A Method for Connecting Traverses to GNSS Controls Eliminating Troublesome Short GNSS Orientation Lines

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ABSTRACT

Global Navigation Satellite System (GNSS) surveys are used to establish long baseline control networks. Further breaking down of the controls are accomplished using total station traversing connected to the GNSS networks. Auxiliary stations are established at relatively short distances to each GNSS main station for traverse azimuth orientation. If the GNSS azimuth reference lines are short, the allowable uncertainties in the GNSS determined coordinates heavily encumber the accuracies of the azimuths derived from them. This is the problem with connecting traverses to GNSS controls via azimuth reference lines that are short. Reorientation traversing can solve the short GNSS azimuth reference line problem by running control traverses linked to GNSS controls without referencing the short GNSS azimuth lines. Four reorientation traverses of total traverse lengths of 1.4Km to 5.1Km were run between GNSS network stations to demonstrate the validity of the new method. A corresponding traditional traverse was run to compare with each of the reorientation traverse cases. Some t-distribution tests established that there were no statistical differences between the coordinates determined by the reorientation traverses and the corresponding traditional traverses coordinates at the 99% confidence level. P-value tests revealed that there were no significant probabilities of an extreme occurrence in which the coordinates from the two methods of traversing may be statistically different at the $P < 0.01$ confidence level. The research results thus show that reorientation traversing is a valid procedure that may be used to avoid the use of short GNSS reference lines.

Key Words: Global Navigation Satellite Systems (GNSS), Reorientation Traversing, Azimuth, Controls

1. Introduction

Ground control densification has gained some interesting developments as a result of the evolution of high-tech digital ground surveying equipment in the recent decades. Generally the efforts to bring in control across great distances using traversing has given way to the use of GNSS for speed and accuracy. On the other hand traversing between GNSS control stations to fill in controls over shorter distances is conducted with the total stations for accuracy purposes (Hill, 2008).

The GNSS survey measurements result in point positions and thus the uncertainties in the measurements are positional factors of longitude and latitude, ellipsoidal height (λ, ϕ, h) or eastings and northings and elevation (E, N, el). Standards for GNSS surveys allow maximum uncertainties in horizontal positioning varying from 5mm + 5ppm to 10mm + 10ppm at 95% confidence level for the topmost class of control surveys depending on the use (Surveys Division, Arkansas Highway and Transportation Department (2013), Permanent Committee on Geodesy (PCG) (2014)). Over a distance of 3km uncertainties of 20mm and 40mm respectively will be allowed by these provisions. The import is that the relative precision of the azimuth between two such GNSS positions improve as distances between the points increase since the effect of the uncertainty on the line vectors is inversely proportional with length. And thus the position uncertainties distort the line vectors more over shorter distances. On the other hand Total Stations directly measure line vectors of distances and directions and can achieve such good precisions in distances at between 1mm + 1ppm to 2mm + 2ppm and in directions at between 1” to 2” (Integrated Land Management Bureau 2009). These level of precision are not yet possible to be achieved from vectors derived from GNSS coordinates with best values at over shorter distances. For example Royal Institution of Chartered Surveyors (RICS) 2010 reports that the high-precision static GNSS used for the most sensitive analyses such as National/international networks and reference frame survey, geodetic surveys to establish transformation parameters, crustal/tectonic plate monitoring surveys yield point positions of 5mm to 10mm, implying that uncertainties in the derived distances and directions will be of the magnitude of 10mm to 20mm and 21” to 41” respectively over a 100m distance. The GNSS surveys thus provide higher accuracies over greater distances, while total station traversing which directly measure terrestrial vectors of distances and direction yield better accuracies over shorter distances.

The present practice requires that traverses be connected to GNSS control stations. To accomplish this auxiliary GNSS control reference stations are established near the main network stations to provide for azimuth reference from the GNSS control network to the traverse. If these reference lines are short they introduce unacceptable azimuthal uncertainties from the GNSS surveys into the traverse.

The important consideration of the magnitude of orientation error as a function of the length of an orientation reference mark from the GNSS control network station arises. Errors in azimuths derived from GNSS coordinates will be of larger magnitudes over shorter distances. Considering that if the coordinates at both ends of a line is in positional error of a maximum of ±30mm as shown in Figure 1, the azimuth derived from this 150m length will be in error by 82.5” while the error in the distance is 0.060m and the linear accuracy of the determination of the distance will be 1/2,500.

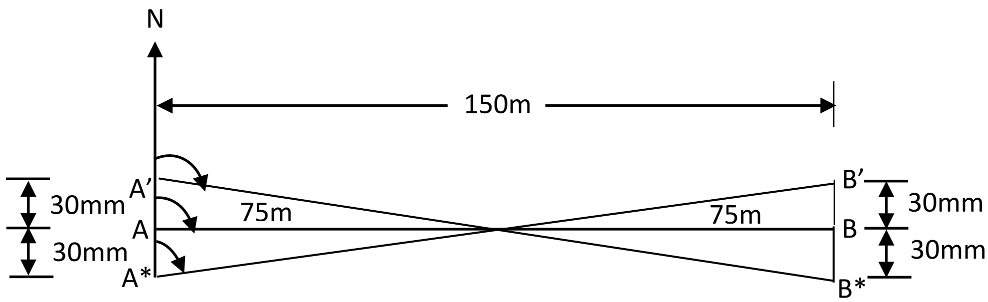

Figure 1. Illustration of the error in the traverse orientation due to GNSS station positional error

Ayers (2011) listed estimates of orientation errors in azimuths derived from Real Time Kinematic (RTK) GNSS survey relative to the length of the orientation line as in Table 1 considering the coordinates of the orientation mark to be in error by ±20mm. Even though Table 1 lists a so called “orientation error best case scenario”, practically the so called “orientation worst case scenario” is the valid case since these errors are uncertainties and no one can say which of the cases is applicable in any case.

Table 1: Estimates of Traverse Orientation Errors as a Function of the GNSS Reference Orientation line distance. Source: Ayers (2011)

Distance	Orientation Error Worst Case Scenario	Orientation Error Best Case Scenario
10m	412"	103"
30m	137"	35"
100m	41"	10"
300m	14"	3"
1000	4"	1"

In practice so far, different survey establishments handle the issue of providing traverse controls from GNSS surveys differently. A sample of such surveys carried out by multinational and local companies in Nigeria show the varying approaches. In a technical Report GPS, SA (1989) on GNSS control survey of the XSV control station series in Nigeria, the orientation line distances sampled for control stations XSV-18, XSV-19, XSV-20, XSV-22 and XSV-23 ranged from 35m to 49m. In another survey by the China National Petroleum Corporation for the Shell Petroleum Development Company, the distances of the orientation lines ranged from 75m to 360m (China National Petroleum Corporation, 2007). Another GPS campaign technical report by TOPNAV for Chevron Nigeria Ltd reports that the orientation line distances ranged from 183m to 413m (TOPNAV, 1998).

United States Department of Agriculture - Forest Service and United States Department of the Interior - Bureau of Land Management (2001) instructs that station pairs used as azimuth or bearing reference for total station survey should be included in a network or measured with a minimum of two independent vectors using the RTK techniques in which the acceptable standard at the 95% confidence level of the local accuracy for cadastral project control application is less than 0.050m. Gardner (2013) provides that if GNSS is used to provide the reference line of a control traverse the line must be over 500m in length.

The effort to use an appropriate length of orientation lines established by GNSS could suffer from quite some environmental constraints due to high rise buildings in urban areas or canopies in forested areas and so on. The aim of this research is to demonstrate a method that may be used to connect traverses to GNSS controls when appropriate lengths of GNSS control reference lines are not implementable in the circumstances.

The method of reorientation traversing is a resected traverse method that excludes setting up on the takeoff and/or closing control stations (Chukwuocha et al, 2017). The advantage of this method is that the reorientation traversing method will enable running of accurate traverses between reliable takeoff and closing single GNSS controls without encumbering the traverse with heavy loads of orientation error from short GNSS reference lines.

Reorientation traverses are run beginning with a setup of the angle measuring instrument on the first new station to be coordinated and a back sight taken to the takeoff GNSS control station to measure angle to the next traverse station. All the traverse distances are measured until the instrument is set up on the last traverse station to be coordinated with a fore sight to the closing GNSS control station. Indeed the reorientation traverse takes its orientation by indirect computations of the azimuth of the traverse from the line linking the two end GNSS controls using the geometry of the traverse figure as measured in the field.

Chukwuocha (2017) explains the computing of the reorientation traverse in two stages. In the first stage, an arbitrary azimuth is imposed on the first leg of the traverse and the traverse figure is computed beginning with the correct coordinates of takeoff control station giving arbitrary coordinates to all the traverse stations. Then the two angles at the control stations not observed in the field are computed from the coordinates of the ends of the appropriate lines. These derived angles are then used to compute the traverse in proper azimuth orientation using the coordinates of the end points of the control line.

2. Materials and Methods

Four different second order traverses were run with legs in the range of 450m to demonstrate the feasibility of the reorientation traversing of connecting traverses of diverse total lengths to GNSS control stations. All the controls used in the traverses were established using dual frequency GNSS receivers in the fast static mode. Trimble Business Center™ GNSS software (Trimble Engineering and Construction Group 2011) was used to process the GNSS data in the fixed solution mode and all the coordinates were determined in the projected Nigeria (modified) Transverse Mercator map system after network adjustment. The results are presented in Table 2. The orthometric heights were determined on the OSU 91A geoid and by a determined constant for Owerri they are here produced with reference to the Lagos mean sea level datum.

Table 2. GNSS Controls for the Traverses

Point ID	**Easting (m)**	**Easting Error (m)**	**Northing (m)**	**Northing Error (m)**	**Elevation (m)**	**Elevation Error (m)**
GPS C1	503936.956	0.004	161089.006	0.004	69.280	0.011
GPS C1A	504917.646	0.004	161426.626	0.003	67.822	0.009
GPS C4	504787.264	0.008	162189.820	0.009	69.383	0.035
GPS C4A	505067.433	0.005	161520.890	0.003	67.917	0.009
GPS C6	504437.592	0.009	163067.269	0.008	69.029	0.024
GPS C6A	505332.343	0.009	163452.723	0.008	69.487	0.024
GPS C9	505723.540	0.008	163605.980	0.007	71.769	0.022
GPS C9A	506398.844	0.014	163884.699	0.010	70.604	0.031
GPS C12	505173.739	0.010	164903.402	0.008	69.738	0.024
GPS C12A	506259.136	0.012	165315.878	0.010	70.779	0.028

The shortest traverse had two new stations and a traverse length of about 1.41Km. The second traverse was of five new stations and had a total traverse length of about 2.8Km. The third of seven new stations had a length of about 3.75km. Since it is becoming standard that traverses should not exceed 5Km (Survey Department Ministry of Development Brunei Darussalam, 2016), the longest traverse was designed to test the feasibility of the reorientation traversing at the longest traverse length and it had a total of ten new stations and was of a distance of 5.1Km.

The traverse field observations were made using a 2” total station on two reflector targets. Apart from the general atmospheric correction factors set for the electronic distance meter (EDM) of the total station, distance measurement was set on refinement mode of average of 3 readings. Field observation of angles and distances were by the forced centering method. Angles were estimated by an average of a minimum of 5 zeroes on both faces. Grid distances on the Nigerian Transverse Mercator map projection system were determined from the mean of field measured distances. Table 3 presents the field observed quantities used in the computation of the traverses. The traverses were computed using the Bowditch traverse computation method and also by least squares adjustment using the adjust software.

The least squares adjustment were carried out using the ADJUST software provided with Ghilani (2010). The adjustment used parametric equations in which an observation equation was written for each distance, direction, or angle. Since these equations are all non-linear in the coordinates of the stations, they were linearized. The traverses in this research were closed by linking the traverses from one set of two control stations to another. In the link closed traverse cases there are n number of sides for (n - 1) number of unknown stations each having two unknown coordinates in the system making 2(n - 1) unknowns. Where as in the same link closed traverses there are n measured distances and (n + 1) number of angles. From the foregoing for these link closed traverses of n sides, there were r = (n + n + 1) – 2(n – 1) = 3 redundant equations.

Table 3. Final Estimates of Field Observations of the Traverses

First Traverse of 2 Unknown Stations

S/N	Stations	Traverse Angle	Grid Distance (m)
1	GPS C9	269°45'43.2"	453.569
2	RT10	178°54'19.1"	450.581
3	RT11	181°14'15.0"	505.006
4	GPS C12	271°43'03.4"	
R.T Takeoff Angle (Ref. GPS C9A) = 269°45'39.0"			
R.T Closing Angle (Ref. GPS C12A) = 271°43'07.7"			

Second Traverse of 5 Unknown Stations

S/N	Stations	Traverse Angle	Grid Distance (m)
1	GPS C6	359°50'30.9"	460.126
2	RT7	181°19'33.6"	451.151
3	RT8	179°33'24.7"	482.999
4	RT9	89°55'02.3"	453.569
5	RT10	178°54'19.1"	450.581
6	RT11	181°14'15.0"	505.006
7	GPS C12	271°43'03.4"	
R.T Takeoff Angle (Ref. GPS C6A) = 359°50'25.1"			
R.T. Closing Angle (Ref. GPS C12A) = 271°43'09.2"			

Third Traverse of 7 Unknown Stations

S/N	Station	Traverse Angle	Grid Distance (m)
1	GPS	181°27'27.7"	462.771
2	RT5	179°05'47.7"	481.810
3	RT6	268°42'26.2"	460.126
4	RT7	181°19'33.6"	451.151
5	RT8	179°33'24.7"	482.999
6	RT9	089°55'02.3"	453.569
7	RT10	178°54'19.1"	450.581
8	RT11	181°14'15.0"	505.006
9	GPS	271°43'03.4	
R.T Takeoff Angle (Ref. GPS C4A) = 181° 27'23.4"			
R.T Closing Angle (Ref. GPS C12A) = 271°43'07.7"			

Fourth Traverse of 10 Unknown Stations

S/N	Stations	Traverse Angle	Grid Distance (m)	S/N	Station	Traverse Angle	Grid Distance (m)
1	GPS C1	325°34'30.5"	460.078	7	RT7	181°19'33.6"	451.151
2	RT2	180°19'13.6"	452.614	8	RT8	179°33'24.7"	482.999
3	RT3	182°35'20.5"	478.643	9	RT9	89°55'02.3"	453.569
4	RT4	119°14'39.5"	462.771	10	RT10	178°54'19.1"	450.581
5	RT5	179°05'47.7"	481.810	11	RT11	181°14'15.0"	505.006
6	RT6	268°42'26.2"	460.126	12	GPS	271°43'03.4"	
R.T Takeoff Angle (Ref. GPS DC1A) = 325°34'28.7"				R.T Closing Angle (Ref. GPS DC12A) = 271°43'06.8"			

Note: R.T. = Reorientation Traversing

3. Results and Discussions

The results of the linear accuracy estimation of each of the Bowditch computation of traverses are presented in Table 4. In all the cases the reorientation traversing resulted in better linear accuracies. Much is not to be made of the huge differences in the linear accuracies of some of the cases. The least squares adjustment provided more reliable comparisons of the results of the traverses.

Table 4. Results of the Traverse Cases Determined by Bowditch Adjustment

U. St.	L (m)	R.T. Accuracy	T.T. Accuracy	U. St.	L (m)	R.T. Accuracy	T. T. Accuracy
2	1409.16	1/83,150	1/76,830	7	3748.01	1/257,510	1/62,460
5	2803.43	1/219,510	1/100,530	10	5139.344	1/623,590	1/44,230

Note: R.T. – Reorientation Traverse; T.T. = Traditional Traverse; L = Total Traverse Length; U. St. = Number of Unknown Stations

Local accuracy values indicates the uncertainty of the position of a control point relative to the adjacent stations measured in the same system as an indicator of the level of reliability or quality of the control coordinates at the 95-percent confidence level. However an average value is reported for the set of control points adjusted in the same network. (Crown Registry and Geographic Base Branch, GeoBC (2009), Surveys Division, Arkansas Highway and Transportation Department (2013)).

Surveys Division, Arkansas Highway and Transportation Department (2013) provides a system for computation of the maximum allowable local accuracy of a traverse based on the class of the traverse and the total length. The maximum local accuracy for the highest accuracy type of control traverse, the Secondary Control (1-99) is 1cm + 10ppm. For other lower accuracy controls it provides for a maximum local accuracy of 2 cm + 50ppm, while property survey traverses are allowed local accuracy of 2cm + 100ppm. So for a second order control of 3km length, the maximum local accuracy value allowed will be 2cm + 10(300,000/1,000,000)cm = 2cm + 3cm = 5cm or 50mm. Other Standards will include Crown Registry and Geographic Base Branch, GeoBC (2009) and Washington State Department of Transport (WSDOT) (2005).

The results of the traverses computed by the least squares adjustment together with their error statistics are presented in Table 5. The first traverse cases of total lengths of 1.4Km, 2.8Km, 3.7Km and 5.1Km returned 11mm, 11mm, 22mm and 33mm respectively for the traditional traverses and 11mm, 14mm, 22mm and 33mm respectively for the reorientation traversing. All these were successful results since each case was lower than the maximum allowed by the standards of 34mm, 48mm, 57mm and 71mm respectively. It is thus demonstrated that the reorientation traversing method is suitable for linking the GNSS control stations with traverses of up to 5Km without the use of the short GNSS reference lines which would have encumbered the traverses.

Table 5: Adjusted Coordinates and Statistics Resulting from the 450m Legs Reorientation traverses and Traditional Traverses

Number of New Stations	Station	Reorientation Traverse					Traditional Traverse					Local Accuracy
		X	Y	Sx	Sy	r(95%)	X	Y	Sx	Sy	r(95%)	
2	PT10	505,548.751	164,024.5231	±0.0047	±0.0036	0.0104	505,548.752	164,024.5241	±0.0047	±0.0036	0.0104	Reorient. 0.0107
ΣD = 1.4km	PT11	505,367.199	164,436.915	±0.0049	±0.0036	0.0109	505,367.199	164,436.915	±0.0049	±0.0036	0.0109	Trad. Trav: 0.0107
	RT7	504,859.667	163,250.491	±0.0043	±0.0051	0.0117	504,859.668	163,250.486	±0.0032	±0.0038	0.0088	Reorient.: 0.0141
5	RT8	505,277.555	163,420.513	±0.0058	±0.0066	0.0153	505,277.556	163,420.508	±0.0044	±0.0049	0.0114	
	RT9	505,723.523	163,605.991	±0.0065	±0.0068	0.0162	505,723.522	163,605.989	±0.0049	±0.0051	0.0121	Trad. Trav. 0.0106
ΣD = 2.8km	RT10	505,548.743	164,024.532	±0.0068	±0.0069	0.0153	505,548.739	164,024.529	±0.0051	±0.0042	0.0115	
	RT11	505,367.197	164,436.921	±0.0055	±0.0042	0.0122	505,367.193	164,436.918	±0.0041	±0.0031	0.0091	
7	RT5	504,619.404	162,621.075	±0.0065	±0.0037	0.0139	504,619.407	162,621.076	±0.0065	±0.0037	0.0139	
	RT6	504,437.580	163,067.260	±0.0119	±0.0062	0.0254	504,437.584	163,067.262	±0.0119	±0.0062	0.0254	
ΣD = 3.7km	RT7	504,859.656	163,250.479	±0.0133	±0.0061	0.0271	504,859.660	163,250.479	±0.0133	±0.0061	0.0271	Reorient.: 0.0220
	RT8	505,277.544	163,420.503	±0.0134	±0.0065	0.0274	505,277.547	163,420.503	±0.0134	±0.0065	0.0274	Trad. Trav. 0.0220
	RT9	505,723.510	163,605.983	±0.0124	±0.0069	0.0265	505,723.513	163,605.985	±0.0124	±0.0069	0.0265	
	RT10	505,548.732	164,024.526	±0.0095	±0.0054	0.0203	505,548.732	164,024.526	±0.0095	±0.0054	0.0203	
	RT11	505,367.191	164,436.917	±0.0062	±0.0036	0.0132	505,367.189	164,436.917	±0.0062	±0.0036	0.0132	
	RT2	504,211.126	161,458.469	±0.0051	±0.0044	0.0123	504,211.128	161,458.468	±0.0051	±0.0044	0.0123	
	RT3	504,482.881	161,820.421	±0.0100	±0.0082	0.0244	504,482.883	161,820.419	±0.0100	±0.0082	0.0244	Reorient.: 0.0334
	RT4	504,787.263	162,189.813	±0.0147	±0.0123	0.0365	504,787.264	162,189.812	±0.0147	±0.0123	0.0365	
10	RT5	504,619.413	162,621.071	±0.0188	±0.0119	0.0416	504,619.412	162,621.069	±0.0188	±0.0119	0.0416	
ΣD = 5.1km	RT6	504,437.581	163,067.302	±0.0201	±0.0237	0.0544	504,437.578	163,067.296	±0.0201	±0.0237	0.0544	Trad. Trav: 0.0334
	RT7	504,859.662	163,250.508	±0.0200	±0.0174	0.0461	504,859.658	163,250.503	±0.0200	±0.0174	0.0461	
	RT8	505,277.554	163,420.519	±0.0191	±0.0108	0.0395	505,277.549	163,420.516	±0.0191	±0.0108	0.0395	
	RT9	505,723.523	163,605.991	±0.0173	±0.0087	0.0369	505,723.518	163,605.989	±0.0173	±0.0087	0.0369	
	RT10	505,548.744	164,024.532	±0.0125	±0.0066	0.0267	505,548.739	164,024.53	±0.0125	±0.0066	0.0267	
	RT11	505,367.197	164,436.92	±0.0072	±0.0041	0.0155	505,367.193	164,436.919	±0.0072	±0.0041	0.0155	

Note: ΣD is total length of traverse

3.1. Statistical Tests of the Quality of Coordinates Determined by the Proposed Method

The proposed method to connect control traverses to GNSS control stations without the short GNSS base lines has proved feasible since all the lengths met the required local accuracy standards. In furtherance tests were carried out to determine if there were any statistical differences between the coordinates as derived in the method of reorientation traversing and the corresponding coordinates derived by the traditional method. The statistical tests carried out in this research were comparison tests between the coordinates determined by the reorientation traversing process as the sample means, against the corresponding coordinates resulting from the already established method of traditional traversing as the population mean.

The t-distribution test is applied to testing if the sample mean is either statistically greater or less than the population mean. The t-distribution single-tail test is applied in such tests where the null hypothesis is set to find if the sample mean is statistically greater or less than the population mean. In the case being studied, the t-distribution two-tail test is applicable since the null hypothesis is set to find if the sample mean is within a prescribed confidence interval. The null hypothesis was whether the difference in the two means was equal to zero versus its alternative, of the difference not being equal to zero. The test statistic based on the difference in two means detailed by Ghilani (2010) for the t-distribution test was used. Since all the reorientation and traditional traverse cases contained the minimum number of observations, that is, $n_1 = n_2$, the redundancies, $v_1 = v_2 = 3$, the critical t0.005,6 value was 3.707 (Anglia Ruskin University, 2008) for all the traverses. The tests were carried out at the 99% confidence level.

The P-value test of the probability of observing a more extreme test statistic in the direction of the alternative hypothesis than the one observed. In the case of this present study the P-value tests were carried out to determine the probability of observing a more extreme test statistic of the coordinates derived from reorientation traversing differing from the traditional traversing. If the P-value is less than (or equal to) α, then the null hypothesis is rejected in favour of the alternative hypothesis. And, if the P-value is greater than α, then the null hypothesis is not rejected (Penn State Eberly College of Science, 2018). The p-value tests were carried out at the 99% confidence level. Online calculator provided by Stangroom (2018) was used to calculate the P-values for the statistical tests.

Table 6 presents the results of the t-distribution and the P-value tests of the coordinates resulting from the traverses. The statistical tests results of the t-distribution indicates that the test failed to reject that the two means derived by the reorientation method and traditional method were different at a 0.01 level of confidence. The results show that not only does the reorientation traversing succeed in running a traverses that connect to single GNSS control stations at the required order of local accuracy it further shows that there is no statistical difference between the resulting coordinates of the traverses.

Table 7. 2-tail t-distribution and P-Value Tests of the Resulting Coordinates

Traverse	Station	n1 = n2	ΔX	ΔY	S_x^2	S_y^2	t-Values ($t_{0.005,6}$ = 3.707)		P-Values for P < 0.01	
							t for X	t for Y	P-value of X	P-value for Y
First 2 Unknowns	PT10	7	0.000	0.000	0.00002209	0.00001296	0.000	0.000	1.0000	1.0000
	PT11	7	0.000	0.000	0.00002401	0.00001296	0.000	0.000	1.0000	1.0000
Second 5 Unknowns	RT7	13	-0.001	0.005	0.00001436	0.00002022	-0.673	2.835	0 .5492	0.0659
	RT8	13	-0.001	0.005	0.00002650	0.00003378	-0.495	2.193	0.6546	0.1159
	RT9	13	0.001	0.002	0.00003313	0.00003612	0.443	0.848	0.6878	0.4587
	RT10	13	0.004	0.003	0.00003612	0.00003262	1.697	1.339	0.1883	0.2730
	RT11	13	0.004	0.003	0.00002353	0.00001362	2.102	2.072	0.1263	0.1300
Third 7 Unknowns	RT5	17	-0.003	-0.001	0.00004225	0.00001369	-1.346	-	0.2710	0.4882
	RT6	17	-0.004	-0.002	0.00014161	0.00003844	-0.980	-	0.3994	0.4166
	RT7	17	-0.004	0.000	0.00017689	0.00003721	-0.877	0.000	0.4450	1.0000
	RT8	17	-0.003	0.000	0.00017956	0.00004225	-0.653	0.000	0.5603	1.0000
	RT9	17	-0.003	-0.002	0.00015376	0.00004761	-0.705	-	0.5316	0.4602
	RT10	17	0.000	0.000	0.00009025	0.00002916	0.000	0.000	1.0000	1.0000
	RT11	17	0.002	0.000	0.00003844	0.00001296	0.940	0.000	0.4166	1.0000
Fourth 10 Unknowns	RT2	23	-0.002	0.001	0.00002601	0.00001936	-1.330	0.771	0.2756	0.4969
	RT3	23	-0.002	0.002	0.00010000	0.00006724	-0.678	0.827	0.5464	0.4689
	RT4	23	-0.001	0.001	0.00021609	0.00015129	-0.231	0.276	0.8322	0.8005
	RT5	23	0.001	0.002	0.00035344	0.00014161	0.180	0.570	0.8686	0.6086
	RT6	23	0.003	0.006	0.00040401	0.00056169	0.506	0.859	0.6477	0.4535
	RT7	23	0.004	0.005	0.00040000	0.00030276	0.678	0.974	0.5464	0.4019
	RT8	23	0.005	0.003	0.00036481	0.00011664	0.888	0.942	0.4400	0.4157
	RT9	23	0.005	0.002	0.00029929	0.00007569	0.980	0.780	0.3994	0.4923
	RT10	23	0.005	0.002	0.00015625	0.00004356	1.356	1.028	0.2681	0.3796
	RT11	23	0.004	0.001	0.00005184	0.00001681	1.884	0.827	0.1561	0.4689

4. Conclusion and Recommendations

This research has demonstrated the feasibility of the reorientation traversing method to connect control traverses beginning at single GNSS control stations and ending at single GNSS control stations devoid of short GNSS reference lines. The reorientation traversing method thus reduces the need for the problematic short azimuth orientation reference lines. Thus it provides a seamless interface between GNSS established control networks and the densification of controls using the optical methods of horizontal control establishment.

The t-distribution test revealed that for all the cases of the traverses of up to 5Km, which standards provide for as the longest distances that traverses should be run, there is statistically no difference between the coordinates resulting from the reorientation traversing and the traditional traversing at the 99% confidence level. The P-value test proved that there is no significant probability of observing a significant difference between the coordinates resulting from reorientation traversing and the traditional traversing.

It is recommended that Surveyors should pay more attention to the implication of short GNSS azimuth reference lines in traversing. The use of the method of the reorientation traversing for control extension from GNSS controls where appropriate reference line distances are not possible is strongly recommended since this research has shown that the same quality of results will be derived by the new method as with the traditional traversing method.

Additionally, the method of reorientation traversing recommends itself in cases where the control stations are orphaned either by obstructions that eliminate intervisibility between the main control stations and the auxiliary reference stations or by uprooted of auxiliary stations. The method is also applicable when forest canopies or urban facilities inhibit planting of the controls at appropriate distances.

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