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An investigation of sociomathematical norms perceived by students regarding the legitimacy of solutions

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With the study reported on here we aimed to determine what learners perceived as normative in the mathematics classroom. For this reason, we focused on negotiation of the problem solutions and we attempted to determine the sociomathematical norms perceived by learners (SNPS). Audio recordings of dialogues among learners, individual reports, and interviews were used as data collection instruments. The research participants were learners in the seventh grade. The study was conducted over a period of 10 weeks covering the second semester of the academic year. Three SNPS (functionality, inclusiveness, connectivity) regarding the legitimacy of the solutions were determined. The determined norms contributed to the understanding of learners' mathematical preferences, thus bringing more inclusive and complementary understanding about the norms perceived by the learners to the literature. It has been observed that learning opportunities emerging through the negotiation of norms contribute to collective mathematics learning by shaping the interaction among class members. In this context, it was deemed necessary to continue research on norms perceived by learners to create general ideas of mathematics learning and teaching.

Keywords: learner perception; learning opportunities; problem solutions; sociomathematical norms

Introduction

Today, mathematics is not only information dependent on cognitive activities but has also become a form that makes sense of social and cultural activities (Sánchez & García, 2014). Many mathematics scholars (Fukawa-Connelly, 2012; Lopez & Allal, 2007; Voigt, 1995) admit that knowing and learning mathematics cannot be understood sufficiently if the sociocultural activities are ignored. The common behaviours performed in the class, and the interaction pattern among the class members (teacher and learners) build the sociocultural activities of the class, in other words, the microculture of the class. According to Sekiguchi (2005), each microculture establishes, develops, or excludes various structures such as rules, expectations, and obligations. These structures that constitute the microculture of the classroom can be called norms. Norms are common unwritten structures that govern the actions and discourses of class members. They can be thought of as a kind of grammar system that regulates the ideas of class members (Bicchieri, 2006). The norm system determines what is acceptable or what is not acceptable in a class as is the case in grammar (Uçar, 2016).

Yackel and Cobb (1996) classify the normative aspects specific to mathematical activities of individuals in the classroom as sociomathematical norms under the sociological perspective. This study has brought sociomathematical norms into focus in mathematics teaching and has led to an increasing number of studies on this subject. While some of the studies state that pre-service teachers could solve problems by sharing the same views on how to interact (Tatsis & Koleza, 2008), some investigated how sociomathematical norms negotiated in two small group discussions play a role in learners' problem-solving strategies (Partanen, 2011). Some studies focused on the role of the teacher in establishing norms (Kang & Kim, 2016), while others associated sociomathematical norms with teachers' professional perspectives (Van Zoest & Stockero, 2012). The above studies used a didactic perspective towards problem-solving for sociomathematical norms. Although these studies are important in terms of norms enacted in the classroom, Levenson, Tirosh and Tsamir (2006) state that learners preserve their personal preferences in the problem-solving process. Therefore, SNPS have become important. In other words, it highlights that only examining the "norms endorsed or enacted by the teacher" (Levenson et al., 2006:340) is insufficient for understanding sociomathematical norms.

With this study we aimed to understand what learners perceive as normative in the classroom. For this purpose the focus was on negotiation of the problem solutions, and an attempt was made to determine the learners' mathematical preferences. In other words, we aimed to determine SNPS regarding the legitimacy of solutions in a secondary school mathematics class. We also aimed to contribute to the literature by obtaining complementary insights into how SNPS are enacted and developed. In addition, the effect of negotiation of probable sociomathematical norms to be determined on the interaction among class members was also examined in detail. Thus, the relationship between norms and the learning opportunities that take place in this microculture was clarified.

Also, some studies (Akyüz, 2014; Güven & Dede, 2017) concerning sociomathematical norms were conducted in Turkey so this study is important and necessary because it determines the frame of norms in mathematics classes of secondary grades in Turkey. Thus, a comparative data set can be generated for research in different cultures on perceived sociomathematical norms in the classroom.

Conceptual Framework

According to the interpretive framework (Yackel & Cobb, 1996) approach, in which sociological and psychological perspectives are coordinated in the analysis of common structures in the classroom the norms constitute the sociological perspective. While social norms are related to general understandings, sociomathematical norms are related to mathematical understandings. While the presentation of different solutions is within the scope of social norms, the understandings that make a mathematical difference are within the scope of the sociomathematical norms. The values and beliefs of class members regarding both norms constitute the psychological perspective in this approach. With this study we focused on SNPS without ignoring the importance of sociological and psychological perspectives in classroom microculture.

Sociomathematical norms are influenced by the contexts in the classroom. For example, in the differential equations class, sociomathematical norms based on the interpretation of rates of change were enacted by the teacher (Yackel, Rasmussen & King, 2000), whereas in another class related to data analysis, sociomathematical norms based on data interpretation rather than calculation procedures were endorsed (Cobb, Stephan, McClain & Gravemeijer, 2001). In this respect, focusing on the negotiation of problem solutions in the interaction among learners in our study may reveal SNPS.

Interaction and cognitive processes are different manifestations of the same phenomenon (Sfard, 2008). Kiwanuka, Van Damme, Van den Noortgate, Anumendem and Namusisi (2015) report that learners' good perception of microculture is a predictor of their mathematics achievement. "As the individual does not have direct access to what other people think in mathematics lessons, the progress of learning and teaching activities depends on the norms shaped in microculture. Norms are expressed as meta-discursive rules that are widely enacted and endorsed within the community" (Sfard, 2008:183). In light of this theoretical framework, the sociomathematical norms that emerged during the discourses of the learners in the problem-solving process were negotiated.

How often discourses are used is important to determine norms. Park (2015) states that discourses should be observed in at least three different lecture sessions. In our study, we noted that discourses, which were accepted as the norm, should manifest themselves sufficiently and clearly in the dialogs in the classroom. However, it was not seen as an inevitable precondition to search for norms clearly stated in discourses (Sánchez & García, 2014). On the contrary, implicit expressions were also considered. Besides, situations that contradict the discourses accepted as the norm were taken into consideration while developing the hypotheses about the norms.

We questioned why solutions were accepted and why they were preferred, in other words, why they were considered legitimate (Elliott, Kazemi, Lesseig, Mumme, Carroll & Kelly-Petersen, 2009; Tatsis & Koleza, 2008). Thus, determined sociomathematical norms were negotiated. Yackel, Cobb and Wood (1991) state that the negotiation process encourages learning opportunities. The learning opportunities were evaluated as a process of creating individual meanings compatible with curricula (Partanen, 2011). We framed learning opportunities not only as an individual meaning-making process but also as insights that shape preferences. Analysing learning opportunities makes it possible to acquire knowledge about mathematical preferences. Therefore, we examined learners' mathematical preferences in depth by relating them to the learning opportunities that were revealed in the negotiation process of sociomathematical norms.

Method

This study was qualitative research and followed an interpretative approach as it focussed on social interactions.

Participants and Content

The participants were seventh grade learners (11–12 years old) who preferred the mathematics practices course (MPC), an elective course in a secondary school in Turkey. There were 24 learners (9M–15F) and a teacher in this course. The learners had the opportunity to use and improve their knowledge through problem-based activities, which included percentages, ratios, and angles (Ministry of National Education [in Turkey] [MNE], 2015).

Eighth grade learners were not preferred as participants because of the high school entrance exam at the end of the semester. Fifth grade learners were also not chosen due to difficulties (such as increased course and teacher diversity, etc.) in transitioning from primary school to secondary school.

Since sixth grade mathematics scores of the participant learners were 82/100, it can be said that they were above the average in terms of readiness (MNE, 2015).

The teacher, on the other hand, had 10 years of professional experience and had experience in the implementation of the course. In addition, he supported methods in which problem solutions were questioned or discussed.

Data Collection Procedures and Teaching Activities

In this study, in which problem-based mathematics activities were applied in the second semester, four heterogeneous groups were formed. The sessions were recorded on video devices. In the first 20 minutes of each session, each learner in the groups was asked to examine the given problem and record their thoughts about the solutions on their reports. In the last 20 minutes of the sessions, the learners were asked to express their discourses and preferences regarding the legitimacy of the solutions.

In this study, where discourses and expressions were the main research object, the data were transcriptions of the dialogues of the above-mentioned teaching activity. The data collected up to the 10th week were constantly compared. During this time, when similar norms were negotiated and associated with learning opportunities, we concluded that saturation had been reached and ended the data collection process. Lincoln and Guba (1985) state that long-term interaction with data collection sources increases credibility.

In addition, unstructured interviews lasting approximately 30 minutes were conducted with three learners in different groups at different academic levels (below average, average and above average). This was done in the third week, sixth week and ninth week. During the interviews with the learners, the basic features of the problem solutions were questioned and the factors determining the mathematical preferences were emphasised. Thus, learners' perspectives on mathematical solutions were examined.

One factor that contributed to the trustworthiness of the study was the variety of data collection tools (video recordings, individual reports and interviews) (Yackel et al., 1991).

Data Analysis

In the first cycle of coding, the dialogues of the course members were coded with capital letters and transcribed. The discourses in the transcript were examined according to two features, namely, "narratives and the use of words" (Sfard, 2008:185). The narratives and mathematical words shown in bold in the narratives included the propositions that learners accepted about the legitimacy of problem solutions. We labelled these propositions and the problematic ones were discussed and approved.

In the second cycle, the characteristics of the approved ones were determined.

In the third cycle, SNPS were inferred when different narratives shared the same characteristics regarding the legitimacy of learners' problem solutions. An example of the inferred SNPS and its defined characteristics are shown in Table 1.

In the last cycle, during the negotiation process of sociomathematical norms, learners' defensive, affirming or rejecting discourses were analysed descriptively and evaluated as a learning opportunity. Learners' participation in discussions or listening to other learners who created new meanings were used to decide on the formation of learning opportunities (Partanen, 2011).

Ethics

Necessary permissions have been obtained for our study. The interviews were conducted on a voluntary basis. Before the interviews, the learners were given detailed information about the research and it was stated that they could end the interview whenever they wanted. In addition, there was no conflict of interest between the authors.

Results

Three SNPS regarding the legitimacy of solutions were determined. One of these norms was *an effective solution must be functional*. One of the expressions of this norm is shown in Table 1. The dialogs in the table relate to the calculations of percentage in the activity in Figure 1. In this activity, the learners examined the problem solutions and explained which solution they would prefer with their justifications.

Table 1 Examples of endorsed narratives about the functionality

Examples of endorsed narratives (Class members discuss solutions about calculating the percentage of money)	Identified features	Inferred norm
Dialogue section-1 F: *My teacher, I preferred the second solution because there are many calculations in the first one.* Teacher (by turning to class): *If there aren't lots of calculations as your friend is saying, in your opinion, can the first solution be more effective?* *Class: Nooo* Teacher: *So we need to explain what makes a solution more effective than the other.* O: *My teacher, in the first solution, all the money is calculated, then the desired percentage is calculated.* Teacher: *So, does it benefit us to calculate all the money?* F: *I think the* ***more practical solution is better****....* Teacher: *What do you mean by practical?* F: *As in the second solution, it is more easily by the proportion without the need to calculate all money. Since proportion was our previous topic, we would use it here and repeat it....* R: ***Two birds with one stone****....*	What makes a solution effective is not that it involves too many calculations, but that it is practical and reminds of previous learning.	An effective mathematical solution must be functional.
Dialogue section-2 Teacher: *Well, in which solution is all the money calculated?* Class (loudly): *In the first solution.* Teacher: *Is it important to calculate all the money?* I: *I think it varies from question to question.* Teacher: *What do you mean?* I: *We do not need to calculate all the money as the problem is asked for profit, so the first solution is unnecessary, waste of time, I think the second will be.* N: *Not all money was not asked, there is no need for the first solution because it asks for the profit.* O: *...all money does not provide us anything, it only makes a confusing,* ***the simple is better****....* Teacher (nods): *Hmm….*	Simple solutions are not a waste of time, so they are more effective.	

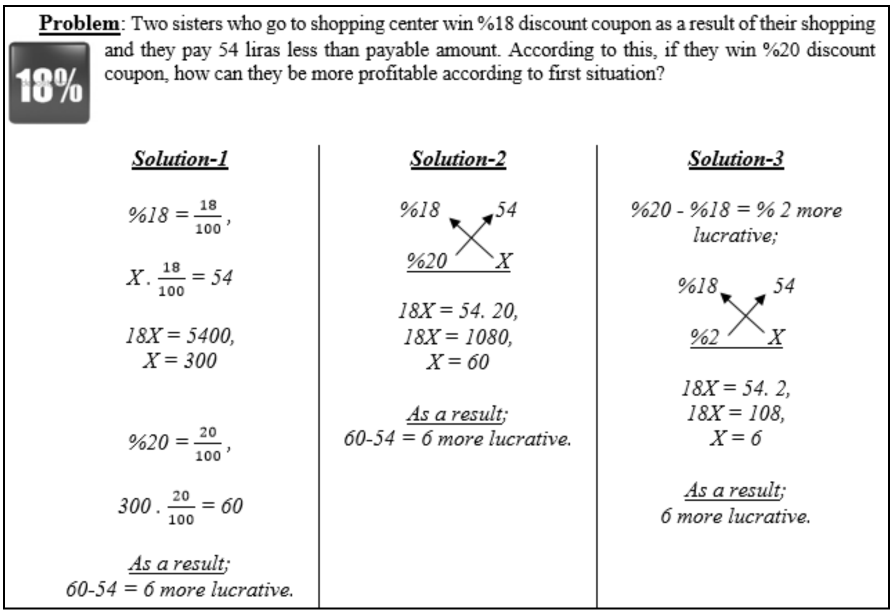

Figure 1 An activity developed to calculate the percentage of money

When dialogue section-1 was examined, F stated that he preferred the second solution because there were too many calculations in the first solution. However, other learners did not consider F's preference as valid. It should not be overlooked that the teacher asking the question sentence negatively had influenced the formation of this reaction. The teacher's negative use of the question sentence could be considered as a discourse that encouraged learners to present new reasons. Besides, this action, which was frequently exhibited in the sessions, could be regarded as an attempt to ensure the continuity of classroom discussions. If we go back to dialogue section-1, when the reason for F was not accepted, the discussion of the effectiveness of a solution in the classroom began. The question, "What do you mean by practical?" prompted F to provide a new justification. F stated that he both used what he had learned about proportion and solved the problem more simply. This discourse shows that while learners choose a solution, it was important for it to be useful. R summed up this situation by saying "*Two birds with one stone*." The Turkish meaning of this idiom is to get many useful results with one action. Therefore, learners accept functional qualities as a common understanding in a classroom where mathematical solutions are discussed. In dialogue section-2, which was the continuation of the discussion, the learners stated that they both saved time by avoiding excessive calculations and tended to stick to simple solutions that minimised the possibility of making mistakes. In this section, arguments that provide functionality in preferred solutions, time and simplicity came to the fore. Besides, the "*Hmm…*" response given by the teacher showed that the arguments presented were approved by the authority in the classroom.

Some of the learners cared about time, while others cared about simplicity. However, functionality was a common reason why learners preferred one or the other. In other words, even if the reasons for being functional in discussions about the legitimacy of problem solutions changed, the actions and discourses showing the existence of this norm were shared. This situation revealed the nature of the functionality.

Another aim of our study was to examine the effects of negotiation of sociomathematical norms on learning. In the first interviews, the learners stated that they got used to the practice of problem solving using known methods (see Table 2).

Table 2 Learners' statements about problem solutions in the interviews

Learner 1: *For my solutions, I usually use the methods we have learned before....*
Learner 2: *First I list the necessary steps one by one, then I make it ...?*
Learner 3: *We learned the methods we will use in solutions in previous lessons. Our teacher taught us everything about this subject.…*

Despite their discourses in the first interviews, it was observed that in the sessions where the functionality norm was negotiated, learners tended to create creative and effective answers instead of applying certain procedures. The following excerpt from E's report about his solution supports this finding: "*... instead of giving numerical values to equal angles, we used equations. We gave values of x and a to equal angles and we solved the problem accordingly.*"

The statement *"... instead of giving numerical values to equal angles, we used equations*" indicates that learners prefer to go beyond the procedure of solving by giving numerical values. In other words, the learners tried to bring a more creative approach to problem-solving. E's statements in the interview that "*listing the process steps required for a solution is time-consuming and unnecessary*" and that "*they have reached a consensus on this issue within the group*" indicate that the learning opportunity for creative and effective solutions was adopted.

Another sociomathematical norm determined in the study was that *an ideal solution should be inclusive*. This norm, which we can call inclusiveness, expresses the common understanding of an ideal mathematical solution to cover different situations. The first narrative in which this norm was determined is shown in the dialogue section in the activity in Figure 2, with which we aimed to discover that the multiplication of inversely proportional multiplicities is constant (see Table 3). In this activity, learners were expected to explain the best solution representing the weekly data table showing the number of workers and time.

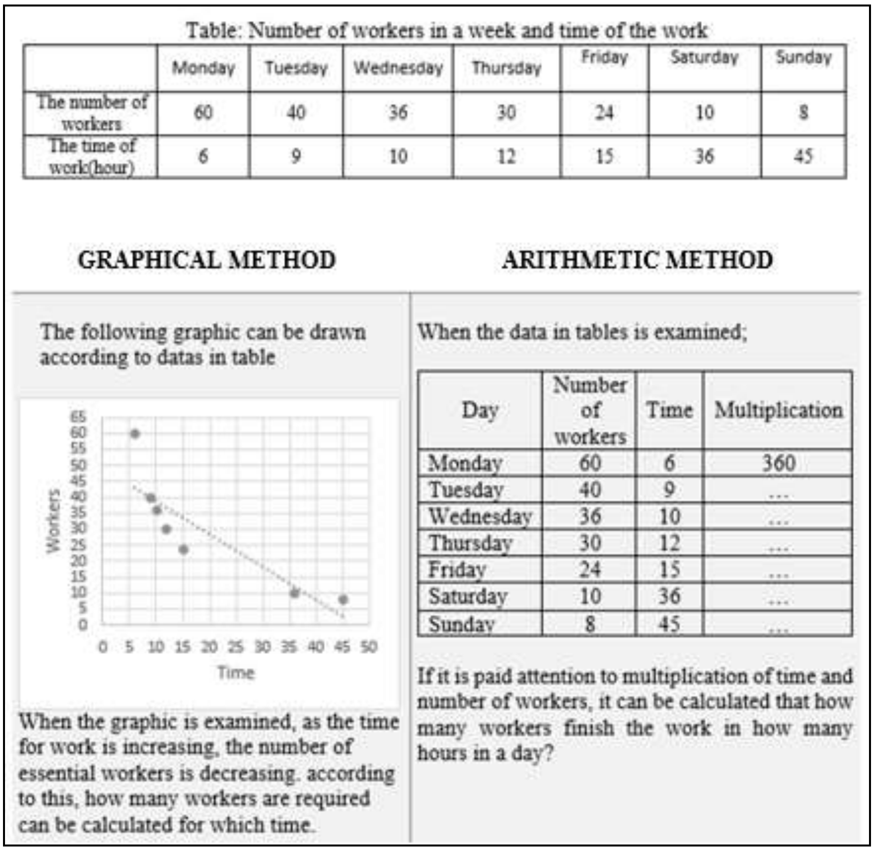

Table: Number of workers in a week and time of the work

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
The number of workers	60	40	36	30	24	10	8
The time of work(hour)	6	9	10	12	15	36	45

GRAPHICAL METHOD

The following graphic can be drawn according to datas in table

When the graphic is examined, as the time for work is increasing, the number of essential workers is decreasing. according to this, how many workers are required can be calculated for which time.

ARITHMETIC METHOD

When the data in tables is examined;

Day	Number of workers	Time	Multiplication
Monday	60	6	360
Tuesday	40	9	...
Wednesday	36	10	...
Thursday	30	12	...
Friday	24	15	...
Saturday	10	36	...
Sunday	8	45	...

If it is paid attention to multiplication of time and number of workers, it can be calculated that how many workers finish the work in how many hours in a day?

Figure 2 An activity on inversely proportional multiplicities

Table 3 Example of endorsed narratives about the inclusiveness

Example of endorsed narratives (Class members discuss solutions about inversely proportional multiplicities)	Identified features	Inferred norm
Dialogue section-3 Teacher: *Which solution did you choose?* İ: *I choose the arithmetic one because we cannot determine the numbers in the graphical method.* Teacher: *(Summarises the explanation of İ and says that his choice is arithmetic)....* R: *The arithmetic makes more sense in my opinion because I had difficulty understanding the graphic. I believe arithmetic is more practical and logical.* S: *I also choose the arithmetic one.* Teacher: *Hı hı....* S: *Because* ***we cannot always draw graphs.*** Teacher: *What do you mean?* S: *So* ***we cannot use graphs for every problem****.*	Each solution should be questioned for situations other than the current situation. For example, graphical solutions may not be suitable for every problem.	An ideal mathematical solution should be inclusive.

When dialogue section-3 was examined, participant İ stated that he preferred the arithmetic solution because he could not determine the coordinates of the points given in the graphical solution. Therefore, the teacher summarised İ's statements in a way that everyone could understand and said that his choice was arithmetic. Repeating a learner's discourse can be regarded as an attempt to get other learners to evaluate the explanations presented. The teacher performed this action many times in the lesson sessions. Returning to the dialogue section, R included in the discussion that the uncertainty in the graphical solution created difficulties for him. However, the teacher's discourse "*Hı hı…*" created a feeling that the authority in the classroom was not satisfied. S pointed out that a graphical solution was not always possible for situations outside of the current situation. This discourse triggered the learners to provide verification in problem solutions for situations other than the current situation. Thus, it contributed to the negotiation of inclusiveness.

Another example of an endorsed narrative in which the norm of inclusiveness was determined and negotiated is shown in the dialogue section of the activity in Figure 3, which involved sharing a land proportionally to the age of two brothers (see Table 4).

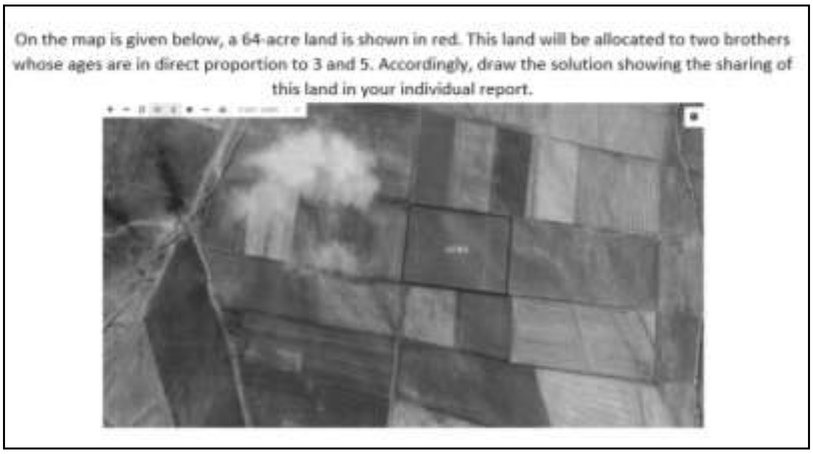

Figure 3 An activity related to ratio

Table 4 Another example of endorsed narratives about the inclusiveness

Example of endorsed narrative (Class members discuss the solutions in Figure 4 about the sharing of land)	Identified features	Inferred norm
Dialogue section-4 Teacher: *So, what can we say in terms of seeing the proportional ages (by showing the first drawing). What does the drawing here mean by being directly proportional to 3 and 5?* E: *We can determine who takes which place, but the second is not always possible.* Teacher: *What do you mean?* E: *If the siblings' total ages were 9 instead of 8, we would not be able to use the 64-acre area in the second drawing ...* ***Because 64 decares cannot be divided by 9.*** Class (mixed voices): *Exactly….*	An ideal solution should be verified for different situations. For example, 64 decares cannot be divided by 9.	An ideal mathematical solution should be inclusive.

When dialogue section-4 was examined, the teacher included proportional ages in the discussion. However, E's statement of "*not always possible*" for the second drawing shows that this drawing was questioned. The teacher's request for E to explain his comment contributed to the negotiation process. E and his friends in the group confirmed by trying different conditions for the second drawing one by one (see Figure 4).

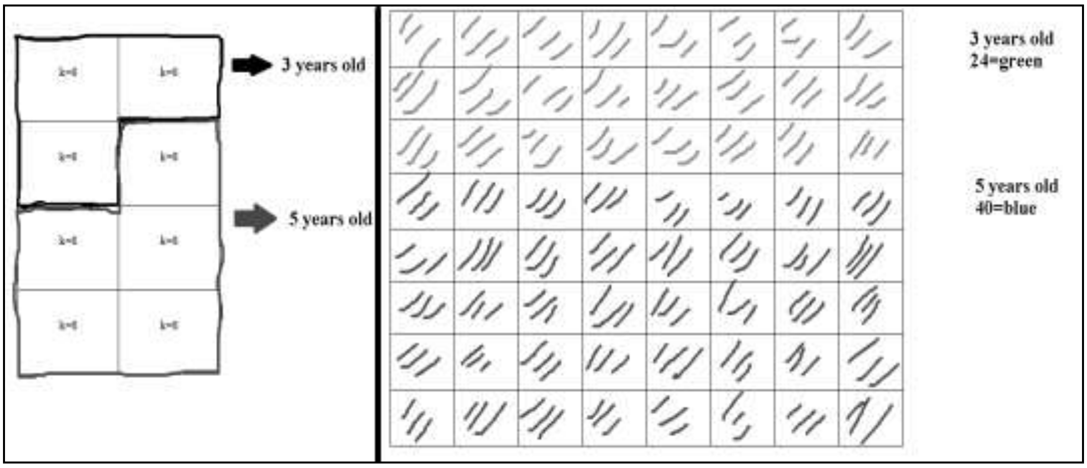

Figure 4 Examples of drawings showing learners' solutions

The discourse that E used during the negotiation process "*...if the total ages were 9 instead of 8, then we would not be able to use a region of 64 in the second drawing ... Because it is not completely divisible ...*" was an indication of the common understanding that an ideal solution covers all situations.

Another understanding that learners emphasised problem solutions in the interviews made in the first weeks was that the solution consisted of a single number or statement (see Table 5).

Table 5 Learners' statements about problem solutions in the interviews

Learner 1: *Every problem has only one solution....*
Learner 2: *While solving problems in the classroom, I just try to find the right solution....*
Learner 3: *The correct solution for a problem is obvious, other solutions are wrong....*

Despite the above statements in the interviews, the learners stated that the negotiation of the norm of inclusiveness allowed them to test different solutions. The following excerpt from O's individual report of the drawings in his group supports this finding: "*... as a group, we tried to validate all solutions and concluded that the most ideal solution was the most inclusive*."

The above discourse shows that the common understandings that defined the norm of inclusiveness contributed to the development of learners' verification skills. The learners' testing of solutions indicated acceptance of learning opportunities about verification.

Another sociomathematical norm determined in the study was *mathematical solutions should connect with previous experiences*. This norm shows the connections between mathematics and life. Experiencing mathematical solutions contributed to both their preference and legitimation.

Examples of endorsed narratives (see Table 6) in which this norm, which we can call connectivity, is determined are shown in the dialogues of the activity in Figure 5, in which the parallelism of lines was questioned. In this activity, learners were expected to explain in which solutions the bold lines were parallel to each other.

Table 6 Examples of endorsed narratives about the connectivity

Example of endorsed narratives (Class members discuss solutions about parallel lines)	Identified features	Inferred norm
Dialogue section-5 Teacher: *Which of the four situations given are parallel?* İ: *There is no parallelism in the first drawing.* Teacher: *Because….* İ: *the angles must be equal, the angles in the first drawing are not equal so they are not parallel.* Teacher: *Well, before starting this activity, I asked 'How do you know that the two lines are parallel?' What was your explanation for the question?* İ: *I said, 'If the lines do not intersect along with their extensions, we will understand that they are parallel.'* Teacher: ***In your previous explanation**, you said that the lines should not intersect to understand whether the lines are parallel, but in this activity, you evaluated whether the lines are parallel or not by looking at their angles. Why did you make a change like this?* N: *Because we cannot extend the lines here forever….* Teacher: *Which ones are parallel?* M: *Third and fourth....* Teacher: *Are there equal angles in them?* R: *I think there is….* Teacher: *How did you understand?* R: ***We learned in previous lessons....*** Teacher: *Come show us.* R: *(comes to the board and forms equal angles.)*	The mathematics used in the solutions should be compatible with previous learnings.	Mathematical solutions should connect with previous experiences.
Dialogue section-6 Teacher: *Which one did you prefer?* M: *I chose the third one, teacher because **I haven't encountered the second one**.* Teacher: *Is it the second one parallel?* E (scratch his head): *I guess not.* Teacher: *Do you understand how your friend determines equal angles?* E: *Yes, but since **I did not encounter** the second one, I did not use it and I am not prone to the second one....* Y: *I also chose the third one.* Teacher: *Why?* A: ***We always use the third one in class....***	Previous experiences positively affect the perspective on mathematical solutions and make them preferred.	

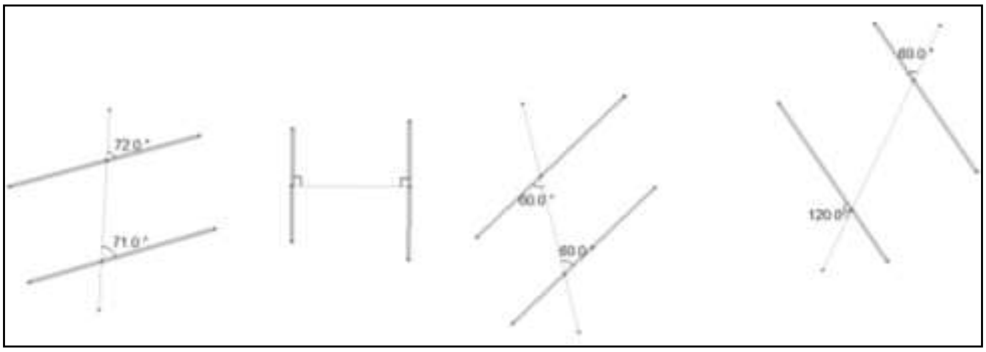

Figure 5 An activity in which the parallelism of two lines is questioned

When dialogue section-5 was examined, the teacher formed an unfinished sentence starting with "*Because....*" This discourse shows that the teacher expected İ to justify his explanation. Therefore, İ stated that in some solutions there were no equal angles and he related the parallel lines to find the equal angles. The teacher, on the other hand, tried to establish a connection between the discourses by questioning the learners' thoughts before starting the activity. This action contributed to the negotiation of the connectivity. Thereupon, R said that although the equal angles were not given directly in the solutions, they learned to find the equal angles in their previous lessons. Thus, it contributed to the enactment of the norm of connectivity in the classroom. In dialogue section-6, which was the continuation of the dialogue, the learners stated that they did not prefer the solutions they had not encountered before. The teacher continued to inquire about other solutions. E stated that he preferred the third solution because he did not solve any questions with the second solution. The teacher contributed to the negotiation of the norms of connectivity by asking how equal angles were determined. E's emphasis on the discourse of "*I am not inclined*" has made this norm clear in the classroom. Similarly, Y based his preference on his experiences in the classroom. These dialogues show that in the choice of mathematical solutions, actions and discourses of being connected were accepted as a common understanding in the classroom.

During the sessions in which the norm of connectivity was discussed, learners stated that they had the opportunity to question the similarities or differences of mathematical solutions. This situation was clearly stated in Y's report: "*... discussing the connections of the solutions in the group facilitated the math we did, in my opinion, the second and third solutions were the same and the others were different.*"

The above discourse shows that determining the connections of the solutions positively affected learners' perspectives on mathematics. Indeed, recent interviews with learners supported this finding (see Table 7).

Table 7 Learners' statements about problem solutions in the interviews

Learner 1: *I liked finding the differences in the solutions....*
Learner 2: *Finding the connections between the solutions was easy for me....*
Learner 3: *Every solution started to seem like each other, it means that many things in mathematics are related....*

These data show that the negotiations of the norm of connectivity were effective in creating learning opportunities for learners to identify their similarities or differences by comparing the solutions.

Discussion

The results of our study broaden the work of authors such as Clarke, Goos and Morony (2007), Elliott et al. (2009), Sekiguchi (2005), and Toscano Sánchez and García (2019) who are interested in defining and analysing sociomathematical norms perceived in the classroom. In this context, it is understood that the perceived sociomathematical norms in a secondary school class where problem solutions are discussed are related to why and how the solutions are accepted rather than how the problems are solved or which solution methods are used. The acceptable explanations about the legitimacy of solutions were perceived as normative in the classroom. This result is important in terms of forming the framework of the context in which SNPS are enacted.

Considering that the participants were secondary school learners, another contribution of our study is that the researcher-teacher did not have an extra purpose for the emergence of predetermined or applied norms in the classroom. This mindset of the teacher increased the interaction among the learners and made them try to understand one another while explaining their preferences. Therefore, direct access to learners' expressions about their mathematical preferences was obtained.

When the sociomathematical norms determined in the study are evaluated, the norm that *an effective solution should be functional* is important in terms of putting mathematics in a pragmatic framework. Although there are no predetermined criteria for what constitutes an effective mathematical solution in the classroom (Yackel & Cobb, 1996), the learners found the less time-consuming, simple, and easy solutions acceptable. Besides, the various everyday language structures used by the learners in the negotiation process (two birds with one stone, etc.) helped them express the effectiveness of the solutions. Levenson, Tirosh and Tsamir (2009) state that secondary school learners found mathematics-based (using the official language) rather than practice-based (using everyday language) explanations acceptable. Kiwanuka et al. (2015) state that classroom characteristics explain an important part of the school-level variance. They also reported that class-level characteristics have a greater impact on learners' behaviour and perceptions. However, with their silent message, major social constitutions such as the education system have a significant impact on the microculture in the classroom (Planas & Gorgorió, 2004). Therefore, the linguistic structures in education systems shape the norms perceived by learners in classrooms. In this context the norm of being functional can be regarded as important to express the connection that learners establish between language and mathematics. Moreover, norms determined in our study may be considered as a general framework for secondary school mathematics classes in Turkey that care about the everyday language structures.

The norm of inclusiveness is a combination of common preferences for generalisation. In other words, the factor that determines the legitimacy of a solution is the verification in different situations. Güven and Dede (2017) report that pre-service teachers put forward norms that a few verifications are not sufficient for generalisation. The norm of inclusiveness, on the other hand, is not relevant to the scarcity or abundance of numerical examples. On the contrary, they are common understandings to question whether there are overlaps with different situations. This result may be due to the differences in grade levels. In this context, studies on negotiating the norm of inclusiveness at different grade levels contribute to the field.

The sociomathematical norm of *mathematical solutions should connect with previous experiences* shows that previous experiences are also questioned when choosing solutions. Considering that the connectivity overlaps with the principles of familiarity and similarity (White & Mitchelmore 2010), which are critical for abstraction, this norm also reflects the nature of mathematics written by deductive method. In other words, deductive mathematical arguments are preferred for the acceptance of solutions in the classroom. Therefore, if we take the mathematical preferences of the learners as reference, it may be a good start to establish the understanding of problem-solving in secondary mathematics classes. This is the first step in the transition from arithmetic to algebraic reasoning by negotiating the connectivity.

Regarding the enactment of the norms perceived in the classroom, the researcher-teacher did not give direct guidance on what was acceptable and only encouraged the learners to present their explanations with their reasons. This situation both limited the effect of the teacher on the produced discourses (Tatsis, 2007) and shaped the interaction between teacher and learner. Dixon, Andreasen and Stephan (2009) state that teachers should be pioneers in establishing norms in the classroom, while Tsai (2007) claims that teachers should regulate the discourse in the classroom. The results show that the teacher plays a role in developing norms enacted in the classroom rather than establishing the norms. For example, although the teacher did not have information about the learners' previous thoughts about being inclusive, he tried to understand that a solution was inclusive in the sense they perceived. This situation enabled the norms perceived by learners to be established and enacted independently from the authority. Besides, the teacher applied various strategies such as making negative statements, repeating a learner's discourse, and using unfinished sentences to improve the norms enacted in the classroom. The use of sentences not ending with a negative question ensured the continuity of the discussions while repeating a learner's utterance was considered an attempt to verify the presented explanations.

Each class, from the most traditional to the most innovative, had its own norms. What makes a mathematics classroom different from other classes is the nature of the norms, not the presence or absence, or number thereof (Gülburnu, 2019). Therefore, it is more correct to evaluate the determined norms by considering their effects on the interaction and learning in the classroom. In this context, especially when the interaction structure among the learners is examined, they have displayed actions such as resisting or challenging the explanations during the negotiation process. Considering that the meta-rules enforced by the authority (Ben-Yehuda, Lavy, Linchevski & Sfard, 2005) indicated how the actors should be according to them, negotiating the norms perceived by the learners in the classroom instead of the norms approved by the teacher may have contributed more to mathematical autonomy. At the same time, it was observed that the negotiation of the sociomathematical norms determined in the study was effective in creating learning opportunities for creativity, verification, similarity, and determination of differences. Besides, they were able to identify similarities or differences by comparing the discourses of solutions applying their minds during classroom discussions. During the negotiation process of norms, while learners structured their learning, they also supported the formation of collective learning by contributing to the argumentation in the classroom. As a result, negotiating the norms perceived by the learners was effective in gaining learning opportunities (Partanen & Kaasila, 2015) that facilitated learning mathematics.

Conclusion

The results of the research have given both researchers and field educators a perspective on what interactions take place in the secondary school mathematics course, where problem solutions are questioned and discussed. This was possible because it provided direct access to sociomathematical norms related to the mathematical preferences of secondary school learners. More research is needed to determine whether the identified sociomathematical norms are transferred to learners' future interactions. Considering that this study was limited to the education system in Turkey, conducting the same study in different cultures is important to compare the perceived norms in the classroom in other settings.

Defining the sociomathematical norms that learners perceive and the conditions in which they emerge will be useful for teachers who want to provide a similar environment in their classrooms. Considering the strategies that the researcher-teacher used, it is important for teachers to review their roles in the problem-solving process. Teachers should adopt an attitude that allows them to develop learners' problem-solving skills rather than being normative or transmitting.

The main argument of a reformist mathematics course is not only to produce solutions to problems, but also to question and discuss the solutions obtained according to the problem situation. Thus, learners can present their actions and discourses about mathematics activities by integrating them with their preferences in the classroom discussion environment. They can contribute to the formation of norms and learning opportunities that help structure the microculture of that classroom. Therefore, we need to continue research on learners in order to present a general model of the effects of sociomathematical norms on mathematics learning and teaching.

Finally, we would like to state that this study was not concerned with generalising in any situation and contributed to the results of other theoretical approaches.

Authors' Contributions

MG wrote the manuscript and collected data, RG conducted all statistical analyses, all authors reviewed the final manuscript.

Notes

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