى 20 MPa تم استعماله على جميع أطراف العينة. تظهر النتائج أن الضغط له تأثير بالغ على معامل شدة الاجهادات في الكيفية I، أما فيما يخص الكيفية II ، فان التأثير معدوم. المقارنة بين وضعتي الاحتباس واللااحتباس تبين أن معامل شدة الاجهادات الفعلي للكيفيات المختلطة يكبر مع زيادة قيمة الضغط مهما كان نمط الاختلاط في تحميل العينات.

الكلمات المفتاحية: ضغط الاحتباس؛ الانكسار المختلط؛ معامل شدة الاجهادات؛ قرص برازيلي؛دراسة رقمية.

Résumé

A des profondeurs importantes de la terre, la rupture dans les masses de roches se produit sous l'influence de la pression de confinement. Cependant, la plupart des études précédentes de rupture de roche traitent seulement dans des conditions ambiantes et ignorent les effets des efforts multiaxiaux induits en raison de l'application de la pression de confinement. Dans cette étude, l'influence de la pression sur les facteurs d'intensité de contrainte en modes mixtes (K_I et K_{II}) est étudiée pour un spécimen d'essai. Un grand nombre de modèles finis de l'élément 3D ont été analysés pour étudier l'effet de divers niveaux de confinement de pression sur le comportement de rupture du spécimen brésilien sous la forme de disque. Pour les modèles d'éléments finis, une pression uniformément distribuée dans l'intervalle de 0 à 20 MPa est appliquée sur tous les côtés du spécimen d'essai. Il est montré que la pression de confinement a un effet significatif sur le facteur d'intensité de contrainte en mode I, mais celui de mode II est complètement indépendant de la pression de confinement appliquée. Une comparaison entre les conditions de confiné et non confiné indique que le facteur d'intensité de contrainte en mode mixte augmente avec la valeur de la pression de confinement quelque soit le mode mixte considéré.

Mots clés: *pression de confinement; mode de rupture mixte; facteurs d'intensité de contraintes; Disque brésilien ; méthode numérique.*

Abstract

At great depths of earth, fracture in rock masses occurs under the influence of confining pressure. However, most of the previous rock fracture studies deal only with ambient conditions and ignore the effects of multiaxial stresses induced because of applying the confining pressure. In this paper, the influence of confining pressure on the mixed mode stress intensity factors (KI and KII) is investigated for a test specimen. A large number of 3D finite element models were analyzed for investigating the effect of various confining pressure levels on the fracture behaviour of brazilian disc specimen. In the finite element models a uniformly distributed confining pressure in the range of 0 MPa to 20 MPa is applied to all sides of the test specimen. It is shown that the confining pressure has a significant effect on the mode I stress intensity factor, but the mode II stress intensity factor is completely independent of the applied confining pressure. A comparison between the confined and unconfined conditions shows that the effective mixed mode stress intensity factor increases by increasing the value of confining pressure for all mode mixities.

Key words: *confining pressure; mixed mode; stress intensity factors; brazilian disc; numerical method.*

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1. INTRODUCTION

Many rock structures like the underground mines and tunnels are often created at great depths of earth and therefore, these structures are usually subjected to complex and multiaxial states of stress. For strength and stability analysis of such structures, attention to the effects of these multiaxial loads is essential. As a practical example, suitable stimulation of gas and oil reservoirs using hydraulic fracturing process needs a good knowledge of rock fracture behaviour at great depths and in-situ conditions. The amount of energy required for creating and extending artificial fractures in hydraulic fracturing depends strongly on fracture toughness of formation rocks in the petroleum reservoir. These rock masses are usually subjected to very high confining pressures at in situ conditions (especially in cases with typical depths of 7000-8000 m). Fracture toughness is an important design parameter showing the rock's resistance to crack initiation and propagation. However, most of the theoretical and experimental fracture investigations on rock materials have focused on the ambient conditions and hence ignore the effects of confining pressure that exists practically for in-situ stress conditions. Very few theoretical and experimental studies can be found in the literature dealing with the effects of confining pressure on fracture toughness of rock materials. For example, Schmidt and Huddle studied the effect of confining pressure on mode I (crack opening mode) fracture toughness of Indiana limestone [9]. Meanwhile, Muller has obtained the fracture toughness (K_{Ic}) of sandstone and granite under various confining pressures [8]. More recently, Funatsu et al. has investigated the fracture toughness of confined clay bearing rocks (Kimachi sandstone and Tage tuff) using two laboratory test specimens under mode I loading. Balme et al. have also conducted similar studies on igneous rocks [3,5]. While crack growth and fracture of rocks at in situ conditions usually occurs under a combination of tension and shear loading (mixed mode I/II), the influence of confining pressure on mixed mode fracture toughness of rocks has been rarely investigated in the past. In this paper, the effect of confining pressure on the crack tip stresses is investigated numerically for the centrally cracked Brazilian disc (BD) specimen using the finite element method. The influence of confining pressure on the mode I and mode II (crack in-plane sliding) stress intensity factors is studied. In the forthcoming sections, first the test specimen is described and then the influence of confining pressure on the BD specimen is explored.

2. BD SPECIMEN SUBJECTED TO CONFINING PRESSURE

The BD specimen is a circular disc of radius R with a central crack of length $2a$ and subjected to a diametral compression load F. By changing the crack orientation angle ~ with respect to the applied load F, different combinations of mode I and mode II deformations can be provided including pure opening, pure sliding, combined tension–shear, and combined compression–shear loading. The specimen has been widely used for mixed mode fracture tests in rocks and geo-materials under ambient and atmospheric conditions [1,4,6,7]. For simulating the confining pressure, a constant distribution of pressure (P_m) is applied on all of the external surfaces of the Brazilian disc specimen. The loading set up is shown schematically in Figure 1.

The stress intensity factors are the most important parameters for describing the crack tip stress field. These parameters define the severity of stress singularity around the crack tip and can be used for determining the fracture load of a cracked body. The mode I and mode II stress intensity factors (K_{I} and K_{II}) for the BD specimen at ambient conditions are written as [2]:

$$K_{\mathrm{I}} = Y_{\mathrm{I}} \frac{F}{Rt}\sqrt{\frac{a}{\text{®}}} \qquad (1)$$

$$K_{\mathrm{II}} = Y_{\mathrm{II}} \frac{F}{Rt}\sqrt{\frac{a}{\text{®}}} \qquad (2)$$

where Y_{I} and Y_{II} are the geometry factors corresponding to mode I and mode II, respectively and t is the specimen thickness.

The finite element method was used for calculating K_{I} and K_{II} in the BD specimen for various crack orientation angles (~) and under both diametral compression and confining pressure loads. The finite element simulation for the mentioned loading conditions is described in the next section.

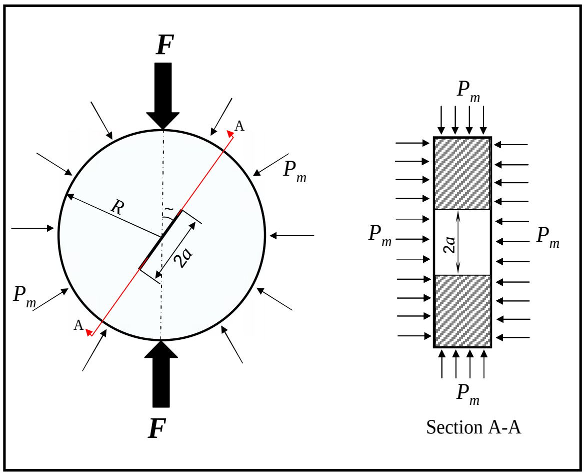

Figure 1. *Loading setup for the cracked Brazilian disc specimen subjected to confining pressure P_m.*

3. FINITE ELEMENT MODELING

Figure 2 shows the finite element model of BD specimen. In this model, the following geometry and loading parameters are considered: R = 50 mm, a = 10, 20, 30 mm, t = 25 mm and F = 10 KN. The total number of 21000 twenty-node cubic solid elements was used for 3D modelling of BD specimen. For simulating crack tip stress singularity, the singular elements were also considered at the first ring of elements around the crack tip. The confining pressure P_{m} was applied in a range from zero to 20 MPa. Also for introducing different mode mixities, crack orientation angle was varied from zero to 80^{o} with increments of 4 degrees. Material properties were selected from a typical rock with modulus of elasticity E =53 GPa and Poisson's ratio ν = 0.23. The ABAQUS code was used for analyzing the models. The J-integral based method was used for calculating the stress intensity factors K_{I} and K_{II}. The results obtained for the stress intensity factors are presented in the following for various crack angles and confining pressures.

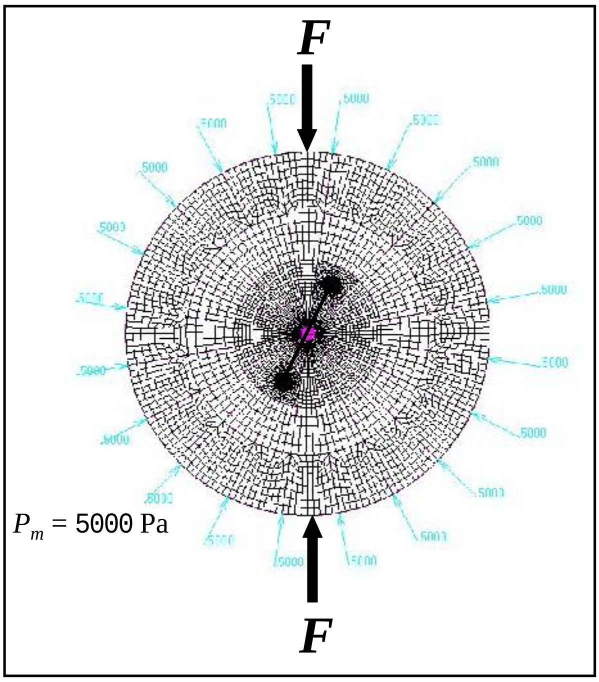

Figure 2. *Finite element model for the cracked Brazilian disc specimen subjected to both diametral compression and confining pressure P_m.*

4. RESULTS AND DISCUSSION

The variations of K_I and K_{II} with ~ for various values of confining pressure P_m are shown in Figures 3 and 4 for a/R = 0.2, 0.4 and 0.6. It is seen from these Figures that the mode I stress intensity factor reduces by increasing ~. However, the mode II stress intensity factor increases when ~ increases and reaches a maximum value (typically at ~ between 28^o to 44^o) depending on a/R. Based on the results shown in Figures 3 and 4, while the confining pressure has a significant effect on K_I, the mode II stress intensity factor K_{II} is independent of P_m.

As mentioned earlier, rock structures and rock masses are usually subjected to multiaxial states of stress. In real conditions, cracked rock masses experience a confining pressure that is applied from all directions to the rock. Hence the influence of confining pressure on fracture toughness and crack growth must be taken into account. The numerical results obtained in this research showed that the confining pressure has significant influences on the fracture behaviour of cracked rock masses located at great depths. However, in most of the rock fracture studies the ambient conditions (P_m = 0) are considered. As shown in Figure 3, the mode I stress intensity factor for all crack orientation angles or mode mixities depends strongly on the magnitude of P_m. In general, there is a significant reduction in K_I when P_m increases. This is because the compressive confining pressure applied to the cracked specimen tends to close the crack and compress the crack faces to each other. Consequently for such high confining pressures, K_I will decrease. This implies that for fracturing the cracked body under confining pressure, a larger fracture load F is required. The theoretical finding obtained in this research is in agreement with the reported experimental studies.

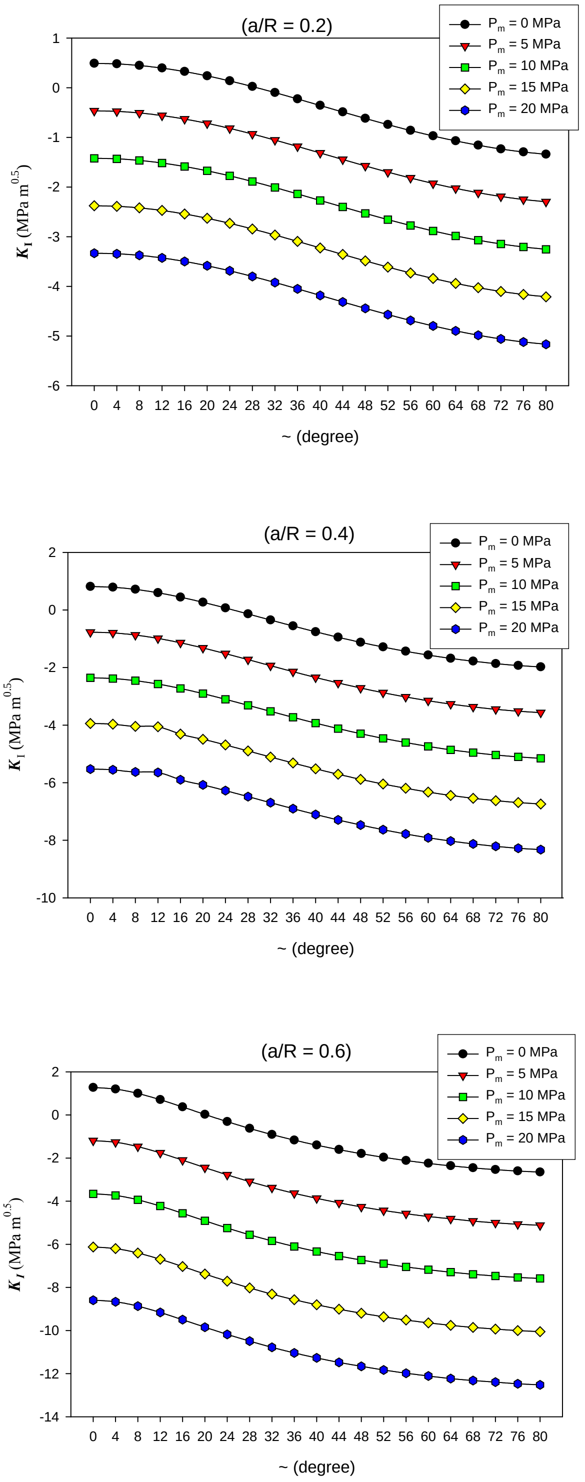

Figure 3. *Variations of mode I stress intensity factor K_I with crack angle ~ for various crack length ratios and confining pressures P_m in BD specimen.*

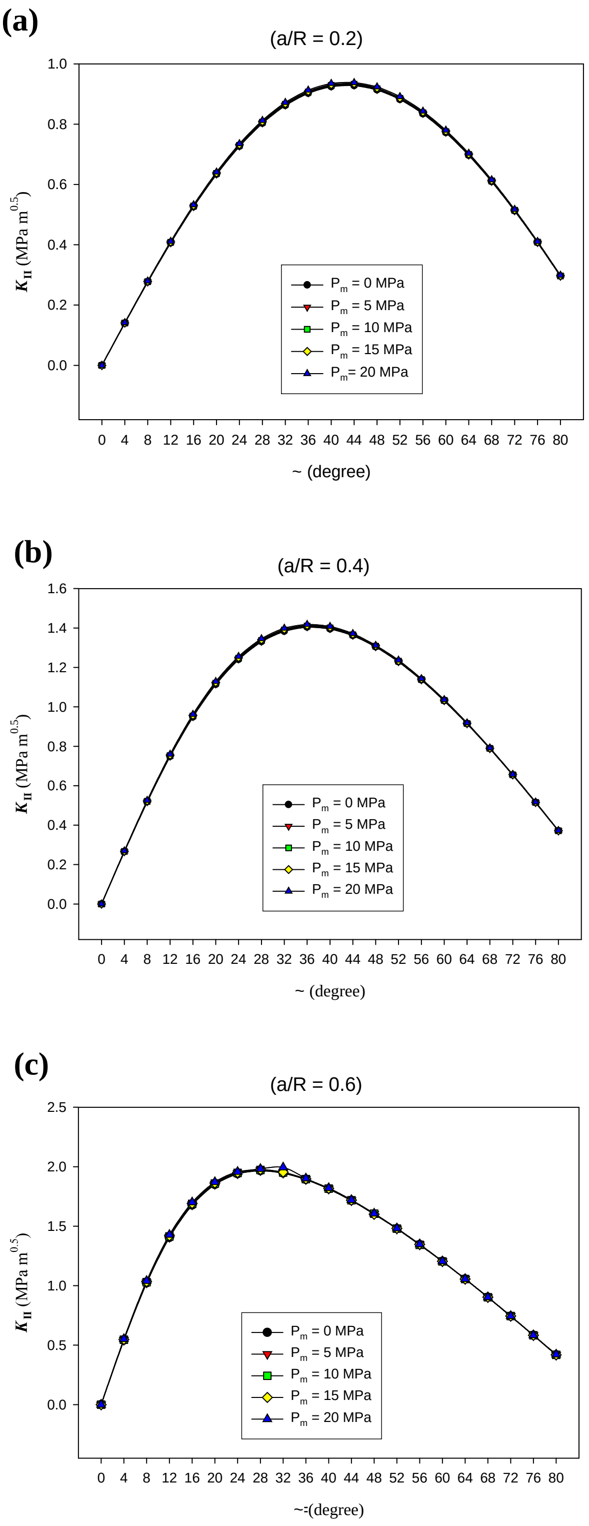

Figure 4. *Variations of mode II stress intensity factor K_{II} with crack angle ~ for various crack length ratios and confining pressures P_m in BD specimen, (a) a/R=0.2, (b) 0.4, (c) 0.6 and Pm in the range 0-20 MPa.*

Based on the experimental data available in the literature for fracture toughness in several types of rocks under various confining pressures [3,5,8,9], the fracture resistance of the tested rocks increases by increasing the confining pressure. Meanwhile, the finite element results showed that the application of confining pressure has no effect on mode II or shear deformation of Brazilian disc specimen. Since the applied confining pressure acts like a hydrostatic stress, it can affect only K_I and not K_{II}.

Also a comparison between different crack length ratios (*a/R*) in Figure 3 shows that the influence of confining pressure is more pronounced for larger cracks. In Figure 5, the effective mixed mode stress intensity factor K_{eff} is plotted versus the crack inclination angle ~ for various values of P_m and *a*/*R*. K_{eff} represents the effects of both mode I and mode II stress singularities and is defined

$$K_{eff} = \sqrt{K_I^2 + K_{II}^2} \tag{3}$$

It is seen from this Figure that for all mode mixities the effective stress intensity factor increases for higher values of P_m. Also, K_{eff} increases for larger crack length ratios *a/R*. These diagrams in association with mixed mode fracture criteria can be used as design curves for estimating the load or energy required for creating and extending fracture (in such cases as hydraulic fracturing) under confining pressure at great depths.

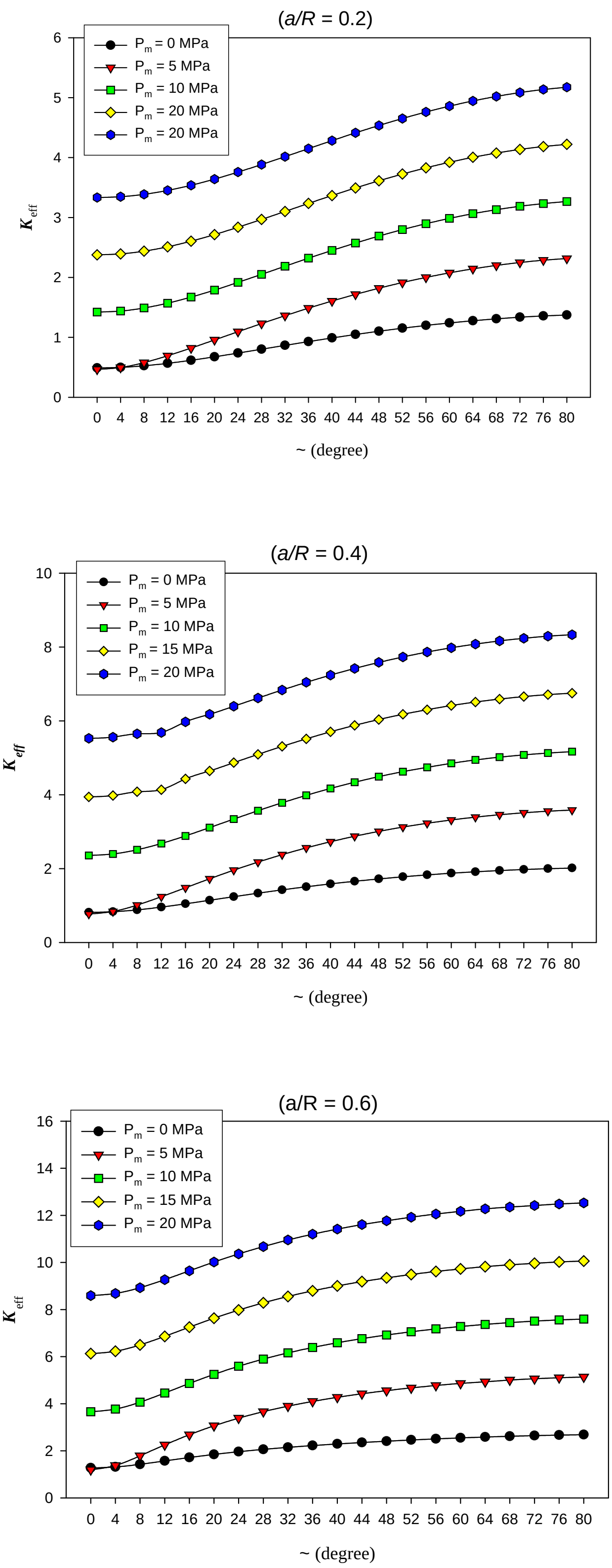

***Figure* 5.** *Variations of effective mixed mode stress intensity factor (K_{eff}) with crack angle (~) for various crack lengths and confining pressures P_m in BD specimen.*

5. CONCLUSION

The main conclusions of this study are as follows:

1- The effects of confining pressure which usually exists at in-situ stress conditions must be taken into account on fracturing process of cracked rock masses.

2- Based on the finite element results obtained for the centrally cracked Brazilian disc specimen subjected to different confining pressures (P_m), there is a significant reduction in the mode I stress intensity factor when P_m increases. However, application of confining pressure has no influence on the mode II stress intensity factor.

3- The effective mixed mode stress intensity factor (K_{eff}) strongly depends on confining pressure for all combinations of mode I and mode II. Generally K_{eff} increases for deeper cracks.

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