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# A NEW APPROACH TO THE STUDY OF GOLF PUTTING

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***ABSTRACT***

*The aim of this study was to apply non-linear techniques in the analysis of golf putting performance. How players adapt to the variability that emerges from the putting execution and how they self-organize their performance toward the task constraints was investigated. The sample consisted of 10 adult male golfers (33.80±11.89 years) who were volunteers, right-handed and experts (10.82±5.40 handicap), including the European champion of pitch and putting (season 2012/2013). The putting movement was analysed using auto tracking methodologies by autonomously comparing the current frame with the previous frame using a MatLab software program. The results indicated that golf putting performance can be described as a non-linear, stable and regular system in which each player discovers active solutions to overcome the constraints of the task. It was concluded that non-linear techniques, like approximate entropy and Lyapunov exponent are extremely useful for analysing human movement within a sport context.*

**Key words:** Non-linearity; Variability; Golf putting; Motor control; Performance.

## INTRODUCTION

The human body is seen as a non-linear system that is exposed to the instability and disturbances the environment offers (Araújo *et al*., 2004). In this sense, while non-linear systems perform continuous energy exchanges with their surroundings and use that same energy to self-organise, closed systems maintain their characteristics unchangeable and exchange nothing with their environment (Davids *et al*., 2008; Harbourne & Stergiou, 2009).

Non-linear techniques have been used in the field of human motor behaviour to explain the intrinsic variability of biological systems[1]. These techniques provide qualitative information on the tendency of the motor system by viewing different patterns of response. Unlike cognitive theories that support traditional motor control models, which consider variability to

[1] For a more detailed description refer to Harbourne and Stergiou (2009).

be a negative factor for learning, the non-linear perspective shows that 'noise'[2] and 'chaos' are necessary to establish new coordinative patterns (Stergiou *et al*., 2004; Harbourne & Stergiou, 2009).

To describe the variability of human motor behaviour in the context of sport performance, it has been established that non-linear techniques, such as the approximate entropy and Lyapunov exponent, allow unravelling of the structure of a mathematical representation of a given sport movement, like golf putting. In spite of non-linear techniques quantifying the motor performance of athletes through the mean, standard deviation and coefficient of variation, they take in consideration the individual characteristics of players and are mostly based upon statistical effects to characterise the learning and training of motor skills (Stergiou *et al*., 2004).

Faced with such arguments, it seems that the problem of 'individuality' in sport is not confined to ideal, linear or standardised techniques. In fact, it has to do with the implementation of a wide variety of exercises that contribute to the self-organisation of the motor system. As a result, similar to other researchers, human movement is seen as a non-linear system capable of producing solutions to solve motor problems (Schöllhorn *et al*., 2008).

With regards to golf skills, which support the main goal of this work, no study is known, where non-linear techniques are used to analyse this sport. Such confirmation deserves special attention and in-depth research, since each golf player has different morphological and functional characteristics that represent a determined performance profile, 'signature' or 'digital fingerprint' (Pelz, 2000; Couceiro *et al*., 2013; Dias *et al*., 2013). In that sense, it seems difficult to study the variability that characterises the motor performance of golfers in putting performance, based only upon traditional statistical results (mean, standard deviation and coefficient of variation), as is common procedure in most studies that have analysed this movement in laboratory context, as well as in training and competition (Schöllhorn *et al*., 2008).

The Professional Golf Association (PGA Tour) shows that golf putting[3] represents almost 40% of the total amount of strikes performed during a game (Pelz, 2000; Alexander & Kern, 2005; Dias *et al*., *in press*). However, there is no reference in the literature to an analysis of golf putting from the perspective of non-linear techniques (approximate entropy and Lyapunov exponent). This has motivated the scope of this research around golf putting which, although described as a simple motor execution movement, is quite complex and comprises of many variables.

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[2] 'Noise' is considered as random fluctuations that incorporate a certain spectrum of action. Thus, several types of noise are well known in the literature (pink, white, brown and black). The pink noise is related to the study of the human heart rate while the white noise can be measured on electromyography signals (Stergiou *et al*., 2004).

[3] Short shot carried out in the green (Pelz, 2000).

## PURPOSE OF THE STUDY

Considering that non-linear techniques allow for the tracking of the motor system in different behaviour patterns (regularity, stability, complexity or chaoticity), the present study aims to apply these techniques to the analysis of golf putting and, consequently, describe the performance of expert players. In order to do so, this study has as basis the studies conducted by Pincus *et al*. (1991), Stergiou *et al*. (2004) and Harbourne and Stergiou (2009), which show that approximate entropy and Lyapunov exponent are robust tools to analyse human movement within a sports context.

## METHODOLOGY

### Participants

Ten male golfers (age: 33.8±11.89) were tested over 3 experimental studies and they were volunteers, right-handed and experts (10.82±4.05 *handicap*), which included the European champion of *pitch* and *putting* (season 2012/2013). All participants signed a university-approved ethical consent form. All tests were conducted in accordance with the ethical guidelines established by the University of Coimbra, Portugal.

### Task and apparatus

The participants executed the task on an indoor rectangular green carpet, replicating a fast putting surface able to provide a ball speed up to 10m/s. The carpet was 10m long and 2m wide with a thickness of 4mm (Dias *et al*., 2014). Four circles, the size of a golf ball, were drawn on the carpet to point the exact location for the execution of the putting trials, 1, 2, 3 and 4 metres away from the hole. For the second and third studies, a slope, where its legs measured respectively 1m and 100mm, was placed beneath the carpet. In that sense, the golf slope gradient was 20%. A platform with a length of 4m was placed attached to the slope. Finally, 2 circles were drawn on the left and right side of the carpet at 25° in relation to the hole (Figure 1).

### Data recording

To perform this study, a digital Casio Exilim/High Speed EX-FH25 camera was used. It was shooting at 210fps (frames per second) with a resolution of 480x360 pixels and a focal length of 26mm. The digital camera was placed 550mm above the ground heading forward and 4m away from the experimental device, in front of the subject. As the digital camera's lens provides a considerable depth of field, a reference in the same plane of the analysed movement was necessary in order to perform the conversion to m/sec. This reference was the putt's metallic part length of 585mm. Note that this introduces some minor errors due to the declination angle of the putter that varies from player to player.

However, and since the motion of the putter's head is always confined to the same distance from the camera (defined by the ball's position), one can reduce the calibration inaccuracy.

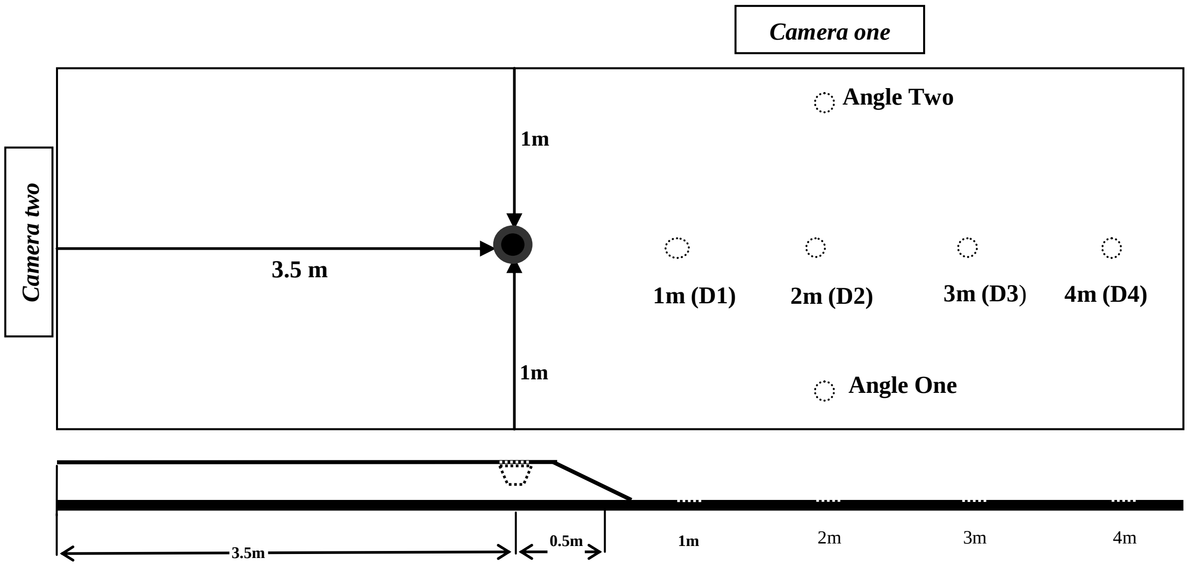


***FIGURE 1*:** **TOP AND SIDE VIEWS OF THE EXPERIMENTAL APPARATUS**

For a more detailed approach about the calibration and acquisition method, please refer to Dias *et al*. (2013) and Couceiro *et al*. (2013). Digital camera recordings provide information about golf putting movements in distinct stages: 1) *back swing*; 2) *down swing*; 3) *ball impact*; and 4) *follow-through*. The putting movement was analysed using auto tracking methodologies by autonomously comparing the current frame with the previous frame using a MatLab script (Couceiro *et al*., 2013; Dias *et al*., 2013).

**Procedures**

The following procedures were followed for studies one, two and three. All the experiments were performed in the same set-up (Figure 1).

1. In Study One, 4 circles were drawn to identify the spots where the ball should be at the beginning of the trials. The circles were aligned with the centre of the hole, 1m away from each lateral extremity of the device (centred).
2. In Study Two, the same experimental device from Study One was used, but a 1m long ramp (slope) was placed under the carpet to elevate the surface by 100mm. This ramp made the ball rise to the level of the hole entrance. Next to the ramp, there was a 4m long and 2m wide platform that worked as a source of additional variability of 'noise' in the performance of a task.
3. In Study Three, the same experimental device from the 2 previous studies were used, and players carried out the putting 2m away, in an ascending trajectory with a 25º angle on the left side of the centre of the hole. Subsequently, the players carried out the putting with the same 25º angle, but on right side of the centre of the hole.
4. For the first practice condition, 30 trials were carried out at each distance of 1, 2, 3 and 4 metres from the hole in a no-slope condition (total of 120 trials). In the second practice condition, the participants performed 30 putts at a distance of 2, 3 and 4 metres from the hole under a slope constraint (total of 90 trials). Finally, the players performed 30 putts at a distance of 2m with 25° to the left of the hole (Angle 1) and 30 putts at a distance of 2m with 25° to the right of the hole (Angle 2), with a constraint imposed by a slope (total of 60 trials).

**Detection algorithm**

The methodology used to detect players' movements, as well as the data analysis techniques is described in this section. As it was a controlled environment, a simple colour detection algorithm, described in Figure 2, was used in order to detect the putter's head through the red marker according to the RGB (Red-Green-Blue components) range values defined (Couceiro *et al*., 2013). The digital cameras' lenses provided a considerable depth of field, a reference on the same plane of the analysed movement. Such procedure was necessary to perform the conversion to m/sec. (Dias *et al*., 2013).

The grey dots in the chart presented in Figure 3 represent an example of a point cloud that represents the detected position, in the horizontal plane, of a golf club during putting execution. Figure 3 shows that the detection algorithm's output have some missing data. This happens when the algorithm is unable to accurately identify the red colour of the marker. In such cases, the detection is skipped in the corresponding time instant to avoid the introduction of errors.

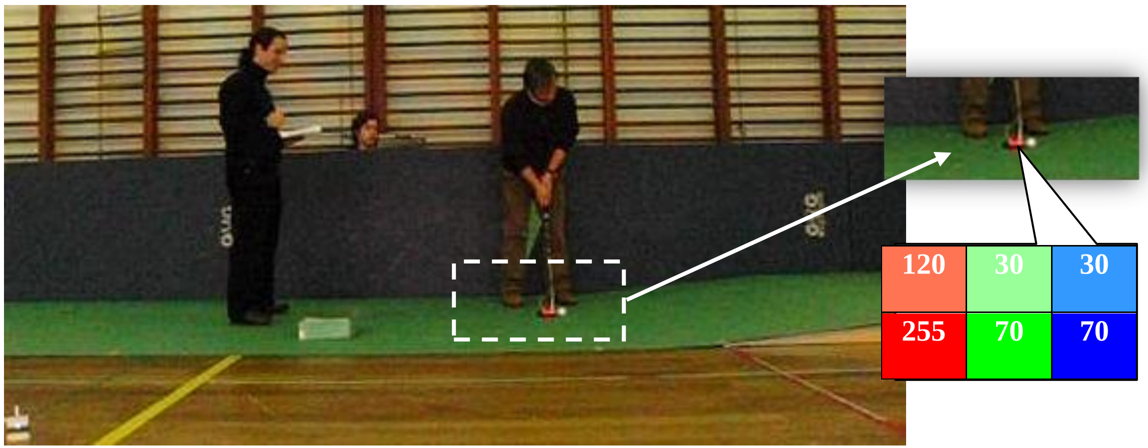


***FIGURE 2*:EXAMPLE OF A REGISTERED SCENE AND RANGE OF COLOUR INTENSITIES TRIGGERING THE DETECTION ALGORITHM** (adapted from Couceiro *et al*., 2013)

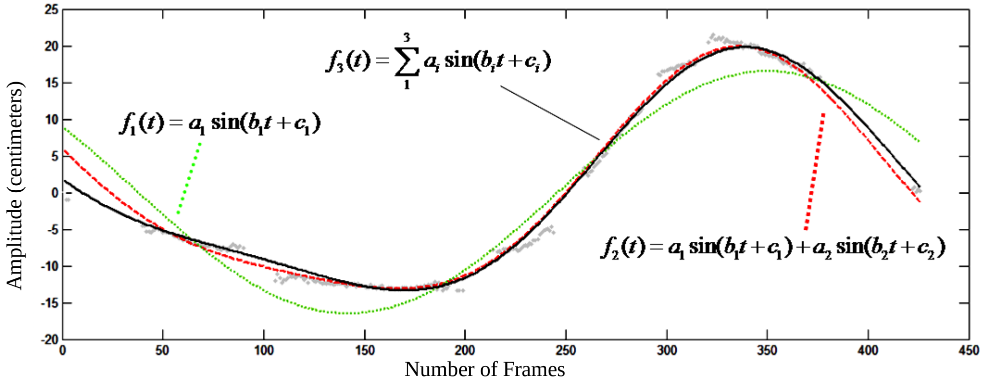


***FIGURE 3*****:** **FITTING SINUSOIDAL FUNCTIONS TO POINT CLOUD REPRESENTS POSITION OF GOLF CLUB DURING PUTTING EXECUTION** (one trial) (adapted from Couceiro *et al*., 2013).

In order to classify the point cloud, linear and non-linear estimation techniques were studied to fit the acquired points of the cloud to a sinusoidal function, thus obtaining a mathematical model to describe the putter's position during the execution of the play. In the next section, the Particle Swarm Optimization (PSO) (Kennedy & Eberhart, 1995) and Darwinian Particle Swarm Optimization (DPSO) estimation techniques are discussed (Tillett *et al*., 2005).

## Estimation algorithms

From the analysis of the shape of various point clouds given by the detection algorithm, it was clear that to model the putter's horizontal position in time, a sinusoidal-like function should be used (Figure 3). Nevertheless, a function composed of only 1 sinusoid was not precise enough to describe the movement, as it is clear in function $f_1$ of Figure 3, which in this case resulted in a mean squared error (MSE) of 2.6568 units. This is due to the amplitude, angular frequency and phase of the descending half-wave corresponding to the player's back swing and down swing, which is usually different than the ascending half-wave, which corresponds to the ball's impact and follow-through (Couceiro *et al*., 2013).

These disparities could not be represented using solely 1 sinusoidal wave. Hence, to obtain a more precise model a sum of sinusoidal waves was used. A compromise between precision and complexity of the problem had to be assumed, as each sinusoid adds 3 more dimensions to the estimation problem. These dimensions are amplitude, angular frequency and phase of the corresponding sine wave. In order not to let the complexity of the problem grow, a function composed of the sum of 3 sinusoids was used (function $f_3$ of Figure 3), due to its precision, with a MSE of 0.6926, when compared to using solely a sum of 2 sinusoids, with a MSE of 0.7124 (function $f_2$ of Figure 3). Although this may be considered a small difference for this particular case, in the course of the several trials function $f_3$ presented highly accurate and more stable results than function $f_2$, without significantly increasing the computation complexity of the model.

The following mathematical model was used to represent golf putting (Couceiro *et al*., 2013):

$$f(t) = a_1 sin\,(b_1 t + c_1) + a_2 sin\,(b_2 t + c_2) + a_3 sin\,(b_3 t + c_3) \qquad (1)$$

Having the estimation function defined as a sum of 3 sine waves, each of the 3 parameters of each wave needs to be estimated, resulting in a 9-dimension estimation problem, which attempts to minimise the mean squared estimation error for every experiment in order to obtain a precise function that describes the horizontal position of the golf club during putting execution.

The *Darwinian Particle Swarm Optimization* (DPSO), first introduced by Tillett *et al*. (2005) and further evaluated in Couceiro *et al*. (2013) in the golf game context, was used. The DPSO extends the original *Particle Swarm Optimization* (PSO) presented by Kennedy and Eberhart (1995), to determine if natural selection (Darwinian principle of survival of the fittest) can enhance the ability to escape from local optima. The aim is to run many simultaneous parallel PSO algorithms, each one a different swarm, on the same test problem and a simple selection mechanism is applied. When a search tends to be a local optimum, the search in that area is simply discarded and another area is searched instead (Couceiro *et al*., 2013; Dias *et al*., 2013).

**Data pre-processing**

After obtaining the parameters of the mathematical model of each trial, it was necessary to generate a single model representing the planar trajectory of the putting over time in all the trials carried out. It would then be possible to numerically calculate the metrics of the non-linear analysis temporally by concatenating the putting trajectory of trials *T* (Figure 3) carried out in a practice condition (Figure 4).

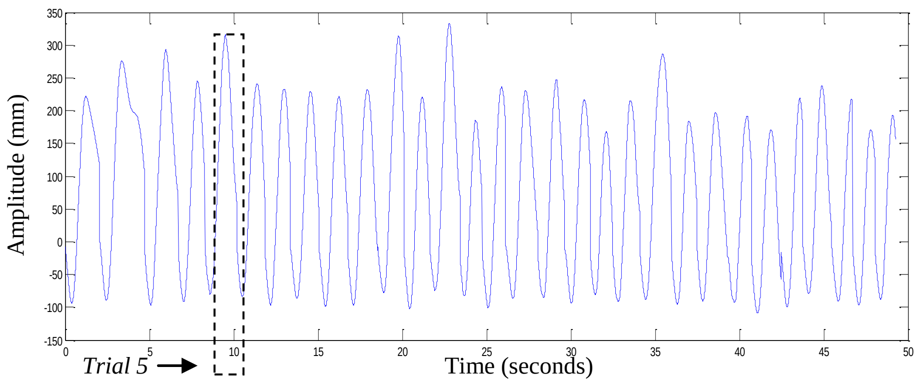


***FIGURE 4*: EXAMPLE OF THE CONCATENATION OF 30 TRIALS**

This mathematical model represents the time series characteristic of a player's movement in a determined practice condition. In the case represented in Figure 4, it is possible to confirm that the player presents some variability at the level of putting execution (the amplitude and duration of the movement diverge throughout the series). The representation between the golf stick position at each instant and speed was used to better observe the movement (Figure 5).

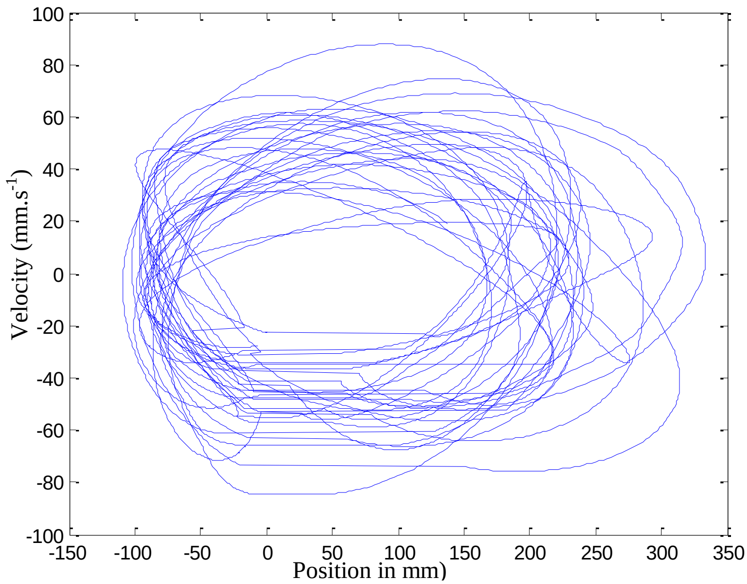


***FIGURE 5*: ATTRACTOR RESULTING FROM STRING CONCATENATION OF 30 TRIALS**

Figure 5 confirms that the movement is placed between the periodic (circular image around a point) and the chaotic (distortion in amplitude and shape of the image). However, it is difficult to quantify the variability of the player. Using non-linear methods that allow for the characterisation of the variability of the player in a determined practice condition becomes important (Harbourne & Stergiou, 2009).

## Non-linear methods

Both the approximate entropy and the largest Lyapunov exponent will be used to further understand the variability of golf players. Throughout the years, several different methods were proposed to calculate both the approximate entropy and the largest Lyapunov exponent (Stergiou *et al*., 2004). The next sections present the chosen approaches based on a preliminary assessment of the related work applied to human movement.

### *Approximate entropy*

Pincus *et al*. (1991) described the techniques for estimating the Kolmogorov entropy of a process represented by a time series and the related statistics approximate entropy. In this sense, consider that the whole data of the $T$ trials is represented by a time-series as $u(1), u(2), \dots, u(N) \epsilon \mathbb{R}$, from measurements equally spaced in time, which form a sequence of vectors $x(1), x(2), \dots, x(N-m+1) \in \mathbb{R}^{1\times m}$, defined by:

$$x(i) = [u(i) \quad u(i+1) \quad \cdots \quad u(i+m-1)] \in \mathbb{R}^{1\times m}.$$

The parameters *N*, *m* and *r* must be fixed for each calculation. *N* is the length of the time series (number of data points of the whole series), *m* is the length of sequences to be compared and *r* is the tolerance for accepting matches. One can define:

$$C_i^m(r) = \frac{number\ \ of\ j\ such\ that\ \leq r}{N-m+1}, \quad (2)$$

for $1 \leq i \leq N-m+1$. Defining $d\big(x(i), x(j)\big)$ for vectors $x(i)$ and $x(j)$, and based on the work of Takens (1983), it results in:

$$d\big(x(i), x(j)\big) = \max_{k=1,2,\ldots,m}[|u(i+k-1) - u(j+k-1)|]. \quad (3)$$

From the $C_i^m(r)$, it is possible to define:

$$C_i^m(r) = (N-m+1)^{-1}\sum_{i=1}^{N-m+1} C_i^m(r), \quad (4)$$

and

$$\beta_m = \lim_{n\to 0\ N\to\infty} \frac{\ln C_i^m(r)}{\ln r}. \quad (5)$$

The assertion is that for a sufficiently large $m$, $\beta_m$ is the correlation dimension. Such a limiting slope has been shown to exist for the commonly studied chaotic attractors. This procedure has frequently been applied to experimental data. Researchers seek a 'scaling range' of *r* values for which $\frac{\ln C_i^m(r)}{\ln r}$ is nearly constant for large $m$, and they infer that this ratio is the correlation dimension (Grassberger & Procaccia, 1983). Some researchers have concluded that this procedure establishes deterministic chaos (Pincus *et al.*, 1991; Pincus & Singer, 1998; Stergiou *et al*., 2004).

The following relation is defined:

$$\phi^m(r) = (N-m+1)^{-1}\sum_{i=1}^{N-m+1} \ln C_i^m(r). \quad (6)$$

One can define the approximate entropy as:

$$ApEn(m, r, N) = \Phi^m(r) - \Phi^{m+1}(r) \quad (7)$$

On the basis of calculations that included the theoretical analysis performed by Pincus *et al*. (1991), a preliminary estimate showed that choices of *r* ranging from 0.1 to 0.2 of the standard deviation of the data would produce reasonable statistical validity of $ApEn(m, r, N)$. As a consequence, values of approximate entropy close to zero characterise a periodical signal/system of high regularity, low variability and little complexity. Following this line of thought, values of approximate entropy equal to or above 1.5, qualify as a signal/system of high variability, low complexity and little regularity (Pincus *et al*., 1991; Pincus & Singer, 1998; Harbourne & Stergiou, 2009).

**Lyapunov exponent**

Using the Lyapunov exponent, it is possible to quantify the sensitivity of initial conditions of dynamical systems. Within the golf context, the spectrum of Lyapunov exponent can classify the divergence of putting trajectories. This concept relates to the spectrum of Lyapunov exponent by considering a small $n$ dimensional sphere of initial conditions, in which $n^{n}$ is the number of equations used to describe the system (Rosenstein *et al*., 1993). The Lyapunov exponent may be arranged so that:

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n, \tag{8}$$

where $\lambda_1$ to $\lambda_n$ correspond to the most rapidly expanding and contracting principal axes, respectively. Hence, one needs to recognise that the length of the first principal axis is proportional to $e^{\lambda_1 t}$, so that the area determined by the first 2 principal axes is proportional to $e^{(\lambda_1+\lambda_2)t}$ and the volume determined by the first $k$ principal axes is proportional to $e^{(\lambda_1+\lambda_2+\cdots+\lambda_k)t}$. Therefore, the Lyapunov spectrum can be defined so that the exponential growth of a $k$-volume element is given by the sum of the $k$ largest Lyapunov exponents. The largest Lyapunov exponent can then be defined by using the following equation, where $d(t)$ is the average divergence at time $t$ and $C$ is a constant that normalises the initial separation:

$$d(t) = Ce^{\lambda_1 t}. \tag{9}$$

In order to improve the convergence (with respect to $i$), Sato *et al*. (1987) proposed the following equation:

$$\lambda_1(i,k) = \frac{1}{k \cdot \Delta t} \cdot \frac{1}{M-k} \sum_{j=1}^{M-k} \frac{\ln\left(d_j(i+k)\right)}{d_j(i)}, \tag{10}$$

where $M$ is the number of axes being analysed. The golf putt can be described by analysing only the horizontal axis, *x*-axis, $M = 1$. From the definition of $\lambda_1$ given in equation (10), we assume that the $j^{th}$ pair of nearest neighbours diverges approximately at a rate given by the largest Lyapunov exponent:

$$d_j(i) \approx C_j e^{\lambda_1 (i \cdot \Delta t)}, \tag{11}$$

where $C_j$ is the initial separation.

By taking the logarithm of both sides of Equation (12) the following is obtained:

$$\ln d_j(i) \approx \ln C_j + \lambda_1(i \cdot \Delta t). \tag{12}$$

Equation (13) represents a set of approximately parallel lines (for $j = 1,2, \ldots, M$), each with a slope roughly proportional to $\lambda_1$. The largest Lyapunov exponent is easily and accurately calculated by using a least-squares fitting to the 'average' line defined by:

$$y(i) = \frac{1}{\Delta t \, \langle \ln d_j(i) \rangle}, \tag{13}$$

where $\langle ln\, d_j\,(i) \rangle$ denotes the average of $ln\, d_j\,(i)$ over all values of $j$. This process of averaging is the key to calculating accurate values of $\lambda_1$ using small, noisy data sets.

The calculus of the largest Lyapunov exponent included the values obtained in the study by Harbourne and Stergiou (2009). In this sense, values close or inferior to zero (0) characterise a periodic signal/system with high periodicity and regularity. On the other hand, values close to 0.1 qualify chaotic signals/systems with high variability and complexity, where values equal to or above 0.4 characterise a system with low regularity and high variability.

Although being evaluated in the golf putting context, by applying the proposed methodology one can characterise any type of human movement in terms of regularity and stability. As such, this methodology can be used to assess the performance of an individual, by comparing it with the typical expected outcome provided by the approximate entropy and Lyapunov exponents. Moreover, as these measures allow classifying the chaos of a given human process, it may shed some light into a closer relationship between process and product variables.

## RESULTS

This section presents the applicability of the previously presented non-linear methods after the detection, estimation and pre-processing steps.

### Approximate entropy

Table 1 presents the average of approximate entropy for the motor execution of the putting of each player in the 3 studies.

***TABLE 1*: AVERAGE OF APPROXIMATE ENTROPY FOR MOTOR EXECUTION OF PUTTING OF EACH PLAYER IN THREE STUDIES**

| | Var | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | Ave per St |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Study 1 | 1m | 0.042 | 0.056 | 0.071 | 0.055 | 0.056 | 0.068 | 0.057 | 0.073 | 0.047 | 0.057 | 0.042 |
| | 2m | 0.042 | 0.056 | 0.068 | 0.053 | 0.052 | 0.071 | 0.048 | 0.065 | 0.044 | 0.051 | 0.042 |
| | 3m | 0.046 | 0.053 | 0.068 | 0.043 | 0.043 | 0.055 | 0.047 | 0.061 | 0.041 | 0.046 | 0.046 |
| | 4m | 0.044 | 0.045 | 0.060 | 0.037 | 0.041 | 0.052 | 0.056 | 0.056 | 0.043 | 0.049 | 0.044 |
| Study 2 | 2m | 0.040 | 0.053 | 0.062 | 0.062 | 0.040 | 0.058 | 0.051 | 0.069 | 0.043 | 0.065 | 0.040 |
| | 3m | 0.033 | 0.048 | 0.064 | 0.054 | 0.039 | 0.066 | 0.064 | 0.058 | 0.044 | 0.048 | 0.033 |
| | 4m | 0.036 | 0.041 | 0.064 | 0.044 | 0.040 | 0.051 | 0.049 | 0.054 | 0.036 | 0.046 | 0.036 |
| Study 3 | A1 | 0.040 | 0.055 | 0.056 | 0.051 | 0.042 | 0.059 | 0.053 | 0.075 | 0.054 | 0.055 | 0.040 |
| | A2 | 0.050 | 0.066 | 0.054 | 0.061 | 0.065 | 0.083 | 0.070 | 0.076 | 0.053 | 0.056 | 0.050 |
| | Ave. per P | 0.041 | 0.053 | 0.063 | 0.051 | 0.046 | 0.063 | 0.055 | 0.065 | 0.045 | 0.052 | 0.053 |

P= Player A= Angle Ave= Average St= Study Var= Variable

The average of approximate entropy obtained by the 10 players in each study and respective distance of shot shows values that vary between 0.033 and 0.050. In distances of 3 to 4m (Study Two), entropy reached minimal values. On the other hand, the maximum value was reached in Study Three, more specifically in Angle Two. As a result, Players 1 and 9 proved to be the most consistent (present the lowest approximate entropy), whereas Players 3, 6 and 8 presented the highest levels of entropy. In addition, when calculating the average of all the values of approximate entropy for each data set, the average value of approximate entropy for putting performance in expert players was 0.053. This is a very stable, regular and periodic value. Through the values obtained for the average of approximate entropy, Figure 6 shows a pattern of regularity and stability of players in the motor execution of putting throughout the 3 studies.

Players 1, 5 and 9 were found to be the most consistent, with player 1 being the most stable of all participants throughout the 9 practice conditions.

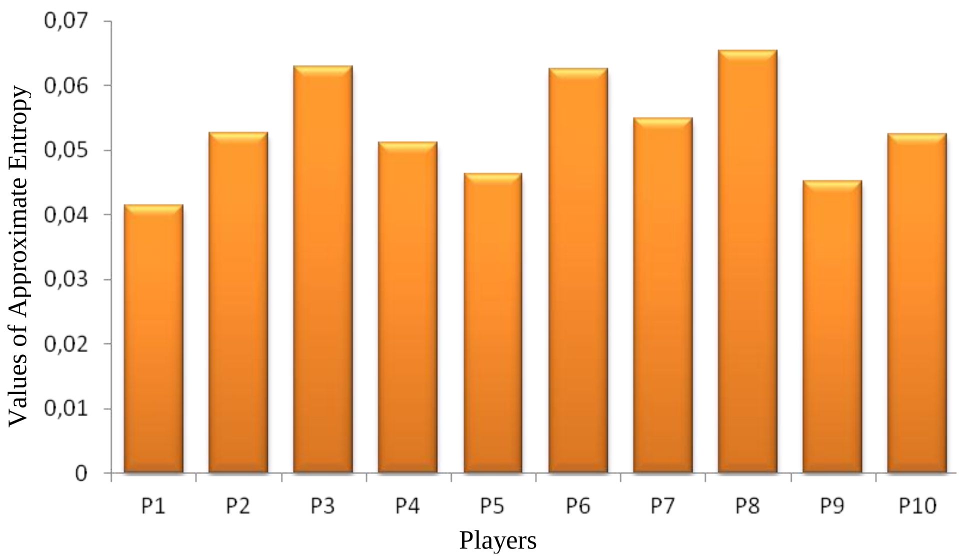


***FIGURE 6*: AVERAGE OF APPROXIMATE ENTROPY FOR MOTOR EXECUTION OF PUTTING OF EACH PLAYER IN THREE STUDIES**

**Lyapunov exponent**

Table 2 presents the median of the Lyapunov exponent for the motor execution of the putting of each player in the 3 studies. The choice to analyse the data shown in Table 2 fell on the median, bearing in mind that the Lyapunov exponent can show extreme and negative values that influence the mean. Moreover, unlike the mean, which can disguise the results obtained, the median is a measure of central tendency that is more consistent and suitable to analyse the Lyapunov exponent. In other words, considering the central value of data distribution, it was concluded that 50% of the values are below to the median and the other 50% are above it (Stergiou *et al*., 2004).

***TABLE 2*: AVERAGE OF APPROXIMATE ENTROPY FOR MOTOR EXECUTION OF PUTTING OF EACH PLAYER IN THREE STUDIES**

| | Var | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | Med per St |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Study 1 | 1m | 0.000 | -0.001 | 0.002 | 0.002 | 0.001 | 0.000 | 0.004 | 0.000 | -0.002 | 0.002 | 0.001 |
| | 2m | 0.000 | -0.002 | 0.000 | -0.002 | -0.003 | -0.001 | 0.003 | 0.000 | -0.002 | 0.003 | 0.000 |
| | 3m | 0.002 | 0.005 | 0.007 | 0.012 | -0.007 | 0.007 | 0.000 | -0.003 | -0.002 | 0.004 | 0.003 |
| | 4m | -0.001 | 0.003 | 0.000 | 0.011 | 0.001 | 0.010 | 0.004 | 0.001 | -0.008 | 0.010 | 0.002 |
| Study 2 | 2m | 0.002 | -0.004 | -0.002 | 0.000 | 0.002 | 0.007 | -0.001 | -0.002 | -0.012 | 0.000 | -0.001 |
| | 3m | 0.002 | 0.003 | 0.011 | 0.003 | -0.004 | -0.010 | -0.004 | -0.002 | -0.009 | 0.010 | 0.000 |
| | 4m | 0.000 | 0.006 | 0.000 | -0.002 | 0.001 | 0.009 | 0.004 | 0.003 | 0.001 | 0.004 | 0.002 |
| Study 3 | A1 | 0.002 | 0.000 | 0.001 | 0.004 | 0.004 | 0.008 | 0.000 | -0.001 | -0.006 | 0.002 | 0.002 |
| | A2 | 0.002 | 0.000 | 0.000 | 0.000 | 0.001 | 0.000 | 0.001 | 0.000 | -0.001 | 0.001 | 0.000 |
| | Med per P | 0.002 | 0.000 | 0.000 | 0.002 | 0.001 | 0.007 | 0.001 | 0.000 | -0.002 | 0.003 | 0.001 |

P= Player A= Angle Med= Median St= Study Var= Variable

The median of the Lyapunov exponent per study and respective distance of shot revealed values that were between -0.001 and 0.003. It reached the maximum value in the 3m distance (Study One). In this sense, Player 9 presented a lower Lyapunov exponent, whilst Player 6 reached the highest value. Moreover, considering the median of all the values of the Lyapunov exponent for each set of data, the resulting value from this non-linear tool for putting performance in players was 0.001.

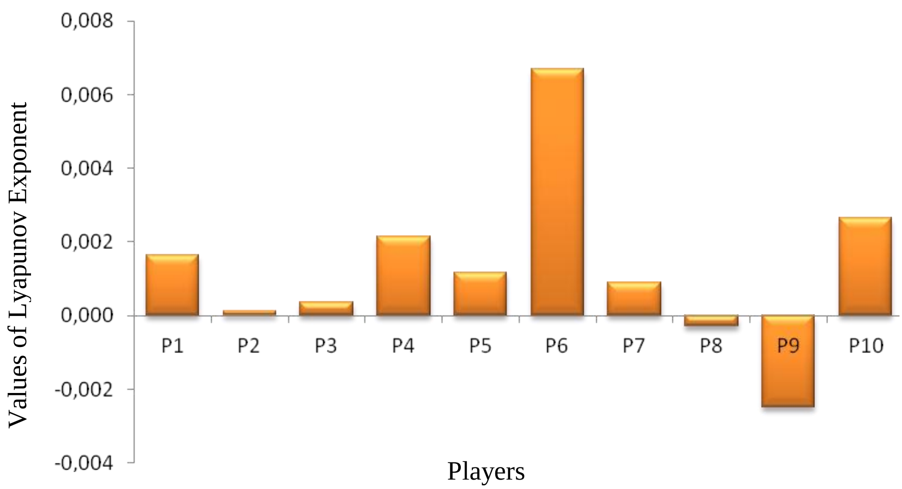


***FIGURE 7*: MEDIAN OF LYAPUNOV EXPONENT FOR MOTOR EXECUTION OF PUTTING OF EACH PLAYER IN THREE STUDIES**

Figure 7 presents the median of the Lyapunov exponent for the motor execution of each player's putting action throughout the 3 experimental studies. As with approximate entropy it was possible to identify a pattern of regularity and stability of the players throughout the entire research. Player 9 presented the lowest and most stable Lyapunov exponent throughout the 9 practice conditions in the 3 studies. Moreover, Player 8 also showed negative values of the Lyapunov exponent.

## DISCUSSION

The main goal of this work was the application of non-linear techniques in the analysis of golf putting performance. The aim was to confirm if this movement can be described as a non-linear system in which each player discovers active solutions to realise the goals of the task. Knowing that non-linear techniques are extremely useful to study the variability of the systems of human movement, approximate entropy and the Lyapunov exponent were used throughout the three studies to analyse the variability of golf putting (Stergiou *et al*., 2004; Harbourne & Stergiou, 2009).

The results showed that the approximate entropy values found throughout the three studies in the longest distances (4m: Study 1) reached minimal values. In addition, contrary to what was expected, the maximum value of approximate entropy was reached in Study Three (Angle 2), 2m away from the hole, when players had to apply a curvilinear trajectory in order for the ball to overcome the ramp, thus being under a large amount of 'noise' and variability. Moreover, the results confirmed that the values of the Lyapunov exponent found in putting performance were between -0.001 and 0.003 (Harbourne & Stergiou, 2009). Thus, unlike approximate entropy, the value with the most 'noise' and variability was reached 3m away (Study 1). However, the median of all the values of the Lyapunov exponent for each set of data (player-study) presented a putting performance value of 0.001, which was below the general approximate entropy of the three studies. Similarly to approximate entropy, it was also possible to follow the motor performance of players and confirm that the putting was an extremely regular, periodic and stable movement (Pelz, 2000; Harbourne & Stergiou, 2009; Dias *et al*., 2013).

By tuning into a non-linear approach and crossing the border into dynamic and chaotic systems, it was possible to confirm that the players adapted to the variability and 'noise' that emerged from putting execution, and self-organised their performance towards the goal of the task (Davids *et al*., 2008). In this sense, the variability that results from motor performance can constitute a 'digital fingerprint' or 'putting signature' that is exclusive to each golfer (Couceiro *et al*., 2013; Dias *et al*., 2013).

## PRACTICAL APPLICATION

Non-linear applications can be used in the study of the variability of systems of human movement by complementing classical linear techniques which are normally used to quantify the performance of motor skills. However, it should be highlighted that this is not about underrating the important role that linear techniques have in the research of systems of human movement, but rather about deepening their study in harmony with non-linear tools (Stergiou *et al*., 2004; Harbourne & Stergiou, 2009).

## CONCLUSION

The variability caused by the manipulation of the task led to the emergence of solutions adjusted to each player within the context of the action. In this sense, the golfers that did not carry out the putting through linear trajectory facing the hole, had to adapt to the difficulties that the experimental device presented. As a result, the high values of approximate entropy obtained in Study Three are justified.

By drawing an analogy between this work and the model proposed by Schöllhorn *et al*. (2008), it is considered that the characteristics of the players (morphological and functional), their level of performance and the complexity inherent in putting execution are important to find substantial differences between the values of approximate entropy and Lyapunov exponent. Consequently, the authors believe that the problem with individuality is not limited to ideal or standardised techniques, but contemplates a wide variety of non-linear strategies that can be implemented according to the specificity of each player.

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