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A NON-LINEAR UNDERSTANDING OF GOLF PUTTING

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ABSTRACT

The aim of this study was to investigate both golf putting precision and accuracy. A new approach is proposed using error ellipses and Fourier series to analyse product variable tendencies in golf putting performance. The sample consisted of 10 male golfers, adults (age=33.80±11.89 years), and volunteers, right handed and experts (10.82±5.40 handicap). Within this context, a ranking of all players, based on the precision and accuracy scores, was carried out, and the best three players with the highest performance, were highlighted. The results indicated that both precision and accuracy of putting performance was adjusted based on the variability conditions and task constraints. It is also noteworthy that the higher ranked players were very regular and stable in their performance even under different practice conditions of variability. These methods may be applied to other sports that require the simultaneous analysis of the precision and the accuracy of a particular movement or motor skill.

Key words: Golf putting; Performance analysis; Fourier series; Error ellipses.

INTRODUCTION

The Professional Golf Association Tour (PGA), shows that golf putting represents almost 40% of the skills performed during the game (Alexander & Kern, 2005; Mackenzie & Evans, 2010). Both accuracy and precision are the main factors that may allow predicting the final outcome (Pelz, 2000). As accuracy is the degree of closeness to a predefined reference, it may be represented by the radial error to the hole (Dias & Mendes, 2010; Dias *et al*., 2011). Precision, on the other hand, is the degree to which repeated measurements under unchanged conditions presents the same result, which may be represented by the dispersion of the ball around the hole (JCGM, 2008). Therefore, a golf player may be accurate but not precise, precise but not accurate, neither, or both (Pelz, 2000).

No research, until now, analysed both accuracy and precision of golf players within the putting movement, while only a few in other sports were reported (Mendes *et al*., 2012; Dias *et al*., 2013). A way of measuring the precision and accuracy of a golfer during putting performance is by applying mathematical techniques (Vicente *et al*., 2010). One of the most promising techniques applied to sport sciences is the Fourier series, which allows representing tendencies through temporal series (Vicente *et al*., 2010).

Many Fourier applications in different areas of investigations have been presented in the literature, namely in studies of heart rate, auditory and visual systems, temperature of the Earth's crust (Verbesselt *et al*., 2010), non-linear systems (Delignières *et al*., 2003), locomotion patterns (Hsiao-Wecksler *et al*., 2010), or team sport games (Kokubun *et al*., 1996).

Within golf research, Vicente *et al*. (2010) applied the Fourier series to analyse the putting performance of five novice players. The authors concluded that the Fourier series analysis allows understanding players' signatures, i.e. behaviour tendencies. Moreover, using the Fourier series transformed how some golf studies analysed the putting phases, namely the backswing, downswing, ball impact and follow-through, as well as the amplitude, duration, velocity and acceleration of movement (Mackenzie & Evans, 2010; Vicente *et al*., 2010).

Pelz (2000) claimed that a golfer who participates in the PGA Tour faces several constraints, being susceptible to a high variability of practice conditions that require constant adaptations. Therefore, the player is faced with multiple possible ball trajectories (either linear or curvilinear, i.e. angle), slopes (either ascending or descending), adverse weather conditions (sun, rain, wind and snow) and even different greens (short grass, high grass, ill-treated grass, grass with holes and sand, among others) (Newell, 1986; Couceiro *et al*., 2013; Dias *et al*., 2013).

AIM OF THE RESEARCH

The aim of this study was to analyse golf putting precision and accuracy. To do so, we first describe the adaptation of relevant putting variables regarding the distance to the hole and the addition of a slope and angle as constraints. Afterwards, a new approach using the error ellipses and Fourier series to respectively analyse both precision and accuracy are proposed regarding the product variables so as to assess the golf putting performance of expert players.

METHODOLOGY

Participants

The sample consisted of 10 volunteer male golfers who were adults (33.80±11.89 years), right handed and experts (10.82±5.40 handicap). By analysing the precision and accuracy scores of all players, it was possible to establish a ranking and, consequently, the three players with the better performance could be selected based on the radial error (Table 1). In that sense, the best three players, namely with lower radial error, were selected for an individual analysis in the results section.

***TABLE 1*: RANK OF BEST THREE PLAYERS BASED ON RADIAL ERROR**

		Radial error (mm)									
(S)	**(D)**	**P1**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10**
S1	D1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
S1	D2	40.13	87.80	31.30	35.00	210.57	215.53	378.67	705.63	62.27	320.80
S1	D3	0.00	283.27	455.30	20.80	489.97	359.17	455.83	812.80	35.53	365.60
S1	D4	0.00	281.37	337.93	174.20	352.03	560.17	459.37	1973.93	12.90	1174.63
S2	D2	155.43	105.03	53.10	37.17	404.53	70.90	115.40	363.60	79.27	123.33
S2	D3	211.97	231.93	39.87	41.70	531.73	364.43	437.50	979.83	333.20	67.77
S2	D4	279.70	356.83	175.83	203.40	979.20	424.17	1027.43	1368.83	348.40	109.53
S3	Ang. 1	433.60	342.83	304.17	540.03	480.73	280.33	522.93	606.30	515.13	457.47
S3	Ang. 2	994.60	413.60	773.17	468.43	528.37	333.23	604.50	1264.20	621.50	710.50
	Mean	235.05	233.63	241.19	168.97	441.90	289.77	444.63	897.24	223.13	369.96

(S)= Study (D)= Distance P=Player mm= Millimetres 0.00 = zero error

This study was conducted within the guidelines of the American Psychological Association and the protocol received approval from a local university ethics committee. All players signed a university-approved ethical consent form respecting the Helsinki Declaration. All tests were conducted in accordance with the ethical guidelines set by the University of Coimbra.

Task and apparatus

The participants executed the task on an indoor rectangular green carpet, which produced a fast putting surface (with an approximate stimp of 10). The carpet was 10 m long and 2 m wide with a thickness of 4 mm (Dias *et al.*, 2011).

Four circles with the size of a golf ball were drawn on the carpet to point the exact location for the execution of the putting trials (1, 2, 3 and 4 m away from the hole). For the second and third studies, a slope, where its legs measured respectively 1 metre and 10 cm, was placed beneath the carpet. In that sense, the golf slope gradient was 20%. A platform of 4 m in length was attached to the slope. Finally, 2 circles were drawn on the left and right side of the carpet at 25 degrees in relation to the hole (Figure 1).

Data recording

Players' putting executions and final outcome were captured using 2 digital cameras (Casio Exilim/High Speed EX-FH25). Camera 1 was placed in a frontal position at a distance of 4 m from the player. Camera 2 was used to capture a lateral and superior view of the experimental apparatus to record the radial error and the ball's trajectory (Figure 1).

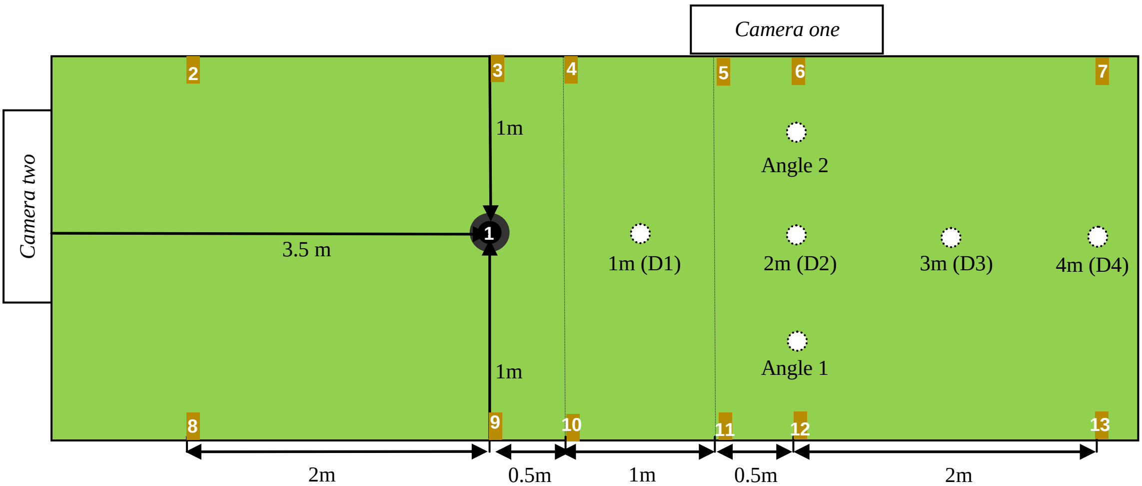


***FIGURE 1*: UPPER VIEW OF EXPERIMENTAL APPARATUS AND REFERENCE POINTS**

Both cameras were static, at the same positioning, height and angle. Images were processed at 210 Hz (camera 1), which allowed a detailed analysis of the putter movement (Delay *et al*., 1997; Coello *et al*., 2000), and 30 Hz (camera 2) to analyse the trajectory of the ball (Dias *et al*., 2013). To support the data analysis, 13 reference points were marked on the green carpet, corresponding to the real coordinates of the experimental device (Figure 1). The centre of the hole is the reference for the adopted coordinate system, with the point (0.0).

Procedures

The following procedures served in conducting the research:

- The task was performed in the gymnasium;
- The players were analysed one-by-one;
- Each player was informed about the main goal of the study;
- Each participant performed 3 adaptation trials at a distance of 2.20 m;
- 30 Trials were performed at each distance of 1, 2, 3 and 4 m for study 1;
- 30 Trials were performed at each distance of 2, 3 and 4 m for study 2 (with a slope);
- 30 Trials were performed on the left and right side of the slope to 25 degrees in relation to the hole for study 3;
- The three studies for each player were performed in the same session.

Analysis of data

Radial error

The radial error is an important form of quantitative evaluation of a player's error during practice in the laboratory or field, namely self-learning situations (Dias & Mendes, 2010). In that sense, recent works about golf putting adopted the radial error as part of the player's performance analysis (Dias *et al*., 2011; Couceiro *et al*., 2012; Mendes *et al*., 2012). In this study, the radial error was obtained using Pythagoras' theorem, as it is the hypotenuse of the right triangle relating to both legs defined by lateral error and longitudinal error (Couceiro *et al*., 2012).

Error ellipses

The centre of the ellipse was calculated using the radial error of the 30 trials with MatLab (Vicente *et al*., 2010). Afterwards, by analysing the ellipse's size and area, one can quantify the accuracy and precision of the golf putting considering the hole (Couceiro *et al*., 2012; Dias *et al*., 2013).

Fourier series

By using the Fourier series, it was possible to analyse the putting performance tendencies of the players. Any tendency may be approximated by a truncated Fourier series for the *n* degree, adjusted to the non-linear least squares (equation 1). In this study, the Fourier series were used to analyse both the maximal velocity of the putting performance and the radial error over the 30 trials for each practice condition (Maor, 2002; Ardito *et al*., 2008):

$$q_n(t) = a_0 + \sum_{j=1}^{n}\left(a_j \cos\left(\frac{j2\pi}{T}t\right) + b_j \sin\left(\frac{j2\pi}{T}t\right)\right) \qquad \text{(Equation 1)}$$

Thus, T (Trials) = 30, the coefficients $a_0, \dots, a_n$ e $b_1, \dots, b_n$ are obtained using the *Trust Region* method, solving the problems of the non-linear least squares (Maor, 2002; Ardito *et al*., 2008).

The choice regarding the process variable selected was supported by Pelz (2000), that considers the maximal velocity as one of the most important variables on golf putting (Vicente *et al*., 2010; Couceiro *et al*., 2013). The maximal velocity was retrieved directly from the direct acquisition of the golf club using Camera 1 (cf., Data Recording section) (Couceiro *et al*., 2013)**.** For that purpose, an auto-tracking methodology that automatically compares the current frame with the previous one was developed under MatLab (Dias *et al*., 2013). Based on the work of Couceiro *et al*. (2013), the pixel/frame value of the putter movement was converted to metric units (m/s).

RESULTS

The results regarding the accuracy and precision performance for each player will be presented in this section.

Error ellipses

The length and width error ellipses of the players' performance could be seen on the Figure 2 and Table 1.

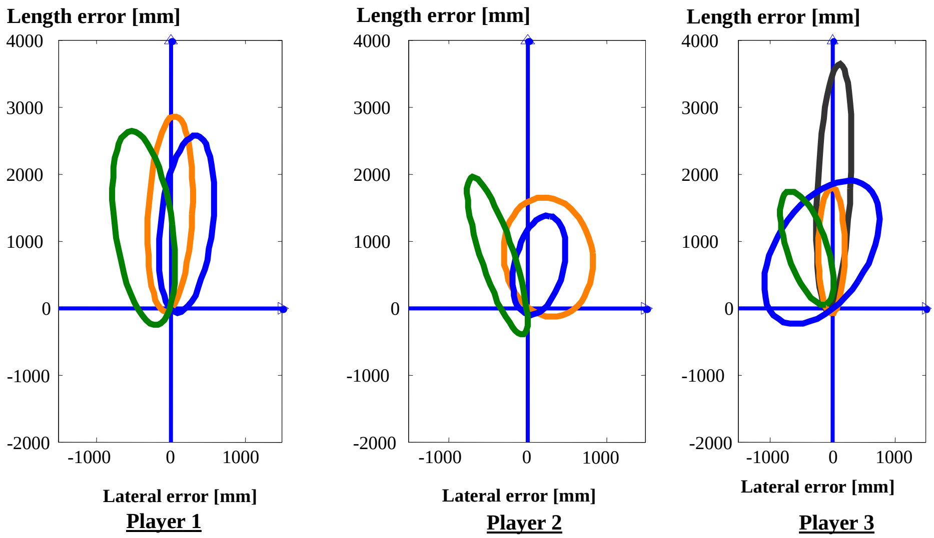


***FIGURE 2*: LENGTH AND LATERAL ERROR OF PLAYERS 1, 2 AND 3 ON THREE STUDIES**

Table 2 presents the accuracy and precision performance for each player, while also showing the lowest number of the ellipses obtained and their lower dimensions of all practice conditions.

***TABLE 2*: ACCURACY AND PRECISION METRICS OF ERROR ELLIPSES OF PLAYERS 1, 2 AND 3 ON THREE STUDIES**

Study (S)	**Player 1**			**Player 2**			**Player 3**		
	area [m^2]	e_C [m^2]	θ_C [°]	area [m^2]	e_C [m^2]	θ_C [°]	area [m^2]	e_C [m^2]	θ_C [°]
S1_1m	0.0	0.0	90	0.0	0.0	90	0.0	0.0	90
S1_2m	0.0	0.0	90	0.0	0.0	90	0.0	0.0	90
S1_3m	0.0	0.0	90	0.0	0.0	90	0.0	0.0	90
S1_4m	0.0	0.0	90	0.0	0.0	90	0.0	0.0	90
S2_2m	0.0	0.0	90	0.0	0.0	90	0.0	0.0	90
S2_3m	0.0	0.0	90	0.0	0.0	90	1.53	1.79	88
S2_4m	1.35	1.40	90	1.58	0.80	70	0.61	0.85	90
S3_ Angle 1	1.46	1.27	80	0.74	0.65	76	2.76	0.84	101
S3_ Angle 2	1.77	1.24	107	0.81	0.86	116	0.89	0.98	114

S= Study m= metre area [m^2] – Ellipse area e_c [m^2] – Ellipse centre θ_C [°] – Angle of the ellipse centre for the hole

Considering the error ellipses, the results suggest that expert players were very regular and stable even when performing under different practice conditions of variability. For example, Player 2 showed higher accuracy (ellipse centre closer to the hole) and precision (average of the ellipse area was lower than the others, thus meaning an inferior dispersion). Nevertheless, in this specific case beyond the absolute error, an attempt was made to represent a stability and regularity of the golf players. In that sense, the ellipses allow the identification of a kind of individual 'signature' of the player.

Fourier series

Using the Fourier approach can possibly identify some individual signatures of the players crossing 2 kinds of important and different information (Kokubun *et al*., 1996). Analysing only the velocities would possibly result in some misinformation about the whole characteristic of the player, because 2 players could have the same patterns in the velocity behaviour but have a different kind of final result, for example radial error (Dias *et al*., 2011).

Analysis of Study 1 for the better three players

Each study corresponds to a specific distance, maximal velocity and radial error. Therefore, a specific legend is presented for the figure about the data. In the case without radial error, it is not possible to perform the curve fitting. Thus, it is normal that in some cases possibly only 2 lines would be seen.

Player 1

For distance 1, Player 1 was completely effective not having missed a single trial, thus obtaining an error of zero. Considering the maximal velocity (Figure 3), the player showed a high decreasing tendency up to trial 8, which increased then up to trial 18 and decreased again at the end (trial 30).

E1=Study 1, D1,D2,D3,D4=Distance, Vmax=Maximal velocity

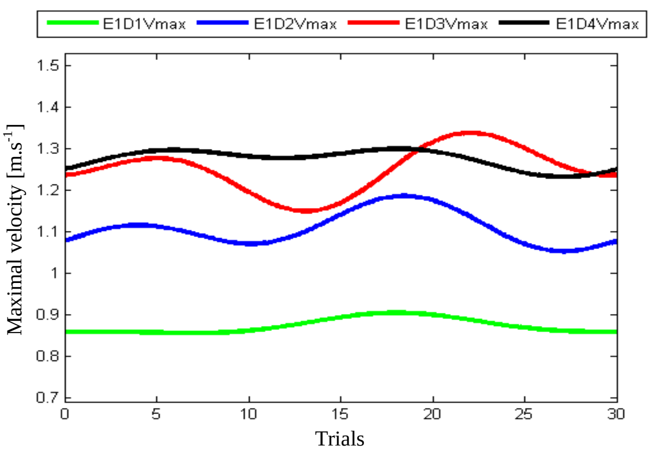


***FIGURE 3*: MAXIMAL VELOCITY [m.s^{-1}] TENDENCY OF PLAYER 1 IN STUDY 1**

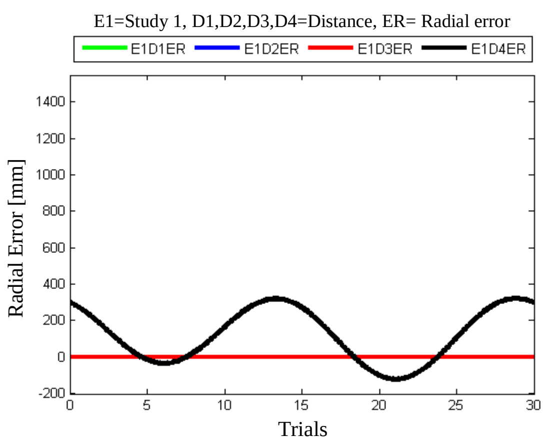
For distance 2, 29 trials were performed without any error. The maximal velocity changed during the 30 trials, with an increase up to trial 5, a decrease up to trial 10, an increase again up to 19 and a decrease during trial 20 through to the end. For distance 3, all trials were performed without any error. The maximal velocity increased up to trial 5, decreased up to trial 13, increased again up to trial 22 and decreased to the finish. Similar to distance 3, in distance 4, the error was zero. Finally, the maximal velocity changed during the 30 trials, increasing up to the 6th, decreasing up to trial 12, increasing again to trial 19 and decreasing to the finish.

Player 2

For distance 1, Player 2 was completely effective not having missed any trial, thus obtaining an error of zero. Considering the maximal velocity (Figure 4), the player showed a constant tendency up to trial 6, an increase to trial 15, a decrease to trial 24 and an increase to the finish (trial 30).

At distance 2, 28 trials were performed without any error. The maximal velocity changed during the 30 trials, decreasing up to trial 7, increasing up to trial 15, decreasing again up to the 23 and finished with an increase. For distance 3, the player missed one single trial. One singular mistake was not enough to change the tendency, thus maintaining the error close to zero. Considering the maximal velocity, Player 2 showed a decreasing tendency up to trial 8, an increase up to trial 13, a decrease to trial 19 and an increase to the finish.

For distance 4, the player showed a decreasing tendency of the radial error up to trial 5, an increase to trial 14. This was followed by a decrease of the error up to trial 21 and an increase up to the finish. Considering the maximal velocity of Player 2, there were decreases until trial 10, increasing up to trial 21, and then decreasing again to the finish.



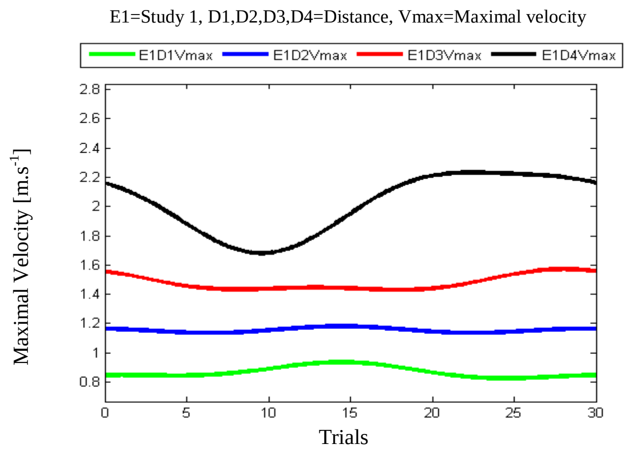
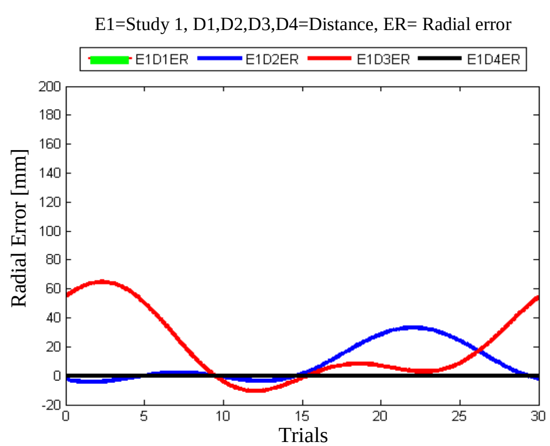


***FIGURE 4*:** **RADIAL ERROR TENDENCY [mm] AND MAXIMAL VELOCITY [m.s^{-1}] OF PLAYER 2 IN STUDY 1**

Player 3

For distance 1, Player 3 just missed one trial. Once again, one singular mistake was not enough to change the tendency, thus maintaining the error close to zero. Considering the maximal velocity, the player showed an increasing tendency up to trial 11, a decrease to trial 17, an increase to trial 23 and a decrease to the finish (Figure 5).

For distance 2, the player missed three trials. Nevertheless, only three mistakes were not enough to change the tendency, thus maintaining the error close to zero. It is possible to observe an increasing tendency of the radial error between trial 15 and 22, which afterwards decreased to the finish. Considering the maximal velocity, the player showed a decreasing tendency up until trial 18, then an increase to the finish.



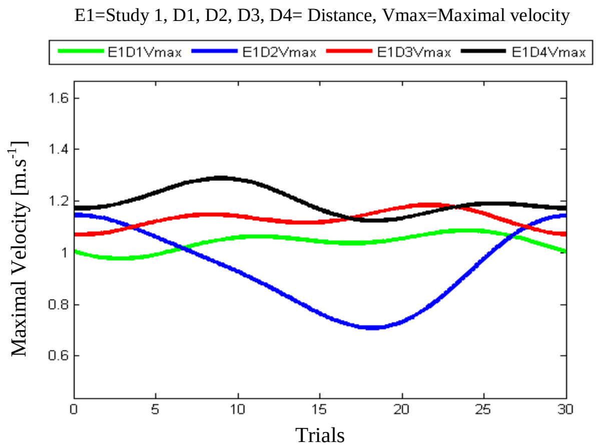


***FIGURE 5*: RADIAL ERROR TENDENCY [mm] AND MAXIMAL VELOCITY [m.s-1] OF PLAYER 3 IN STUDY 1**

For distance 3, the player showed a decreasing tendency of the radial error until trial 10, maintaining close to zero until trial 24. In the last trials, the radial error tendency increased until the finish. Considering the maximal velocity, Player 3 showed an increase until trial 8, and a decrease to the finish. In distance 4, the error was zero. The maximal velocity changed during the 30 trials, increasing until trial 9, decreasing to trial 18 and finishing by an increase.

Analysis of Study 2 for the better three players

Player 1

For distance 2, Player 1 increased the radial error until trial 7, decreasing to trial 12 and maintaining this until the end. The maximal velocity showed a decreasing tendency until trial 22 then increased to the end (Figure 6).

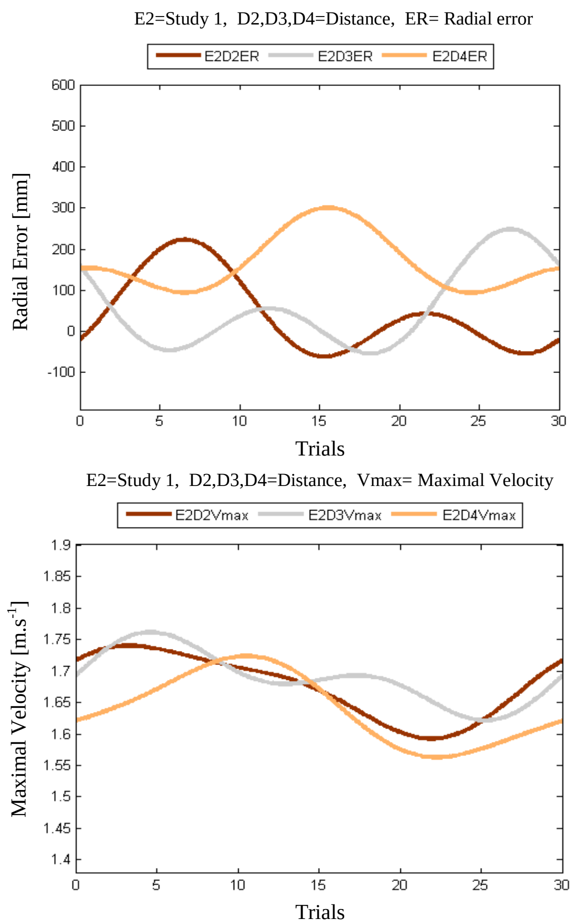


***FIGURE 6*: RADIAL ERROR TENDENCY [mm] AND MAXIMAL VELOCITY [m.s^{-1}] OF PLAYER 1 IN STUDY 2**

For distance 3, Player 1 decreased the radial error until trial 5, increased it to trial 12 and decreased it until the end. The maximal velocity showed an increasing tendency to trial 5, decreased to trial 13, maintaining the velocity until trial 18, which decreased again to 25 and finishing with an increase. Considering the radial error on distance 4, the player showed a slow decreasing tendency to trial 6, an increase until trial 16, a decrease to trial 25 and finishing with an increase. The maximal velocity showed an increasing tendency to trial 11, a decrease until 22 and finished with an increase.

Player 2

The radial error for distance 2 showed a decreasing tendency to trial 6, an increase to trial 14, a decrease until trial 21 and finished with an increase. The maximal velocity showed a slow increasing tendency until trial 3, a decrease to trial 10, an increase until trial 18, a decrease again to trial 23 and finished with an increase (Figure 7).

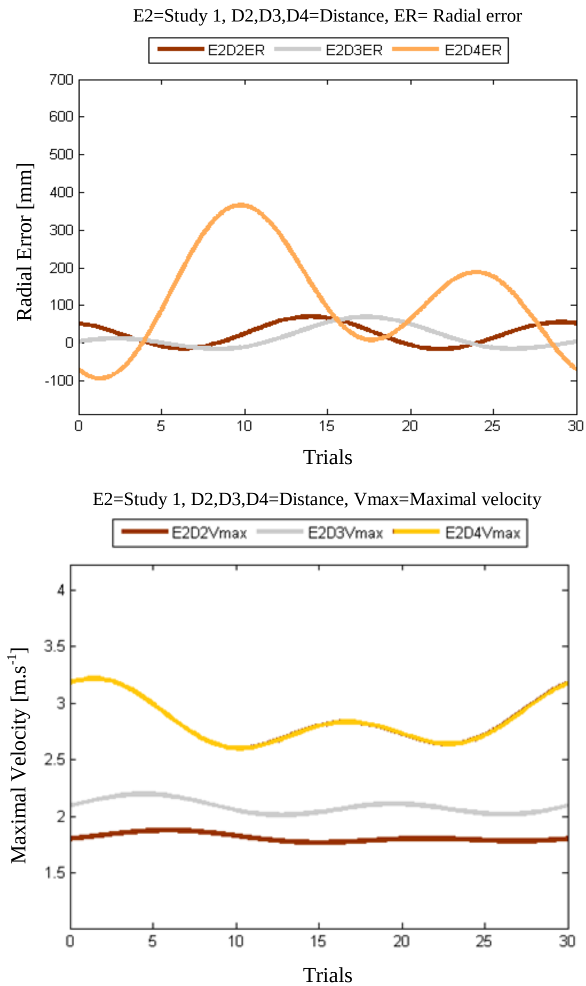


***FIGURE 7*:** **RADIAL ERROR TENDENCY [mm] AND MAXIMAL VELOCITY [m.s^{-1}] OF PLAYER 2 IN STUDY 2**

For distance 3, the radial error maintained almost nil until trial 12, increased to trial 17, decreased to trial 23 and finished by almost maintaining nil. The maximal velocity showed an increasing tendency to trial 5, a decrease until trial 13, an increase to trial 20, a decrease again to trial 26 and finished with an increase. For distance 4, the player showed a radial error increase until trial 10, a decrease to trial 17, an increase again to trial 14 and finished with a decrease. The maximal velocity decreased to trial 10, increased until trial 17, decreased to trial 13 and finished with an increase.

Player 3

During distance 2, Player 3 showed a decreasing radial error until trial 6, an increase to 14, a decrease until trial 12 and increased to the finish. The maximal velocity slowly increased to trial 7, decreased to trial 14, increased to trial 21 and finished with a decrease (Figure 8).

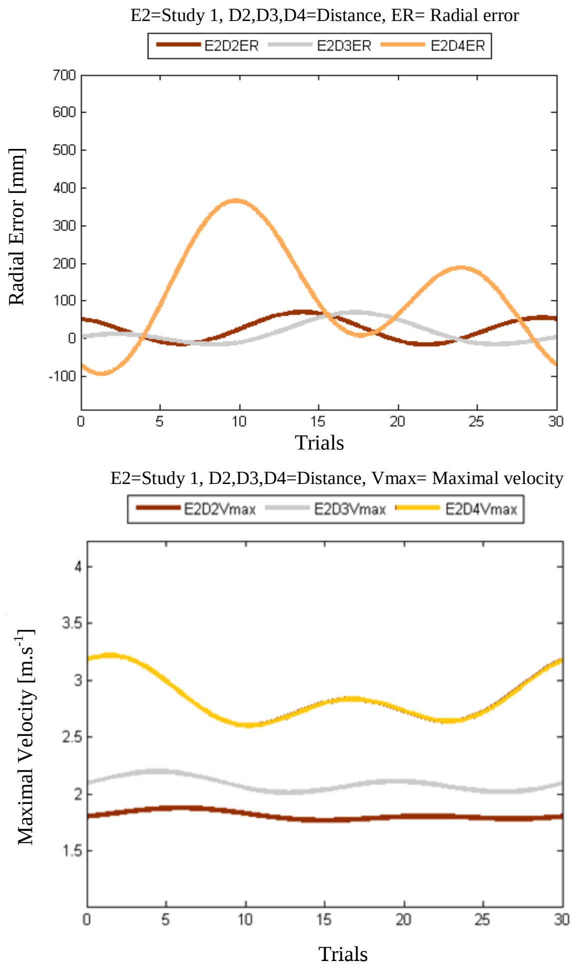


***FIGURE 8*:** **RADIAL ERROR TENDENCY [MM] AND MAXIMAL VELOCITY [m.s^{-1}] OF PLAYER 3 IN STUDY 2**

For distance 3, the player showed an almost nil radial error until trial 12, which increased to trial 17, decreased to trial 23 and maintained almost nil until the finish. The maximal velocity increased to trial 5, decreased to trial 13, increased again to trial 20, decreased until trial 26 and finished increasing. During distance 4, the player showed an increasing tendency of radial error up to trial 10, a decrease to 17, an increase to 24 and finished decreasing. Finally, the maximal velocity decreased to trial 10, increased to 17, decreased to 23 and an increase to the finish.

Analysis of Study 3 for the better three players

Player 1

For angle 1, the player showed a decreasing tendency of radial error to trial 10, an increase up to trial 18, a decrease to trial 24 and an increase to the finish. The maximal velocity increased to trial 10, decreased to 17, an increase to 26 and a decrease to the finish (Figure 9)[1].

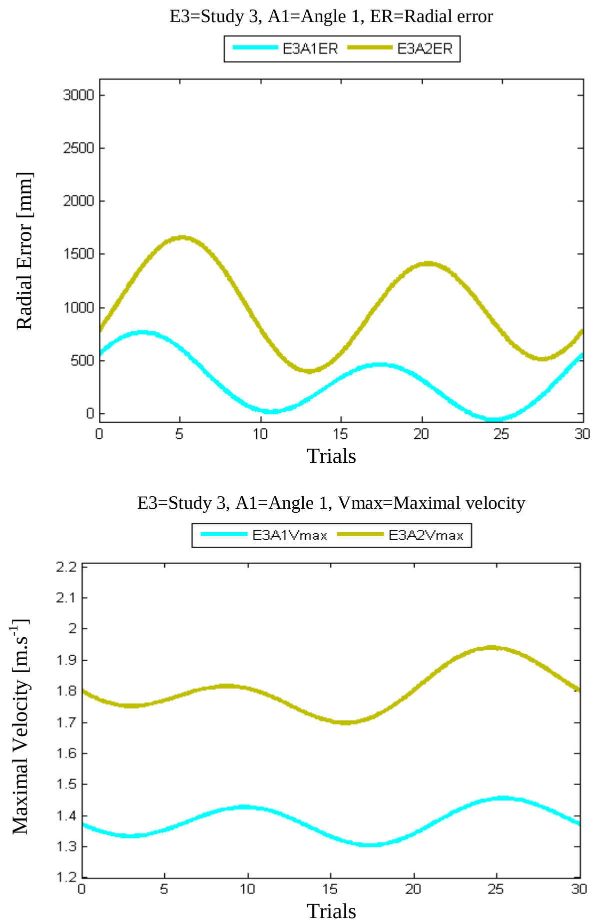


***FIGURE 9*: RADIAL ERROR TENDENCY [mm] AND MAXIMAL VELOCITY [m.s^{1}] OF PLAYER 1 IN STUDY 3**

[1] When the curve achieves negative values (below to 0) is assumed a great number of the success trials (radial error equal to 0). When this happens, it is possible to observe a constant zero tendency over the interval where the curve achieves negative values.

For angle 2, the player showed an increasing tendency of radial error until trial 5, a decrease up to trial 13, an increase to trial 21, a decrease again up to trial 27 and an increase to the finish. The maximal velocity increased to trial 9, decreased to trial 16, increased again up to trial 25 and finished with a decrease.

Player 2

For angle 1, Player 2 showed a decreasing tendency for radial error up to trial 13, which increased up to 23 and maintained it to the finish. The maximal velocity slowly increased up to trial 5, decreased to 11, increased to 18, decreased until 26 and finished with an increase (Figure 10).

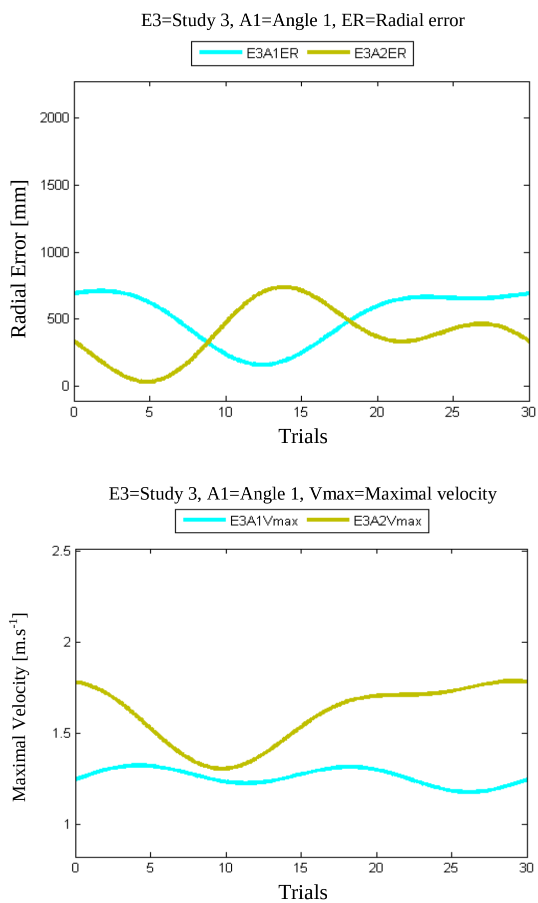


***FIGURE 10*: RADIAL ERROR TENDENCY [mm] AND MAXIMAL VELOCITY [m.s^{-1}] OF PLAYER 2 IN STUDY 3**

For angle 2, the player showed a radial error decreasing tendency up to trial 5, an increase to 14, and a decrease up to trial 21, which increased again up to trial 27 and finished with a decrease. The maximal velocity showed a decreasing tendency up to trial 10 and finished with an increase.

Player 3

For angle 1, Player 3 showed a decreasing tendency for radial error up to the trial 6, followed by an increase to trial 13, a decrease up to trial 20 and finished with an increase. The maximal velocity decreased up to trial 3, increased to trial 11, decreased again up to trial 18, increased up to trial 26 and decreased to the finish (Figure 11). Despite the velocities, there was an adjustment over the practice conditions.

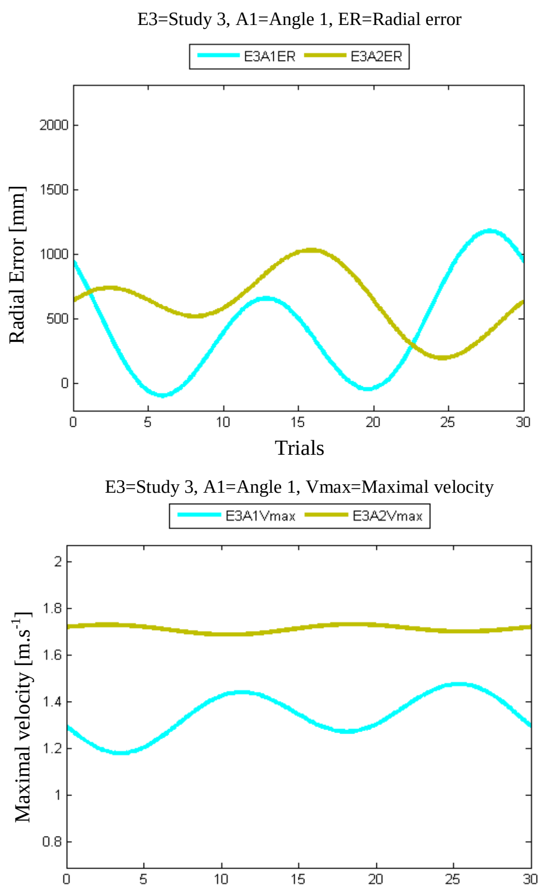


***FIGURE 11*:** **RADIAL ERROR TENDENCY [mm] AND MAXIMAL VELOCITY [m.s^{-1}] OF PLAYER 3 IN STUDY 3**

Regarding the second angle, the player tended to decrease the radial error up to trial 8 and increased it up to trial 16. Henceforth, the error decreased up to trial 25 and increased once again until the end of the trials. Maximum velocity presented a downward trend up to trial 10 and slightly increased up to trial 18. Afterwards, the maximum velocity slowed down until trial 25 and increased again until the end of the trials.

DISCUSSION AND CONCLUSION

No research, until now, has analysed both accuracy and precision of golf players within the putting movement, while only a few have been reported in other sports (Mendes *et al*., 2012; Dias *et al*., 2013). A way of measuring the precision and accuracy of a golfer during putting performance is by applying mathematical techniques (Vicente *et al*., 2010). With reference to the latter, one of the most promising techniques applied to the sport sciences is the Fourier series, which allows representing tendencies through a temporal series (Kokubun *et al*., 1996; Vicente *et al*., 2010).

Even under those conditions, there is a large 'void' in the literature regarding the performance analysis of golf putting using non-linear methods to understand the accuracy and precision of top-ranked players (Kokubun *et al*., 1996; Mackenzie & Evans, 2010). Hence, one could speculate that what actually matters in golf is simply to put the ball into the hole without any concern about the motor performance or the ball's trajectory (Pelz, 2000; Wulf & Su, 2007; Dias *et al*., 2011).

The combination of the aforementioned aspects led this research to implement new performance analysis methods that include the Fourier series and the error ellipses. Operationally, and based on the mathematical models of Maor (2002), Ardito *et al*. (2008) and Vicente *et al*. (2010), the analysis of both movement velocity and radial error was investigated (Pelz, 2000; Dias *et al*., 2013). Therefore, the expectation was that, even when facing different practice conditions of variability, the players were able to retain their precision and accuracy during the performance (Pelz, 2000; Davids *et al*., 2008; Dias & Mendes, 2010).

It is noteworthy that the 'motor variability' aspect was envisaged considering that the game of golf may require a constant stability and performance of golfers (Perkins-Ceccato *et al*., 2003; Poolton *et al*., 2006). For instance, players are under several competing pathways (linear or curvilinear) and slopes (ascending or descending), adverse weather conditions (sun, rain, wind and snow) and different greens (short grass, tall poorly treated with holes and sand) (Pelz, 2000; Dias *et al*., 2011, 2013).

Under these assumptions, the results of this study indicate that it was possible to observe four tendencies in maximum velocity during 30 trials. For instance, during Study 1 the radial error was close to zero for all three of the players measured. This maximal velocity fluctuation, which was similar to the radial error tendency, may suggest a level of self-organisation over the practice conditions, by trying to adjust their movement to improve the output (Davids *et al*., 2008; Dias *et al*., 2013). Thus, these outcomes suggest that studying only the product variables may not be enough for a deeper understanding of sports (Mendes *et al*., 2012; Couceiro *et al*., 2013; Dias *et al*., 2013). The Fourier analysis allows for an observation of the variation over time, thus explaining, to some extent, the unpredictability of human movement behaviour (Kokubun *et al*., 1996; Dias *et al*., 2011).

The experimental results also suggest that expert players are very consistent and stable even when performing under different practice conditions of variability as the ellipses and the Fourier series show. One could assume that the precision and accuracy of golfers were adjusted to the practical conditions and constraints of the task (slope and angle) (Newell, 1986; Davids *et al*., 2008). Moreover, the error ellipses can be considered a supplementary method to further understand the accuracy and precision of the practice condition and possible patterns (Mendes *et al*., 2012). Hence by using these two methods (Fourier series and Error Ellipses), it is feasible to suggest that the information provided is important and gives a new kind of feedback about golf putting performance (Kokubun *et al*., 1996; Mackenzie & Evans, 2010).

Finally, it is concluded that the performance analysis on golf putting should benefit from the development of mathematical methods that would allow assessment of the precision and the accuracy of players simultaneously. Henceforth, these methods could also be applied to other sports that require the simultaneous analysis of the precision and the accuracy of a particular movement or motor skill.

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