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Price-mediated contagion model – extension of the framework of Greenwood et al.[1]

This spillover model is described here in detail:

Algorithm: Assume an initial exogenous shock hits the banking system, triggering the following process[2]:

1) *Direct exposure*: In time t, every bank holding the shocked assets incurs direct losses which can be quantified by

$$a_{i,t} \sum_{k} w_{i,k,t} f_{k,t} \text{ for bank } b_i \tag{1}$$

where $f_{k,t} \in$ [-1, 0] is the devaluation shock on asset k The bank can be hit with shocks on multiple asset classes, which is why the product of the portfolio weight and the shock value per asset class is summed up before multiplying by total assets $a_{i,t}$. This impact on bank's assets reduces equity on the liability side, which leads to an increase in the bank's leverage ratio. An important assumption of the model is *leverage targeting*, i.e. banks maintain a constant leverage ratio over time.

2) *Liquidity buffer*: Greenwood et al.[1] assume that banks immediately pay off debt to return to their initial leverage ratio l_i in response to the direct losses. A convenient modelling feature that follows from their assumption is that portfolio weights of the k assets are held constant, i.e. banks sell assets in the manner that keeps their portfolio composition the same throughout the de-leveraging phase. However, it is more realistic to assume that banks first use their liquidity buffer to pay off their debt before liquidating assets. Thus, portfolio weights are allowed to fluctuate in our model. The critical value determining the shortfall $\Gamma_{i,t}$ that bank i needs to cover is given by

$$\Gamma_{i,t}: \quad d_{i,t} - \left(l_i \ \max\{e_{i,t} - a_{i,t} \sum_{k} w_{i,k,t} f_{k,t}; 0\}\right) \tag{2}$$

$$\text{with } \Gamma_{i,t} \in [0, d_{i,t}] \quad \text{and}$$
$$\Gamma_{i,t} > 0 \quad \text{if} \quad f_{k,t} < 0$$
$$\Gamma_{i,t} = 0 \quad \text{if} \quad f_{k,t} = 0$$

The intuition behind equation 2 is as follows. If the direct exposure is 0 because the shock is 0%, the shortfall bank i needs to cover is also 0. This is because in the absence of a shock on balance sheets, the composition of the liability side does not change, i.e. equity does not change and the difference between the previous period's debt and next period's debt is also 0. If the shock is negative, the shortfall will be larger than 0 with its maximum at the previous period's level of debt. One should note here that $f_{k,t} \in$ [-1, 0]. It is theoretically possible that equity is wiped out entirely by a very large shock; thus the max operator limits losses to 0, i.e. there is no negative equity.

3) *Fire-sales*: For an individual bank i, the algorithm checks two conditions that can occur in the face of a shock $f_{k,t}$ on its balance sheet. If the shock is too large and liquidity buffers are depleted, bank i starts selling assets immediately in proportion to its weights $w_{i,k,t}$. In the second case, if the bank is able to absorb the shock, neither fire-sales nor spillover to other banks occur, but the balance sheet composition changes in response to transactions. At the bank level, if the individual shortfall is larger than the bank's liquidity buffer, the total bank's de-leveraging amount is determined by the product of its leverage and its direct exposure:

$$\Omega_{i,k,t} = \begin{cases} \underbrace{\tilde{w}_{i,k,t}}_{\text{weight for asset k}} \; \underbrace{l_i}_{\text{leverage}} \; \underbrace{a_{i,t} \sum_k w_{i,k,t} f_{k,t}}_{\text{direct exposure}} & \text{if } \Gamma_{i,t} > \underbrace{a_{i,t} w^c_{i,k,t}}_{\text{liquidity buffer}} \\ 0 & \text{else} \end{cases} \tag{3}$$

with $\tilde{w}_{i,k,\cdot}$ being the adjusted portfolio weight for asset k after cash operations are being taken into account. We sum up the bank-level selling volumes for asset k across all banks to get to the system-wide fire-sales for asset k:

$$\textit{Asset sales}_{\,k,t} = \sum_i^n \Omega_{i,k,t} \tag{4}$$

Note that the first term $\tilde{w}_{i,k,\cdot}$ in 3 and 4 contains the intermediate adjusted weights that follow from cash operations. We define their derivation in equation 8, however, first in the law of motion is the adjustment of the liability side as described below.

How are balance sheets adjusted?

Whenever liquidity buffers are used, weights are adjusted proportionately according to the new total assets of bank i, which in turn depend on how equity and debt are affected by the direct exposure and the pay-off of debt obligations. Equity and debt in $t+1$ are defined by:

$$e_{i,t+1} = max\{e_{i,t} - a_{i,t} \sum_k w_{i,k,t} f_{k,t}; 0\} \tag{5}$$

$$d_{i,t+1} = max\{l_i \;\; e_{i,t+1} \;\; ; 0\} \tag{6}$$

The sum of adjusted equity and updated debt gives total assets of bank i in $t+1$ as

$$a_{i,t+1} = \max\{d_{i,t+1} + e_{i,t+1}, 0\} \tag{7}$$

On the asset side, cash is reduced by how much of the shortfall $\Gamma_{i,t}$ can be covered. In $t+1$, its value is determined by debt pay-offs transactions. The maximum amount that is payable is $\Gamma_{i,t}$, hence new cash positions in $t+1$ amount to:

$$c_{i,t+1} = \begin{cases} 0 & \text{if} \quad \Gamma_{i,t} \geq \underbrace{a_{i,t} w^c_{i,k,t}}_{\text{cash liquidity buffer}} \\ c_{i,t} - \Gamma_{i,t} & \text{else} \end{cases}$$

$$\text{with } c_t = a_{i,t} w^c_{i,k,t}$$

In the case that the cash buffer is not sufficient to de-leverage, $c_{i,t+1}$ is 0. Alternatively, the new cash position is the difference between the previous period's amount and Γ_i, t.

The next step is the intermediate update of portfolio weights $\sum_k w_{i,k} = 1$. As in Greenwood et al.[1], we assume that asset weights determine how much of each asset is sold in the deleveraging process. This assumption is a drastic simplification as selling behaviour is more complex in real markets. However, it is a necessary building block which helps to gauge the extent of overlapping portfolios in the sector, while still being reasonably simple to allow for data calibration. While in Greenwood et al.[1], weights are constant, we allow for fluctuations due to cash transactions. The update process takes place between t and $t+1$, which is why 'intermediate' adjusted weights are denoted with $\tilde{w}_{i,k,\cdot}$. Starting with cash, the intermediate portfolio weight is given by the ratio of the target positions:

$$\tilde{w}^c_{i,k,\cdot} = \frac{c_{i,t+1}}{a_{i,t+1}} \tag{8}$$

Since $\tilde{w}^c_{i,k,\cdot}$ is smaller than $w^c_{i,k,t}\ \forall f_{k,t} < 0$, the difference needs to be accounted for so that $\sum_k w_k = 1$. For sake of simplicity, we distribute the difference proportional to the existing weights. Consider the correction factor $\tau = \frac{w^c_{i,t} - \tilde{w}^c_{i,k,\cdot}}{k-1}$, so that the remaining intermediate weights are given by

$$w_{i,k,\cdot} = w_{i,k \neq c,t} + \tau \qquad \forall f_{k,t} < 0 \tag{9}$$

To re-iterate the law of motion, the intermediate weights are used in the determination of fire-sale volumes in the deleveraging process described in equations. Once transactions materialised overnight, the intermediate weights become the new weights for the period $t+1$.

System-wide de-leveraging

We now turn to the spillover effects that arise from system-wide deleveraging. Recall from equation 4 that the amount of asset k that is sold across all banks is given by

$$\Omega_{k,t} = \sum_i \tilde{w}_{i,k,\cdot} \quad l_i \quad a_{i,t} \sum_k w_{i,k,t} f_{k,t}$$

The direct exposure of bank i is multiplied by its leverage to determine the shortfall bank i needs to cover by asset sales in case liquidity buffers are depleted. This shortfall is multiplied by asset k's portfolio weight $w_{i,k,t}$ to determine the proportional amount that bank i sells of asset k. The sales are summed up over all banks, leading to a total amount $\Omega_{k,t}$, i.e. the system-wide fire-sales of asset k following the initial shock $f_{k,t}$. The equity of bank i is reduced by direct exposure $a_{i,t} \sum_k w_{i,k,t} f_k$, while debt is paid off according to $l_i(a_{i,t} \sum_k w_{i,k,t} f_{k,t})$.

4) *Price impact*: The cumulative sales lead to a price effect $\upsilon(\rho_k, \Omega_{k,t})$ which depends on the liquidity parameter ρ_k and the selling volumes $\Omega_{k,t}$. The assumption is that an exogenous buyer steps in to accommodate the selling volumes at the fire-sold price.

5) *Spillover losses*: The price effect leads to further losses on banks' balance sheets. These are the *indirect* spillover losses arising from common asset holdings. Our analysis is particularly concerned with these kind of spillover losses as they represent the amplification mechanism in the centre of the fire-sale contagion channel. It is possible to describe total spillover losses for asset k by

$$SP_{k,t} = \sum_i (a_{i,t} \sum_k \tilde{w}_{i,k,\cdot}) \underbrace{\left[\rho_k \Omega_{k,t}\right]}_{f*} \tag{10}$$

where the expression inside the square brackets can be interpreted as second round shock f_k^* on asset k. The routine from *3.* is repeated to determine the system-wide losses $SP_{k,t}$ for asset k which result only from the second round fire-sale price-shock f_k^*. Summing up second-round sales across all asset classes gives us the system-wide spillover losses

$$\lambda_t = \sum_k SP_{k,t} \tag{11}$$

In the next step, we capture the fragility of the banking system to fire-sale spillovers by putting λ_t in relation to pre-shock banking sector equity $E_= \sum_i e_i$:

$$AV_t = \frac{\lambda_t}{E_{t-1}} \tag{12}$$

Greenwood et al.[1] call this the *Aggregate Vulnerability* of the banking system to the preceding shock. It is further possible to break down AV into every bank's contribution to the overall losses in the banking system attributable to *indirect spillover losses*, i.e. $AV_t = \sum_i S_{i,t}$.

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