Art. #2279, 19 pages, https://doi.org/10.15700/saje.v44n1a2279

Self-control and self-monitoring behaviour of gifted learners in the mathematical problem-solving process: A case study

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In the study reported on here we used a qualitative case study design to examine the self-control and self-monitoring behaviour of gifted learners in problem-solving processes. We selected 3 gifted secondary learners using the purposeful sampling method. For the study, each learner completed 10 individual problem-solving sessions. A think-aloud protocol as well as observations and interviews were used in each problem-solving session. The gifted learners displayed various and intertwined self-control and self-monitoring behaviour to read, understand, and solve the problems, and to find and verify the answer. They also displayed this behaviour much more frequently in problems that required using visual drawings and/or had long texts. The gifted learners left or adapted self-control behaviour when the behaviour did not work for solving mathematical problems. They made decisions regarding self-control behaviour by means of the self-monitoring process. The participants presented insistent, quick, flexible, and fluent actions for both self-control and self-monitoring processes. Based on our findings, we propose a portrait of gifted learners' self-regulative behaviour in the mathematical problem-solving process.

Keywords: case study; gifted learners; mathematical problem-solving; qualitative research; self-control; self-monitoring; self-regulated learning

Introduction

Several qualities are usually associated with gifted learners, including curiosity, being smart, being highly motivated, and focusing on success. Depending on the country's policies and cultural values, a person may be described as gifted in different ways (Davis & Rimm, 2004; Milne & Mhlolo, 2021; Oswald & De Villiers, 2013). For instance, in Turkey, the Ministry of National Education (MoNE) describes gifted learners as those who display high performance in their intelligence, creative, artistic, or leadership capacity, or in specific academic fields, and who require services that are not provided by the school (Millî Eğitim Bakanlığı, 2015). Various academic disciplines have differing views on intelligence. While no generally accepted description for giftedness exists in these fields or in literature, a high result on a standardised intelligence test is traditionally viewed as a criterion. The use of such tests has often been criticised since they do not have a cultural basis. Consequently, several researchers have examined the concept of intelligence and have attempted to explain giftedness by naming it ability rather than intelligence (Gagné, 2003; Zeidner & Stoeger, 2019). However, it remains a mystery why gifted learners achieve more than others (Greene, Moos, Azevedo & Winters, 2008). Numerous researchers also assert that successful learners might have effective self-regulated learning behaviour (Boekaerts & Corno, 2005; Otto & Kistner, 2017). Gifted learners are also assumed to be self-regulated learners. In most cases, they display individual and independent work; they prefer to monitor and direct their studies rather than being guided by a teacher. Gifted learners engaged in tasks are often highly motivated and persistent, which are self-regulated learning elements (Risemberg & Zimmerman, 1992; Ruban & Reis, 2006). Consequently, the definition of giftedness has been expanded to include structures of self-regulated learning theories. Self-regulation is also one of the topics receiving more prominence in literature on the gifted (Efklides, 2019). There is a common assumption that gifted learners do not need any treatment or intervention for realising their intellectual potential in educational settings. But this erroneous assumption has resulted in less attention on gifted education (Singer, Sheffield & Leikin, 2017). "Reaching excellence in a talent domain requires the optimization of numerous learning processes, and this optimization depends on self-regulation" (Stoeger, Fleischmann & Obergriesser, 2015:258). Self-regulation is crucial for gifted learners to reach their own potential in learning environments. Therefore, we believe that it is valuable to analyse the learning process of gifted learners through self-regulated learning.

Self-regulated Learning Theory

Based on the social cognitive approach, self-regulated learning theory is a broad framework for understanding the processes of learners becoming active agents in their learning (Zeidner, Boekaerts & Pintrich, 2000). A constructive and dynamic process is one of the main characteristics of self-regulated learning theory. It involves setting learners' goals for learning; controlling, monitoring, and regulating their cognition, motivation, and behaviour. Learners' goals and contextual factors in the environment guide and constrain their actions in this active process (Pintrich, 2000). Forethought, performance, and self-reflection are three cyclical phases of this theory (Zimmerman, 2000). During the forethought phase, self-regulated learners initiate their metacognitive knowledge and prior content knowledge to cognitively analyse the task. Strategic planning and goal-setting are also elements of the forethought phase. Self-efficacy, perception of task difficulties, goal orientation, and beliefs of interest/task value are motivational factors involved in this phase. The performance phase involves the selection and adoption of cognitive and motivational strategies in order to achieve their goals (self-control). During the performance phase, self-regulated learners take notice of both motivational and cognitive strategies and monitor these strategies (self-monitoring). During the self-reflection phase, self-regulated learners make judgments about and evaluate their process against a standard, and assign causal attributions to behaviour connected to their performance. Finally, the cycle is complete as a result of these self-reflection processes having an impact on their future forethought phases (Pintrich, 2000; Zimmerman, 2000, 2013).

In this study we specifically investigated gifted learners' performance phase of self-regulated learning in the process of mathematical problem-solving actions; thus, we explain the elements of the performance phase in this article. The performance phase processes, i.e. self-control and self-monitoring, are activated during efforts to learn (Zimmerman, 2013). Self-control consists of choosing to use cognitive, metacognitive, and motivational strategies such as rehearsing and self-talking. Learners choose and use the strategies to reason, solve problems, and think. Moreover, self-monitoring guides the efforts towards self-control. To clarify, self-monitoring pays attention to self-control strategies. Self-monitoring also gives information about the learning process according to the specific goals in the forethought phase and motivates change in behaviour, if necessary. For example, learners could persist to regulate or leave their self-control behaviour by monitoring their goals and task difficulties.

Measuring self-regulated learning may be an event or an aptitude (Winne & Perry, 2000; Zimmerman, 2008). Aptitude is the attribution of a person that predicts future behaviour (e.g. self-report scales). The event is like a snapshot that freezes activity in motion and spans time (e.g. think aloud). Recently, studies that measure self-regulated learning as an event have been presented in the literature (Bannert, Reimann & Sonnenberg, 2014; Cleary, Velardi & Schnaidman, 2017; McCardle & Hadwin, 2015; Zeidner, 2019). However, a considerable number of studies is still being carried out with aptitude measures such as self-report and Likert-type instruments. Nevertheless, the aptitude approach has been criticised for assessing decontextualised self-regulated behaviour and perceptions in learning settings (Cleary, Callan & Zimmerman, 2012; Dinsmore, Alexander & Loughlin, 2008). On the other hand, event measures "ultimately target self-regulation as a contextualized event" (Cleary et al., 2012:2). Namely, an event approach provides an opportunity to investigate the context-based dynamic nature of the self-regulated learning process. There is still a need to conduct studies to investigate self-regulated learning in a specific context such as mathematical problem-solving (Zimmerman, 2008). In this study, we measured the performance phase of self-regulated learning as an event in the problem-solving process which is also recommended for future studies on gifted learners (Zeidner, 2019).

Self-regulated Learning and Problem-solving

Problem-solving is one of the crucial components of mathematics learning and teaching since it supports mathematical understanding in a meaningful way. Researchers tried to conceptualise the problem-solving process that includes methods and approaches applied to solve the problems. Polya (1954) suggests a problem-solving model which is one of the well-known studies in the literature. This model consists of four stages: (i) understanding, (ii) devising a plan, (iii) carrying out the plan, and (iv) looking back. Schoenfeld (1985) proposed a prominent problem-solving model based on Polya's work in which the stages are reading, analysis, exploration, planning, implementation, and verification. Some researchers revised these problem-solving models involving other processes such as metacognitive activities. For instance, Garofalo and Lester (1985) adjusted the stages of the problem-solving model to orientation, organisation, execution, and verification. With regard to these notable studies it is widely accepted that the problem-solving process does not only involve solving of mathematical problems. It also involves complex activities such as problem awareness and comprehension, reforming a mental model, implementing previous knowledge and experience, developing and implementing a plan, overcoming obstacles in the process, knowing when and how to use other plans, evaluating, and changing the solution path (Davidson & Sternberg, 1998; Lester, 1980; Lester & Kehle, 2003; Schoenfeld, 1985). Due to these acts being included in components of self-regulated learning, problem-solving and self-regulation are seemingly closely linked (Kramarski, Weisse & Kololshi-Minsker, 2010). "Problem solving is perhaps the area of mathematics in which self-regulation is most apparent" (Pape & Smith, 2002:95). In this regard, mathematical problem-solving appears to be a rich area where self-regulative behaviour can be investigated – especially in challenging problems (Marcou & Philippou, 2005; Pape & Smith, 2002). In this study we explored self-regulative behaviour in a problem-solving process adapted from the literature which includes (i) reading and understanding the problem, (ii) obtaining the solution and, (iii) finding and verifying the answer.

Self-regulated learning in mathematics assists learners in planning, guiding, and monitoring their thoughts when confronted with difficult tasks. It also supports solving problems effectively by providing mathematical understanding. Since the 1980s the focus of many research studies has been on mathematical problem-solving and aimed at improving the abilities of learners featured in self-regulated learning theories. As we stated before, problem-solving that requires applying multiple abilities is seen as a rich area for further research in self-regulated learning (De Corte, Verschaffel & Op't Eynde, 2000; Panaoura & Philippou, 2003). Several subcomponents of self-regulated learning are found in the literature, for example examining learners' use of strategy, investigating the interaction between motivational beliefs and problem-solving – mostly in an experimental context (Marcou & Philippou, 2005; Montague, 1991; Pajares, 1996; Pajares & Miller, 1994; Pape & Wang, 2003; Schunk & Hanson, 1985; Weinstein & Mayer, 1986). Factors that influence self-regulated learning components and problem-solving training, which include these components in the classroom context, have been analysed (Gidalevich & Kramarski, 2017; Leidinger & Perels, 2012; Pape, Bell & Yetkin, 2003; Perels, Gürtler & Schmitz, 2005; Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaerts & Ratinckx, 1999; Wilburne & Dause, 2017). Much research has also been done on problem-solving in which the focus was on metacognitive activities and strategic behaviour, both of which are crucial components of self-regulated learning (Artz & Armour-Thomas, 1992; Kramarski & Friedman, 2014; Kramarski, Weiss & Sharon, 2013; Pugalee, 2001; Schoenfeld, 1985; Sontag & Stoeger, 2015; Tachie, 2019). Studies in which gifted learners are compared with average learners also appear in the literature on problem-solving (Chen, Chang & Kuo, 2016; Garofalo, 1993; Heinze, 2005; Montague, 1991). It is known that successful learners could have the potential to solve challenging problems. Despite research from the last 30 years demonstrating that self-regulated learning enhances academic learning, gifted learners' self-regulatory behaviour has not been adequately researched (Dresel & Haugwitz, 2006; Stoeger et al., 2015). A need exists to reveal the relationship between sub-processes of self-regulated learning in gifted learners (Zeidner, 2019). Therefore, investigating gifted learners' self-regulative behaviour (i.e. self-control and self-monitoring behaviour) in a domain-specific context could have the potential to result in better understanding of the use of self-regulation in the gifted population and contribute to the accumulating literature on self-regulated learning (Garn & Jolly, 2014). In this regard, we investigated the following question in this study: "What are self-control and self-monitoring behaviours of gifted learners, and how do these self-regulative behaviours interact in the context of mathematical problem-solving?"

Method

Research Model

This qualitative study was designated to examine gifted learners' self-control and self-monitoring behaviour in the mathematical problem-solving process (Denzin & Lincoln, 1998). A case study, which is one of the qualitative research designs, not only allows for a detailed analysis of an event or a notion but also an extensive description. Focus on extensive understanding is one of the goals of case study (Merriam, 1998). As a result, a holistic multiple case study was used in this study. Each gifted learner was evaluated as a case. These cases were analysed holistically on their own and then compared to others (Yin, 1994).

The Participants

Gifted and other learners in Turkey mainly attend the same classes and follow the same mathematics curriculum. But for learners who perform well on tests and in class, some private schools may offer weekend or after-school courses that include extra-curricular activities. Private schools in Turkey are self-funded through school tuition fees paid by each learner. These courses most often cover mathematical topics that are more advanced than the average level in class topics. Additionally, the learners with higher scores in a standard placement test (a test used for enrolment in high school) are placed in science high schools and Anatolian high schools. They receive more in-depth education based on their test results (Sak, 2011). Since 1995, gifted learners have also been receiving after-school instruction in science and art centres affiliated to the MoNE. In all 81 cities in Turkey, the Ministry of Education provides an enrichment programme for gifted children in over 275 science and art centres. Nationally, these centres support the gifted learners' education (Shaughnessy & Sak, 2015). The educational programme at these centres follows a five-stage sequential process that differs from the national mathematics curriculum. The stages include orientation, supporting education, noticing individual abilities, developing special talents, and project production/management (Millî Eğitim Bakanlığı, 2015). Learners with an intelligence quotient (IQ) of greater than 130 may attend these centres.

We chose our participants from one of these centres in a major Turkish city. In this centre, the first author conducted unsystematic observations for one semester. She designed a year-long programme for gifted learners in the "developing special talents" stage. These gifted learners attended this centre as extra-curricular activity complemental to their primary school education. When we interviewed the learners' mathematics teachers at the centre, they pointed out that they systematically observed all the learners in the first two stages of the education programme. They expressed that teachers in different fields collaboratively designed lessons in these stages. For instance, when making non-systematic observations in the centre, the first author attended such a lesson in which mathematics, literature and music teachers worked together with gifted learners with the aim of composing a triangle song according to a poem that contains properties of triangles. After having observed in these stages, teachers and counsellors in the centre guided gifted learners according to their individual talents considering the learners' needs and intentions. The participants in this study mainly attended science and mathematics courses under supervision of the centre once their individual talents have been noticed. We can say that the majority of these learners were good at and interested in both science and mathematics. Four middle and four secondary gifted learners were involved in the programme on developing special talents designed by the first author. Observations were done in order to do purposeful sampling in order to obtain in-depth and rich data (Patton, 2002). We chose three gifted learners from the 10th grade for this study: Ahmet, Demir, and Ege (all pseudonyms). The three learners were the only ones who voluntarily participated in this study. Ahmet and Demir were 16 years old and Ege was 17 years old. These learners were chosen for the study because they had no trouble expressing their thoughts related to various subjects and communicating with the first researcher throughout this programme. When we asked for their IQ scores, the principal of the centre stated that they were not allowed to state the scores publicly. We provide more information about the participants' views on mathematical problem-solving. Common characteristics among participants were a love of mathematics, verbalising their experiences while solving problems, and regarding themselves as good problem solvers. Ahmet perceived the changes in his thinking process while he begun to engage in mathematics at age 7 by saying: "*I was able to analyse the situations easily with separating their causes and effects*." Ahmet stated that while solving a problem, he thought about the problem and tried to restate and symbolise the problem by evaluating all of the possibilities. He also added that improving his skills and finding several solutions to a problem was one of his goals. Demir participated in mathematical Olympiad courses in his high school and specified that he really enjoyed solving mathematical problems. He noted that he liked problems that challenged him and required of him to think deeply. Demir also stated that he had no problem concentrating to solve problems and added: "*if I am not able to solve a problem, I think on that problem for more than an hour without getting out of my seat*." Similarly, Ege expressed that when he was confronted with challenging problems that he enjoyed, he could spend approximately 1 hour for solving the problem. He also emphasised that having fun was one of the goals of solving mathematical problems. Ege stated that he paid attention to the written style in which the problem was presented, as he experienced some confusion while reading the problems.

Data Collecting Process and Tools

Each gifted learner participated in ten 40-minute problem-solving sessions that were videotaped and recorded each week. The research lasted roughly 4 months. This long-term interaction that started approximately 1 year before the study also served as one of the criteria for the trustworthiness of the study. Firstly, we selected 20 non-routine problems from a variety of sources for the purpose of the study (Gardiner, 1987; Krantz, 1996; Posamentier & Krulik, 1998; Posamentier & Salkind, 1988). The non-routine problem-solving process demands and has potential to reveal self-regulative behaviour since these problems require high-level cognitive skills from the learners such as flexible and systematic thinking, converting knowledge or procedures into new contexts, combining skills, and associating different mathematical concepts (Gidalevich & Kramarski, 2017; Pape & Smith, 2002). We selected the mathematics problems that provided a rich context to reveal the participants' self-regulative behaviour. We selected non-routine problems from various resources (Gardiner, 1987; Krantz, 1996; Posamentier & Krulik, 1998; Posamentier & Salkind, 1988). Since we did not aim to evaluate the participants' content knowledge, we took into account that the problems did not contain any concepts or abilities from the curriculum. To determine whether or not the chosen problems were appropriate for exploring self-control and self-monitoring behaviour of gifted learners, the problems were assessed by four experts (two professors and two Doctor of Philosophy [PhD] candidates) in mathematics. The 10 problems were chosen based on two pilot interviews and the opinions of the experts. The problems are presented in Appendix A.

The participants solved the problems using a think-aloud protocol, which is one of the effective ways of evaluating self-regulated processes (Zimmerman, 2008). A think-aloud protocol reveals gifted learners' mental process and thinking pathways while they engage with the problems. Prior to the study we had a pilot run of the think-aloud protocol with each of the participants. In each problem-solving sessions we requested participants to read the problem. After having read the problem, we requested them to solve the problem while thinking aloud. When they stopped talking or did calculations, we asked them questions such as, "What are you thinking now?" "What is on your mind now?" "Could you please share your thoughts with me?" While solving the problems, we took some notes related to the participants' problem-solving acts. These notes were observations that included the participants' methods and strategies. In some problem sessions, because of quick thinking, gifted learners only expressed their thoughts verbally without reflecting any of their ideas on paper. When the participants said that they had finished solving the problem, we asked them to explain how they did it and to clarify any operations/drawings that were unclear. In this way, we aimed to collect information about the reasons for self-regulated behaviour. For example, if the participant quit the calculation during the solving process and read the problem again, we asked questions such as, “Why did you turn back and read the problem? Why did you feel the need to turn back?” If the participant claimed not to understand the problem, we asked questions such as, “Why do you think you were unable to understand it? Do you always turn back while solving problems?” When the gifted learner changed the method in solving the problem, we asked questions like, “Why did you give up on the solving method that you were using? How did you decide to switch to the other method?” In order to enhance the trustworthiness of the study, we also confirmed whether we correctly observed their actions by asking gifted learners’ questions about their intentions with these actions that we labelled as self-regulative behaviour.

The Data Analysis

For data analysis, qualitative content analysis was used (Auerbach & Silverstein, 2003). We transcribed the video-recorded problem-solving sessions into written documents. Before beginning the data analysis, a coding protocol was developed based on the literature concerning self-regulation and problem-solving (Gidalevich & Kramarski, 2017; Pintrich, 2000; Polya, 1954; Schoenfeld, 1985; Wilburne & Dause, 2017; Zimmerman, 2000). We chose this approach to elaborate on the existing self-regulation processes (i.e. self-control and self-monitoring) within the context of mathematical problem-solving (Auerbach & Silverstein, 2003). Table 1 includes descriptions of codes and behaviour indicating the codes. In order to determine the similarities and differences between each case, cross-case analysis was used (Yin, 1994). Only the findings from the cross-case analysis are presented.

Table 1 Descriptions of codes and indicators of the behaviour

Self-regulation behaviour	Descriptions	Indicators
Self-control	Refers to choose or use actions that helps searching, finding a solution or changing a method for the solution during the problem-solving process	*For reading and understanding the problem*: Actions to understand the meaning of the problem *For obtaining the solution:* Using several strategies to solve the problem Changing strategies while solving the problem according to their monitoring behaviour *For giving the answer:* Being able to find an answer to the problem
Self-monitoring	Refers to paying attention or making evaluations about self-control behaviour that results in implementing or adjusting the behaviour during the problem-solving process	*For reading and understanding the problem:* Actions to ensure and evaluate whether they understand the meaning of the problem *For obtaining the solution:* Assessing the problem-solving process with by paying attention to self-control behaviour Deciding to change or adapt the self-control behaviour *For verifying the answer:* Actions to check and re-examine the problem-solving process after having found the answer.

We examined the self-regulation behaviour in the performance phase according to two main processes: self-control and self-monitoring. We also considered learners’ purposes in the mathematical problem-solving process as (i) reading and understanding the problem, (ii) obtaining the solution to the problem, and (iii) finding and verifying the answer (Polya, 1954; Schoenfeld, 1985). To clarify, if the learner displayed self-control behaviour such as “going back and reading the problem again” to understand the problem, we coded the behaviour as a “self-control behaviour for understanding the problem.” Similarly, if the learner displayed self-monitoring behaviour such as “thinking about the solution based on previous knowledge” to find the solution, then we coded this behaviour as “self-monitoring behaviour for obtaining the solution to the problem.” We requested a PhD student who took a self-regulated learning course to code three problem-solving sessions corresponding to 10% of the raw data. Our coding revealed an 85% agreement with her coding of the data. Lastly, we introduced a coding section from one of the participants' (Ahmet) actions during the solving-process of Problem 1 (cf. Appendix B). This might help to make sense of self-control and self-monitoring behaviour and the relation between these processes.

Findings and Discussion

The gifted learners displayed a wide variety of self-control and self-monitoring behaviour while solving problems (cf. Table 2). They mostly demonstrated this self-regulative behaviour in order to obtain the solutions. One has to display considerable self-control behaviour in order to understand the problems. The gifted learners used several instances of self-monitoring for verifying their answers but we observed less self-monitoring behaviour for understanding the problem. This may be due to the participants being certain about understanding the problems. We observed that the gifted learners displayed noticeably more self-regulative behaviour for the problems that required the drawing of a visual model or contained long texts (i.e., Problem (P) #5, 6, 7, 8, 9, 10). We describe some examples of self-regulative behaviour according to problem-solving processes, namely (i) reading and understanding the problem (ii) obtaining the solution and (iii) finding and verifying the answer. We firstly refer to self-regulative behaviour used mostly by all of the participants, then we give examples of less frequently used behaviour in each subsection.

Table 2 Self-control and self-monitoring behaviour during problem-solving sessions

Self-regulative behaviour		P#1	P#2	P#3	P#4	P#5	P#6	P#7	P#8	P#9	P#10
Self-control											
Reading and understanding the problem	Ahmet	4	3	-	3	1	7	8	2	3	-
	Demir	2	3	-	-	-	6	6	2	3	3
	Ege	1	1	-	1	-	3	4	2	3	3
Obtaining the solution	Ahmet	4	3	8	2	5	2	2	6	11	8
	Demir	2	4	2	1	2	8	4	7	4	11
	Ege	2	3	2	1	4	7	2	3	6	4
Self-monitoring											
Reading and understanding the problem	Ahmet	-	1	-	-	-	5	3	-	-	-
	Demir	-	-	-	-	-	6	2	-	-	-
	Ege	1	-	-	1	-	-	1	-	-	-
Obtaining the solution	Ahmet	5	3	13	4	3	1	3	2	2	8
	Demir	1	3	2	-	1	5	13	-	4	11
	Ege	-	-	2	-	5	-	12	-	8	3
Verifying the answer	Ahmet	1	-	4	-	4	-	1	4	1	-
	Demir	2	1	1	2	1	1	1	5	-	3
	Ege	1	-	1	-	-	-	-	2	1	-

Self-control

In this study the self-control process includes behaviour that the gifted learners choose to use during the mathematical problem-solving process. We defined self-control behaviour that indicates selecting or using actions to help to search or find a solution to the problem and change a solution method in the problem-solving context. Since problem-solving is basically a cognitive act, the learners especially displayed cognitive behaviour. The participants mostly displayed self-control behaviour like choosing and using mathematical problem-solving strategies as expected. The process consists of two sub-processes: self-control behaviour for understanding the problem and self-control behaviour for finding the solution to the problem.

Self-control behaviour for reading and understanding the problem

All of the participants envisioned the problem in their minds after having read some of the problems (Ahmet, P#1; Demir, P#2; Ege, P#1). Demir said that he envisioned the 10 straight lines given in Problem 2 as a flower bouquet. Also, the learners went back and read the problem again to make sure what was asked (Ahmet, P#2; Demir, P#8; Ege, P#8). To understand the problem, they used visual drawings based on the given information and also read the problem by dividing it into sections (Ahmet, P#2; Demir, P#8; Ege, P#8). Ahmet and Demir paraphrased the problem by using their own words for understanding the problem better (Ahmet, P#4; Demir, P#10).

Ege, differing distinctly from the other learners, followed the words with a pen while reading the problem to accurately understand the information given in the problem (Ege, P#2). Similarly, Ege said that he "*underlined the information given in the problem*" to understand exactly what was demanded and not to miss any information (Ege, P#9). As we only observed this behaviour while he was solving Problem 9, we asked him whether he generally underlined the information in other problems. He explained as follows: "*I don't underline a single line problem, but I need to* *underline a problem with 5-6 lines.* [..] *It allows me not to miss any words and to connect the words for understanding the sentence in my mind*" (Ege).

Self-control behaviour for finding the solution to the problem

Since the mathematical problem-solving strategies such as reducing methods, examining every possible case, helping to find a solution or to correct the solution methods, we regarded these problem-solving acts as self-control behaviour. Therefore, we present some examples of these strategies which we mostly observed in the problem-solving sessions. The problems, in which the learners displayed varied self-control behaviour, required researching the numbers which satisfied a specific condition (P#1, 2, 5, 9). In such problems the learners firstly considered a smaller number than the original number in the problem. Thus, they decided to solve the problem by using the reducing method. Moreover, they selected other numbers such as a bigger or odd numbers and searched for the conditions for these numbers given in the problem. They tried to find a pattern by determining whether an order existed among the numbers which were smaller than the numbers in the problem. When they managed to find a pattern, they made a generalisation by considering the same for the original numbers in the problem (Ahmet, P#1, 9; Demir, P#1, 2; Ege, P#2, 5, 9). For example, Demir stated that he used symbols for people in Problem 1 and considered the number of people shaking hands as two.

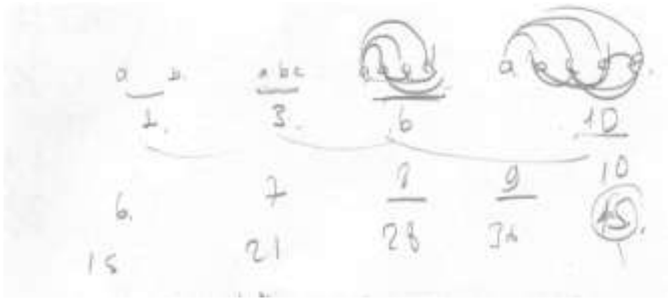

Figure 1 Demir's operations for Problem 1

In other words, Demir started the solution by reducing the number of people shaking hands to two and tried to solve the problem by finding a pattern. As demonstrated in Figure 1, Demir tried to model the hand shakings among people by drawing a line between each letter; and he tried to find a pattern by writing the numbers of handshakes for each case under his drawings. Then he said that the answer was 45. Demir, determining that the number of handshakes between two people was one and the number of handshakes among three people was three, he made a generalisation that "*as the number of people shaking hands increase by one, the number of handshakes also increases by the number of people in the previous handshakes*."

In some of the problems that required researching the numbers satisfying a specific condition, we observed that they examined every possible case that satisfied the condition (Ahmet, P#3, 10; Demir, P#3, 10; Ege, P#9, 10). They systematically examined the numbers to be more organised and to discover all of the numbers, while researching for the numbers satisfying a specific condition (Ahmet, P#3; Demir, P#3, 5; Ege, P#3, 5). For instance, while solving Problem 3, Demir firstly examined every possible number that could be placed in each digit by indicating them using horizontal lines (cf. Figure 2). But he then stated that the problem could not be solved with this method and started to systematically examine the numbers by saying: "*Yes, when hundreds digits is 1, there is 9, no there is 8; for 2, there is 7; for 3, there is 6. I mean* [number pairs are] *4-5, 5-4, 6-3- 7-2, 8-1 and 9-0 when hundreds digit is 1*." Demir then stated that there could be nine possible numbers for the hundreds digit and according to this situation there could be nine possible number pairs for the units and 10s digits.

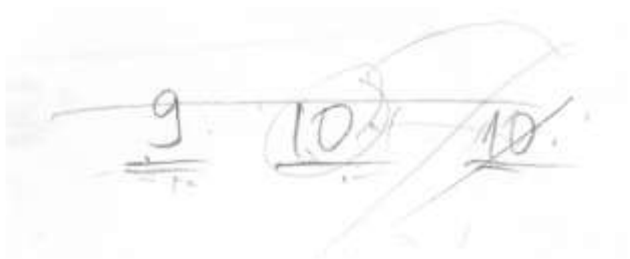

Figure 2 Demir's operations for Problem 3

The learners used visual drawings to describe the solutions they planned in their minds about the information given in geometry problems (Ahmet, P#1, 3; Demir, P#6, 7; Ege, P#2). Similarly, they used symbols in order to determine what was asked in the problems (Ahmet, P#6, 10; Demir, P#1, 2; Ege, P#7, 10) and set up equations (Ahmet, P#8; Demir, P#7, 8; Ege, P#7). The participants also tried to associate the problem with mathematical subjects since they thought that it would help solve the problem (Ahmet, P#1; Demir, P#9; Ege, P#6). For instance, Ahmet reduced the number of people to three, four, and five; then used visual drawings in Problem 1 (cf. Figure 3). He quickly tried to associate with another subject namely convex polygons: "*These geometric figures came up. I take this into consideration because if we create a closed object like a square and all points touch each other, we also count the diagonals*" (Ahmet). However, when he could not remember the formula for polygons' number of diagonals, he tried to find a pattern by considering his own drawings. We observed that Ahmet was able to establish connections among different mathematical cases. We have also seen that when he could not remember the formula, he immediately changed his self-control behaviour. We can say that Ahmet tried to apply his knowledge about polygons in the context of Problem 1.

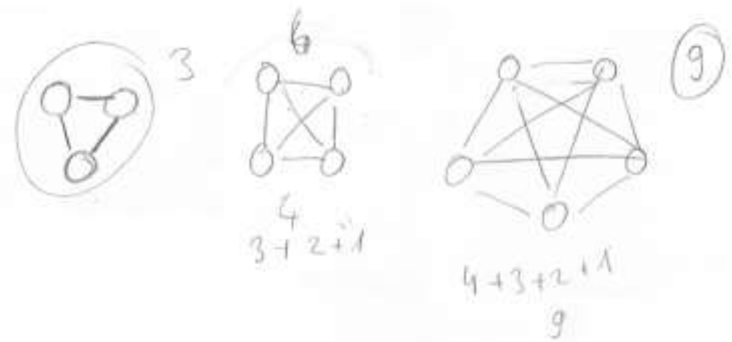

Figure 3 Ahmet's operations for Problem 1

The gifted learners used working backwards when they could not find the solution (Ahmet, P#10; Demir, P#6, 7, 10; Ege, P#6). For example, Ege assumed that the condition, which was "[BP] *should be parallel to bisector of angle A*" was correct in Problem 6 and tried to solve the problem. Ege was unable to obtain a result by using that way, then he tried to solve the problem by examining a specific case, in other words, by assuming that the measure of one acute angle of a triangle was more than two times the measure of the other acute angle.

Ahmet also tried to support his ideas by associating them with another scientific area (Ahmet, P#4). In order to support his solution in Problem 4, Ahmet considered his knowledge about forces and motion from physics. It can be said that Ahmet had a broad and comprehensive knowledge and a rich schema which are indicators for good mathematical problem-solvers (Lester, 1994). Furthermore, while researching palindrome numbers in Problem 5, Ahmet tried to associate palindrome numbers with the mirror concept, in other words; he tried to make an analogy (Polya, 1945).

Self-monitoring

In this study, the self-monitoring process included the behaviour that guided gifted learners to pay attention to their self-control behaviour in the problem-solving process. We defined self-monitoring behaviour that signifies implementing or changing self-control behaviour and making judgements about self-regulative behaviour in the problem-solving context. Within self-monitoring behaviour, the gifted learners dynamically examined their self-control behaviour and decided whether the behaviour was useful or not for solving the problem. The self-monitoring process includes three sub-processes: (i) self-monitoring behaviour for understanding the problem, (ii) self-monitoring behaviour for obtaining the solution to the problem, and (iii) self-monitoring behaviour for verifying the answer.

Self-monitoring behaviour for understanding the problem

We observed that the gifted learners thought about the conditions given in the problems to better understand the problems with long texts and to make the drawings correctly for the problems requiring drawings (Ahmet, P#7, 9; Demir, P#10; Ege, P#8). For example; Ahmet realised that he did not understand what PC meant in Problem 6. He said: "*so, from where do I take the PC? From right or left?*"*;* and then thought about the conditions:

> *I'm trying to figure out how to demonstrate that it is right or parallel now.* [after 2–3 seconds] *Is it right or parallel? Because we draw two BP's...* [thinks for 5 seconds] *Well, since here it says right or parallel, we assume that it's either right or parallel according to BP straight line, but why doesn't it tell us that it is parallel?*

We observed that Ahmet tried to figure out how to represent the conditions given in the problem.

The participants also confirmed whether they accurately transferred the information given in the problem by rethinking their acts under the conditions (Ahmet, P#6; Demir, P#6, 7; Ege, P#4). For instance, Demir asked himself: "*Did I correctly draw the problem?*" after drawing the triangle given in Problem 6 and then compared his drawing to the conditions. Demir displayed similar behaviour in Problem 7, and in the interview, he stated that "*I talk to myself while solving the problem, and it is because I want to approve myself*." We observed that Demir consciously checked the cognitive activities during the problem-solving process and intensely displayed metacognitive activities (Montague & Applegate, 1993). Also, Ahmet and Ege realised that they did not understand some parts of the problem, so they went back to the problem and read it again (Ahmet, P#6; Ege, P#10). Besides, Ege said that he mostly returns to the problem and reads it again in order to check whether he made any mistakes related to the information in the problem (Ege, P#1, 6). Ahmet tried to understand the problems by associating them with his prior mathematical knowledge (Ahmet, P#2, 7).

Self-monitoring behaviour for obtaining the solution to the problem

We observed that the gifted learners thought about their solutions to guide themselves in finding the solution and to decide whether these solutions were practical. They also thought about their solutions to see whether the self-control behaviour that they used worked or not (Ahmet, P#1, 3, 7, 10; Demir, P#1, 9; Ege, P#5, 6, 7). They also thought about the solution based on their previous knowledge to overcome being stuck in a situation (Ahmet, P#9; Demir, P#7; Ege, P#6). The learners asked questions and talked to themselves to produce ideas, to avoid making mistakes, to check the accuracy of their operations, and to produce ideas regularly when the method did not work (Ahmet, P#5, 9, 10; Demir, P#5, 6, 10; Ege, P#5, 6, 7). When the learners were unable to proceed with self-control behaviour, they wondered whether they did not notice a condition and went back to the problem and read it again (Ahmet, P#10; Demir, P#7, 10; Ege, P#7, 10).

One of the most striking examples of the learners adjusting their self-control behaviour was observed in Demir's self-monitoring behaviour on Problem 7. Demir firstly drew an isosceles triangle. Then, while doing operations related to the interior angles of the other triangles in the isosceles triangle, he found that an angle is -40 degrees. He realised that there was a mistake, thus, he turned back to the problem and read it again. Demir realised that he misunderstood the problem while reading it and drew a new triangle after making the required adjustments. Therefore, he regulated using a visual drawing, which is self-control behaviour, by reading the problem again, which is self-monitoring behaviour. He tried to remember his previous knowledge related to angle relationships of the triangle by considering the triangle he drew, but he was unable to do so; in other words, he could not adjust his self-control behaviour. Later on, he confirmed his solution with the knowledge that "*the sum of the interior angles of a quadrilateral is 360 degrees*"; and continued to solve the problem by setting up equations, which is also self-control behaviour. He explained what he did about setting up equations by saying "*the method I used did not help me solve the problem, but it enabled me to crosscheck, I shall* [thinks for 5 seconds] *draw this triangle again*." Demir was unable to obtain a solution, and then he thought about solving the problem by using his content knowledge about the properties of an isosceles triangle: "*it seems that there is nothing useful with the right angle* [thinks for 2 seconds] *let's just say median* [thinks for 3 seconds] *will bisect it*." Here, Demir again tried to adjust the self-control behaviour. After showing that "[BP] *is parallel to the bisector of angle A*" in Problem 6, he said the following:

> *But how do I* [stops for 1 second] *show that it is right?* [stops for 1 second] *I have no idea. That is right. I mean BP to A angle and A angle's* [goes over PC] [thinks for 5 seconds] *how come BP is right to A angle?* [thinks for 3 seconds] *To bisector of A angle?* [thinks for 5 seconds] *I have no idea about that, but it seems to me I had shown that it should be parallel* [thinks for 3 seconds, playing with his hair] *but how consistent is the trial and error method?* [thinks for 7 seconds, playing with his hair] *That's*

it. I cannot show that it is right. How could it be right after all, based on the given information?

Demir talked to himself and asked himself questions about how to satisfy the condition of "[BP] *being right to the bisector of angle A*." In the interview, Demir said that he was aware of self-monitoring behaviour that helped him produce ideas for the solution.

The participants went back to the problem and read it again in order to search for alternative solution methods when they had trouble with producing solutions (Ahmet, P#10; Demir, P#10; Ege, P#10). When the learners were unable to remember the knowledge required for a solution or solutions to similar problems, they tried to remember their prior knowledge (Ahmet, P#10; Demir, P#7; Ege, P#7). In the Problem 10, Ahmet quit evaluating all possible cases about the game results, which is self-control behaviour, by going back to the problem and reading it again. Instead, he used other self-control behaviour which is working backwards. However, when he was unable to obtain the solution by working backwards, he thought about the table he created for 10 seconds; and said "*let me read this from the beginning again*" and read the problem again for 25 seconds. Later, he tried to solve the problem by reducing the number of games given in the problem. However, he still could not solve the problem; and went back to the problem, thought about the given conditions and tried to remember his previous knowledge by asking: "*I just thought about these in the class, actually we thought about them and we also did these. I can't remember right now. How did we do it?*" Nevertheless, Ahmet decided that he could not obtain any solution and then resolved the problem by accepting the assumption that "*one of the teams will win more games or lose more games*." Thus, Ahmet's self-monitoring behaviour enabled him to produce a solution to the problem. Ahmet tried to use his previous knowledge while solving the problem (Sriraman, 2003).

Ahmet and Demir thought about changing their solutions when they realised that their solution method was too long and challenging or when they doubted whether their solution method was accurate or not (Ahmet, P#3; Demir, P#3). In Problem 3, Ahmet represented the place values of the numbers by drawing small boxes side by side. Afterwards, he thought about the solution and then changed his self-control behaviour. In the following interview, Ahmet stated: "*Firstly I needed to draw a lot of small boxes which would take a long time, and secondly it was not very, I mean, very practical*." Hence, Ahmet thought about and revised his solution method. This is in accordance with Krutetskii's (1976) observation that gifted learners mostly look for the most elegant solution methods which are cleaner, simpler and shorter than other methods. Furthermore, we have seen that Ahmet revised his solution method while systematically examining three digit numbers; and thus tried to obtain a solution by clarifying his operations.

Ege, however, displayed motivational self-monitoring behaviour. Motivational self-monitoring behaviour includes clearing his mind to increase his concentration by leaning back and moving away from the problem (Ege, P#7). When we asked whether he was aware of his behaviour, he answered: "*it is just to reset myself, to clear my mind; that's why I leaned back and looked at the whole figure and thought to myself, maybe I can do this operation*." Ege constantly scratched his hair when he realised his mistakes in solving Problem 3. While we asked him whether he remembered the reasons behind pulling his hair, he explained as follows: "*Well, I mean, when I make a mistake while solving a problem, I scratch my head or when I need to think, I scratch my head, it makes me concentrate, I think*." Other motivational self-monitoring behaviour was scratching his hair to increase his concentration when he made a mistake or he had to think about something.

Self-monitoring behaviour for verifying the answer

We observed that the gifted learners reviewed their solutions to ensure whether they correctly solved the problem or not (Ahmet, P#3, 5; Demir, P#6, 8; Ege, P#1). For example, Ahmet stated that he was going to revise his solution because he was not sure of the total number of the three-digit numbers, of which the digits added up to 10 in Problem 3. While revisiting the problem, he realised that he had some missing numbers while revising it. Afterwards, he switched back to the solution phase, used self-control behaviour, and corrected his answer. Other self-monitoring behaviour is trying to justify the solution through a second method (Demir, P#1; Ege, P#8). In Problem 8, Ege thought that 98, which was the extreme value of two-digit numbers, did not satisfy any conditions. In other words, he justified the answer that he had obtained through a second method. Ahmet and Demir went back to the problem and read it again to verify the answer and made adjustments in their solutions when necessary (Ahmet, P#5; Demir, P#2, 5, 10). When Demir went back to the problem and read it again to check if he correctly determined the answer to Problem 10, he said "*hmm, here it asks how many games did they win, not the total of games played*"; thus, he realised that he misunderstood the problem and solved it again.

Conclusion and Suggestions

We measured self-regulated learning as an event with gifted learners (specific population) and elaborated the components of self-control and self-monitoring in a specific context (mathematical problem-solving). We do not claim that we revealed the mechanisms and actions of gifted learners' self-regulative problem solving behaviour. Because we used a limited number of participants and problems in the study limited, our findings can be considered to be an initial step in presenting self-regulative problem-solving processes for gifted learners. We firstly coded all of the problem-solving sessions as we clarified in Appendix B, then we focused on how the flow of the behaviour occurred from a wide perspective. Based on the research findings, the interaction between the self-control and the self-monitoring behaviour of gifted learners during the problem-solving process is generalised as shown in Figure 4. In Figure 4, boxes (A, B, and C) represent self-control behaviour, and ovals (A', B' and C') represent the self-monitoring behaviour that we found during the problem-solving process in terms of (i) reading and understanding the problem, (ii) obtaining the solution, and (iii) finding and verifying the answer. Dashed arrows also represent possible pathways through self-regulative behaviour that we observed in the study. As we stated before, the problem-solving process begins with reading and understanding the problem, and the learners mostly aimed to understand the problem using both processes: self-control and self-monitoring (box A and oval A' in Figure 4). When these gifted learners thought that they understood the problem, they continued finding the solution through self-control and self-monitoring behaviour (box B and oval B' in Figure 4). Lastly, the learners finalised the problem-solving process by finding the answer if possible (box C in Figure 4). As the learners were able to find the answer, they generally verified the answer through self-monitoring behaviour (oval C' in Figure 4). A key aspect of the interaction between these three phases does not have a linear ordering (boxes A, B, and C in Figure 4). For example, when the learners believed that they have provided the correct answer but realised their mistake as a result of self-monitoring behaviour (path C↔C' in Figure 4), then they could either go with reading and understanding the problem (path C→A in Figure 4) or obtaining the solution (path C→B in Figure 4). The learners could also reach the answer phase without passing through any self-monitoring processes (path A→B→C in Figure 4). We also observed an interaction among the self-monitoring processes: if the learners realised that it was problematic to verify the answer, then they might have followed self-monitoring process to obtaining the solution (path C'↔B' in Figure 4). We also showed this interaction in Appendix B. The reason why we didn't observe the other possible interactions might be due to either our participants' characteristics or the structure of the problems used in this study. In future studies one may investigate these interactions in the self-monitoring process more thoroughly by increasing the number of participants and also the problems. We explain these interactions among the self-regulative processes focusing on self-control and self-monitoring.

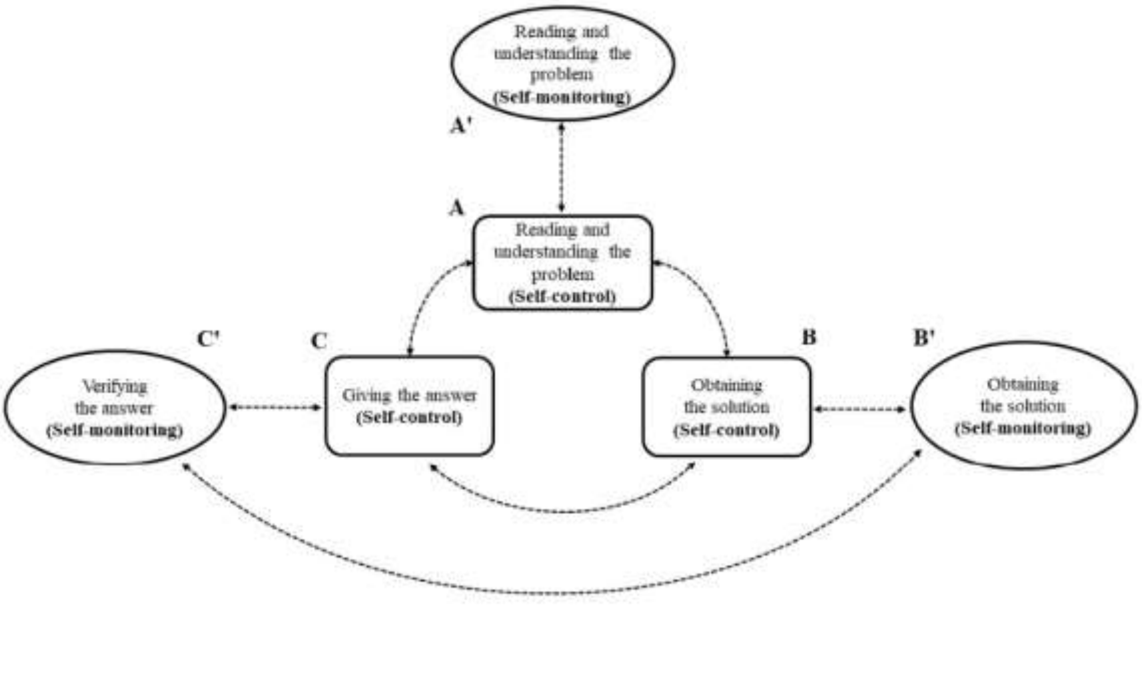

Figure 4 Self-regulated behaviour of gifted learners during the problem-solving process

After having read the problem, the gifted learners firstly displayed self-control behaviour for understanding the problem (box A in Figure 4). We observed this behaviour mostly in the problems that required the use of drawings and/or had long texts, in other words, which were more difficult for the learners to understand. They went back to the problem and read it again; used visual drawings, and read by dividing the problem into sections to confirm what was asked in the problem and to check whether their drawings were appropriate. In the interviews we determined that the learners envisioned the problem in their minds in different contexts. Visualisation might help learners to re-form the mathematical information before solving the problem. The results from our study are thus in line with literature which states that gifted learners have the ability to visualise problems and/or relations (Paz-Baruch, Leikin & Leikin, 2016; Presmeg, 1986; Van Garderen, 2006).

The participants revised self-control behaviour for understanding the problem through self-monitoring behaviour and adjusted the self-control behaviour when necessary (path B ↔ B' in Figure 4). They tried to understand the problem by thinking about the conditions given in the problems which had long texts or required them to make drawings. Furthermore, they contemplated whether they were proceeding according to the conditions given in the problem, went back to the problem, and read it again. These results are consistent with other studies in which it is argued that gifted learners display self-control and self-monitoring behaviour in solving problems more extensively than other learners (Arancibia, Boyanova & González, 2016; Montague, 1991; Montague & Applegate, 1993). This monitoring behaviour of the gifted learners is to create links with their prior knowledge that might help them to explore the information in the problem, and think about solutions to the problem (Arancibia et al., 2016; Davidson & Sternberg, 1998; Zimmerman & Moylan, 2009).

After understanding the problem, the gifted learners displayed self-control behaviour for obtaining the solution to the problem (box B in Figure 4). We concluded that when the participants were familiar with the problems, they quickly decided on self-control behaviour without any problems and used less self-control behaviour than in other problems. When they could not remember the formulae and/or rules that would enable them to find the solution, they thought of alternative solutions and accordingly displayed self-control behaviour instead of trying to remember them. Similarly, they either quit or re-arranged the self-control behaviour which did not take them to the solution or help them proceed while solving the problem. Our findings confirm that gifted learners' decision-making during problem-solving could be very quick (Sriraman, 2003). Krutetskii (1976) states that in mathematics, one of the qualities of gifted learners that sets them apart from other learners is the flexibility that they display while solving problems. In this manner, participants' quick decision-making may also be linked to their flexibility in problem-solving. The self-control behaviour of the gifted learners in our research supports finding in other studies (Chen et al., 2016; Hekimoglu, 2004). Our claim is that flexibility in applying self-regulative acts (i.e. switching between self-control behaviour) could facilitate gifted learners' problem-solving acts.

In the problems that required finding numbers meeting certain conditions, the learners displayed more self-control behaviour aimed to use problem-solving strategies such as trying to find patterns and/or obtaining generalisations. We also found that these gifted learners could realise mathematical patterns and think reversibly. The behaviour is described as higher-order cognitive abilities that are presented as gifted learners' characteristics in the literature (Amit & Neria, 2008; Krutetskii, 1976; Miller, 1990; Sheffield, 2003; Sriraman, 2003, 2005). We noticed that our participants quickly displayed regulating behaviour, especially in the problems for which they struggled finding solutions. This behaviour may be related to being persistent, which could affect gifted learners' choice of self-control behaviour in the problem-solving process.

The participants displayed self-monitoring behaviour that was intertwined with self-control behaviour for obtaining the solution to the problem (oval B' in Figure 4). These gifted learners switched quickly between self-control and self-monitoring behaviour (path B↔B' in Figure 4). They used monitoring behaviour for obtaining solutions to the problems which they had not seen before, which they had considered to be difficult, and for which they had struggled to produce solutions. However, we observed that they did not display or displayed less self-monitoring behaviour in problems that they were familiar with and in which they could easily produce ideas. Thus, it is possible to say that they used self-monitoring behaviour to overcome the obstacles during the problem-solving process. This is consistent with the findings of various research studies showing that difficult problems activate self-monitoring behaviour (Arancibia et al., 2016; Montague & Applegate, 1993; Pugalee, 2001; Yimer & Ellerton, 2006).

We concluded that the gifted learners thought of a variety of solution methods at the same time and decided on the most appropriate one through using self-monitoring behaviour. In the interviews, we heard that the learners consciously checked and regulated cognitive processes, which is regarded as good problem-solving in the literature (Carlson & Bloom, 2005; Lester, 1994; Montague & Applegate, 1993). It can be said that self-monitoring behaviour enabled the gifted learners in our study to understand the problem and select the operations correctly. The findings include examples suggesting that the learners were looking for a more elegant solution (Krutetskii, 1976; Tjoe, 2015). It is likely that the learners' views emerged with the help of self-monitoring in the problem-solving process. Furthermore, the participants also exhibited behaviour such as regulation of motivation and consciously increasing concentration (Miele & Scholer, 2018; Moseki & Schulze; 2019; Wolters, 1998; Wolters & Benzon, 2013). The self-monitoring behaviour probably helped them to focus on the problems and/or the solution methods.

When the gifted learners were able to find the answer (box C in Figure 4), they mostly displayed self-monitoring behaviour to verify the answers to the problems (oval C' in Figure 4). The learners tried to ensure that their answers were correct or not while solving the problems, and criticised the problem-solving processes more frequently – which is also stated in the literature (Fehrenbach, 1991; Heinze, 2005; Sriraman, 2003). In this research, the gifted learners displayed self-monitoring behaviour such as revising and re-reading the problem, both of which were included in previous studies (Pativisan & Neiss, 2007; Sriraman, 2003; Yimer & Ellerton, 2010). Consequently, the gifted learners constantly and fluently monitored and checked their problem-solving process.

The self-regulative behaviour of going back to the problem and reading it again appeared often in this study because the learners displayed this behaviour as self-control and as self-monitoring behaviour. Self-regulative behaviour was the most used behaviour in the problem-solving process. When the learners tried to figure out what was asked in the problem, they displayed it as a self-control behaviour for understanding the problem. This action of re-reading the problem developed into self-monitoring behaviour for understanding the problem whereas the gifted learners wanted to check whether they considered all the information in the problem. While participants had doubts about finding the solution to the problem (self-control behaviour), they used it as monitoring behaviour for obtaining the solution to the problem. Re-reading the problem as self-monitoring behaviour for verifying the answer was displayed when the learners were about to finalise their solution process. We observed that the gifted learners used this behaviour for different purposes (Yimer & Ellerton, 2010). It might be because of the role of the problem itself as orchestrating the whole solving process. This self-regulative act of re-reading the problem also indicates that the gifted learners might be experts (unlike novices) who moved back and forth between phases in problem-solving (Schoenfeld, 1985).

The participants were more persistent in the problems in which they displayed more self-regulated behaviour and they also enjoyed that process more. As the problems provided opportunities for gifted learners to realise their own potential, to cognitively and metacognitively develop and to motivate themselves, the problems could be useful in their education (Diezmann & Watters, 2002; Stoeger et al., 2015). From the results of our study, we believe that problems should enable gifted learners to make generalisations and have potential to associate with other scientific areas (such as physics); moreover, the problems should be open to questioning and interpreting to support gifted learners to showcase their existing abilities and self-regulative behaviour. By considering gifted learners' self-regulated behaviour the teachers of non-gifted learners could design activities that would develop learners' problem-solving behaviour.

We observed that the participants used their advanced content knowledge frequently while solving problems. The more content knowledge the gifted learners had, the more they were likely to involve these in the problem-solving process. Hence, the mathematics curriculum to be developed for gifted learners should have a richer content than the standard programmes in order to be beneficial for their cognitive development.

There is a need to research specific elements such as meta-cognitive and meta-motivational strategies, goal orientation, self-monitoring, and self-reinforcement, etc. rather than higher-order constructs (Zeidner, 2019). Thus, with this study we attempted to fill a gap in the literature by revealing the interaction between self-control and self-monitoring processes of gifted learners. It is reasonable to think that the number of participants was too small to make conclusions related to self-regulative behaviour in the problem-solving process. Despite this limitation, it should be noted that this study might be a step in investigating the self-regulative behaviour of gifted learners as an event in the problem-solving context. More research is needed to learn to measure self-control and self-monitoring processes as an event with using not only the think-aloud method but also using other methods such as microanalysis, which have the potential to elaborate the role of the processes in mathematical problem-solving (Zimmerman, 2008). As a limitation, the problem-solving models that we considered (e.g. Garofalo & Lester, 1985; Polya, 1945) focussed on cognitive and metacognitive processes in solving problems. Our study provided only a few insights regarding motivational and emotional aspects of the self-regulated learning theory. Future research could investigate these aspects from a broader perspective. Future research could also examine gifted learners' problem-solving processes in different settings such as classroom or competition contexts that might expand our view on both gifted learners' self-regulative behaviour and the problem-solving process itself. More refined research can be done in examining self-control and self-monitoring in the problem-solving process such as taking mathematical modelling or real-life problems into account. Thereby, the self-regulated mathematical problem-solving behaviour examined in this research could be expanded.

Authors' Contributions

Both authors contributed equally to the publication. Both authors drafted and reviewed the final manuscript.

Notes

ii. DATES: Received: 11 August 2021; Revised: 20 November 2023; Accepted: 1 February 2024; Published: 29 February 2024.

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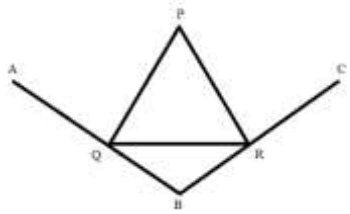
Appendix A: Problems

1) In a room with 10 people, everyone shakes hands with everybody else exactly once. How many handshakes are there?
2) How many pairs of vertical angles are formed by 10 distinct lines, concurrent through a point?
3) If you take the digit-sum of a number, that is, you add the digits in the number, how many three-digit numbers will have a digit-sum of 10? (For example, 262 is one, since 2 + 6 + 2 = 10; 505 is another, since 5 + 0 + 5 = 10).
4) In Figure, m <ABC = 120°, and ΔPQR is equilateral and has vertices Q and R on AB and BC, respectively. As equilateral triangle PQR changes size and moves, with Q and R remaining on the rays of <ABC, what is the path taken by point P?



5) A palindrome is a number that reads the same forward and backward, such as 747 or 1991. How many palindromes are there between 1 and 1,000 inclusive?
6) The measure of a line segment $[PC]$, perpendicular to hypotenuse $[AC]$ of right ΔABC, is equal to the measure of leg $[BC]$. Show $[BP]$ may be perpendicular or parallel to the bisector of <A.
7) ΔABC is isosceles with CA = CB. m <ABD = 60, m <BAE = 50, and m <C = 20. Find the measure of <EDB.
8) The number of 12 is equal to exactly four times the sum of its digits. So is 24.
   (i) Can you find a whole number which is equal to exactly twice the sum of its digits? Is your answer the only possible answer?
   (ii) Can you find a whole number which is equal to exactly three the sum of its digits? Is your answer the only possible answer?
   (iii) Which numbers other than 12 and 24 are equal to exactly four times the sum of its digits?
9) Every number can be written in several ways as a sum of 1's and 2's. For example, 3 = 2 + 1 and 3 = 1 + 1 + 1.
   (i) In how many ways can the number 11 be written as a sum of 1's and 2's? In how many ways can the number 73 be written as a sum of 1's and 2's? Find a general rule and explain why it works.
   (ii) We would not usually treat 3 = 1 + 2 and 3 = 2 + 1 as different. But if we do, then there are three different ways of writing 3 as a sum of 1's and 2's. What are they? And if we count in the same way, how many different ways are there of writing 11 as a sum of 1's and 2's. Investigate!
10) Three teams play a round robin tournament. The team from New York sits out the first game. After that, the loser of any particular game sits out the following game. A total of 11 games are played. Each team won a different number of games, and New York lost the last game. What are the won-lost records for each of the three teams?

Appendix B: A Coding Section from Ahmet' Actions During Problem 1

Observed actions of Ahmet	Analysis
[he reads the problem]	Ahmet firstly *read the problem (Self-control)*
"*It asks the total handshakings, each one them handshake once.*"	After reading the problem, Ahmet *turned the problem again and read it again* in order to understand the problem (*Self-monitoring*)
"*It says that including itself there is 10 people. I will reduce it to three people.*"	Ahmet started with *choosing and using one of problem solving strategies that is reducing method* in order to obtain the solution (*Self-control*)
"*I will draw three rounds here, then I will construct a diagram among them.*"	Ahmet continued *with using visual drawing* (Figure 3) in order to obtain the solution (*Self-control*)
"*1, 2, 3 … There are three handshakings among three people.…*"	In the interviews had done after solving the problem, Ahmet stated that for understanding the problem better he *envisioned the problem within a context in his mind*. He gave a detailed information about the context in which there were people standing in a line as if they were shaking hands for the feast a closed room (*Self-control*)
"*There are one handshaking between 2 people*. [he calculates for 10 seconds] *There are … 1, 2, 3, 4, 5, 6 … There are six handshaking among four people.…*"	Ahmet continued *with making a generalization* in order to obtain the solution (*Self-control*)
"*In the meanwhile, objects showed up. I am taking them into consider, because if we done these as closed objects, I mean, if each points contacts to other points, then we can count the diagonals. But I don't remember the formula for the number of diagonals.…*"	After drawing a visual model (Figure 3) and trying to make a generalization, Ahmet *associated the problem with one of the other mathematical subjects* that is diagonals of the geometric figures. He tried to remember the formula for sum of diagonals of the polygons (*Self-control*)
"But I can guess. [he thinks for 5 seconds while his hand was on his chin] *Ok … It appears slowly. After doing the first, second, third, fourth, fifth, I will not do it again. Because when I done the forth, but example was not enough. I mean it is not necessary to be 10 people, it would be even 15 or 20 people. I will construct it for the fifth person. The diagonals of the polygon are more different, at least the number is odd, its diagonals produced different figures, I will look for it, then I will make decision.…*"	After Ahmet tried to construct the formula for sum of diagonals of the polygons, he *thought over his solution method* which aimed to obtain for the solution of the problem. He was also aware of which parts of the method was not enough for obtaining the solution. He verbalized in detail how to get the formula and to relate this possible outcome with the solution of the problem *(Self-monitoring)*
"*There are four lines among five points. The diagonals among these five points* [he draws; 15 seconds passes] *The star is formed… The star is consisting of five lines. 9, 5 plus 4, 9.* [he thinks while looking at the figure]. […] *There are 10 people. I think that there will be one star for 10 people. 9, 8*" [he thinks for 15–20 seconds].	Ahmet still *associated the problem with one of the other mathematical subjects* that is diagonals of the geometric figures (Figure 3). He tried to remember the formula for sum of diagonals of the polygons and relate the figure with the star *(Self-control)*
"*The first person handshake with four people. The second person handshake with three people. The third person handshake with two people. The fifth person handshake with one person. And it is finished. So there is four digits. According to this, there must be nine digits. I have found the answer a permutation of nine. I think so.…*"	Ahmet calculated the sum of handshakings with the concept of permutation while *getting the answer* (*Self-control*).
"*I have finished. But the answer is not the correct answer. Hah, sorry, I have found as nine factorials. It's not permutation, I said it wrong.*"	After saying that he finalized solving the problem, he suddenly realized the he made a mistake with *reviewing the answer* in order to revise his answer. Then he corrected the concept of permutation as factorial concept (*Self-monitoring*)
"*Sorry, sorry! There is addition among them. Sorry, 1 plus 9, it is 10. 1, 2, 3, 4. Isn't it 5, the mid of 9?* [he asks the question to himself, he calculates for 15–20 seconds and uses his hands while calculating]."	Ahmet realized that there must be addition between the numbers. For obtaining the solution, he *asked a question and talked to himself* while calculating the sum of the numbers (*Self-monitoring*)
"*I found the answer as 45.*"	Ahmet stated that he *get the answer* (*Self-control*)