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Evaluation of recent Global Geopotential Models over South Africa

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Abstract

This study evaluates the performance of three recent global geopotential models (GGMs) — WHU-SWPU-GOGR2022S, GOSG02S, and Tongji-GMMG2021S — over South Africa by comparing both height anomalies and free-air gravity anomalies derived from these models to data from 141 GPS/levelling points and 105,408 gravity data stations, respectively. The comparison method is crucial as it directly relates the model outputs to precise geodetic measurements, thereby providing a clear picture of model accuracy and effectiveness. Specifically, the Tongji model, developed using GOCE data, exhibited the smallest bias (3.9 cm), with a standard deviation of ±31,7 cm, thereby demonstrating the most accurate fit among the evaluated models with an RMSE of $\pm 31{,}8cm$ *. Additionally, the free-air gravity anomalies comparison yielded biases of -1,74 mGal, -1,69 mGal, and -1,74 mGal for the WHU, Tongji, and GOSG02S models, respectively, with corresponding standard deviations around ±19 mGal. These comparisons not only validate the models against the established South African quasigeoid model, CDSM09A, but also highlight areas for potential refinement. The method employed enhances the contribution of the study to transitioning to a geoid-based vertical datum, thereby improving the accuracy of height and gravity measurements across South Africa and underlining the utility of these models in regional geophysical applications.*

Keywords: *Global Geopotential Model, Quasigeoid Model, Geoid Model, Height Anomaly, CDSM09A*

1. Introduction

Heights measured by space-based instruments are considered purely geometric, meaning they are derived in relation to the ellipsoid, a mathematically-defined reference shape of the Earth. However, these geometric heights lack physical significance, rendering them inadequate for surface analyses, such as waterflow prediction. To attain heights that have physical meaning, it is essential to incorporate Earth's gravity field data into the height measurement process. This involves the use of differential levelling to determine accurate, physically meaningful heights. In differential levelling, the geometric height differences between benchmarks are adjusted by a correction factor derived from gravity data. Spirit levelling is employed to accurately measure the geometric height differences between two benchmarks.

Nonetheless, to establish absolute heights, a height datum is necessary. This datum acts as a reference point, representing the surface at zero height, to which all levelling heights are compared. Essentially, the geoidal height (N) serves as a crucial transformation factor, converting geometric heights (h) into physically meaningful heights (H). The geometric relationship defined by equation (1) below assumes that the geoid and mean sea level coincide. However, this assumption is not accurate (Heiskanen & Moritz, 1967).

$$N = h - H \tag{1}$$

To achieve a high-resolution geoid model, it is pivotal to use the best-fitting global gravimetric model (GGM). Thus, evaluating the recently published GGMs is of paramount importance in establishing the most suitable GGM for geoid modelling. This study aims to assess the performance of the latest published GGMs over South Africa, as detailed in Table 1. These models are accessible through the International Centre for Global Earth Models (ICGEM) (*https://icgem.gfz-potsdam.de/tom_longtime*), and are being evaluated with the intention of informing future geoid modelling efforts. This will significantly contribute to the transition towards a geoid-based vertical datum. Additionally, this evaluation plays a vital role in aligning with the International Height Reference System (IHRS), thereby enhancing global consistency and accuracy in height measurement.

The study of the continental geoid in Africa, while relatively recent, has seen significant developments. In Southern Africa, three regional geoids were introduced: UCT2003, UCT2004, and UCT2006. Additionally, in 2003, a continental quasi-geoid model, known as the African Geoid Project of 2003 (AGP2003), was developed by Merry (2003), aiming to support infrastructural development across Africa. An official hybrid geoid model for South Africa, SAGEOID10, was introduced in 2009 (Merry, 2007; Chandler & Merry, 2010), serving the specific purpose of facilitating the conversion of GPS-derived ellipsoidal heights to spheroidal orthometric heights on the Land Levelling Datum (LLD). Furthermore, a more recent geoid model for the African continent, AFRgeo2019, was developed in 2019 by Abd-Elmotaal et al. (2020) with the goal of providing a unified reference surface for the entire continent. While the SAGEOID10 model serves an important function within the borders of South Africa, it is limited to this geographical area and does not address the broader challenges associated with the LLD such as instability and inconsistency.

The geoid is defined as the equipotential surface of the Earth's gravity field that generally aligns with mean sea level (MSL). The ellipsoid, on the other hand, is a mathematical model of the Earth, characterised by its semi-major axis at the equator and semi-minor axis at the poles. The difference between the geoid and the ellipsoid, known as geoidal height, is crucial for converting GPS-measured heights above the ellipsoid to heights relative to a vertical datum. While mean sea level is often considered a practical approximation of the geoid, it represents a surface of gravitational equilibrium and is not identical to the geoid. However, along coastlines, the geoid and mean sea level are typically regarded as equivalent. As per Merry, (2009), the relationship between the geoid and the ellipsoid is critical for geospatial measurements. The interactions between these reference surfaces over South Africa are further detailed by Mphuthi and Odera (2022).

This study uses a diverse array of data, encompassing gravity anomalies, recently released high-resolution global geopotential models, and GPS/levelling data. The primary objective of this study is to identify the most appropriate GGM for geoid modelling in South Africa. The most suitable GGM identified is further compared against the SAGEOID10 hybrid geoid model to confirm the consistency of this comparison.

2. Data Sources and Methodology

Geoid models are constructed using a variety of data sources, including gravity measurements, the density model, the global geopotential model, and surface topography data. Satellite-derived data are particularly valuable for mapping large areas, including oceans, through a process known as satellite altimetry surveys. As highlighted by Capra & Gandolfi, (1999), this method is noteworthy for its ability to gather data independent of the Earth's gravity field.

Geoid modelling leverages gravity data, which are segmented into terrestrial and satellite-based datasets to produce the short-wavelength elements crucial for gravimetric geoid models. An instance of this application is seen in the UCT geoid model, which integrates data from the UCT gravity database to develop its short-wavelength component (Merry, 2007). Additionally, GPS/levelling measurements, obtained from South Africa's primary levelling network, are integral to both constructing and validating geoid models. These measurements are used to determine the accuracy of the GGMs by comparing the height and gravity anomalies computed from the GGMs with those from GPS/levelling and gravity data, respectively. This process aims to ascertain the most accurate GGM for geoid modelling purposes.

2.1. Gravity Data

The primary gravity dataset is from the South African Council for Geoscience (previously known as the South African Geological Survey) and is accessible at www.geoscience.org.za. South Africa has around 105,408 gravity data stations, with their distribution illustrated in Figure 1. These stations adhere to the 1971 International Gravity Standardisation Net (IGNS71) system, with gravimetric observation precision ranging from ±0,02 mGal to ±0,5 mGal. The positional accuracy of these data points, both horizontally and vertically, emanates from classical topographical maps based on the Cape datum, but these are not, however, integrated with the TrigNet system. While not highly accurate, these data points are deemed suitable for computing Bouguer gravity anomalies. TrigNet, a comprehensive network of continuously operating GNSS base stations across South Africa is managed from the National Geospatial Information control centre.

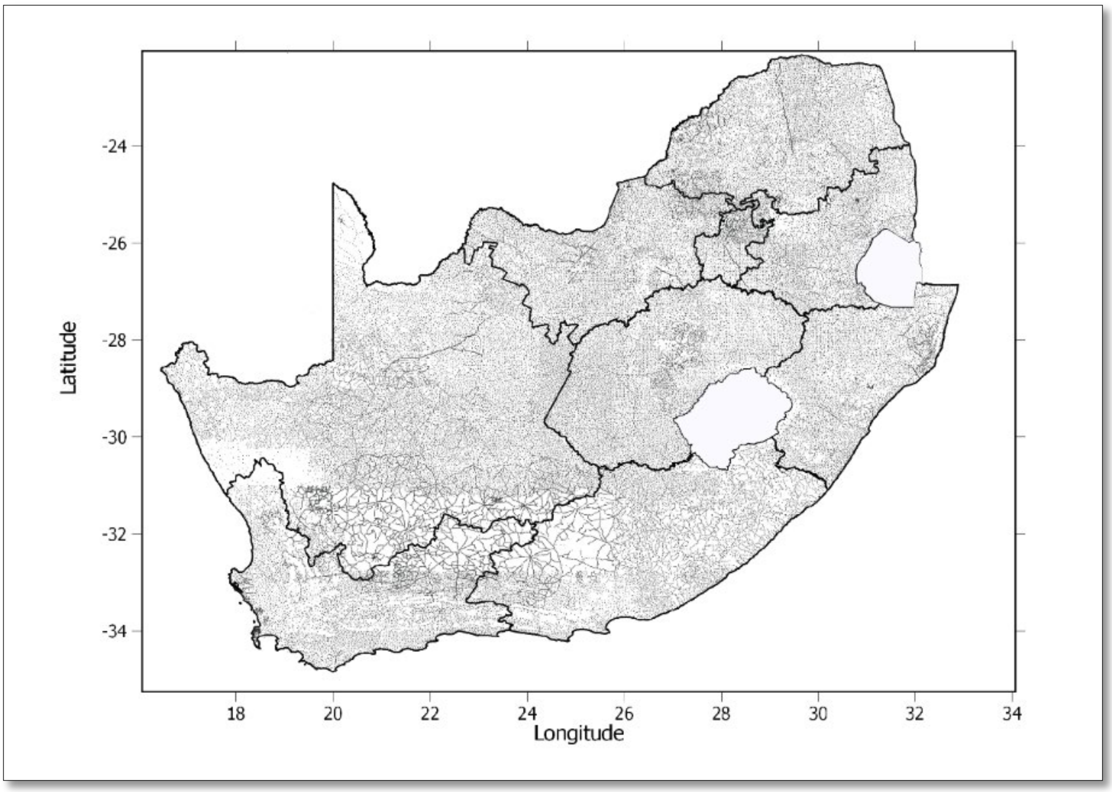

Figure 1: Distribution of terrestrial gravity data over South Africa.

The residual gravity anomalies (Δg_r) are calculated as the difference between the free-air gravity anomalies (Δg_{FA}) obtained from 105,408 observed gravity data stations and the gravity anomalies (Δg_{GGM}) derived from the spherical harmonic coefficients.

$$\Delta g_r = \Delta g_{FA} - \Delta g_{GGM}, \tag{2}$$

The gravity anomalies generated by the spherical harmonic coefficients are evaluated by the applying the following expression (Heiskanen & Moritz, 1967; Moritz, 1980; Yun, 1999; Hofmann-Wellenhof & Moritz, 2005):

$$\Delta g_{GGM} = \frac{GM_g}{a_g . r} . \sum_{n=2}^{n_{max}} (n-1) \left(\frac{a_g}{r}\right)^n . \sum_{m=0}^{n} \left(\Delta \bar{C}_{n,m} \cos m\lambda \, + \, \bar{S}_{n,m} \sin m\lambda\right) . \bar{P}_{n,m}(\sin \bar{\varphi}), \tag{3}$$

Where $GM_g -$ the gravitational mass constant of the geopotential model in m^3/s^3 , as defined from the geodetic model; $r -$ the radial distance to the computational point which is also known as the local elliptic radius in metres (m); $a_g -$ the semi-major axis radius of the geopotential model; $\bar{C}_{n,m}$ and $\bar{S}_{n,m} -$ fully normalised spherical harmonic coefficients, or Stokes' coefficients, of degree *n* and order *m;* $\bar{P}_{n,m} -$ the fully normalised harmonics Legendre function; $\bar{\varphi}$ and $\lambda -$ the geocentric latitude and longitude of the computation point; $\Delta\bar{C}_{n,m} -$ the difference between the full harmonic coefficient $\bar{C}_{n,m}$ and the harmonic coefficient generated by the normal gravity field, $C^*{}_{n,m}$. The resulting gravity anomaly, Δg_{GGM}, computed from the spherical harmonic series expansions ($from\ n = 2\ up\ to\ n_{max}$) is compared to the free gravity anomalies.

2.2. GPS/Levelling Data

The South African height datum, known as the Land Levelling Datum (LLD), is based on tide gauge measurements from over a century ago. These measurements are linked to networks of primary levelling benchmarks, which were adjusted in a piecemeal fashion. However, the mean sea level determination at these tide gauge stations faces various systematic distortions (Merry, 1985). Consequently, distortions are present in both the primary levelling networks and the datum itself. The presence of these systematic distortions in the mean sea level determinations and the primary levelling networks underscores the necessity for a geoid-based vertical datum.

This approach would provide a more accurate and consistent reference for height measurements across South Africa. However, the initial step in establishing a geoid model involves identifying the best-fitting GGM. This foundational phase is crucial to ensure the accuracy and reliability of the geoid-based vertical datum. Approximately 141 GPS/levelling data points, distributed throughout the country and collected by the National Geo-spatial Information (NGI) (https://ngi.dalrrd.gov.za/) are then used to evaluate the GGMs.

2.3. Global Gravimetric Models

A total of three of the latest published GGMs developed over the last two years are used in this study. The height anomalies derived from the spherical harmonics of the GGMs to be compared with the height anomaly derived from the GPS/levelling data (expressed by equation 1), are computed as follows (Heiskanen & Moritz, 1967; Moritz, 1980; Rapp, 1994; Yun, 1999; Hofmann-Wellenhof & Moritz, 2005):

$$\zeta_{GGM} = \zeta_0 + \frac{GM_g}{r\gamma} . \sum_{n=2}^{n_{max}} \left(\frac{a_g}{r}\right)^n . \sum_{m=0}^{n} \left(\Delta\bar{C}_{n,m} \cos m\lambda \ + \ \bar{S}_{n,m} \sin m\lambda\right) . \bar{P}_{n,m}(\sin\bar{\varphi}) \tag{4}$$

Where γ represents normal gravity, and ζ_0 represents a zero-degree harmonic term contribution to the GGM geoid undulations with respect to the WGS84 reference ellipsoid, $\zeta_0 = \frac{GM_g - GM_0}{R\,\gamma} - \frac{W_O - U_0}{\gamma}$ (Heiskanen & Moritz, 1967; Hofmann-Wellenhof & Moritz, 2005). W_O and U_0 represent the gravity potential value on the global geoid and the normal gravity potential on the reference ellipsoid, respectively. The WGS84 ellipsoidal normal gravity field parameters have been used for this computation. This evaluation is then able to provide an estimation of the correction height anomaly $\zeta_{GGM} - (h - H) = \delta$ (Tenzer et al., 2006). The GGMs encompass a broad spectrum of models, each characterised by different combinations of input data, including altimetry data, satellite tracking data, and terrestrial gravity data. Additionally, all three models used in this study, WHU-SWPU-GOGR2022S, GOSG02S, and Tongji-GMMG2021S, are defined to 300 degrees and orders. A brief description of each model is provided, and detailed specifications can be found in Table 1 below:

1. **WHU-SWPU-GOGR2022S (300)**: This is a static gravity field model that achieves completeness up to the spherical harmonic degree and order of 300. It is developed by merging normal equations from the GOCE and GRACE satellite missions, thereby enhancing the model's gravitational field representation (Zhao et al., 2023).
2. **GOSG02S (300)**: This is a static gravity field model completed to a spherical harmonic degree and order of 300. It uses Satellite Gravity Gradiometry (SGG) data and Satellite-to-Satellite Tracking (SST) observations from the GOCE mission, employing least-squares analysis for model derivation (Xu et al., 2023).
3. **Tongji-GMMG2021S (300)**: This is a high-precision static GRACE-only global Earth gravity field model, achieved through refined data-processing strategies. It offers detailed analysis and recovery of the static gravity field, particularly focusing on the reprocessed GOCE Level 1b Gravity Gradient Observations (Chen et al., 2022).

Table 1: Global Gravimetric Model Data (https://icgem.gfz-potsdam.de/tom_longtime)

Model	**Year**	**Degree**	**Data**	**References**
WHU-SWPU-GOGR2022S	2023	300	S (Goce), S (Grace)	Zhao, Yongqi et al 202
GOSG02S	2023	300	S (Goce)	Xu, Xinyu et al 2023
Tongji-GMMG2021S	2022	300	S (Goce), S (Grace)	Chen, J. et al, 2022

2.4. The South African Quasigeoid Model (CDSM09A)

The South African Hybrid Geoid Model (SAGEOID10) is a hybrid geoid model for South Africa, created by integrating GPS/levelling data into a quasigeoid model (CDSM09A) (Chandler & Merry, 2010) . This model incorporates long-wavelength components from the EGM2008 geopotential model (up to 360 degrees) and adds medium and short-wavelength components, based on land and marine gravity anomalies with terrain corrections. The geometric geoid model used 79 benchmarks across the country, enabling it to calculate height anomalies by determining the difference between the ellipsoidal and spheroidal-orthometric heights (Chandler & Merry, 2010). Using the WGS84 ellipsoid as the reference, a correction surface was established from the comparison of the geometric and quasigeoid model to correct existing biases and tilts in the quasigeoid model, thus forming the SAGEOID10 hybrid geoid model. As noted by Chandler and Merry (2010), this model was developed and validated using subsets of GPS/levelling data points, thereby achieving a standard deviation of 7cm at the validation points. However, in this study, height anomalies from the GPS/levelling data, the CDSM09A quasigeoid model, and the SAGEOID10 were compared with height anomalies derived from the GGMs.

3. Data Analysis

The three global geopotential models, WHU-SWPU-GOGR2022S (WHU), GOSG02S, and Tongji-GMMG2021S (Tongji), were evaluated by comparing the height anomalies calculated from these models against measurements from 141 GPS/levelling data points over South Africa. This comparison was instrumental in identifying discrepancies and evaluating the precision of each model within the country. The observed differences, which are non-zero, can be attributed to the inevitable random errors in data collection, as well as to the systematic biases and differences in computation techniques. This evaluation offers insights into how effectively each Global Geometric Model (GGM) corresponds with the GPS/levelling data across the country. The means and standard deviations of the differences between the GPS/levelling data and the GGMs, implying height anomalies, are illustrated in Table 2, while the means and standard deviations of the differences between the GGMs and CDSM09A height anomalies models are illustrated in Table 3.

Table 2: Comparison of the GGMs with the GPS/levelling data

	GPS/Levelling-WHU (m)	GPS/Levelling-Tongji (m)	GPS/Levelling-GOSG02S (m)
Min.	-0,653	-0,662	-0,650
Max.	1,353	1,304	1,357
Mean.	0,043	0,039	0,046
Std	0,322	0,317	0,321
MAE	0,241	0,239	0,241
RMSE	0,323	0,318	0,324

As evidenced by the mean bias results, Table 2 demonstrates that all three global geopotential models are positioned below the GPS/levelling data. Additionally, it shows that the performance of the three GGMs is comparable when measured against the GPS/levelling data. On average, the WHU, Tongji, and GOSG02S global geopotential models implied height anomalies compared to 141 GPS/levelling data points across South Africa, showing biases of $4.3\text{cm}, 3.9\text{cm}, and\ 4.6\text{cm}$, respectively. The standard deviations of the height anomaly differences for the models were $\pm 32.2\text{cm}$, $\pm 31.7\text{cm}, and \pm 32.1\text{cm}$, respectively. Among these models, the Tongji GGM, developed using GOCE data, provided the best fit to the GPS/levelling data, with a Root Mean Square Error (RMSE) of $\pm 31{,}8\ \text{cm}$, thereby demonstrating greater accuracy compared to xxx the WHU and GOSG02S models. These three models were then compared to the recent South African quasigeoid model (CDSM09A). According to the findings provided in Table 3, the CDSM09A model exhibited biases of

$18{,}5\text{cm}, 18{,}2\text{cm}, and\ 18{,}8\text{cm}$ when compared to the WHU, Tongji, and GOSG02S models, respectively. The standard deviations of the height anomaly differences for the models were $\pm 34{,}5\text{cm},\ \pm 34{,}9\text{cm}, and\ \pm 34{,}4\text{cm}$, respectively.

Table 3: Comparison of the GGMs with the CDSM09A height anomaly model

	CDSM09A - WHU (m)	CDSM09A - Tongji (m)	CDSM09A - GOSG02S (m)
Min.	-1,245	-1,268	-1,242
Max.	1,413	1,384	1,416
Mean	0,185	0,182	0,188
Std	0,345	0,349	0,344
MAE	0,314	0,318	0,315
RMSE	0,391	0,394	0,392

The three GGMs were also compared to the SAGEOID10 hybrid quasigeoid model, developed from the CDSM09A model using GPS/levelling measurements. It is important to recognise that the SAGEOID10 model serves as a practical approximation of the geometric quasigeoid across South Africa and is expected to closely align with the GPS/levelling data. As such, it is anticipated to exhibit behaviour similar to the GPS/levelling data when compared to the three GGMs. The summary statistics of the differences in implied height anomalies between SAGEOID10 and the GGMs across South Africa are detailed in the Table 4 below.

Table 4: Difference between the SAGEOID10 and GGMs height anomalies

	SAGEOID10 - WHU (m)	SAGEOID10 - Tongji (m)	SAGEOID10 - GOSG02S (m)
Min.	-1,106	-1,115	-1,104
Max.	1,550	1,516	1,553
Mean	0,030	0,028	0,033
Std	0,308	0,310	0,308
MAE	0,239	0,240	0,239
RMSE	0,309	0,311	0,310

To further assess where the smallest and largest margins of discrepancy are distributed across the country, trendline scatterplots in Figure *2* below clearly illustrate the differences between the GPS/levelling data and the three global geopotential models, respectively.

The scatterplots depicted in the figure 2 illustrate that the margins of discrepancy tend to decrease with a gradient of 2cm from the south to the north of the country. The decrease could be due to a few reasons, such as, the existing geophysical variations as one moves from the southern to the northern regions of the country, data density, and data quality. The trendlines show similar behaviours across the three GGMs when compared to the GPS/levelling data. To pinpoint where in the country the biases are smallest, refer to Figure 3 below. Note that only the comparison between the GPS/levelling data and the Tongji geopotential model is presented, as it best fits the GPS/levelling data across South Africa.

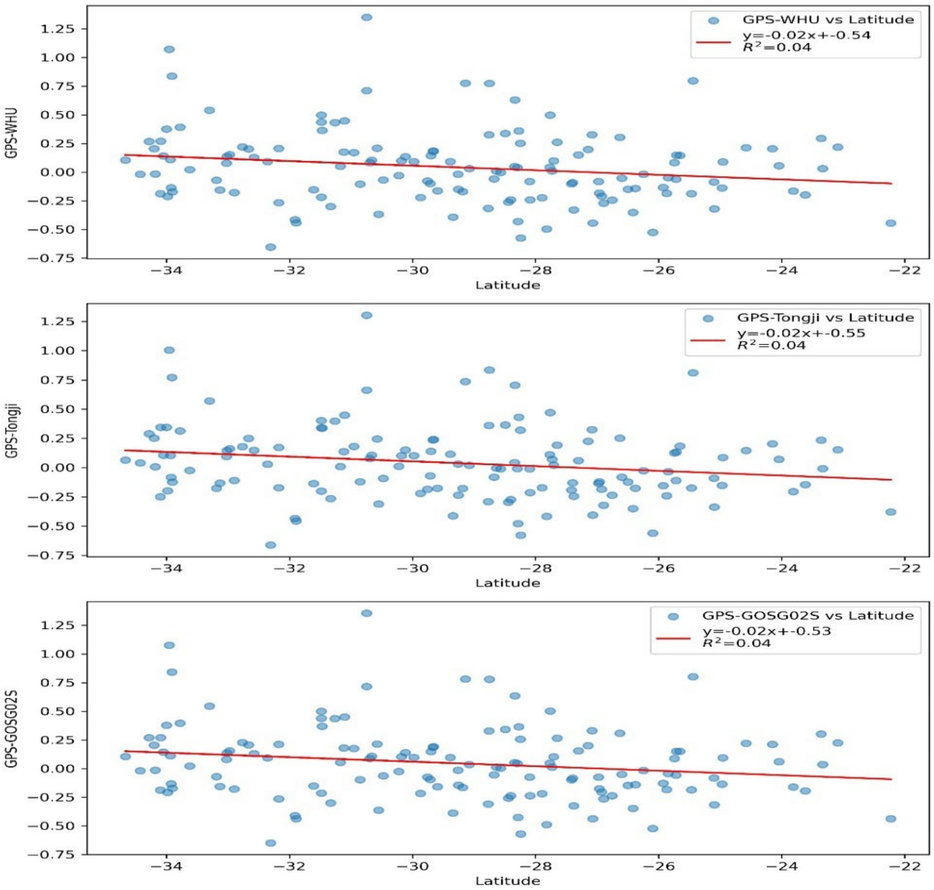

Figure 2: Trendline scatterplots showing the difference between the GPS/levelling data and the height anomalies implied by the three GGMs.

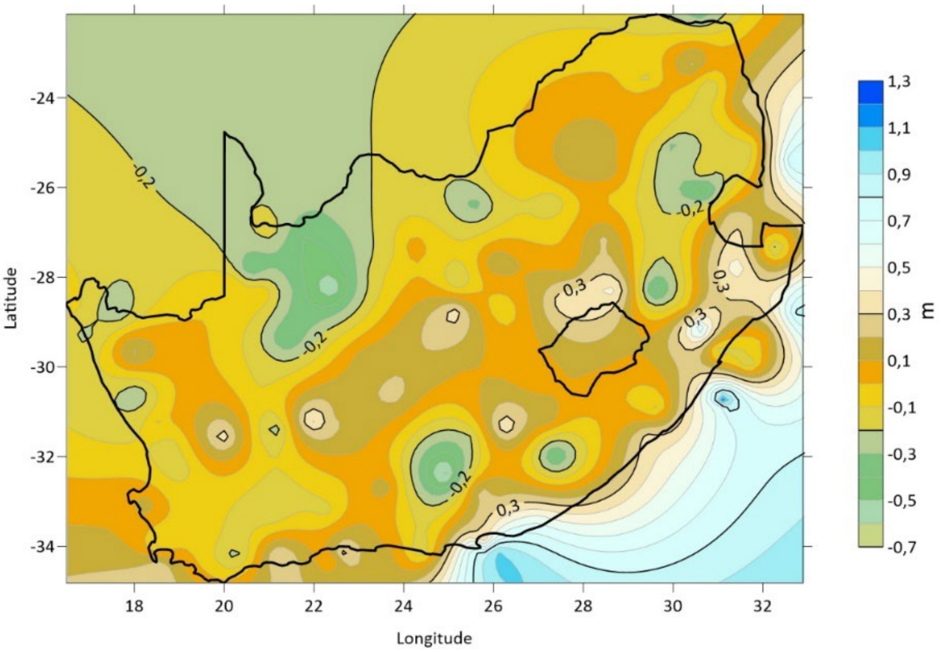

Figure 3: Differences between the GPS/levelling data and the height anomalies implied by the Tongji GGM.

The final phase of this evaluation involved a comparison of the observed free-air gravity anomalies with those implied by the GGMs (WHU, Tongji, and GOSG02S models). This step is crucial when using a specific global geopotential model for deriving accurate free-air gravity anomalies over South Africa. The biases observed in the comparison of the observed and GGM-implied free-air gravity anomalies over South Africa were $-1{,}74\text{mGal}, -1{,}69\text{mGal}, and\ -1{,}74\text{mGal}$ for the WHU, Tongji, and GOSG02S models, respectively. The corresponding standard deviations for the gravity residuals were $\pm 19{,}26\text{mGal},\ \pm 19{,}34\text{mGal}, and\ \pm 19{,}26\text{mGal}$ for each model, respectively.

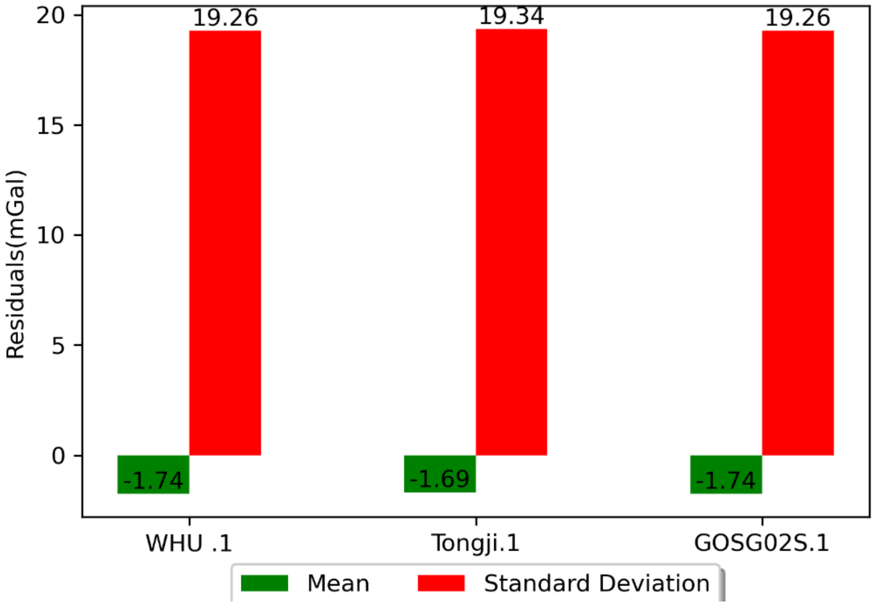

Figure 4: Differences between the observed free-gravity anomalies and the GGM-implied free-air gravity anomalies over South Africa.

Having established the performance of the WHU, Tongji, and GOSG02S models in recovering both height anomalies and free-air gravity anomalies over South Africa, the findings from this section are consolidated in the following section. This examination has not only highlighted the precision and biases of these models but has also underscored their effectiveness in practical geophysical applications.

4. Conclusion and Recommendations

The comprehensive evaluation of the WHU, Tongji, and GOSG02S global geopotential models, as detailed in Table 2 and Table 3, reveals critical insights into their performance in modelling height anomalies across South Africa. The comparison against 141 GPS/levelling data points shows that, compared to the WHU and GOSG02S models, the Tongji model, with the smallest bias of 3.9 cm, provides the most accurate fit, indicating its superior calibration against local geodetic data. This suggests that the Tongji model may be more reliable for applications requiring high precision levels in height anomaly determinations over South Africa.

Furthermore, when these models are assessed against the recent South African quasigeoid model (CDSM09A), all three exhibit larger biases, ranging from 18.2 cm to 18.8 cm, and increased standard deviations, suggesting less consistency in their performance against the model. This discrepancy highlights potential areas for improvement in model adaptation to local geophysical variations and suggests a need for ongoing refinement and validation against more region-specific datasets.

From the comparison of the observed and GGM-implied free-air gravity anomalies using the WHU, Tongji, and GOSG02S models, a conclusion could be that all three models exhibit a consistent representation of the gravity field over South Africa, with their biases and standard deviations being closely matched. Specifically, the biases of -1.74 mGal for the WHU and GOSG02S models, and -1.69 mGal for the Tongji model, indicate a slight underestimation of the gravity anomalies by these models. However, the relatively tight standard deviations of around ±19 mGal suggest that these models are quite precise. This precision demonstrates their utility in geophysical studies in South Africa, making them robust tools in applications such as geodesy, earth science research, and resource exploration. Nonetheless, the presence of biases and residuals points to potential areas for improvement in future model refinements, particularly through the integration of more localised data or enhanced modelling techniques. This analysis affirms the effectiveness of these global geopotential models in accurately recovering gravitational anomalies in the country, thereby underlining their value for both scientific and practical applications.

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