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An Investigation into the Stability of some selected Geodetic Controls in Lagos State of Nigeria using the Strain Analysis Technique

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Abstract

Natural disasters pose global challenges and can result in social, economic, and environmental damage, substantial loss of life, and even pose a threat to geopolitical stability. The study of such disasters through deformation modeling and analyses has found application in the disciplines of Geodesy and Geodynamics. The strain method has in fact been used to model deformation. The strain deformation parameters, namely, dilatancy, total shear strain and differential rotation, of this finite elemental model were calculated by using the baseline ratios of the coordinates of a classical traverse observed using the Global Positioning System (space technique), in the Minna datum platform. Computation was undertaken in a MATLAB programme and a MONTE CARLO environment, after the ill-conditioned triangles in the network were excluded. Statistical analysis was used to determine the significance levels of the respective deformation parameters at the 95%, 97.5% and 99.5% confidence intervals. After the statistical testing of the deformation parameters, it was observed that some of the controls were unstable in terms of their computed dilatancy and their total shear strain values. For the differential rotation of the network, the significance levels at the 95%, 97.5% and 99.5% confidence intervals were found to be 1.8743908, 0.9651796 and 0.4338522, respectively, while, on the other hand, the controls or centroids that did not respond to the network rotation had a mean value of approximately -0.99999.The minimal and maximal principal strain levels occurring at Centroids 11 and 36 with their triangulated station identities were found to be (36-12, 30-84, 43-34A) and (34-30A, 34-32A, 34-36A), respectively. The method adopted for this research proved to be very effective for a deformation study and analysis.

Keywords: *significance, stability, deformation, strain, parameter, dilatancy, shear strain, differential rotation, finite element, baseline ratios, centroid.*

1. Introduction

Natural disasters are a problem of global concern. In attempting to model and analyze such disasters, birth has been given to the notion, deformation. In such a context, deformation might result from earthquakes, landslides, floods, tsunamis, etc. These phenomena make the Earth's terrain very unstable and the consequences can be catastrophic in terms of the danger posed to human lives. It is then mandatory to monitor tectonic movements in those areas threatened by such disasters.

In order to monitor tectonic movements in such areas under study, geodetic controls or benchmarks need to be in place. Of the methods for establishing such controls are the GPS (Global Positioning System), trilateration, triangulation, traversing, and many more.

In attempting to model deformation in this context, problems are encountered when it comes to the availability of geodetic data. Until recently, the Nigeria Geodetic Control Network, established between the 1940s and early 1960s, served as the mapping institute.

Even with the newly established space-based geodetic controls, it is difficult to model deformation. This is due to the fact that at least two epochs or a series of campaigns are necessary for fully fledged deformation studies and analyses.

It is possible to model deformation from space-based and terrestrial geodetic data even though these datasets vary in terms of accuracy, time of observation and the different datum platforms that they present, thus making it necessary, particularly in the last case, to convert from one datum platform to the other.

This paper attempts to discuss an aspect of using the geodetic data obtained from classical traverses and GPSs, both of which are space-based sources of geodetic data and to apply strain analysis techniques, which are free from datum translation and rotation.

1.1. Finite elemental model: mathematical formulation

To achieve the objective of the finite elemental method, the network of controls must comprise finite or discrete elements. One such method to adopt is to establish a network of controls in the form of the Delaunay triangulation. This is a basic step in adopting the finite elemental method.

After the least squares adjustment of the two epochs of data (observations emanating from the classical traverse and the GPS), the linearly extended length of the baselines between the two stations/controls can be written as follows:

$$\varepsilon = \frac{D_2 - D_1}{D_1} \tag{1}$$

Where

D_1 is a distance at the first epoch;

D_2 is the corresponding baseline at a later epoch; and

ε is the strain accumulation.

Linear extension ε with azimuth α of a baseline can be written as follows (Brunner et.al, 1980; Deniz & Ozener, 2010):

$$\varepsilon = e_{xx}\cos^2\alpha + e_{xy}\sin 2\alpha + e_{yy}\sin^2\alpha \tag{2}$$

e_{xx}, e_{xy} and e_{yy} are called strain tensor parameters from which other strain parameters are calculated. These strain tensor parameters are the strain parameters of points of equilibrium or the centre of gravity for each triangle.

Other strain parameters can be calculated from the parameters of a strain tensor as follows (Deniz, 2007; Deniz & Ozener, 2010):

$$\Delta = e_{xx} + e_{yy} \tag{3}$$

$$\gamma_1 = e_{xx} - e_{yy} \tag{4}$$

$$\gamma_2 = 2e_{xy} \tag{5}$$

$$\gamma = \sqrt{\gamma_1^2 + \gamma_2^2} \tag{6}$$

$$\delta\omega = \omega - ½\left(e_{yx} - e_{xy}\right) - \frac{1}{m}\sum_{i=1}^{m}\omega \tag{7}$$

Where

Δ is the dilatancy, also called the trace of the strain tensor,

γ_1 is the principal shear strain or the pure shear,

γ_2 is the engineering shear strain or the simple shear,

γ is the total shear strain (the geometric mean of the pure shear and the simple shear),

ω is one of the non-zero diagonal elements of the anti-symmetric strain tensor,

$\delta\omega$ is the differential rotation of the network,

$\omega_0 = \frac{1}{m}\sum_{i=1}^{m}\omega$;

ω is the global rotation of the network; and

m denotes the number of deformed points.

Principal strain parameters are calculated as follows (Deniz & Ozener, 2010):

$$E_{\text{max}} = \frac{1}{2}(\Delta + \gamma) \quad (8)$$

$$E_{\text{min}} = \frac{1}{2}(\Delta - \gamma) \quad (9)$$

$$\beta = \tan^{-1}\left(\frac{e_{\text{xy}}}{E_{\text{max}} + e_{\text{xy}}}\right) \quad (10)$$

Where; E_{max} is the maximum principal strain,

E_{min} is the minimum principal strain; and

β is the direction of the maximum principal strain arc.

A MATLAB programme was developed and used for the realization and computation of the aforementioned parameters.

1.2. Statistical analysis of the strain deformation parameters

Considering the fact that a deformable body represented by a geodetic network responds differently to a strain influence and in different directions, it becomes important to study the statistical behaviour of the primitive values of the deformation (dilatation, total shear strain and twist or rotation). These three parameters describe the magnitude of the deformation and how it manifests (Michel and Person, 2003).

In this paper, it is assumed that the data thus employed underwent several sets of artificial creation, also known assimulation, in that the MONTE CARLO method, the parameters of which are the standard deviations in respect of the observed values, was used (Michel and Person, 2003). Based on this justification, the mean and standard deviation of the deformation parameters are given as follows:

$$\left.\begin{aligned} \bar{\Delta} &= \frac{1}{\text{sim}}\sum_{i=1}^{\text{sim}} \Delta_i \\ \bar{\gamma} &= \frac{1}{\text{sim}}\sum_{i=1}^{\text{sim}} \gamma_i \\ \delta\bar{\omega} &= \frac{1}{\text{sim}}\sum_{i=1}^{\text{sim}} |\delta\omega|_i \end{aligned}\right\} \quad (11)$$

$$\left.\begin{aligned} \sigma_{\Delta} &= \frac{1}{\text{sim}-1}\sum_{i=1}^{\text{sim}} (\Delta_i - \bar{\Delta})^2 \\ \sigma_{\gamma} &= \frac{1}{\text{sim}-1}\sum_{i=1}^{\text{sim}} (\gamma_i - \bar{\gamma})^2 \\ \sigma_{\delta\omega} &= \frac{1}{\text{sim}-1}\sum_{i=1}^{\text{sim}} (|\delta\omega|_i - \delta\bar{\omega})^2 \end{aligned}\right\} \quad (12)$$

From the definitions of the mean and the standard deviation, the network deformability (which is the ability of a network to respond to any changes in its vation with respect to its standard deviation) can be computed as follows:

$$\left.\begin{aligned} \mathit{\Delta}_{\mathrm{def}} &= \bar{\mathit{\Delta}} + \mathrm{CV}\sigma_{\mathit{\Delta}} \\ \gamma_{\mathrm{def}} &= \bar{\gamma} + \mathrm{CV}\sigma_{\gamma} \\ \delta\omega_{\mathrm{def}} &= \delta\varpi + \mathrm{CV}\sigma_{\delta\omega} \end{aligned}\right\} \tag{13}$$

Where:

CV is the critical value at level α:

CV depends on the statistical distribution used and the level of significance α (Marjetič et al, 2010).

$\mathit{\Delta}_{\mathrm{def}}$, γ_{def} and $\delta\omega_{\mathrm{def}}$ are collectively termed the deformability of the network.

1.3. Significance of the deformation parameters

The known confidence areas for kinematic quantities at each point of the geodetic network can serve in terms of the following relationship (Michel and Person, 2003):

$$\left.\begin{aligned} \textstyle\sum_{\mathit{\Delta}} &= \frac{\mathit{\Delta}-\mathit{\Delta}_{\mathrm{def}}}{\mathit{\Delta}_{\mathrm{def}}} \\ \textstyle\sum_{\gamma} &= \frac{\gamma-\gamma_{\mathrm{def}}}{\gamma_{\mathrm{def}}} \\ \textstyle\sum_{\delta\omega} &= \frac{|\delta\omega|-\delta\omega_{\mathrm{def}}}{\delta\omega_{\mathrm{def}}} \end{aligned}\right\} \tag{14}$$

Therefore, the degree of significance of deformation $\sum_{(\mathit{\Delta},\gamma,\omega)}$ takes on values between (-∞ and +∞).

If $\sum_{(\mathit{\Delta},\gamma,\omega)} \leq 0$, the measured deformation is less than the deformability of the network. We then say that there is no deformation since the measured deformation $\sum_{(\mathit{\Delta},\gamma,\omega)}$, has a smaller magnitude than the deformability of the network. Otherwise, if $\sum_{(\mathit{\Delta},\gamma,\omega)} > 0$, the measured deformation is greater than the deformability of the network, and we say that the deformation is significant.

2. Results and analyses

2.1.1. Results

Figure 1 shows the Delaunay triangulation of the network of stations formed in the ArcGIS environment/ It is based on the second-order harmonized ZTT GPS stations of the Lagos state and the third-order ZTT controls (comprising 76 weighted stations forming a network of 139 well-conditioned triangulations) of the traverse. It should be noted that the units of the coordinates are in metres.

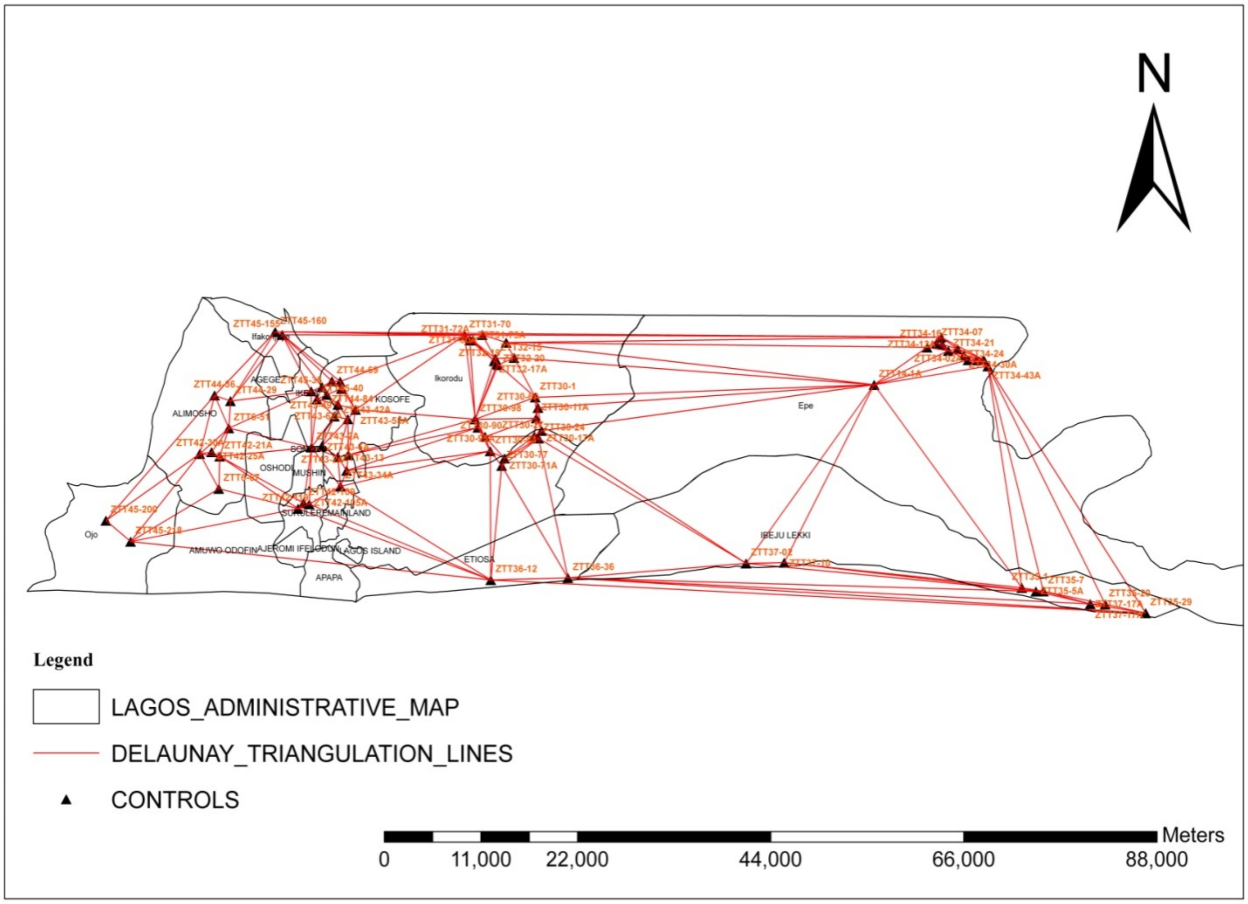

Figure 1: Delaunay Triangle of Controls

Table 1 shows the mean and standard deviation used to compute the network deformability, as well as the significance level of the deformation. Table 2 shows the respective network deformability levels at the 95%, 97.5% and 99% confidence intervals, as computed from Equation 13.

Table 1: Mean and Standard Deviation of Deformation Parameters

	Dilatancy	Total Shear Strain	Differential Rotation
Mean	-.000370	.00352	-.000254
Standard deviation	.0110	.0133	.000260

Table 2: Network Deformability

CONFIDENCE INTERVAL	CONFIDENCE VALUE	DILATANCY	TOTAL SHEAR STRAIN	DIFFERENTIAL ROTATION
95%	1.656	0.017846	0.025545	0.00018056
97.5%	1.9773	0.02138	0.029818	0.0002641
99%	2.3537	0.025521	0.034824	0.00036196

The figures from Figure 2 through to Figure 10 present charts presenting a pictorial representation of the variation in the significance levels of the deformation parameters at the respective 95%, 97.5% and 99% confidence intervals.

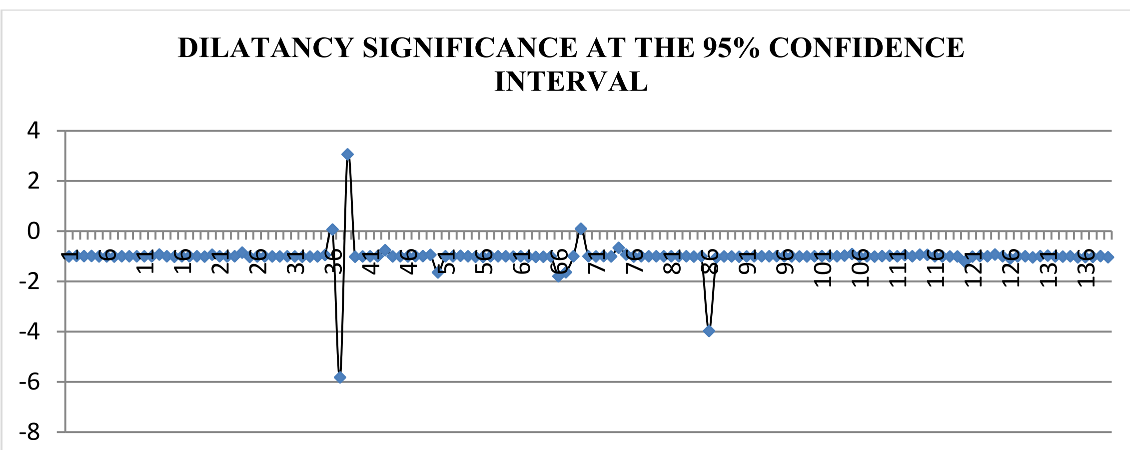

Figure 2: Dilatancy Significance Levels at the 95% Confidence Interval

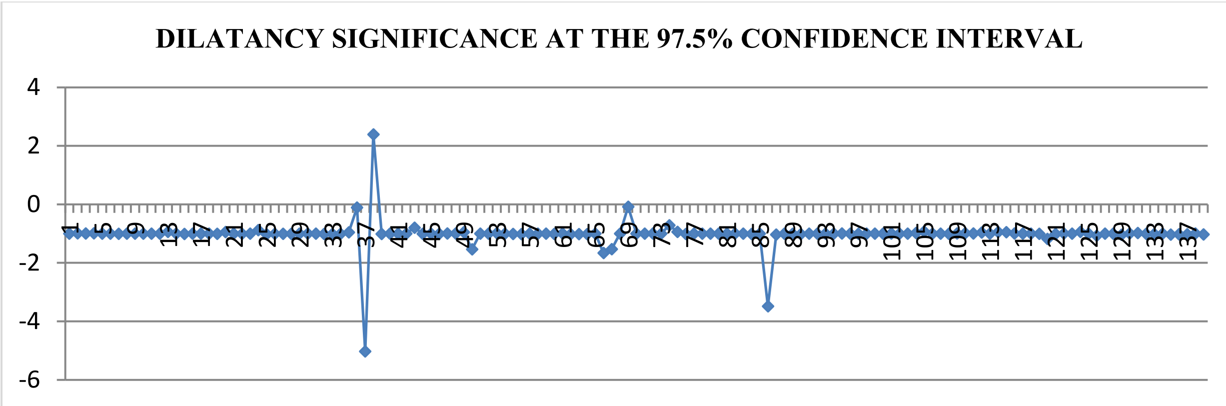

Figure 3: Dilatancy Significance Levels at the 97.5% Confidence Interval

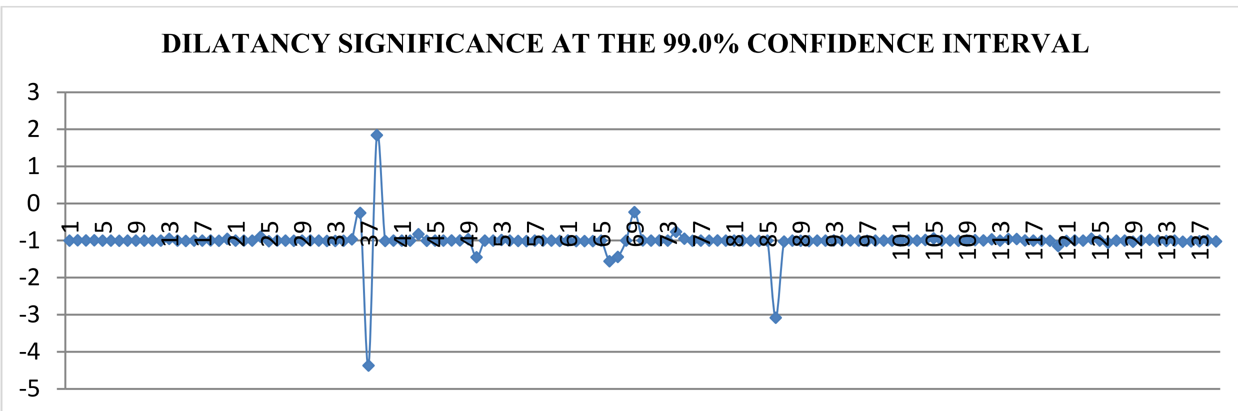

Figure 4: Dilatancy Significance Levels at the 99% Confidence Interval

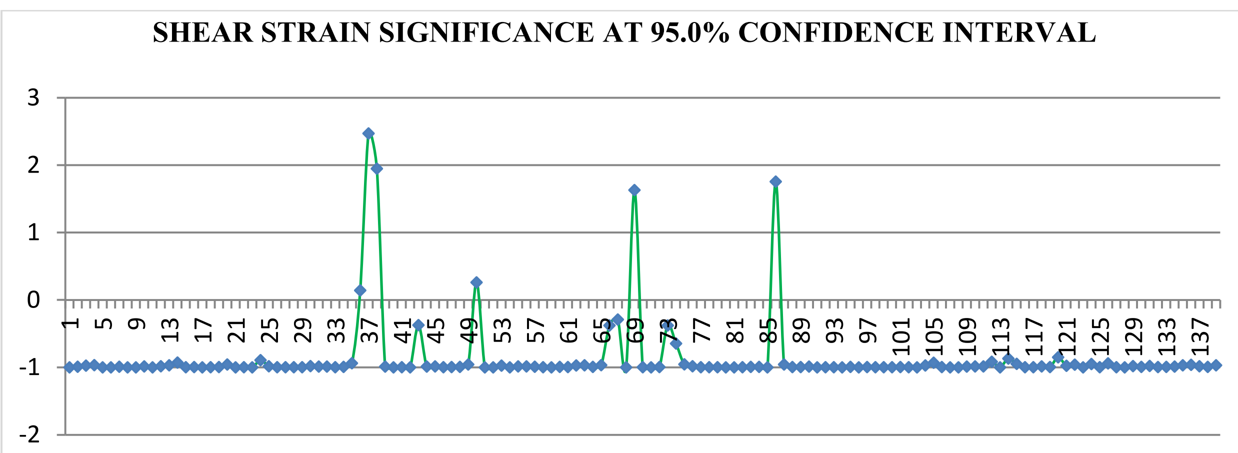

Figure 5: Shear Strain Significance Levels at the 95.0% Confidence Interval

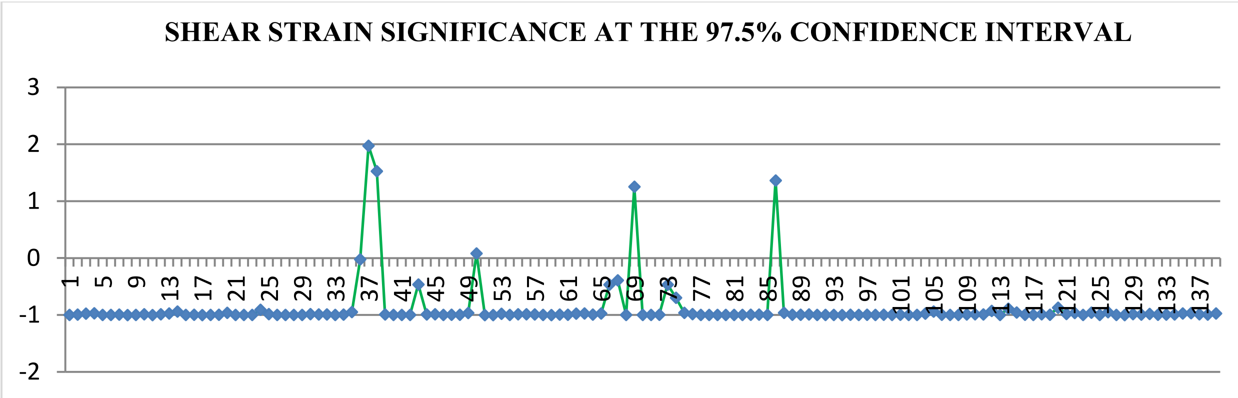

Figure 6: Shear Strain Significance Levels at the 97.5% Confidence Interval

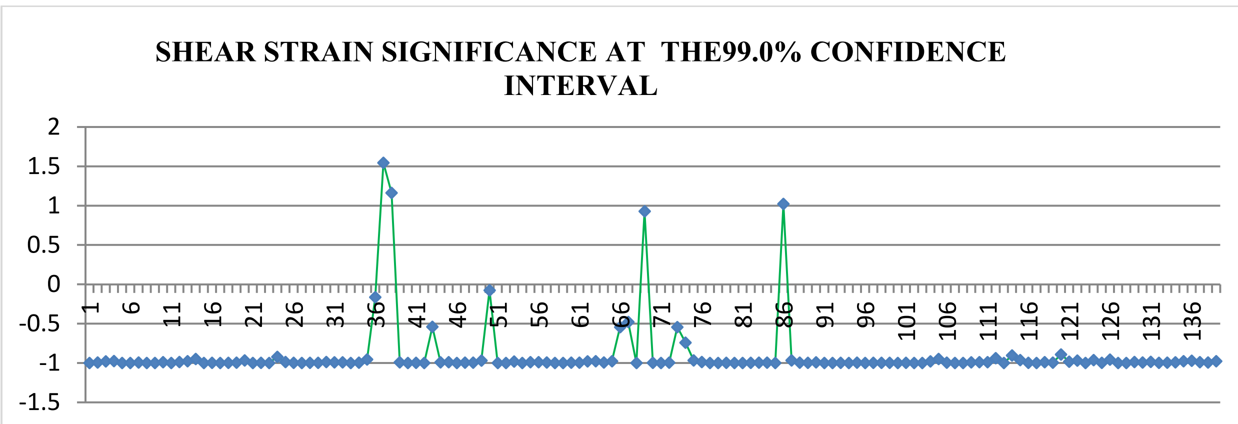

Figure 7: Shear Strain Significance Levels at the 99.0% Confidence Interval

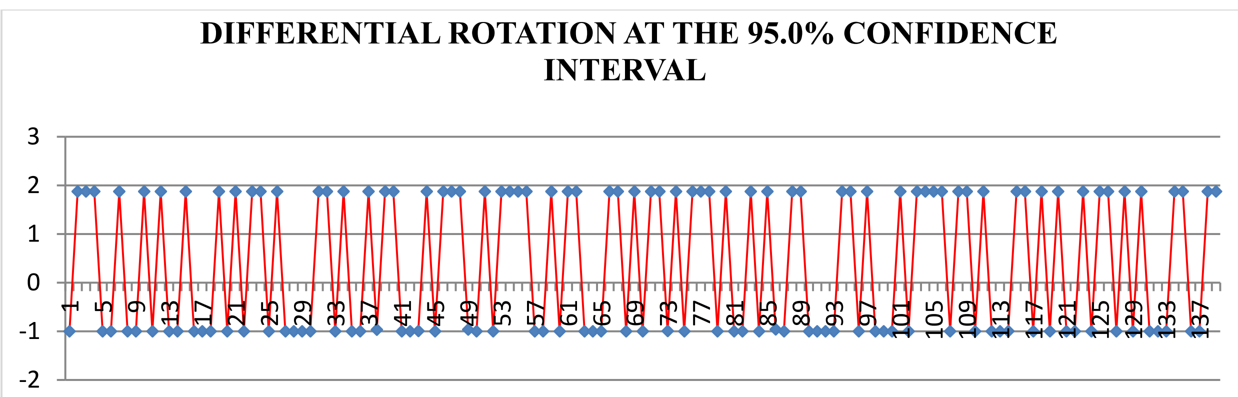

Figure 8: Differential Rotation Levels at the 95.0% Confidence Interval

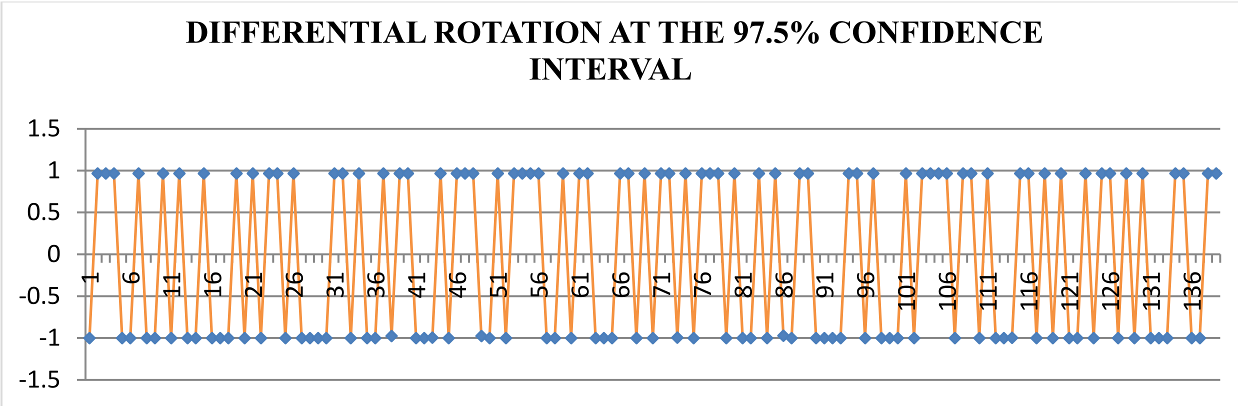

Figure 9: Differential Rotation Levels at the 97.5% Confidence Interval

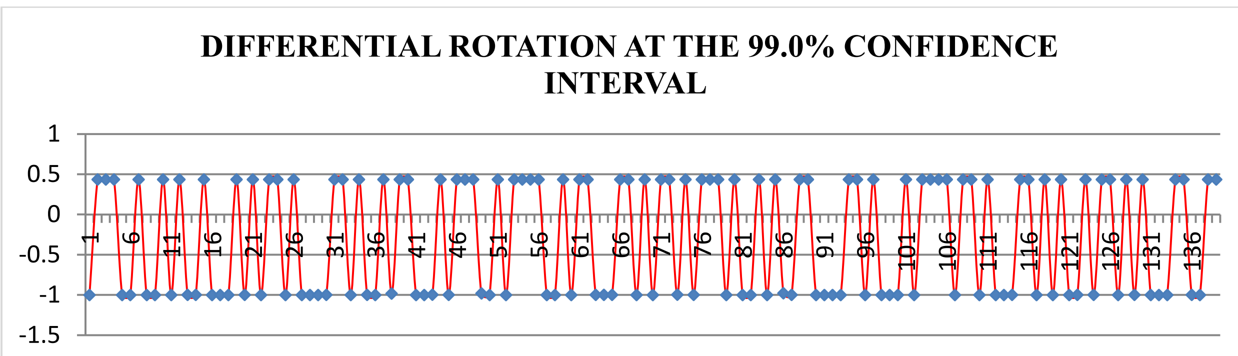

Figure 10: Differential Rotation Levels at the 99% Confidence Interval

2.1.2. Maximum and minimum principal strain

The maximum and minimum principal strain levels, together with their orientations, are displayed in the figure plotted via MATLAB in the form of error ellipses.

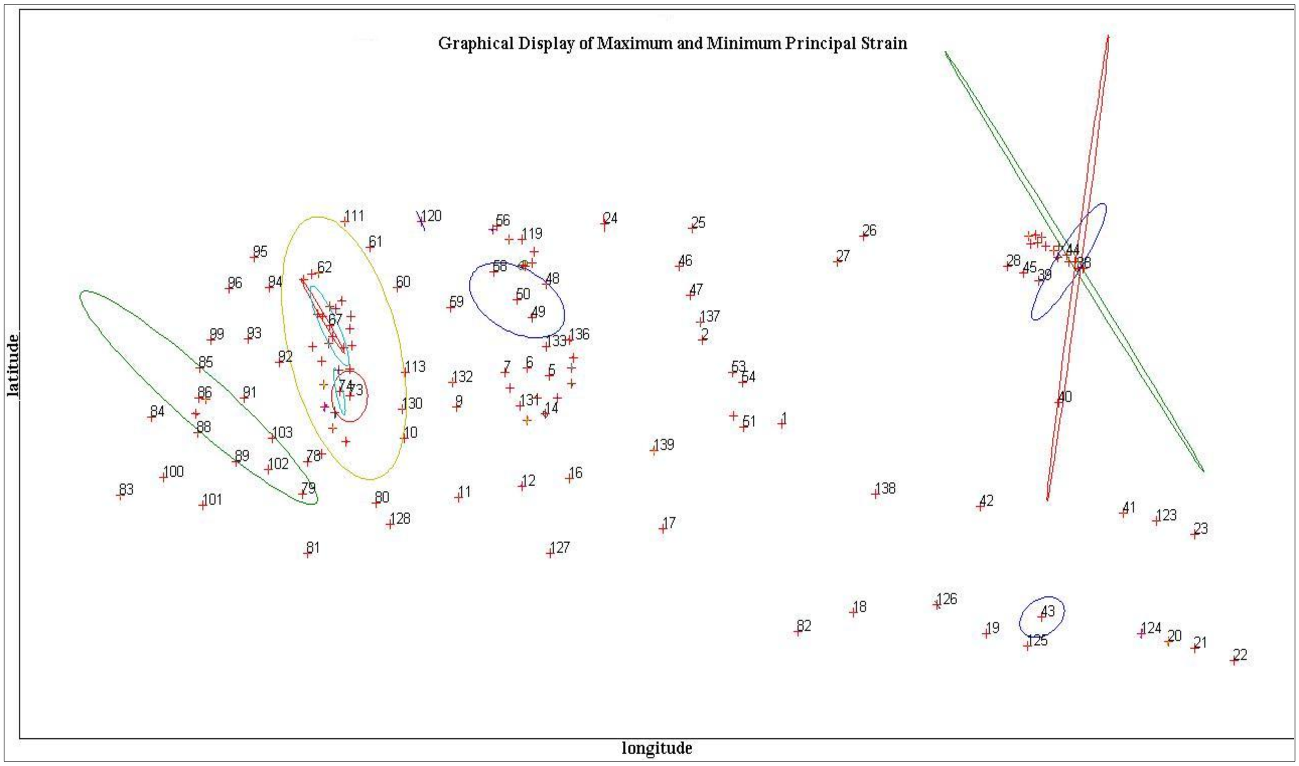

Figure 11: Maximum and Minimum Principal Strain Levels

Table 3: Main statistic constraints in maximum and minimum principal strain levels

	CENTROID ID	E1(μs)	E2(μs)	TRIANGLE
MIN.	11	0.945	0.4	36-12, 30-84, 43-34A
MAX.	36	87372	1289.6	34-30A, 34-32A, 34-36A

2.1.3. Analysis of results

The computed significance levels for the network dilatancy are below zero (0), except for Centroid 38, which is associated with triangle 34-32A, 34-36A, 34-43A and shows values greater than zero, i.e. 3.0637678, 2.392001 and 1.8416932 at the 95%, 97.5% and 99% confidence intervals, respectively. Figures 2 to 4 show the variations in the significance levels at the respective computed confidence intervals. Therefore, any points greater than zero (0), together with the controls associated with them, are said to be unstable or deformed.

In the case of total shear strain, Centroid 36, that is associated with triangle 34-30A, 34-32A, 34-36A at the 95% confidence interval showed that the controls associated to the centroid are unstable, since the value is 0.1418762 but at 97.5% and 99%, the deformation was very significant. At centroid 37(34-30A, 34-36A, 34-39A) the deformation in total shear strain were significant

(2.4708434, 1.9734299 and 1.5459874 at 95%, 97.5% and 99% confidence interval respectively). At centroid 38(34-32A, 34-36A, 34-43A), the deformation in total shear strain were also significant (1.948193, 1.5256816 and 1.1626047, at 95%, 97.5% and 99% confidence interval respectively). At centroid 50 (30-98, 32-20, 32-19), the deformation was significant at 95% (0.2610786) and 97.5% (0.0803509) but insignificant at 99% confidence interval. At centroid 69 (45-49, 43-42A, 43-62A), the deformation is significant at 95% (1.6294588), 97.5% (1.2526258) and 99% (0.9288018). At centroid 86 (6-51, 42-25A, 32-30A) the deformation is also significant at 95% (1.7570386), 97.5% (1.3619219) and 99% (1.0223862). Figures 5 to 7 show the variation in significance at 95%, 97.5% and 99%% confidence interval respectively. Appendix 4 shows the table for the plotted chart.

For network differential rotation, the significance at 95%, 97.5% and 99% confidence interval are 1.8743908, 0.9651796 and 0.4338522 respectively. While centroids or controls that did not respond to network rotation had values approximately -0.99999. Figures 8 to 10 shows variation in significance levels at the 95%, 97.5% and 99% confidence intervals, respectively.

From Figures 11 and 36, it can be seen that the centroid/triangles where the greatest minimal and greatest maximal principal strain levels were being experienced are at Centroid 11 (36-12, 30-84, 43-34A) and Centroid 36 (34-30A, 34-32A, 34-36). For dilatancy, the entire set of the computed significance levels were below zero (0). Only Centroid 38, which is associated with triangle 34-32A, 34-36A, 34-43A, showed values greater than zero, i.e. 3.0637678, 2.392001 and 1.8416932 at the 95%, 97.5% and 99% confidence intervals, respectively. Figures 2 to 4 show the variations in significance at the respective computed confidence intervals. Therefore, any points greater than zero (0), together with the controls associated with them, are said to be unstable or deformed.

In the case of total shear strain, Centroid 36, associated with triangle 34-30A, 34-32A, 34-36A at the 95% confidence interval, shows that the controls associated with it are unstable, since the relevant value is 0.1418762. However, at the 97.5% and 99% confidence intervals, the deformation proved to be very significant. At Centroid 37(34-30A, 34-36A, 34-39A), the deformation levels in total shear strain were significant (2.4708434, 1.9734299 and 1.5459874 at the 95%, 97.5% and 99% confidence intervals, respectively). At Centroid 38(34-32A, 34-36A, 34-43A), the deformation levels in total shear strain were also significant (1.948193, 1.5256816 and 1.1626047, at the 95%, 97.5% and 99% confidence intervals, respectively). At Centroid 50 (30-98, 32-20, 32-19), the deformation levels were significant at the 95% (0.2610786) and 97.5% (0.0803509) confidence intervals, but insignificant at the 99% confidence interval. At Centroid 69 (45-49, 43-42A, 43-62A), the deformation levels were significant at the 95% (1.6294588), 97.5% (1.2526258) and 99% (0.9288018) confidence intervals. At Centroid 86 (6-51, 42-25A, 32-30A) the deformation levels were also significant at the 95% (1.7570386), 97.5% (1.3619219) and 99% (1.0223862) confidence intervals. Figures 5 to 7 show the variations in significance at the 95%, 97.5% and 99%% confidence intervals, respectively.

In terms of the differential rotation of the network, the significance levels at the 95%, 97.5% and 99% confidence intervals were 1.8743908, 0.9651796 and 0.4338522, respectively, while those centroids or controls that did not respond to the network rotation presented with values of approximately -0.99999. Figures 8 to 10 show variations in the significance levels at the 95%, 97.5% and 99% confidence intervals, respectively.

In Figures 11 and 36, it can be seen that the centroids/triangles where the principle strain levels, both minimal and maximal, are at their greatest, are located at Centroid 11(36-12, 30-84, 43-34A) and Centroid 36(34-30A, 34-32A, 34-36A).

Similarly, the error ellipse plots (Figure 11) show the linear and orientation (angular) shifts of the centroids of the Delaunay triangles that were included in the network of stations used in this study. This confirms the noticed significance levels in dilantancy, shear strain and differential rotation of the respective networks of the triangular stations in relation to their centroids. As such, the minimum and maximum principal strain levels (Table 3) summarize the state of deformation of the network of stations. In effect, this can be used to analyze any past, present or future deformation occurrences within the network area.

3. Conclusion

The study shows that the finite elemental method is a promising alternative in the analysis of deformation-prone areas (e.g., landslide areas and those threatened by earthquakes, faults and tectonic movements of the continental plates). This method can be used alongside other methods relating to deformation applications in the engineering realm that are currently being used.

4. Acknowledgment

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