Empirical investigations towards establishing a geoid-based vertical datum over South Africa

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Abstract

This study investigates prospects for establishing a geoid-based vertical datum in South Africa and aligning it with the International Height Reference System (IHRS) to modernise and unify vertical positioning. Employing the SAGEOID10 quasigeoid model alongside 138 GPS/levelling data points, this research evaluates the compatibility of spheroidal orthometric, normal, and orthometric height systems with the current quasigeoid and derived geoid models. The assessment is carried out using vertical datum offsets modelled at 100 and validated at 38 GPS/levelling points by applying a four-parameter planar model. The cross-validation results show that the normal and orthometric height systems provide a best fit, with standard deviations of ±5.1 and ±3.9 cm on quasigeoid and geoid models, respectively. The spheroidal orthometric height system referred to the land levelling datum (LLD) used over South Africa provided a better fit with the quasigeoid (±6.3 cm) than with the geoid (±7.6 cm). In addition, the study determined linear vertical datum offsets between the IHRS and variants of the local vertical datum (LLD, local quasigeoid and local geoid) on four tide gauge benchmarks (TGBMs) around South Africa. Empirical tests on a few benchmarks observed around each TGBM followed. The linear offsets at each TGBM, between each local height system and the global vertical datum (IHRS), revealed similar trends for the quasigeoid and geoid, but not for the LLD. The transformed heights (on the IHRS) were used to determine datum offsets based on benchmarks around each TGBM. Compared to the other three TGBMs (PEL, ELN and DBN), the results show the smallest mean offset around the TGBM in Cape Town. They also indicate that either normal or orthometric height systems should be adopted over South Africa and that the TGBM at CPT should be adopted when transforming a selected local height system to the IHRS.

Keywords: *quasigeoid model, geoid model, height system, international height reference system, SAGEOID10*

1. Introduction

A gravimetric geoid/quasigeoid model is commonly preferred for height unification worldwide because it provides a consistent vertical datum over large areas. In practical terms, geometric geoid undulation/height (N) is the difference between ellipsoidal height (h) and orthometric height (H), $N = h - H$ (Heiskanen & Moritz, 1967; Vaníček, 1976). This definition ignores the existence of a vertical offset, and systematic errors associated with local or global vertical datums (Odera & Fukuda, 2015b). A vertical datum offset (δ) can be expressed as $N - h - H = \delta$ (Vanicek, 1991; Rapp, 1994; Burša et al., 2004; Hofmann-Wellenhof & Moritz, 2005; Odera & Fukuda, 2015b; Singh, 2018). On the other hand, geometric quasigeoid height/ height anomaly (ζ) is the difference between ellipsoidal height (h) and normal height (H), $\zeta = h - H^N$; hence, the vertical datum offset for a quasigeoid surface is expressed as $\zeta - h - H^N = \delta$. The offset indicates that the origin of the local vertical datum does not always coincide with the geoid or quasigeoid surface. This is due to factors such as approximate gravity modelling, adjustment techniques in levelling networks, and systematic changes in mean sea level over time. Therefore, the vertical offset between local vertical datum and local geoid or local vertical datum and global geoid/international height reference system is required in vertical datum transformation. The vertical datum offset generally defines how the geoid or quasigeoid model fits the GPS/levelling data: - in some cases it is referred to as a corrector surface, created to remove existing biases (Chandler & Merry, 2010; Younis, 2017) and to provide an accurate evaluation of a reference surface (either geoid or quasigeoid models). It is worth noting, however, that the computed vertical datum offset from the GPS/levelling data would still contain unavoidable random errors and systematic biases (Amos & Featherstone, 2008).

The current height system over South Africa is referred to as the spheroidal orthometric height system. It is similar to the normal height system. However, no actual gravity functionals are used. Instead, a Vignal approximation is used for the mean normal gravity along the normal plumbline, while the spheropotential/geopotential number is estimated from normal gravity (Merry, 1985; Mphuthi & Odera, 2022). As the world moves towards the global unification of vertical datums, exemplified by the International Height Reference System (IHRS), there is a need to develop a geoid-based vertical datum over South Africa, consistent with the local geoid/quasigeoid model and subsequently with the IHRS. This transition ensures that the vertical positioning is not only precise, but also universally consistent and accessible, similar to the horizontal positioning realised on the International Terrestrial Reference Frame (ITRF). The relationship between the land levelling datum (LLD) and the IHRS has been analysed, resulting in the determination of linear offsets at four benchmarks across South Africa. These highlight inconsistencies and provide a basis for adjustment (Mphuthi & Odera, 2021). The estimated linear offsets between the four TGBMs on LLD and the global vertical datum are: 5.973, -20.647, -26.518, and 21.496 cm for Cape Town, Port Elizabeth, East London and Durban, respectively (Mphuthi & Odera, 2021).

The current study determines vertical datum offsets between the current quasigeoid model (SAGEOID10) and the derived geoid model over South Africa. It uses GPS/levelling data to identify a suitable vertical reference surface and the best corresponding height system for South

Africa. The following height systems have been considered: spheroidal orthometric (H^S), normal (H^N), and orthometric (H^O). Instead of the rigorous orthometric height system, the Helmert orthometric height system (Helmert,1890; Heiskanen & Moritz 1967) is used. However, it should be noted that there are considerable differences between the Helmert and rigorous orthometric height systems (Santos et al., 2005; Odera & Fukuda, 2015a; Foroughi et al., 2017). Furthermore, using a few benchmarks observed around each of the four TGBMs (Cape Town, Port Elizabeth, East London and Durban), this study sets out to determine vertical datum offsets between the local vertical datums (LLD, quasigeoid, and geoid) and the IHRS. Finally, we propose the way forward towards a geoid consistent vertical datum over South Africa.

2. Data and methods

A total of 138 GPS/levelling data points (Figure 1) are used: 100 as model data and 38 as independent test points. The internal accuracy of GPS coordinates is approximately ± 1 and ± 2 cm horizontally and vertically, respectively, while the accuracy of the first-order levelling network in South Africa is estimated at $1.9\sqrt{L}$ mm, with L being the distance of a levelling line in km (Odera, 2019). It should be noted that orthometric (H^O) and normal (H^N) heights have already been derived at 141 GPS/levelling points by Mphuthi and Odera (2022). However, owing to suspected outliers, three of the GPS/levelling points have been excluded from the current study. Spheroidal orthometric heights (H^S) at each of the 138 points are readily available from the national mapping organisation (NGI). For clarity, it should be stated that SAGEOID10 is a quasigeoid model, not a geoid model. The geoid is an equipotential surface of Earth's gravity field, which, ideally, would coincide with mean sea level. The quasigeoid, while similar, differs mainly in its mathematical and practical applications. Accordingly, the separation between the geoid and the ellipsoid is considered a geoid undulation, while the separation between the quasigeoid and the ellipsoid is considered a height anomaly. Corresponding geoid undulation values at the GPS/levelling points were computed in our previous study (Mphuthi & Odera, 2022). The offsets at 100 GPS/levelling points for each height system are modelled using a four-parameter planer model to determine a corrector or conversion surface. The converted heights are tested at 38 test points.

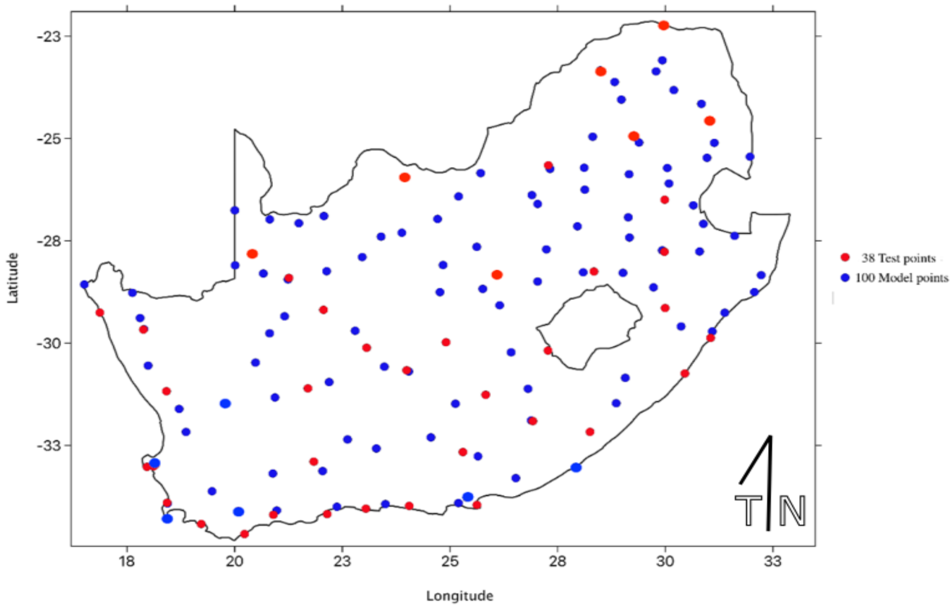

Figure 1: Distribution of the 138 GPS/levelling data points

Statistics of the datum offsets for each reference surface (quasigeoid and geoid) at 138 GPS levelling points are presented in Tables 1 and 2. It is observed that spheroidal orthometric height (H^S) is more compatible with the quasigeoid than with the geoid model. Similarly, the normal height system is more compatible with the quasigeoid than with the geoid model. These empirical results indicate that the quasigeoid model would be the appropriate vertical reference surface over South Africa. Owing to inconsistencies in the definition and establishment of the land levelling datum based on the spheroidal orthometric height system, the normal height system would be preferred over South Africa. Improving the quality of the local quasigeoid model to allow for its adoption as a reference surface would require access to higher-quality data, given that the current gravity data are associated with inaccurate coordinates (Mphuthi & Odera, 2021). However, the geoid is a physically meaningful surface and is sensitive to the density variations within the Earth. On the other hand, the quasigeoid is not a physically meaningful surface and requires integration over the Earth's surface (Vaníček *et al*., 2012). Furthermore, computation of a geoid model (in contrast to that of a quasigeoid model) requires knowledge about the topographic density distribution (which is frequently not well known). This implies that the height anomaly can be computed more accurately than the geoidal height (Sjöberg, 2013). Statistics pertaining to the geoid and quasigeoid offsets for the three height systems at the 100 model data points and the 38 test data points are not provided as they are subsets of the 138 points: hence, practically similar.

Table 1: Statistics of the quasigeoid offset at the 138 GPS stations over South Africa (units are in m). ζ is height anomaly, h is ellipsoidal height, and H^O is orthometric height

Datum offset	Minimum	Maximum	Mean	Standard dev.
$\boldsymbol{\zeta - (h - H^S)}$	-0.267	0.647	0.195	0.227
$\boldsymbol{\zeta - (h - H^N)}$	-0.247	0.783	0.208	0.229
$\boldsymbol{\zeta - (h - H^O)}$	-0.256	0.667	0.425	0.268

Table 2: Statistics of the geoid offset at the 138 GPS stations over South Africa (units are in m). N is geoid undulation, h is ellipsoidal height, and H^O is orthometric height

Datum offset	Minimum	Maximum	Mean	Standard dev.
$\boldsymbol{N - (h - H^S)}$	-0.265	0.763	0.413	0.249
$\boldsymbol{N - (h - H^N)}$	-0.246	1.502	0.643	0.360
$\boldsymbol{N - (h - H^O)}$	-0.247	0.783	0.425	0.251

On the other hand, the linear vertical datum offsets between the IHRS and variants of the local vertical datum (LLD, local quasigeoid, and local geoid) were conducted on the four TGBMs, as described in Mphuthi & Odera (2021), and tested on a few benchmarks observed around each TGBM. The GPS/levelling data points around each TGBM were collected by the chief directorate of National Geospatial Information (CD: NGI) as part of an attempt to readjust the LLD. Local-scaled precise levelling was also conducted on the local network around each fundamental benchmark. The distribution of the benchmarks around each TGBM (Figure 2) is depicted in Figures 3 - 6. The determined linear offsets at each TGBM for LLD, local quasigeoid and local geoid with respect to the IHRS were applied to the spheroidal orthometric (H^S), normal (H^N), and orthometric (H^O) heights, respectively.

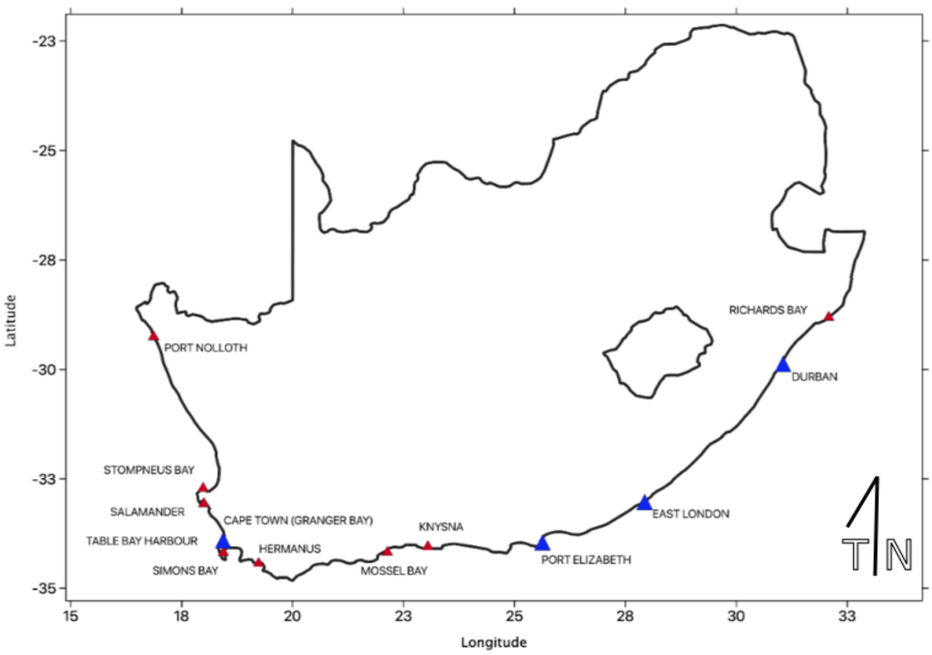

Figure 2: Tide gauge stations along the South African coastline (The blue triangles indicate the tide gauge stations connected to the four fundamental benchmarks, while the red triangles represent the tide gauge stations forming part of the tide gauge network)

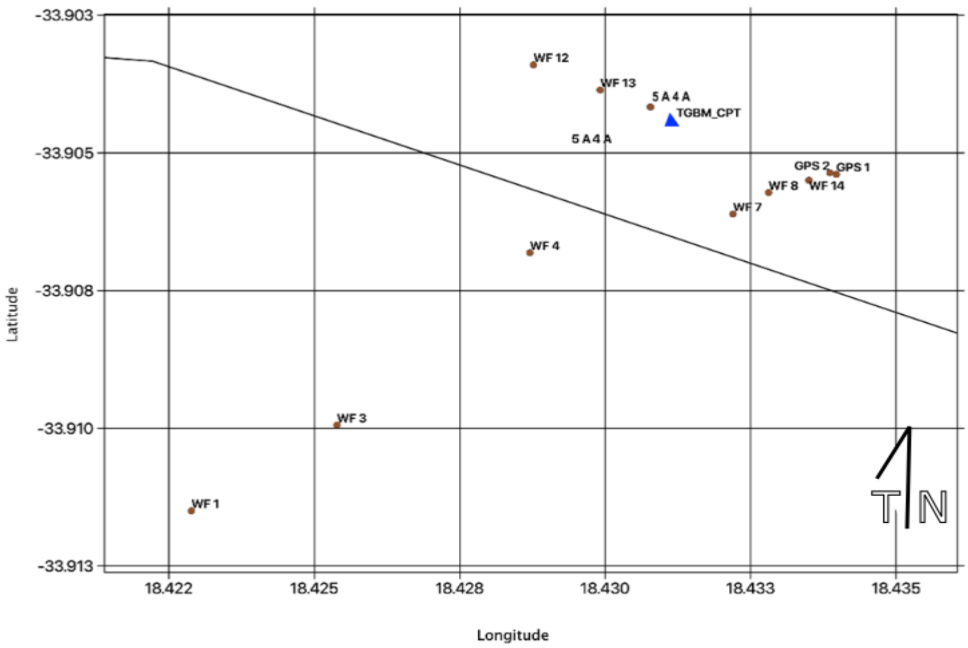

Figure 3: Distribution of benchmarks around Cape Town TGBM_(CPT)

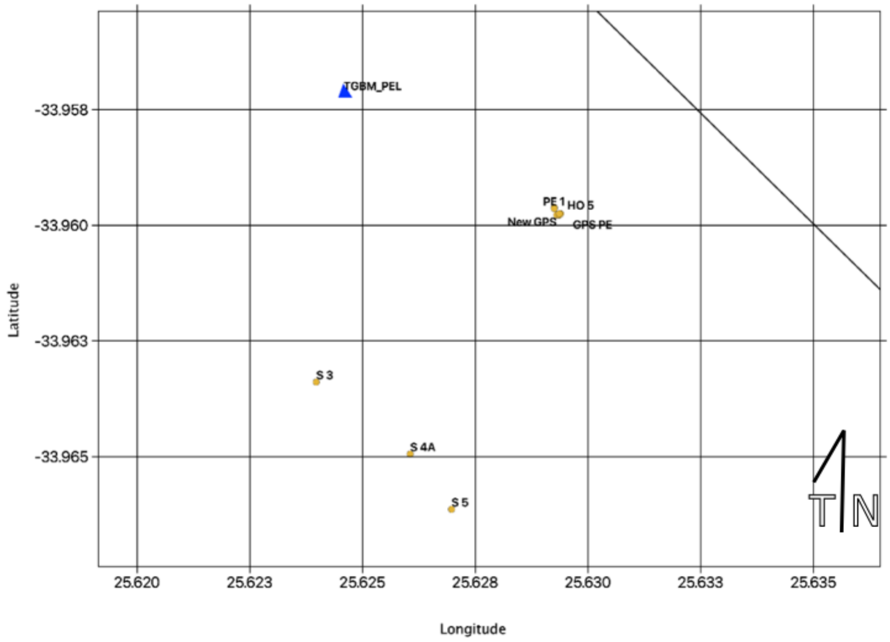

Figure 4: Distribution of benchmarks around Port Elizabeth TGBM_(PEL)

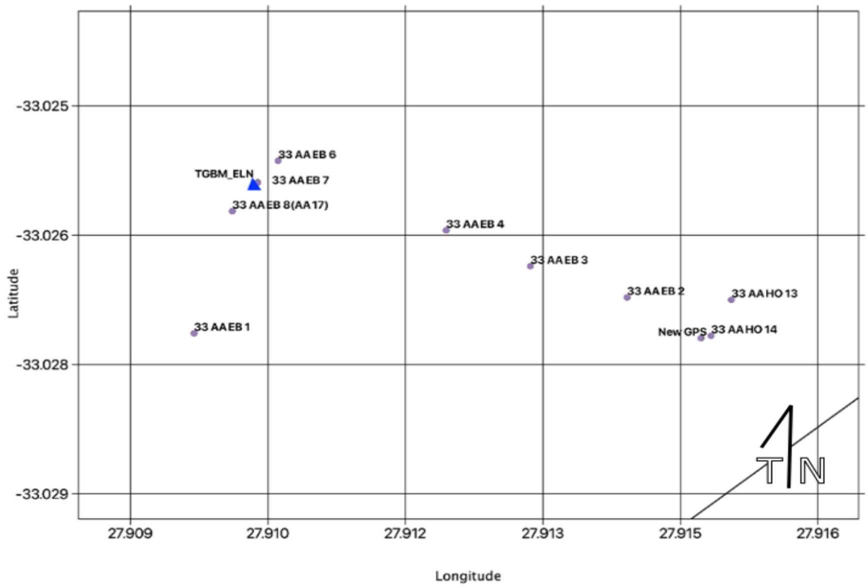

Figure 5: Distribution of benchmarks around East London TGBM_(ELN)

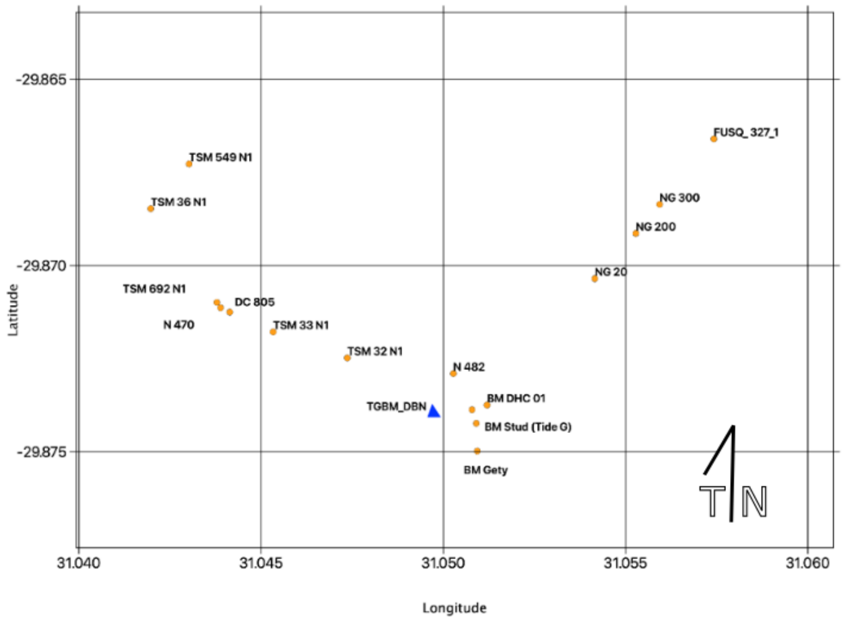

Figure 6: Distribution of benchmarks around Durban TGBM_(DBN)

3. Results and discussion

Cross-validation results[1] for the corrected heights at 38 independent GPS/levelling points (Figure 1) are presented in Tables 3 and 4. The corrected heights are represented by H^{Sc} (corrected spheroidal orthometric height), H^{Nc} (corrected normal height), and H^{Oc} (corrected orthometric height). The results show that the normal and orthometric height systems provide the best fit with their respective reference surfaces, quasigeoid and geoid, showcasing standard deviations of ±5.1 cm and ±3.9 cm, respectively. It is worth noting that the spheroidal height system shows a closer fit with the quasigeoid model (±6.3 cm) compared with that of the geoid model (±7.6 cm). It is clear from the results that quasigeoid and normal orthometric height systems would be preferrable over South Africa.

Table 3: Statistics of the quasigeoid datum offset at 38 GPS/levelling validation points after parametric model fitting (units are in m)

Datum offset	Minimum	Maximum	Mean	Standard dev.
$\zeta - (h - H^{Sc})$	-0.016	0.029	0.047	0.063
$\zeta - (h - H^{Nc})$	-0.037	0.036	0.003	0.051
$\zeta - (h - H^{Oc})$	-0.020	0.067	0.066	0.076

[1] after applying the corrector surface

Table 4: Statistics of the geoid datum offset at 38 GPS/levelling validation points after parametric model fitting (units are in m)

Datum offset	Min. (m)	Max. (m)	Mean (m)	Standard dev. (m)
$\boldsymbol{N-(h-H^{Sc})}$	-0.033	0.041	0.058	0.076
$\boldsymbol{N-(h-H^{Nc})}$	-0.054	0.093	-0.080	0.083
$\boldsymbol{N-(h-H^{Oc})}$	-0.075	0.075	0.003	0.039

The linear offsets at each TGBM, between each local height system and the global vertical datum, are presented in Table 5. The linear offsets from the quasigeoid and geoid maintained similar trends at all four benchmarks (with negative offsets at TGBM DBN only). However, the linear offset from LLD differed (with negative offsets at PEL and ELN TGBMs). It should be noted that the LLD is not well defined or theoretically and practically well established, while the quasigeoid and geoid reference surfaces are well defined − both in theory and practice. The linear offset, as indicated in Table 5, was applied accordingly to transform the H^S, H^N, and H^O of the benchmarks around each TGBM (Figures 2 to 6) to the IHRS. The transformed heights were then used to determine the datum offsets. The statistics of this analysis are presented in Tables 6 - 13.

Table 5: Linear offsets at each TGBM (units are in m)

TGBM	Linear Offsets		
	LLD	Quasigeoid	Geoid
CPT	0.060	0.406	0.386
PEL	-0.203	0.219	0.199
ELN	-0.265	0.064	0.044
DBN	0.220	-0.068	-0.088

Table 6: Statistics of the quasigeoid offsets at 13 benchmarks around the TGBM_CPT (units are in m)

Datum offset	Minimum	Maximum	Mean	Standard dev
$\boldsymbol{\zeta-(h-H^{SIHRS})}$	0.287	0.288	0.287	0.000
$\boldsymbol{\zeta-(h-H^{NIHRS})}$	-0.059	-0.058	-0.059	0.000
$\boldsymbol{\zeta-(h-H^{OIHRS})}$	-0.039	-0.038	-0.039	0.000

Table 7: Statistics of the geoid offsets at 13 benchmarks around the TGBM_CPT (units are in m)

Datum offset	Minimum	Maximum	Mean	Standard dev.
$\mathrm{N} - (h - H^{SIHRS})$	0.279	0.282	0.281	0.001
$N - (h - H^{NIHRS})$	-0.067	-0.064	-0.065	0.001
$N - (h - H^{OIHRS})$	-0.047	-0.044	-0.045	0.001

Table 8: Statistics of the quasigeoid offsets at 11 benchmarks around the TGBM_PEL (units are in m)

Datum offset	Minimum	Maximum	Mean	Standard dev.
$\zeta - (h - H^{SIHRS})$	0.627	0.628	0.628	0.001
$\zeta - (h - H^{Nihrs})$	0.202	0.203	0.203	0.001
$\zeta - (h - H^{Oihrs})$	0.222	0.223	0.223	0.001

Table 9: Statistics of the geoid offsets at 11 benchmarks around the TGBM_PEL (units are in m)

Datum offset	Minimum	Maximum	Mean	Standard dev.
$\mathrm{N} - (h - H^{SIHRS})$	0.707	0.709	0.708	0.001
$N - (h - H^{NIHRS})$	0.282	0.284	0.283	0.001
$N - (h - H^{OIHRS})$	0.302	0.304	0.253	0.001

Table 10: Statistics of the quasigeoid offsets at 11 benchmarks around the TGBM _ELN (units are in m)

Datum offset	Minimum	Maximum	Mean	Standard dev.
$\zeta - (h - H^{SIHRS})$	0.594	0.594	0.594	0.000
$\zeta - (h - H^{Nihrs})$	0.265	0.265	0.265	0.000
$\zeta - (h - H^{Oihrs})$	0.285	0.285	0.285	0.000

Table 11: Statistics of the geoid offsets at 11 benchmarks around the TGBM _ELN (units are in m)

Datum offset	Minimum	Maximum	Mean	Standard dev.
$\mathbf{N}-(\boldsymbol{h}-\boldsymbol{H}^{SIHRS})$	0.537	0.538	0.537	0.001
$\boldsymbol{N}-(\boldsymbol{h}-\boldsymbol{H}^{NIHRS})$	0.208	0.209	0.208	0.001
$\boldsymbol{N}-(\boldsymbol{h}-\boldsymbol{H}^{OIHRS})$	0.228	0.229	0.228	0.001

Table 12: Statistics of the quasigeoid offsets at 16 benchmarks around the TGBM_DBN (units are in m)

Datum offset	Minimum	Maximum	Mean	Standard dev.
$\boldsymbol{\zeta}-(\boldsymbol{h}-\boldsymbol{H}^{SIHRS})$	-0.503	-0.501	-0.503	0.001
$\boldsymbol{\zeta}-(\boldsymbol{h}-\boldsymbol{H}^{Nihrs})$	-0.220	-0.218	-0.220	0.001
$\boldsymbol{\zeta}-(\boldsymbol{h}-\boldsymbol{H}^{Oihrs})$	-0.200	-0.198	-0.200	0.001

Table 13: Statistics of the geoid offsets at 16 benchmarks around the TGBM_DBN (units are in m)

Datum offset	Minimum	Maximum	Mean	Standard dev.
$\mathbf{N}-(\boldsymbol{h}-\boldsymbol{H}^{SIHRS})$	-0.202	-0.200	-0.201	0.001
$\boldsymbol{N}-(\boldsymbol{h}-\boldsymbol{H}^{NIHRS})$	0.081	0.083	0.082	0.000
$\boldsymbol{N}-(\boldsymbol{h}-\boldsymbol{H}^{OIHRS})$	0.101	0.103	0.102	0.000

Compared to the other three TGBMs (PEL, ELN and DBN), the datum offsets from the benchmarks around the TGBM in Cape Town provided the smallest mean datum offset. The spheroidal orthometric height system was consistently shown to have the largest mean datum offset with respect to the quasigeoid and geoid models. We also observed that normal and orthometric heights are more consistent with the quasigeoid and geoid models, respectively. This is expected because the reference surfaces for normal and orthometric heights are quasigeoid and geoid, respectively. These results show that either normal or orthometric height systems should be adopted over South Africa. The results also show that the TGBM at CPT should be adopted when transforming the selected local height system to the IHRS.

4. Conclusion

This study carried out an analysis using the current quasigeoid model (SAGEOID10) and a derived geoid model to determine a consistent height system with a corresponding reference surface for South Africa. By applying a four-parameter planar model, this assessment was carried out using vertical datum offsets modelled at 100 and validated at 38 GPS/levelling points. The cross-validation results show that the normal and orthometric height systems provide a better fit when compared with those of the quasigeoid and geoid models, with standard deviations of ± 5.1 and ± 3.9 cm, respectively. On the other hand, the spheroidal height system provides a better fit with the quasigeoid than that with the geoid model, with standard deviations of ±6.3 and ±7.6 cm, respectively. This means that the LLD is more compatible with the quasigeoid than with the geoid model. It should be noted that, based on cross-validation, the established accuracy of the SAGEOID10 model is approximately 6 cm (Merry, 2009). This value reflects a realistic estimate of the model's accuracy after accounting for various corrections and evaluations against external data.

In addition, the study determined linear vertical datum offsets between the IHRS and variants of the local vertical datum (LLD, local quasigeoid and local geoid) on four TGBMs around South Africa. This exercise was followed by empirical tests on a few benchmarks observed around each TGBM. The linear offsets at each TGBM, between each local height system and the global vertical datum (IHRS), revealed similar trends for the quasigeoid and geoid, but not for the LLD. The transformed heights (on the IHRS) were used to determine datum offsets based on benchmarks around each TGBM. Compared to the other three TGBMs (PEL, ELN and DBN), the results around the TGBM in Cape Town show the smallest mean offset. The spheroidal orthometric height system is consistent in that it has the largest mean datum offset with respect to the quasigeoid and geoid models, while the orthometric height system is even more consistent with respect to the quasigeoid and geoid models.

These results show that either normal or orthometric height systems should be adopted over South Africa. However, the spheroidal orthometric height system over South Africa is closer to the normal height system than to the orthometric height system (Mphuthi & Odera, 2022). Hence, the normal orthometric height system would be the empirically acceptable choice. The other reason for selecting the normal height system is due to the difficulty in determining the integral-mean value of gravity along the plumbline between the geoid and the Earth's surface that is required for establishing the orthometric height system. The results also show that the TGBM at CPT should be adopted when transforming a selected local height system to the IHRS.

Having dealt with the questions on the choice of a height system and the tide gauge benchmark, it is important to note that a precise high-resolution geoid/quasigeoid model is necessary for establishing a geoid/quasigeoid-based height system. Our future studies will concentrate on precise/quasigeoid modelling over South Africa.

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