SINET: *Ethiop. J. Sci.*, 29(1):1–16, 2006

ISSN: 0379–2897

# DESIGN CHART PROCEDURES FOR POLYGONAL CONCRETE-FILLED STEEL COLUMNS UNDER UNIAXIAL BENDING

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**ABSTRACT**: High quality moment-axial-force interaction diagrams have been developed for hexagonal and octagonal steel-concrete composite columns subjected to uniaxial bending. Comparative discussion with the procedures stipulated in relevant building code standards has been presented. A unified approach has been presented for the procedure of establishing design charts for concrete-filled steel tubes under uniaxial bending and valuable charts have been prepared for hexagonal and octagonal shape composite columns. This paper also outlines procedures that will enable preparation of similar design charts for other shapes and material types.

**Key words/phrases: Concrete-filled tubes (CFT), hexagonal/octagonal columns, interaction diagram**

## INTRODUCTION

Hexagonal steel columns and to a limited extent octagonal columns are used to impart aesthetic values to building structures and bridge piers besides their structurally improved buckling properties as compared to rectangular or circular shapes with regard to the steel component of the cross sections. They are also used as flood-light poles and in communications as well as transmission structures. Their strength and other structural properties are enhanced by making use of steel-concrete composite form of these columns.

Composite construction lies between steel-only and concrete-only constructions. It is the most important and most frequently encountered combination of construction materials with applications in multi-storey buildings, industrial buildings and in a variety of large-span building systems. They are also employed in large-span bridge piers and towers.

Composite steel-concrete columns are structural elements that can assume a variety of shapes and compositions depending on, among other things, the loading type and magnitude. Typical cross sections of such columns are shown in Fig. 1.

These structural elements make use of the attractive structural and non-structural features of each of the constituent materials while they minimize their undesirable features and properties

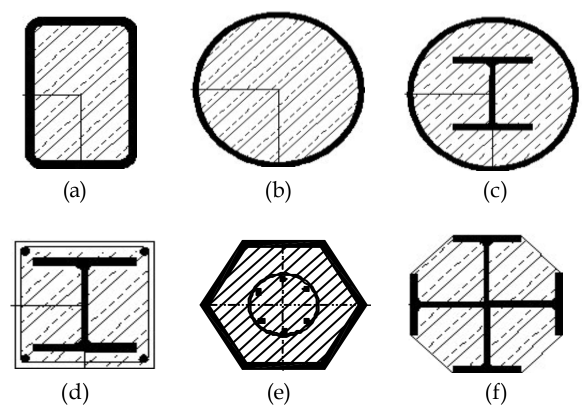


**Fig. 1. Typical cross-sections of steel-concrete composite columns.**

as a result of which their use is enhanced in the construction industry. With respect to their structural properties, the two materials are completely compatible and complementary to each other in that they exhibit an ideal combination of strengths and enhanced stiffness and ductility (Hajjar, 2000; Johansson and Gylltoft, 2001). In this regard, the efficiency of concrete in compression and that of steel in tension is made use of. Furthermore, the restraining feature of concrete to local or lateral-torsional buckling of slender steel elements is also another attractive feature especially in concrete-filled tubes. It has also been shown that such type of columns maintain sufficient ductility when high strength concrete is used (Lahlou *et al.*, 1999). Regarding their non-

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structural behaviour, their combined use in composite construction exhibits minimized potential differential deformation under the low temperature ranges in which steel-concrete composite structural elements are assumed to operate. While the present case of CFT does not benefit from it, the other attractive feature of concrete in composite construction is the fact that it also gives corrosion protection and thermal insulation to the steel at elevated temperatures (Eurocode Notes, 2001).

Further benefits of composite construction are attributed to the structural planning aspect of the design process (Ermiyas Ketema and Shifferaw Taye, 2006). In this respect, the concept of composite construction has given engineers ample opportunity to design steel-concrete composite structural systems to produce more efficient structures when compared to designs using either material alone.

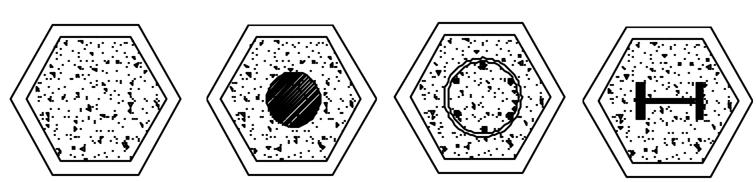
As in all design undertakings, the economy of resulting structures is of great concern. CFT columns can effectively replace other commonly used structural columns such as ordinary reinforced concrete, structural steel with reinforced concrete or structural steel alone with superior performance while at the same time reducing material costs to a minimum especially when both structural and non-structural features are considered in an integrated manner (Viest *et al.*, 1997).

Unlike other forms of steel-concrete composite columns, concrete-filled steel tubes of all shapes may not require additional reinforcement steel especially when the structural action is not significantly large. While generally higher concrete grades provide better results, no concrete grade below C20/25 shall be used in these columns. Likewise, hot-rolled steel tubes are used while cold-formed and welded sections are generally avoided in practice.

There are generally four types of hexagonal and octagonal concrete-filled steel tube columns as shown in Fig. 2 depending on the relative magnitude of moment/axial-force combination to which the member is subjected. In general, larger concentration of the steel component is desirable to resist mainly axial-load systems while distributed placement of the steel component is required in those cases where the dominance of the flexural moment is significant.

Design of composite columns, as in all types of compression members, calls for a procedural approach in which the effects of both axial and flexural stresses are taken into consideration in order to assess the capabilities of the particular member. To this effect, interaction diagrams have been proposed for a variety of structural column systems–concrete (see, for example, EBCS 2 Part 2, 1997) and steel (see, for example, Hofmann, 2002) under various loading conditions including procedures to produce such diagrams for steel-concrete composite columns (EBCS 4, 1995; Eurocode 4, 2002; Bode and Bergmann, 1985).

The purpose of this paper is to propose high quality interaction chart procedures the outcome of which will have dual purpose–enable easy determination of the necessary cross-sectional dimensions and material requirement for a CFT column under a specified set of loads on one hand and to assist in the determination of the capacity of a given cross-section when the size and relevant material properties of concrete and steel are known in advance. The proposed charts may be used both for short and long columns. Utilization of the proposed charts will be facilitated and generalized for any cross-sectional shape if their development is based on non-dimensional parameters. Towards this goal, the capacity equations to be developed and subsequently used to establish the charts will be made non-dimensional.



| | | | | |
|---|---|---|---|---|
| **axial force N** | small | large | moderate | moderate |
| **moment M** | insignificant | insignificant | moderate | large |
| **e = M/N** | small | small | moderate | large |

**Fig. 2. Possible arrangement of composite polygonal tubular columns with reference to loading.**

## COLUMN LOAD CAPACITY

The cross-sectional resistance of a composite column under axial compression and uniaxial bending is given by an M-N (moment-axial force) interaction curve. The interaction curve can be determined point by point by considering different positions of plastic neutral axis in the principal plane under consideration (Hofmann, 2002). The concurrent values of moment and axial resistance are then found from the stress blocks. Fig. 4 illustrates this process for four particular positions of the plastic neutral axis corresponding, respectively to the points A, B, C, D marked in Fig. 3. While the subsequent presentation will be given by referring to hexagonal-shaped columns, the discussion is indeed applicable to any shape and, thus, octagonal columns are also covered by the presentation.

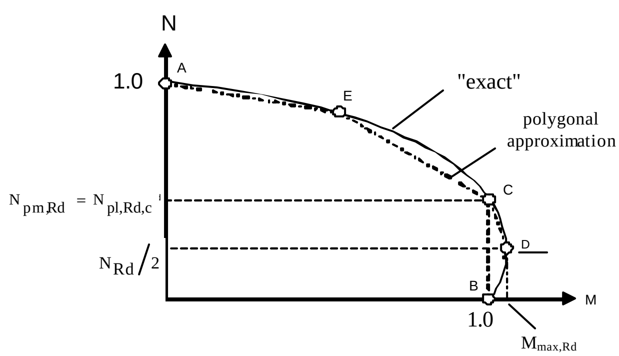



**Fig. 3. M-N interaction curve for uniaxial bending.**

Point A: Axial compression resistance alone:

$$N_A = N_{pl.Rd} \qquad M_A = 0 \quad \ldots\ldots\ldots\ldots (1)$$

Point B: Uniaxial bending resistance alone:

$$N_B = 0 \qquad M_B = M_{pl.Rd} \quad \ldots\ldots\ldots\ldots (2)$$

Point C: Uniaxial bending resistance identical to that at point B, but with non-zero resultant axial compressive force:

$$N_C = N_{pm.Rd} \qquad M_C = M_{pl.Rd} \quad \ldots\ldots\ldots\ldots (3)$$

where $N_{pm,Rd} = A_c f_{cd}$ compressive resistance of the concrete section.

Point D: Maximum moment resistance

$$N_D = 0.5 N_{pm.Rd} = 0.5 A_c f_{cd} \quad \ldots\ldots\ldots\ldots (4a)$$

$$M_D = W_{pa} f_{yd} + 0.5 W_{pc} f_{cd} \quad \ldots\ldots\ldots\ldots (4b)$$

in which $W_{ps}$ and $W_{pc}$ are the plastic moduli of the steel section and the concrete, respectively. Point D corresponds to the maximum moment resistance $M_{max,Rd}$ that can be achieved by the section. This is greater than $M_{pl.Rd}$ because the compressive axial force inhibits tensile cracking of the concrete, thus enhancing its flexural resistance (Petersen, 2001).

Point E: Situated midway between A and C.

The enhancement of the resistance at point E is only insignificantly more than that given by direct linear interpolation between *A* and *C*, and determination of this point can therefore be omitted.

The concurrent values of moment and axial resistance for establishing the M-N interaction diagram of Fig. 3 are then found from this set of stress blocks. Code standards usually substitute the linearized version AECDB (or the simpler ACDB) shown in Fig. 3 for the more exact interaction curve, after carrying out the calculations to determine these points. However, the results tend to be approximate since the entire curve is based only on four control points.

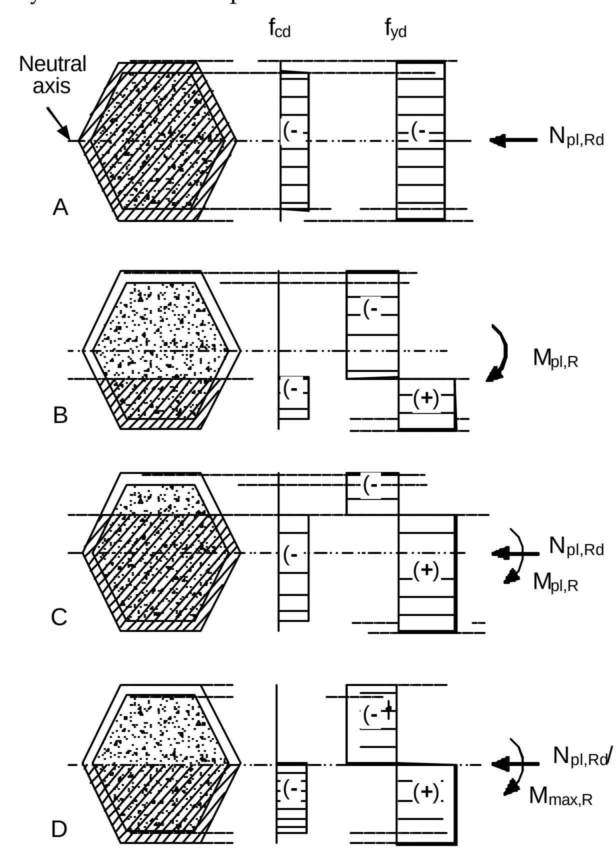



**Fig. 4. Development of stress blocks at different points on the interaction diagram.**

This paper outlines and develops a refined uniaxial interaction chart procedure based on exact formulation of a set of concurrent points.

## CHART DEVELOPMENT

### *Calculation method and scope*

The structural engineering practice calls for a variety of cross-sectional types to be used as compression members and as beam-columns. While the procedure to be proposed in this work is general and may easily be modified for adaptation to other shapes, the cross-sections considered are those that fulfill the criteria for simplified method of analysis given in the EBCS 4 (1995) and Eurocode 4 (2002).

From the permissible steel ratios ω stipulated in the EBCS 4 (1995), those for which $\omega \leq 4.0$ have been selected for drawing the chart as this range utilizes comparatively less amount of steel, thus resulting in more economical sections (compared to those with higher steel ratios).

The national code standard EBCS 4 (1995) recommends applicable structural steel and concrete grades for use in steel-concrete composite constructions. For the purpose of this paper, Steel Grade Fe360 with cross-sectional thickness of up to 40mm and concrete Grade C30 have been implemented. However, the procedure for establishing improved interaction charts and diagrams are fairly general and can, thus, be used to deal with other material properties as well.

### *Fundamental equations*

The fundamental equations to be used for the development of these charts with respect to typical composite cross-section as shown in Fig. 5 are the following:

Steel ratio ω:

$$\omega = \frac{A_s f_{yd}}{A_c f_{cd}} \quad \text{(5a)}$$

Moment capacity $M_u$:

$$M_u = (W_{ps} - W_{psn})\, f_{yd} + (W_{pc} - W_{pcn})\frac{f_{cd}}{2} \quad \text{(5b)}$$

Axial capacity $N_u$:

$$N_u = A_{cc} f_{cd} + A_{s,net} f_{yd} \quad \text{(5c)}$$

where:

$A_s$ and $A_c$ total cross-sectional area of the steel and concrete sections, respectively

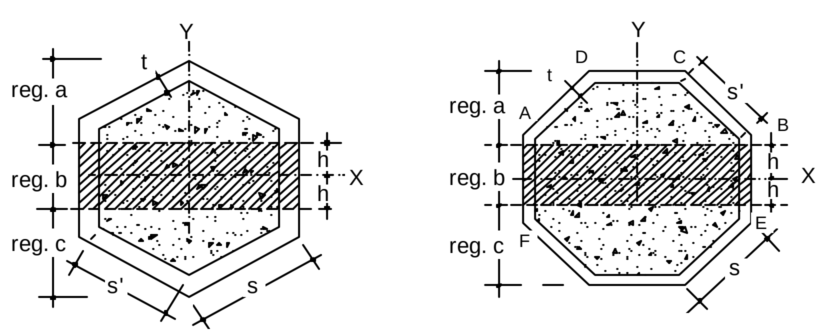
$W_{ps}$ and $W_{pc}$ plastic section modulus of the total steel and concrete section parts, respectively

$W_{psn}$ and $W_{pcn}$ plastic section moduli of the steel and concrete sections, within the shaded region (Region b, see Fig. 5), respectively

$A_{sc}$ and $A_{cc}$ cross-sectional areas of the portion of the steel and concrete sections in compression, respectively

$A_{st}$ cross-sectional area of the portion of the steel section that is subjected to tension

$A_{s,net} = A_{sc} - A_{st}$




(a) Hexagonal section (b) Octagonal section

**Fig. 5. Notations, orientations and regions of composite cross-section for computing section capacity.**

Equations (5) form the basis for the establishment of interaction diagrams and will be referred to frequently in subsequent sections. It will be noted that especially the six parameters $W_{ps}$, $W_{pc}$, $W_{psn}$, $W_{pcn}$, $A_{sc}$ and $A_{cc}$ in Eqs. (5) will play the central role as sources of variation in the establishment of the various components of the interaction diagrams as they are dependent upon the positions of the neutral axis.

### *Computing section capacity for a given neutral axis position*

In hexagonal composite columns, structural response to external actions–direct compression and bending moment–are influenced by the orientation of the axis about which bending takes place. This is due to the fact that the cross-sectional properties of such sections about all possible orthogonal axes may not be similar. In this context, therefore, the determination of section capacity will be carried out separately for bending moment about x and y axes, shown in Fig. 5a.

On the other hand, the perfect symmetry of octagonal sections about any two orthogonal axes as shown in Fig. 5b facilitates the formulation to be carried out only about a single axis.

Formulation of interaction equations will be facilitated by employing non-dimensional parameters. To this goal, we adapt the following relationships between the outer side dimension *s* of the entire column cross section and the thickness *t* of the steel component as shown in Figs. 5 (a &b):

$$\alpha = \frac{s}{t} \quad \text{......(6)}$$

As will be shown in subsequent sections, the term $h/t$, where h is the distance of the neutral axis relative to the centroidal axis of the cross section and t are shown in Fig. 5 appears frequently as a result of substitutions and simplifications. For the sake of brevity, we replace this term as follows:

$$\beta = \frac{h}{t} \quad \text{......(7)}$$

The procedure for establishing the interaction diagrams is enhanced by employing two non-dimensional parameters. To this end, expressions for two non-dimensional parameters $\nu$ and $\mu$, corresponding to the axial force $N_u$ and the bending moment $M_u$, respectively, are formulated through normalization as follows:

$$\nu = \frac{N_u}{A_c\, f_{cd}} \quad \text{......(8a)}$$

$$\mu = \frac{M_u}{A_c\, f_{cd}\, s'} \quad \text{......(8b)}$$

where the various terms have been defined earlier along with Eqs. (5) and s' is as shown in Fig. 5.

Details for the determination of $N_u$ and $M_u$ for the implementation of Eqs. (8) will be presented subsequently. Those equations will then be used to establish $\nu$ and $\mu$ in terms of cross-sectional and material properties and, subsequently, to produce high quality uniaxial bending-axial force interaction diagrams in terms of $\nu$ and $\mu$ for hexagonal and octagonal composite sections.

### *Uniaxial chart procedures for hexagonal sections*

Based on the principles discussed this far, the uniaxial chart procedure for hexagonal CFT will be presented in detail subsequently.

*Determining value of $\alpha$ for a particular steel ratio w*

In Eq. (5a), for a given steel ratio and material properties, the only unknown quantity is the variable $\alpha$ as given by Eq. (6). One can also see that for a hexagonal section, $A_c$=2.598$s'^2$, $A_s$=2.598($s^2$-$s'^2$) and s'=s-1.156t are valid where all terms are defined in Eqs. (5) and the various terms shown in Fig. 5.

Substituting these relationships given into the equation for $\omega$ and dividing the numerator and denominator by $2.598t^2$, one obtains:

$$[\alpha^2 - (\alpha - 1.155)^2]f_{yd} = \omega(\alpha - 1.155)^2\, f_{cd}$$

This can be explicitly solved for $\alpha$ after a series of simplifications and re-arrangement as:

$$\alpha = \frac{2.31\,(f_{yd} + \omega f_{cd}) + \sqrt{[2.31(f_{yd} + \omega f_{cd})]^2 - 5.333\,\omega f_{cd}\,(\omega f_{cd} + f_{yd})}}{2\,\omega f_{cd}} \quad \text{......(9)}$$

This is applicable irrespective of the axis of bending.

*Cross-sectional capacities for different neutral-axis positions–bending about X-axis*

While the general expressions given by Eqs. (5) are valid, one needs to establish major cross-sectional properties when refereeing to bending stresses about different axes. Thus, referring once again to Fig. 6:

Total cross-sectional area $A=2.598s^2$ .......... (10a)

Net area of concrete $A_c=2.598s'^2$ ........ (10b)

Net area of steel $A_s=2.598(s^2-s'^2)$ (10c)

Area of the shaded region DEFG $A' = 3.464\,sh$ .....(10d)

Area of region ABC $A_{abc} = 1.732\,(s-h')^2$ ...(10e)

Total section modulus $W = 1.01\,s^3$ .......... (10f)

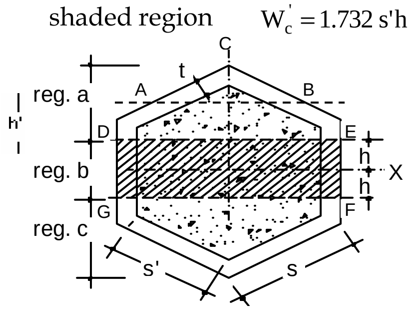
Section modulus of the shaded region DEFG

$W' = 1.732 s\,h^2$ ..... (10g)

Section modulus of region ABC

$$W_{abc} = 0.577(s-h')^2(s-2h') \text{ ..........(10h)}$$

Section modulus of concrete portion in the shaded region $W_c' = 1.732\,s'h^2$ ........ (10i)




h′ is the distance between the neutral axis X and the line A-B.

**Fig. 6. Hexagon of side length *s*.**

The position of the neutral axis varies depending on the relative size of the bending moment and the associate axial force. To establish the desired relationships as was presented in Sec. 2, five different cases of neutral axis position are selected; these cases will be dealt with separately.

Case i: When the entire cross-section is under direct compression

a. Moment capacity

This is the case when only the direct axial force exists and, thus, the whole part is subjected to direct (not flexural) compression. Consequently, no moment-resistance capacity is needed. Under this circumstance,

$$M_u = 0$$

Thus, in this particular instance,

$$\mu = 0 \text{ ..............................(11a)}$$

b. Axial load capacity

$$N_u = N_{pl,rd} \qquad \text{where } N_{pl,rd} = A_c f_{cd} + A_s f_{yd}$$

Taking into account Eqs. (10b) and (10c), and carrying out appropriate substitution into:

$$\nu = \frac{A_c f_{cd} + A_s f_{yd}}{A_c f_{cd}}$$

one gets:

$$\nu = \frac{(\alpha-1.155)^2 f_{cd} + \{\alpha^2 - (x-1.155)^2\} f_{yd}}{(\alpha-1.155)^2 f_{cd}} \text{ ......(11b)}$$

Case ii: More than half the area under compression (Fig. 7) where the position of the neutral axis is given by $\frac{s}{2} \le h \le s-1.155t$

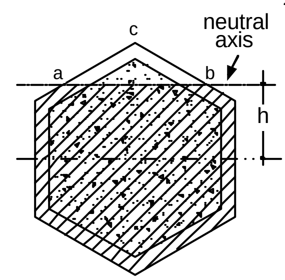



**Fig. 7. Location of neutral axis for Case (ii).**

a. Moment capacity

Referring to Eq. (5b) and taking the relationships and associated description of the notations involved, the following relationships hold for this particular case:

$$W_{ps} - W_{psn} = 2\times W_{abc} = 1.155(s-h)^2(s-2h) - 1.155(s'-h)^2(s'-2h)$$

$$W_{pc} - W_{pcn} = 2\times W_{abc} = 1.155(s'-h_i)^2(s'-2h)$$

Appropriate substitution in Eq. (8b) yields the required relationship for μ as:

$$m = \frac{1.155(s-h)^2(s-2h) - 1.155(s'-h)^2(s'-2h)\, f_{yd} + \{1.155(s'-h)^2(s'-2h)\}\, f_{cd}/2}{2.598 s'^3\, f_{cd}}$$

Now, dividing the numerator and denominator by $t^3$, the non-dimensional parameter μ in terms of the cross-sectional dimensions and material properties becomes:

$$\mu = \frac{1.155\,(\alpha-\beta)^2(\alpha-2\beta) - 1.155\,(\alpha-1.155-\beta)^2(\alpha-1.155-2\beta)\, f_{yd}}{2.598\,(\alpha-1.155)^3 f_{cd}} + \frac{\{1.155(\alpha-1.155-\beta)^2(\alpha-1.155-2\beta)\}\, f_{cd}/2}{2.598\,(\alpha-1.155)^3 f_{cd}} \text{ ..............................(12a)}$$

b. Axial load capacity

The size of the axial force is established by Eq. (5c) taking the following relationships in this case:

$$A_{cc} = A_c - A_{abc} = 2.598s'^2 - 1.732(s' - h)^2$$

$$A_{snet} = A_{sc} - A_{st} = A_s - 2A_{abc} = 2.598(s^2 - s'^2) - 3.464\{(s-h)^2 - (s' - h)^2\}$$

it can be seen that ν assumes the following form:

$$\nu = \frac{\{2.598s' - 1.732(s' - h)^2\}f_{cd} + [2.598(s^2 - s'^2) - 3.464\{(s-h)^2 - (s'-h)^2\}]f_{yd}}{2.598s' f_{cd}}$$

Dividing equation by $t^2$, the non-dimensional parameter ν will attain its final form in terms of the cross-sectional dimensions and material properties as:

$$n = \frac{\{2.598\ (x - 1.155)^2 - 1.732\ (a - 1.155 - b)^2\}\ f_{cd}}{2.598\ (a - 1.155)^2\ f_{cd}}$$
$$+ \frac{[2.598\ \{a^2 - (a - 1.155)^2\} - 3.464\ \{(a - b)^2 - (a - 1.155 - b)^2\}]\ f_{yd}}{2.598\ (a - 1.155)^2\ f_{cd}} \quad \text{....................(12b)}$$

c. Values of β used

In order to sketch the interaction diagram in the given range of the neutral axis, for the various steel ratios ω, four values of β are used: 0.5α, 0.6α, 0.8α and α-1.155.

Case iii: In this case, too, more than half the area under compression (Fig. 8); however, the possible range of neutral axis will be different from that assessed in Case (ii) above. In this particular case, the depth of the neutral axis will vary as $0 \le h \le \frac{s'}{2}$.

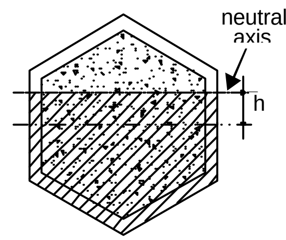



**Fig. 8. Location of neutral axis for Case (iii).**

a. Moment capacity

In this situation, the general expression for $M_u$ as given by Eq. (5b) can be established by substituting the following cross-sectional values:

$$W_{pa} = 1.01(s^3 - s'^3) \qquad W_{pan} = 1.732(s - s')h_i^2 = 2th_i^2$$

$$W_{pc} = 1.01s'^3 \qquad W_{pcn} = 1.732\,s'h_i^2$$

Thus, taking the relationships in Eq. (8b) and associated description of the notations involved, and after appropriate substitutions and simplifications, one obtains the final form of the non-dimensional parameter in terms of cross-sectional and material properties as:

$$m = \frac{[1.01\{a^3 - (b - 1.155)^3\} - 2b^2]\,f_{yd} + \{1.01(a - 1.155)^3 - 1.732(a - 1.155)b^2\}\,f_{cd}/2}{2.598(a - 1.155)^3\,f_{cd}} \quad \text{....(13a)}$$

b. Axial load capacity

The following relationships hold in this particular case:

$$A_{cc} = 1.299s'^2 + 1.732s'h \qquad A_{snet} = 4ht$$

Referring to Eq.(5c) and carrying out appropriate substitutions, the axial load is given by:

$$N_u = [2.598\,s'^2 - 0.5\{(2s' - 1.155\,h) + (s - 1.155\,t)\}\{.866\,s - t - h\}]\,f_{cd} + 4.62\,htf_{yd}$$

The non-dimensional parameter ν is then given by: $\nu = \dfrac{(1.299\,s'^2 + 1.732\,s'\,h)\,f_{cd} + 4htf_{yd}}{2.598\,s'^2\,f_{cd}}$

Dividing the above equation by $t^2$, ν will attain its desired form as function of cross-sectional variables, parameters and material properties as:

$$\nu = \frac{\{1.299(\alpha - 1.155)^2 + 1.732(\alpha - 1.155)\beta\}f_{cd} + 4\beta f_{yd}}{2.598(\alpha - 1.155)^2 f_{cd}} \quad \text{.........................................................(13b)}$$

c. Values of $\beta$ used

Values of $\beta$ used are $0\alpha$, $0.1\alpha$, $0.3\alpha$ and $(\alpha-1.155)/2$

Case iv: Less than half the area under compression (Fig. 9) where the neutral axis position with $0 \le h \le \frac{s'}{2}$

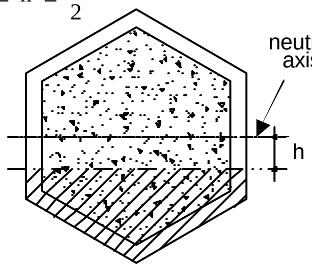



**Fig. 9. Location of neutral axis for Case (iv).**

a. Moment capacity

The moment capacity is given by the same expressions developed for Case (iii) above and, consequently, the non-dimensional parameter $\mu$ will be given by:

$$m = \frac{[1.01\{a^3 - (b - 1.155)^3\} - 2b^2]\, f_{yd}}{2.598\,(a - 1.155)^3 f_{cd}} + \frac{\{1.01(a - 1.155)^3 - 1.732\,(a - 1.155)\, b^2\}\, f_{cd}/2}{2.598\,(a - 1.155)^3 f_{cd}}$$

............................................................................... (14a)

b. Axial load capacity

In this case, the following relationships hold:

$$A_{cc} = 1.299 s'^2 - 1.732 s' h \qquad A_{snet} = -4ht$$

The value-4ht, obtained as the area of steel under compression minus that under tensile stress, will be used in the computational algorithm.

The axial-force will then attain the following form:

$$N_u = 0.5\{(2s' - 1.155h) + (s - 1.155t)\}(0.866s - t - h)\, f_{cd} - 4.62 ht f_{yd}$$

The non-dimensional parameter $\nu$ will then become:

$$\nu = \frac{(1.299 s'^2 - 1.732 s' h) f_{cd} - 4ht f_{yd}}{2.598 s'^2 f_{cd}}$$

Now, dividing the above equation by $t^2$, one obtains the desired expression for $\nu$:

$$\nu = \frac{\{1.299\,(\alpha - 1.155)^2 - 1.732\,(\alpha - 1.155)\beta\} f_{cd} - 4\beta f_{yd}}{2.598\,(\alpha - 1.155)^2 f_{cd}}$$

............................................................................... (14b)

c. Values of $\beta$ used

Values of $\beta$ used are $0\alpha$, $0.1\alpha$, $0.3\alpha$ and $(\alpha-1.155)/2$

Case v: In this case, too, less than half the area under compression (Fig.10), but with position of the neutral axis being given by $\frac{s}{2} \le h \le s'$.

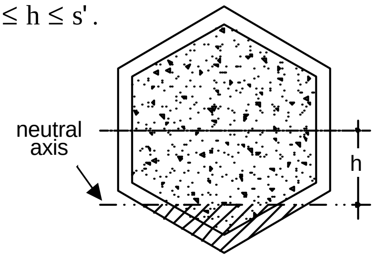



**Fig. 10. Location of neutral axis for Case (iii).**

a. Moment capacity

The moment capacity is the same as that given in Case (ii) and the non-dimensional parameter $\mu$ will be given by:

$$m = \frac{1.155(a-b)^2(a-2b) - 1.155(a-1.155-b)^2(a-1.155-2b)\, f_{yd}}{2.598(a-1.155)^3 f_{cd}} + \frac{\{1.155(a-1.155-b)^2\ (a-1.155-2b)\} f_{cd}/2}{2.598(a-1.155)^3 f_{cd}}$$

............................................................................... (15a)

b. Axial load capacity

Again, the axial-load capacity is given by:

$$N_u = A_{cc} f_{cd} + A_{snet} f_{yd}$$

In this case, the following relationships hold with regard to cross-sectional properties:

$$A_{cc} = A_{tri} = 1.732(s' - h)^2$$

$$A_{snet} = 2.598(s^2 - s'^2) - 2.464\{(s-h)^2 - (s'-h)^2\}$$

The non-dimensional parameter $\nu$ will then be given by:

$$n = \frac{1.732(s'-h)^2 f_{cd} + [2.598(s^2 - s'^2)}{2.598 s'^2 f_{cd}} - \frac{3.464\{(s-h)^2 - (s'-h)^2\}]\, f_{yd}}{2.598 s'^2 f_{cd}}$$

Dividing equation by $t^2$ and making appropriate substitutions, $\nu$ will attain its final desired form as:

$$n = \frac{1.732(a-1.155-b)^2 f_{cd} + [2.598(a^2 - (a-1.155)^2)}{2.598(a-1.155)^2 f_{cd}} - \frac{3.464\{(x-b)^2 - (a-1.155-b)^2\}]\, f_{yd}}{2.598(a-1.155)^2 f_{cd}}$$

............................................................................... (15b)

c. Values of $\beta$ used

Values of $\beta$ used are $0.5\alpha$, $0.6\alpha$, $0.8\alpha$ and $\alpha$-1.155 Equations (11) through (15) will be used to establish the uniaxial interaction chart for hexagonal section as shown in Fig.19a for various $\omega$ values.

*Cross-sectional capacity for different neutral axis positions–bending about Y-axis*

Following the similar procedure adopted to establish the various parameters for bending about x-axis as presented earlier, corresponding quantities will be derived for bending about the y-axis. The cross-sectional properties of these columns frequently referred to in subsequent developments are summarized hereafter. Thus, again, referring to Fig. 11:

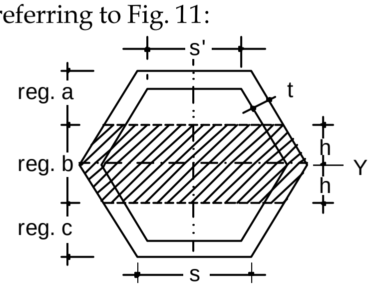



**Fig. 11. Hexagonal section: bending about Y axis.**

- Cross-sectional area of the entire section
  $A=2.598s^2$..........................................................(16a)
- Cross-sectional area of the concrete portion
  $A_c=2.598s'^2$......................................................(16b)
- Cross-sectional area of the steel portion
  $A_s=2.598(s^2-s'^2)$.............................................(16c)
- Total area of the shaded region
  $A' = 3.464s^2 - (1.732s - h)(2s - 1.155h)$ .... (16d)
- Section modulus of the whole section
  $W = s^3$ ............................................................(16e)
- Total section modulus of the shaded region
  $W' = 2s^3 - (1.732s - h)(2s - 1.155h)(0.577s + .667h)$
  ...........................................................................(16f)

As was noted earlier, the position of the neutral axis varies depending on the relative size of the bending moment and the associated axial force. To establish the desired relationships as was presented in Sec. 2, four different cases of neutral axis position are selected; these cases will be dealt with separately.

Case i: When the entire cross-section is under direct compression.

a. Moment capacity

This is the case when only the direct axial force exists and, thus, the whole part is subjected to direct (not flexural) compression. Consequently, no moment-resistance capacity is needed. Under this circumstance,

$M_u=0$

Thus, in this particular instance, the non-dimensional parameter $\mu$ of Eq. (8b) turns out to be zero. Thus,

$$\mu = 0 \quad (17a)$$

b. Axial load capacity

$N_u = N_{pl,rd}$

where $N_{pl,rd} = A_c f_{cd} + A_s f_{yd}$

Now, applying Eq. (8b) and considering the relationships expressed by Eqs (16b) and (16c), one gets:

$$\nu = \frac{(\alpha - 1.155)^2 f_{cd} + [\alpha^2 - (\alpha - 1.155)^2] f_{yd}}{(\alpha - 1.155)^2 f_{cd}} \text{ ...........(17b)}$$

Case ii: When some part of the flange of the steel section is in tension while other parts of the cross-section are in compression and the neutral axis will assume any position in the range given by $\frac{\sqrt{3}s}{2} - t \le h \le \frac{\sqrt{3}s}{2}$ (Fig. 12).

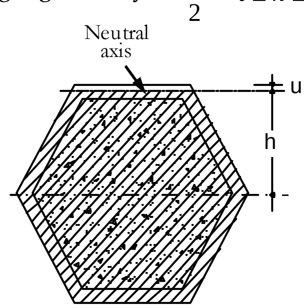



**Fig. 12. Location of neutral axis for Case (i).**

In this case, the moment- as well as axial-load capacities are established as follows:

a. Moment capacity

Referring to Eq. (5b) and nothing in this case the fact that only the steel portion of the composite column contributes for the moment capacity of the cross-section, Eq. (5a) modifies to:

$$M_u = (W_{ps} - W_{psn}) f_{yd}$$

In this case, one can see that the following relationships hold with respect to cross-sectional properties:

$$W_{ps} = s^3$$

$$W_{psn} = 2s^3 - (1.732s - h)(2s - 1.155h)(0.577s + .667h)$$

Thus, applying Eq. (8b):

$$m = \frac{M_u}{A_c f_{cd} s'}$$
$$= \frac{[s^3 - \{2s^3 - (1.732s - h)(2s - 1.155h)(0.577s + .667h)\}] f_{yd}}{2.598 s'^3 f_{cd}}$$

Diving both the numerator and denominator by t³ one obtains the non-dimensional parameter μ in terms of the cross-sectional dimensions and material properties as:

$$\mu = \frac{[\alpha^3 - \{2\alpha^3 - (1.732\alpha - \beta)\}(2x - 1.155\beta)(0.577\alpha - 0.667\beta)] f_{yd}}{2.598(\alpha - 1.155)^3 f_{cd}}$$
........................................................................................ (18a)

b. Axial load capacity

The size of the axial load in this case is given by:

$$N_u = 2.598 s'^2 f_{cd} + [\{3.464 s^2 - (1.732s - h)(2s - 1.155h)\} - 2.598 s'^2] f_{yd}$$

Applying Eq. (8a):

$$n = \frac{2.598 s'^2 f_{cd}}{A_c f_{cd}} + \frac{[\{3.464 s^2 - (1.732 s - h)(2s - 1.155 h)\} - 2.598 s'^2] f_{yd}}{A_c f_{cd}}$$

Dividing equation by t² one obtains the non-dimensional parameter ν in terms of the cross-sectional dimensions and material properties as:

$$n = \frac{2.598(a - 1.155)^2 f_{cd}}{2.598(a - 1.155)^2 f_{cd}} + \frac{[\{3.464 a^2 - (1.732 a - b)\} - (2a - 1.155 b) - 2.598(a - 1.155)^2] f_{yd}}{2.598(a - 1.155)^2 f_{cd}}$$
........................................................................................ (18b)

c. Values of β used

Values of β are 1.732α-1, 1.732α -2/3 and 1.732α -1/3

Case iii: When more than half of the cross-section is under compression while the neutral axis assumes any position in the range given by $0 \le h_i \le \frac{\sqrt{3}s}{2} - t$ (Fig. 13).

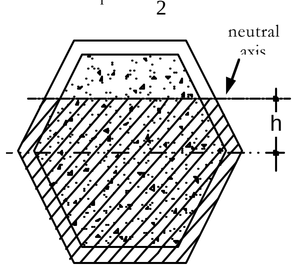



**Fig. 13. Location of neutral axis for Case (iii).**

a. Moment capacity

Referring to Eq. (2) and based on Eq. (5b), one can see that the following relationships are applicable for this particular case,:

$$W_{pa} = s^3 - (s - 1.155)^3$$
$$W_{pan} = 2.31 h^2 t$$
$$W_{pc} = s'^3$$
$$W_{pcn} = 2s'^3 - \{(1.732 s' - h)(2s' - 1.155h)(0.557 s' + 0.667 h)\}$$

Now, making appropriate substitutions into Eq. (8b), the expression for non-dimensional parameter μ in terms of the cross-sectional dimensions and material properties will finally become:

$$m = \frac{\{a^3 - (a - 1.155)^3 - 2.31 b^2\} f_{yd}}{2.598 (1 - 1.155)^3 f_{cd}} + \frac{[-(a - 1.155)^3 + \{1.732(a - 1.155) - b\}\{0.577(a - 1.155) + 0.667b\}] \frac{f_{yd}}{2}}{2.598(1 - 1.155)^3 f_{cd}}$$
........................................................................................ (19a)

b. Axial load capacity

Appropriate substitution and re-arrangement shows that $N_u$ in Eq. (5c) will be given by:

$$N_u = 4.62\, ht\, f_{yd} + [2.598 s'^2 - 0.5\{(2s' - 1.155h) + (s - 1.155t)\}(0.866s - t - h)] f_{cd}$$

Subsequently, using Eq. (8a) for the non-dimensional parameter ν, substituting appropriate cross-sectional dimensions and material properties and, finally, after a series of simplifications, the desired expression for ν becomes:

$$n = \frac{4.62 b f_{yd} + [2.598 (a - 1.155)^2] f_{cd}}{2.598 (a - 1.155)^2 f_{cd}} - \frac{[0.5\{(2(a - 1.155) - 1.155b) + (a - 1.155)\}\{0.866a - 1 - b\}] f_{cd}}{2.598 (a - 1.155)^2 f_{cd}}$$
........................................................................................ (19b)

c. Values of β used

Values of β used are 0, 0.2α, 0.4α, 0.6α, 0.8α, α, 1.3α and 1.5α.

Case iv: When less than half the cross-sectional area is under compression and the neutral axis assumes any position in the range given by $0 \le h_i \le \frac{\sqrt{3}s}{2} - 1$ (Fig. 14).

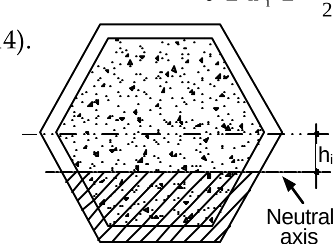



**Fig. 14. Location of neutral axis for Case (iv).**

a. Moment capacity

The moment capacity given by the corresponding expression in Case (iii) above and, consequently, the expression for the non-dimensional parameter μ is identical that given by Eq. (19a).

$$m = \frac{\{a^3 - (a-1.155)^3 - 2.31 b^2\}\ f_{yd}}{2.598\ (1-1.155)^3\ f_{cd}} + \frac{[-(a-1.155)^3 + \{1.732\ (a-1.155) - b\}\{0.577(a-1.155)+0.667 b\}]\frac{f_{yd}}{2}}{2.598\,(1-1.155)^3\ f_{cd}} \quad \text{.......(20a)}$$

b. Axial load capacity

The general expression for axial load for this position of neutral axis is:

$$N_u = 0.5\,[(2s' - 1.155\,h) + (s - 1.155\,t)]\ (0.866\,s - t - h)\ f_{cd} - 4.62\ h\ t\ f_{yd}$$

Subsequently, using the expression for the non-dimensional parameter ν, the desired expression becomes:

$$n = \frac{0.5[\{2(a-1.155) - 1.155 b\} + (a-1.155)(0.866 a - 1 - b)]\, f_{yd}}{2.598(a-1.155)^2\ f_{cd}} - \frac{4.62\ b f_{yd}}{2.598\ (a-1.155)^2\ f_{cd}} \quad \text{.......(20b)}$$

c. Values of β used

Values of β used are 0, 0.2α, 0.4α, 0.6α, 0.8α, α, 1.3α and 1.5α.

Equations (17) through (20) will be used to generate the uniaxial interaction curves for bending about Y axis for different values of ω; this will be given in Fig. 19b.

***Uniaxial charts for octagonal sections***

*Determining value of a for a particular steel ratio w*

In Eq. (5a), for a given steel ratio and material condition, the only unknown is the variable quantity α as given by Eq. (8). but, $A_c = 4.828s'^2$, $A_s = 4.828(s^2 - s'^2)$ where $s' = s - 0.828t$.

Substituting the above equation in equation of ω and dividing both equations by $4.828t^2$

$$[\alpha^2 - (\alpha - 1.155)^2]\, f_{yd} = \omega(\alpha - 1.155)^2 f_{cd}$$

Simplifying the above equation and solving for α:

$$a = \frac{1.656\ (f_{yd} + w\ f_{cd})}{2w\ f_{cd}} + \frac{\sqrt{\{1.656\ (f_{yd} + w\ f_{cd})\}^2 - 2.744\ w\ f_{cd}\,(w\ f_{cd} + f_{yd})}}{2w\ f_{cd}} \quad \text{.......(21)}$$

*Cross-section capacities for different neutral axis positions*

Fig. 5b shows the dimensional parameters used to describe the section and these will be employed subsequently to develop the moment-axial force interaction diagrams.

The following cross-sectional properties will be referred to in subsequent developments:

- Cross-sectional area of the entire section

  $$A = 4.828s^2 \quad \text{.......(22a)}$$
- Area of polygon ABCD

  $$A_{abcd} = 5.328s^2 - 0.5(3.414s - 2h)^2 \quad \text{.....(22b)}$$
- Area of the shaded region ABEF

  $$A_{abef} = 0.5(4.414s - 2h)(1.207s - h) \quad \text{...(22c)}$$
- Section modulus of the entire cross-section

  $$W = 2.545s^3 \quad \text{.......(22d)}$$
- Section modulus of the shaded region ABEF

  $$W = 2\{1.616s^3 - 0.25(3.414s - 2h)^2(0.569s + 0.667h)\} \quad \text{.......(22e)}$$

Case i: The whole cross-section is under compression.

a. Moment capacity

This is the case when only the direct axial force exists and, thus, the whole part is subjected to direct (any, net, not flexural) compression. Consequently, no moment-resistance capacity is needed. Under this circumstance,

$$M_u = 0$$

Thus, in this particular instance,

$$m = 0 \quad \text{.......(23a)}$$

b. Axial load capacity

Noting that $A_c = 4.828s'^2$ and $A_s = 4.828s^2 - A_c$, and substituting these parameters into Eq. (5c) and subsequently into Eq. (8b), one gets:

$$\nu = \frac{A_c f_{cd} + A_s f_{yd}}{A_c f_{cd}}$$

Dividing the above equation by $t^2$ and simplifying

$$\nu = \frac{(\alpha - 0.828)^2 f_{cd} + [\alpha^2 - (\alpha - 0.828)^2]\, f_{yd}}{(\alpha - 0.828)^2 f_{cd}} \quad \text{.......(23b)}$$

Case ii: More than half the area under compression (Fig. 15) wherein the neutral axis can assume any position within the range given by

$$\frac{s}{2} \leq h \leq 1.207s - t$$

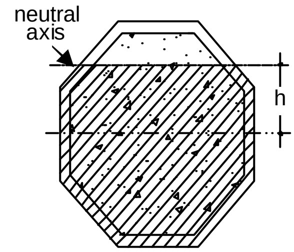



**Fig. 15. Location of neutral axis for Case (ii).**

a. Moment capacity

In this situation, the general expression for $M_u$ as given by Eq. (5b) can be established by substituting the following cross-sectional values:

$$W_{pa} = 2.545(s^3 - s'^3)$$

$$W_{pan} = 2\{1.616s^3 - 0.25(3.414s - 2h)^2(0.569s + 0.667h)\} - W_{pcn}$$

$$W_{pc} = 2.545\ s'^3$$

$$W_{pcn} = 2\{1.616s'^3 - 0.25(3.414s' - 2h)^2(0.569s' + 0.667h)\}$$

Thus, taking the relationships in Eq. (2) and associated description of the notations involved, and after appropriate substitutions and simplifications, one obtains:

$$m = \frac{[2.545\{a^3 - (a-0.828)^3\} - 2\times 1.61\{a^3 - (a-0.828)^3\} - 0.25(3.414a - 2b)^2(0.569a + 0.667b)]f_{yd}}{4.828(a-0.828)^3 f_{cd}}$$

$$+ \frac{0.5\times[\{3.414(a-0.828) - 2b\}^2\{0.569(a-0.828) + 0.667b\}]f_{yd}}{4.828(a-0.828)^3 f_{cd}}$$

$$+ \frac{[2.545(a-0.828)^3 - 2\times 1.616(a-0.828)^3 - 0.25\{3.414(a-0.828) - 2b\}^2\{0.569(a-0.828) + 0.667b\}]\frac{f_{yd}}{2}}{4.828(a-0.828)^3 f_{cd}}$$

..........(24a)

b. Axial load capacity

To establish the axial force $N_u$, it is important to note that the following relationships hold with regard to cross-sectional properties:

$$A_{cc} = A_c - A_{tra} = 4.828s'^2 - 0.5(4.414s - 2h - 3.656t)(1.207s - t - h)$$

$$A_{snet} = A_{sc} - A_{st}$$

$$A_{snet} = 5.328(s^2 - s'^2) - 0.5\{(3.414s - 2h)^2 - (3.414s' - 2h)^2\}$$

The non-dimensional parameter ν will then be given by after a series of substitutions and simplifications:

$$n = \frac{\{4.828(a-0.828)^2 - 0.5(4.414a - 2b - 3.656)(1.207a - 1 - b)\}f_{cd}}{4.828(a-0.828)^2 f_{cd}}$$

$$+ \frac{[5.328\{a^2 - (a-0.828)^2\} - 0.5\{(3.414a - 2b)^2 - (3.414(a-0.828) - 2h)^2\}]f_{yd}}{4.828(a-0.828)^2 f_{cd}}$$

..........(24b)

c. Values of $\beta$ used

Values of $\beta$ used are 0.5$\alpha$, 0.6$\alpha$, 0.8$\alpha$, $\alpha$ and 1.207$\alpha$ -1

Case iii: This is also another case where still more than half the area under compression (Fig. 16). The neutral axis will have the following position with $0 \le h \le 0.5s - 0.414t$

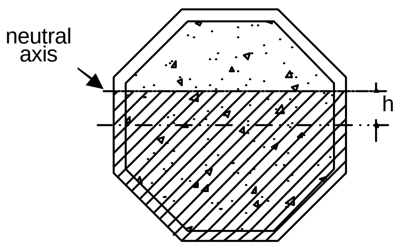



**Fig. 16. Location of neutral axis for Case (iii).**

a. Moment capacity

To establish the $M_u$ as given by Eq. (5b), it is important to note the following relationships for this particular case:

$$W_{pa} = 2.545(s^3 - s'^3) \qquad W_{pan} = 2h^2 t$$

$$W_{pc} = 2.545 s'^3 \qquad W_{pcn} = (2.414s - 2t)h^2$$

After a series of substitutions and simplifications, the non-dimensional parameter in its final form becomes:

$$m = \frac{[2.545\{a^3 - (a - 0.828)^3\} - 2b^2]\, f_{yd}}{4.828\,(a - 0.828)^3 f_{cd}} + \frac{\{2.545(a - 0.828)^3 - (2.414a - 2)\, b^2\} f_{cd}/2}{4.828(a - 0.828)^3 f_{cd}}$$

..........................................................................(25a)

b. Axial load capacity

In this case, the following relationships hold for the establishment of $N_u$:

$A_{cc} = 2.414\, s'^2 + 2h(1.207\, s - t) \qquad A_{snet} = 4ht$

The final form of the non-dimensional parameter $\nu$ becomes:

$$\nu = \frac{\{2.414(\alpha - 0.828)^2 + 2h(1.207\alpha - 1)\} f_{cd} + 4\beta f_{yd}}{4.828(\alpha - 0.828)^2 f_{cd}}$$

..........................................................................(25b)

c. Values of $\beta$ used

Values of $\beta$ used are 0, 0.2$\alpha$, 0.4$\alpha$ and 0.5$\alpha$ - 0.414.

Case iv: When less than half the area is under compression (Fig. 17) and the neutral axis can take any position within the range $0 \le h \le 0.5s - 0.414t$

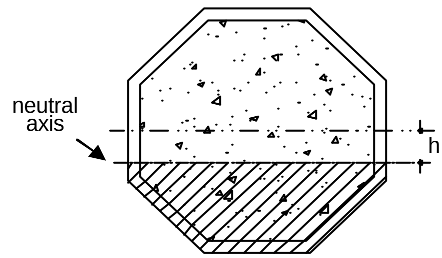



**Fig. 17. Location of neutral axis for Case (iv).**

a. Moment capacity

Expressions for moment capacity and those of the non-dimensional parameter $\mu$ are similar to those for Case (iii). Thus,

$$m = \frac{[2.545\{a^3 - (a - 0.828)^3\} - 2b^2]\, f_{yd}}{4.828\,(a - 0.828)^3 f_{cd}} + \frac{\{2.545\,(a - 0.828)^3 - (2.414\,a - 2)\, b^2\} f_{cd}/2}{4.828\,(a - 0.828)^3 f_{cd}}$$

..........................................................................(26a)

b. Axial load capacity

In order to arrive at the desired result for the non-dimensional parameter $\nu$, we will take the following relationships into consideration for this case:

$$A_{cc} = 2.414 s'^2 - 2h(1.207s - t) \qquad A_{snet} = -4ht$$

Now, making appropriate substitutions into Eq. (8a), $\nu$ will attain the following form:

$$\nu = \frac{\{2.414s'^2 - 2h(1.207s - t)\} f_{cd} - 4ht f_{yd}}{4.828 s'^2 f_{cd}}$$

Dividing equation by ₽, the desired expression for $\nu$ becomes:

$$\nu = \frac{\{2.414(\alpha - 0.828)^2 - 2h(1.207\alpha - 1)\} f_{cd} - 4\beta f_{yd}}{4.828(\alpha - 0.828)^2 f_{cd}}$$

..........................................................................(26b)

c. Values of β used

Values of β used are 0, 0.2α, 0.4α and 0.5α -0.414.

Case v: This is the other case where less than half the area under compression (Fig. 18). Here, the neutral axis may take up any position in range $\frac{s}{2} \le h \le 1.207\,s - t$.

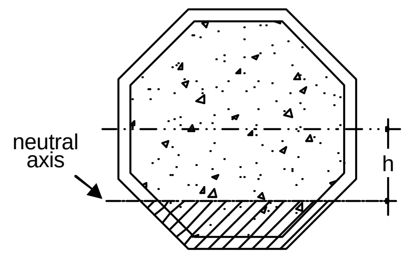



**Fig. 18. Location of neutral axis for Case (v).**

a. Moment capacity

Moment capacity is same as in Case (ii). Thus:

$$m = \frac{2.545[\{a^3-(a-.828)^3\}-2\times 1.616\{a^3-(a-0.828)^3\}-0.25(3.414a-2b)^2(0.569a+0.667b)]f_{yd}}{4.828(a-0.828)^3 f_{cd}}$$

$$+\frac{0.5[\{3.414(a-0.828)-2b\}^2\{0.569(a-0.828)+0.667b\}]f_{yd}}{4.828(a-0.828)^3 f_{cd}}$$

$$+\frac{[2.545(a-0.828)^3-2\times 1.616(a-0.828)^3-0.25\{3.414(a-0.828)-2b\}^2\{0.569(a-0.828)+0.667b\}]\frac{f_{yd}}{2}}{4.828(a-0.828)^3 f_{cd}}$$

..........................................................................................................................................................(27a)

b. Axial load capacity

The following relationships hold in this range:

$$A_{cc} = A_{tra} = 0.5(4.414s - 2h - 3.656t)(1.207s - t - h) \qquad A_{snet} = A_{sc} - A_{st}$$

$$A_{snet} = -[5.328(s^2 - s'^2) - 0.5\{(3.414s - 2h)^2 - (3.414s' - 2h)^2\}]$$

Substituting these into Eq. (8a), dividing the resulting equation by $t^2$, the final form of the non-dimensional parameter ν becomes:

$$n = \frac{\{0.5(4.414a - 2b - 3.656)(1.207a - 1 - b)\}\ f_{cd}}{4.828(a-0.828)^2 f_{cd}} +$$

$$\frac{[-5.328\{a^2-(a-0.828)^2\}-0.5\{(3.414a-2b)^2-\{3.414(a-0.828)-2h\}^2\}]f_{yd}}{4.828(a-0.828)^2 f_{cd}}$$

..........................................................................................................................................................(27b)

c. Values of β used

Values of β used are 0.5α, 0.6α, 0.8α, α and 1.207α -1

All the above relationships will now be used to establish the interaction diagram for concrete-filled steel tubes.

## UNIAXIAL INTERACTION CHARTS

Based on the various relationships developed for the non-dimensional parameters ν and μ in the preceding sections, interaction diagrams have been provided both for hexagonal and octagonal section and these are given in Fig. 19 and Fig. 20, respectively.

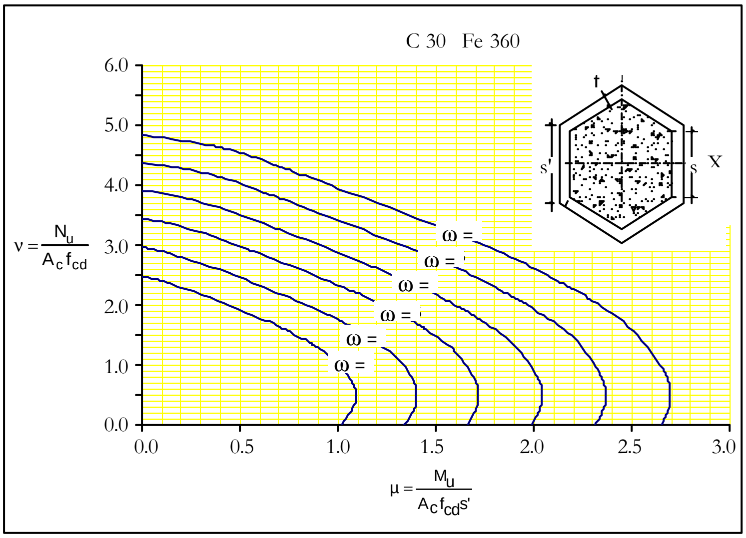



Fig. 19a. Uniaxial interaction diagram for hexagonal section with bending about x-axis.

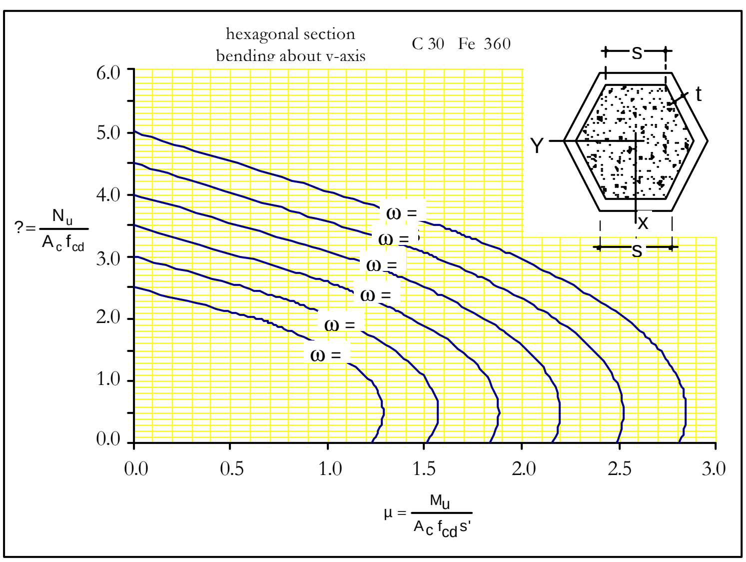



Fig. 19b. Uniaxial interaction diagram for hexagonal section with bending about y-axis.

These charts can be used to assess the validity of hexagonal or octagonal uniaxial steel-concrete composite columns of established cross-sectional and material properties or may be used to propose an appropriate design for a given set of axial compression and/or uniaxial moment.

## CONCLUSION

Concrete-filled steel tubes used as structural columns have significant economic, structural and functional advantages. However, their design procedures stipulated in various code standards have been computationally demanding as they need development of interaction curves for each trial cross-section considered in the design process.

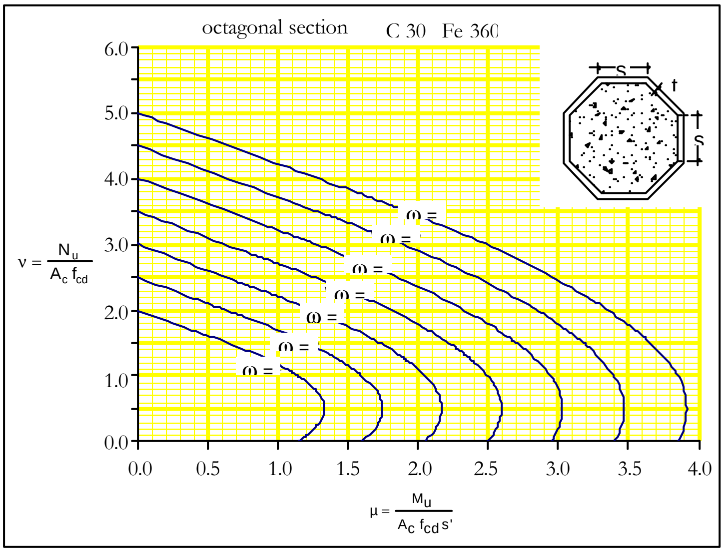



**Fig. 20. Uniaxial interaction diagram for octagonal section.**

To alleviate this problem, normalized charts have been produced that simplify the design calculation. The charts can be used to directly compute the amount of steel required for a given cross-section without resorting to the code-based trial-and-error procedure. In addition to this, they can also be effectively employed to propose a design capable of withstanding a given set of loads. Besides being computationally efficient, the produced charts also provide more accurate results than using the method stipulated in EBCS4(1995).

## ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support of the Office of Research and Graduate Office, Addis Ababa University, to the first author during the preparation of this paper. The continued material support of Alexander von Humboldt Foundation, Federal Republic of Germany, to the second author is also thankfully appreciated.

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