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Mathematics, Devan, and project work

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I analyse the efforts of one learner, Devan, and a student teacher, Sumaiya Desai, in whose class he participated, as she attempted to realise what may be referred to as a social, cultural, political approach to the school mathematics curriculum through the practice of project work. The focus is specifically on the actions and reflections of Devan as he participated in the project work experience and overturned his casting as a "failing" mathematics learner in a Grade 6 mathematics classroom. I theorise the practice of project work through the data related to Devan and the conceptual tools of project work: problem-orientation; participant-directed; inter-disciplinarity; and exemplarity, and use this framework to organise the structure of this paper. Through this conceptual framing, the challenges and possibilities that such classrooms pose, for learners like Devan but also for the theoretical ideas and associated practice, are made visible and I discuss the potentiality that remained unexplored.

Project work in a social, cultural, political approach

This paper is drawn from a larger study which explored what happens when student teachers attempt to realise what may be referred to as a social, cultural, political approach to the mathematics curriculum (Vithal, 1997; 2003). Bringing together at least four different strands that have seen considerable growth and development in the last three decades, this approach includes ethnomathematics (e.g. D'Ambrosio, 1985; 1990; Gerdes, 1996; Powell & Frankenstein, 1997); a political or critical mathematics education (e.g. Mellin-Olsen 1987; Skovsmose, 1994); concerns about race, gender, class (e.g. Shan & Bailey, 1991; Secada *et al*., 1995; Rogers & Kaiser, 1995; Parker *et al*., 1996; Keitel, 1998; Dowling 1998; Walkerdine, 1998), as well as South Africa's own legacy of People's Mathematics for People's Power, a part of the People's Education movement during the apartheid era (e.g. Julie, 1993; Breen, 1986; Adler, 1988). These fields of study and of practice foreground and integrate progressive pedagogical ideas such as equity, social justice, democracy, context and diversity in the teaching and learning of mathematics and have become part of a broad international perspective (e.g. Atweh *et al*., 2001).

A group of student teachers taking mathematics education as their major were introduced to this social, cultural, political curriculum approach and its different associated practices during their pre-service programme and then invited to try them out in the time set aside for practicing teaching, to explore and understand what such ideas could mean in the real life of a mathematics classroom. Although many different kinds of practices exemplify and illustrate this approach, the student teachers chose project work in their attempt to interpret and recontextualise this approach in their classrooms (Vithal *et al*., 1997). In this paper I present data and an analysis of a description of experimentation with this approach by one student teacher, Sumaiya Desai, with special reference to one learner, Devan (not his real name), to point to the tensions and potentials of project work and its associated concepts.

Project work is a well-established educational practice and is interpreted and practiced in many different ways both as a pedagogical practice and an assessment practice. In the new national mathematics curricula reforms in South Africa it appears to be advocated mainly as part of assessment (see e.g. Dept of Education, 2002a). This seems to be confirmed in the Third International Mathematics and Science Study (TIMSS 1999) where South Africa is one of only three countries (out of 38) reported as giving "working on mathematics projects" a major emphasis in the intended official curriculum (Mullis *et al.*, 2000). In the same study, mathematics teachers of approximately 40% of students surveyed in South Africa, the third highest of 38 countries, stated that they sometimes or always assigned mathematics homework based on small investigations and individual or small group projects compared to the international average of 18%. The rise in project work, previously virtually unheard of in South African mathematics classrooms, may be attributed in large measure to the introduction of continuous assessment in the mid 1990s. However in this conception, project work typically involves a small self study on a topic usually chosen by a teacher, and done outside class time for assessment purposes. Even in this rather limited notion of project work and despite it appearing to be a relatively widely implemented assessment practice, virtually no research exists to examine its use and effects. In the study reported in this paper, a much broader conception of project work was investigated to examine both its practice as a pedagogy and the theoretical assumptions underpinning it.

Projects or project work form part of a "progressive" approach to mathematics education that advocates more "open-ended", "problem-centred" activities in which learners are given greater independence in their learning, in contexts relevant to them. Some studies have shown the effectiveness of these less procedural and rule-bound strategies for providing a more meaningful mathematics education (e.g. Boaler, 1997; Cobb *et al*., 1992). However, within the four areas mentioned earlier such approaches have also been used as a means for engaging the social, cultural, political dimensions of mathematics education to provide not only a meaningful mathematics education but also a socially critical one (e.g. Gutstein, 2003; Frankenstein, 1997). But here too it has taken different forms. For example, Bishop (1988) refers to project work for addressing the "societal component" within a cultural approach to the mathematics curriculum, as "a piece of personal research" based on topics that consider society in the past, present and future. One problem in the elaboration of these progressive approaches is that while project work is advocated and the practice itself may be explained or described, it is often under-theorised.

An exception to this is its practice and related research within some European countries, especially Scandinavia, where project work has a long history and substantial theoretical development. In countries such as Denmark it has been institutionalised for several decades. For example, at Aalborg and Roskilde Universities both the educational programmes and the buildings are structured to support the pedagogy and practice of project work (e.g. Olesen & Jensen, 1999). Here project work has been developed and used as a means for organising curricula to integrate broader socio-political concerns and to address the question of "subject matter abundance" (Olesen & Jensen, 1999) that includes the broad range of disciplines and professions, even the teaching and learning of university mathematics (Vithal *et al*., 1995; Niss, 2001). Project work has also been implemented and researched with respect to school mathematics both at primary (Skovsmose, 1994; Nielsen & Simoni, 1994; Nielsen *et al*., 1999) and secondary levels (Christiansen, 1996) and these explorations integrate a critical perspective.

Across these educational settings, project-based studies are theorised with reference to a particular landscape that involves key concepts such problem orientation, inter-disciplinarity, participant directed learning, and the exemplarity principle. Typically this conception of project work involves learners working in groups with the teacher

as supervisor or guide engaging in a research-like process. Historically, "these concepts were developed in contrast to uncritical positivist research, and the traditional teaching approach in Danish Universities, characterised by the almost feudal power of the professors" (Vithal *et al*., 1995:210) as a serious response toward the democratisation of education. Collectively they provide a conceptual framework for analysing mathematics education from a critical perspective, which may be described in terms of concerns such as: preparation of learners for active political life; mathematics as a tool for identifying and analysing critical features of society; mathematics becoming implicated in producing and reproducing inequalities in society, and the communications among teachers and learners as reflections of relations of power (Skovsmose & Nielsen, 1996).

Elsewhere I have argued that it is these concerns and concepts of project work that serve to pull together the four different strands mentioned above of a social, cultural, political approach to a mathematics curriculum (Vithal, 2003). Given the imperatives of post-apartheid society and the new curriculum reforms in South Africa, this diverse literature constituting a social, cultural, political approach as well as exemplars of project work were integrated into the student teachers' mathematics education major course and preparation as beginning teachers of mathematics in the third year of the four-year BPaed (Primary) degree to illustrate and encourage more progressive pedagogies (Vithal, 1997; 2003). The research was designed to follow them into school the following final year of their teacher education programme where they implemented and tried out a ranged of different projects during the period of teaching practice (for project descriptions see Vithal *et al*., 1997; Paras, 1998). In this paper it is the efforts of one of these student teachers, Sumaiya Desai, who attempted to realise this social, cultural, political approach in a mathematics classroom through project work, and the experiences of one of her "failing" learners of mathematics, Devan, that is analysed and theorised with reference to the conceptual tools of project work: problem orientation, inter-disciplinarity, participant directed learning, and the exemplarity principle. These key concepts, which form the basis for the structure of this paper, serve on the one hand as a framework to interrogate the practice of project work but on the other hand also for explaining and critiquing the concepts themselves, thereby to assess the strengths and weaknesses of such a pedagogy in a South African mathematics classroom.

Methodology and context

Inspired by critical research approaches (e.g. Carspecken & Apple, 1992; Kincheloe & McLaren, 1999) and to maintain a resonance between the theoretical mathematics educational and empirical approaches (Skovsmose & Borba, 2000; Vithal, 2003), a wide variety of data were produced with the involvement of the student teacher, Sumaiya Desai as co-teacher/researcher, and myself as teacher educator/researcher. This included videoed classroom observations, interviews, journals, and the written work of the many participants — the class teacher, all the learners in this class, the student teacher and the researcher — over a six week period. These data were organised into what may be referred to as a detailed crucial case description (Vithal, 2003), a description that allows critique of the practice, the theory that underpins and informs the practice, and the research processes and analysis set up to investigate the link between the theory and the practice. (For this approximately 100 page full description of practice see Vithal, 2003). For the analysis presented in this paper I selected from this description the data related to one learner, Devan, to understand and explain his experiences of this project work pedagogy as he interacts with the teacher figures in the classroom and other learners in his group and in the class as a whole.

The context within which Sumaiya attempted to realise this particular approach was a Grade 6 class of thirty "Indian" and "African" students in a still predominantly "Indian" school. The learners in the classroom were organised in five groups of six. Devan's group was the only race and gender mixed group and he was the group leader. When Sumaiya introduced the different ideas for a project, this group chose a project on investigating "How much money is spent on my education". (For description of all projects and groups see Vithal, 2003). Devan was an articulate and deeply reflective student as he engaged the project work activities and participated in this mathematics class. Yet by all accounts, including by his own admission, he was a "failing" mathematics student. By focusing on the data related to this one learner, the following analysis weaves together a narrative about Devan that speaks back to both the theory underpinning a social, cultural, political approach and the practice of project work in terms of the stated key concepts.

Problem orientation

A key foundational principle in this conception of project work is that a curriculum need not be organised according to the structures of a discipline but rather some problem, critical question or theme related to the context or societal conditions and deemed important by the students (Vithal *et al*., 1995; Bastos & Costa, 2000). However narrow or wide context is defined, student interest in the problem is therefore a central concept because learning, as part of critical activity, cannot be forced (Skovsmose & Nielsen, 1996). Learning as action implies that the student owns the reason for learning because acting involves intentions. Intentions relate to both students' backgrounds, and foregrounds — the possibilities a social situation reveals as potentialities for the future (Skovsmose, 1994). For intentionality to exist, the freedom to choose what is to be learnt must be given space even if within the constraints of the structures of schools and classrooms. To provide learners with equitable opportunities for learning, the choice of topic, in this critical perspective, needs to meet several conditions: (1) choosing topics that are known to learners and that can be discussed in non-mathematical terms or natural language; (2) learners need to be able to enter the project problem at different levels irrespective of their ability; (3) the topic must have a value of its own and not merely be a context for demonstrating and developing mathematical content; and (4) working in the project must be generative of mathematical concepts and ideas; about how and where mathematics is used as well as develop mathematical skills (Skovsmose, 1994). Project work in a political mathematics education, for Mellin-Olsen (1987), is the activity of making available to students the thinking-tools of the curriculum that will help them to deal with a project problem in which they have a vested interest.

Sumaiya introduced the learners to project work by listing a number of different project problem ideas, for example, "time spent after school" and "developing a newsletter", which were then all brainstormed by the learners in their groups. Some problems listed such as a "consumer profile" and "traffic count (jam) at the school" were not chosen because they were deemed to be problems of concern to adults. When freedoms are made available to learners they are immediately taken up and acted on. At least one entirely new project problem was suggested by a group of boys about the provision of sport facilities. It was added to the list as a "sports survey. Devan's group comprising Harry, Vikesh, Loresha, Bernard and Mohan, together with one other group chose the project "how much money is spent on my education" from those projects listed by Sumaiya. If student interest and intentionality in learning is to be given any meaning then project work is not simply about solving a problem but also about clarifying and owning the problem, hence each of the problems were "subverted" to address issues of concern to the learners themselves and directly related to matters in school. For instance, the problem of "time spent after school" was re-interpreted by the group of girls who chose this project to lodge a complaint about too much homework; and "money spent on my education" became an investigation into the high cost of school fees and an opportunity to challenge the use of these funds by the school. In this, neither the problem nor the learning gained can be controlled. As Devan's group produced the data for dealing with the project problem, they were learning to categorise and quantify the costs. But at the same time aspects of each others' lives were revealed

and opened to scrutiny wherever they might be on the socio-economic class continuum. At the first presentation of their project ideas Devan explained:

> *We had already written out our school budget. Also getting to know our parents' salary. We are doing monthly work out and trying to find out whether our school fees should be higher or brought down. Some people have very little money to pay for food.*

One of the deeply contested issues in this project was a roof shelter "structure" that the school had built. This debate divided the group. For Mohan, one of the members in Devan's group, this was a strong issue. He challenged the teacher during the final presentations, after all the graphs drawn by each member of the group reflecting how much their parents paid for their schooling costs were presented:

> *I don't think the structure is very important. So much of money is spent on this when our toilet facilities need to be improved; need money for computers. We shouldn't worry how our school looks, rather on our education. We thought ma'am, when they said they going to build a structure, that it was going to be fully closed, it's going to be like a hall.*

But this was not a consensus view of the group and a public disagreement ensued as Devan countered with another view:

> *That was like Mohan's question, he agreed to ma'am, because Harry, Vikesh ... we thought that school fund is quite ok because the things that we get.*

In this the students express a Mündigkeit (Skovsmose, 1994) an important critical and democratic competence of demonstrating the capacity to speak for oneself. Even though they may not succeed in changing the school fees they get to raise their voice and in this, also come to see the limits of their interest and action.

But the theoretical assertion that if learning is organised by prioritising student interest, for example by giving them choice in a project problem, they will choose that which is important or critical even to themselves was not fully borne out, and does not represent the full description of students' reasons for acting and learning. Devan's group took a vote when choosing the problem and hence the decision was in a sense forced on some members. Loresha, the only girl in the group, wrote in her diary:

> *I wanted to do the sports project but we had to vote and we had to do the education project.*

Other factors also featured as can be seen when Devan made explicit his preference and reasons:

> *I thought first that this project will be easy. But when it came to the difficulties, I wanted to do the project that Vasentha and them wanted to do because it involved much more interesting things then us — something like a newsletter. But when our group thought of a newsletter, they said it will be definitely more hard because you have to go through more work.*

Theoretically what is argued is that if students are interested and able to control and shape that interest, they will learn. But what is observed here is that while choice of the project problem is mediated through student interest, that interest must be constituted in a broader sense to include: interest in choosing the shortest or easiest route. How the project comes to be realised is shaped by how co-operating and conflicting interests and intentions get played out as participants jointly direct their participation in the project. Choice and reasons for learning must in a sense include choice or reasons for not learning. This may be characterised as a demonstration of underground intentions on the part of learners (Alro & Skovsmose, 2002), a kind of resistance since learners may in fact choose to disengage from the official classroom activity to express intentions other than those that the teacher expects or is even able to discern — e.g. to complete the task quickly to join friends to play; to work only when the teacher is watching; or to avoid being noticed. Hence student interest in a project problem must include both its positive and negative connotations and feature in how learner participation is explained.

Participant directed learning

The idea that project work is participant directed incorporates and highlights the joint actions and responsibilities of teachers and learners in the project work experience. That is, teachers and learners co-construct the project even if they have different skills and knowledge in relation to the project as well as different vested interests and intentions in the processes and outcomes of the project. In being participant directed, this notion of project work subsumes the concept of learner-centred in that it includes the teacher as participant, and goes beyond it to open the possibility for some kind of democratic life itself to be enacted in the mathematics classroom. Not only can project work teach about democracy, about voting and other democratic ideals, it can teach through democratic classroom life, "influencing how decisions are taken, how topics are discussed between teachers and students, etc." (Nielsen *et al.*, 1999:14), and creating opportunities to learn how knowledge and power are negotiated as the project is worked through. A direct critical "student-citizenship" may be developed in the micro-democratic life of a classroom as preparation for the macro-democratic life of society where many decisions, whether political, economic, even social are supported by mathematically based arguments. "Mathematics has become part of the language of decisions and policy-making. Therefore attitudes towards mathematics have a special influence on the interpretation of the 'language of decision making' " (Nielsen *et al.*, 1999:15). Learning to actively participate in this language is therefore central in a critical mathematics education and may be realised through project work.

The project work in Sumaiya's class was organised in groups and each of the groups was differently constituted in terms of race, gender, English language and mathematical competence, interest and other dimensions of difference. Though the groups were set up by the class teacher and learners were invited by Sumaiya to move, to a different group if desired, none of them did so. Devan was elected or nominated by the group as the leader and evaluated as a competent and democratic leader by the student teachers. He brought the chart paper to school and got each of his diverse group members to draw graphs indicating how much money was spent by parents on different items required for school. Except for Bernard, who arrived late to class, all the group members participated in the final presentation compared to other groups where some maginalising was observed. Despite being cast as failing and unable to do mathematics, Devan's group, according to the data, engaged the most mathematics as they deliberated about how to represent the information and which graphs to select. Moreover, he led the presentation making the clearest link between the graphs and the problem being investigated compared to any other group (see next section); and despite the difficulties and choices in drawing the graphs, he affirmed the efforts and participation of each member:

> *We related this to maths. [(softly) Mohan do you want to talk?] And we done it in hundreds: like hundred, two hundred, three hundred, four hundred. We enjoyed this graph quite a bit. Although, when we started, we didn't know what to do, we just wanted to put it aside. We honestly thought that ma'am would forget about this graph and we won't be able to do it. When we seriously came down to work, it was quite tough because everyone was suggesting their idea. Like Harry wanted to do the bar graph and some wanted to do the line graph. Then we all decided we'll do one graph, the line graph. The paper and things like that didn't cost too much money. We proud to say that we all pitched in this graph, everyone contributed a little. Like Harry's contribution is the art, you can see what marvellous art piece he's done there (points out the colourful bar graph done by Harry). And Mohan, our group really owes a thanks to him because he basically helped us with the drawings of the graphs; and Vikesh also helped us with the drawing. We proud to say that we all worked together.*

Yet in the final presentation Devan came under severe criticism from the other learners and the class teacher for dominating the presentation

as his exchange with Niren from another group investigating the same topic indicates:

Niren: *Why don't you let everybody else like talk? Because all they had to say was like lunch costs... and something like that but you were doing most of the talking. Like you were saying Harry came from KwaMashu or something, he could've said that.*

Devan: *Before we done this, I told them that everyone must have a chance to talk. When you all are asking questions, don't just ask it to me, ask it to everyone because it was a group effort. It wasn't an individual effort. So it's the problem that you all want to ask others the question, if not, then ask others the question.*

These criticisms did have the desired effect in that other members of the group increased their participation in the presentation to explain and defend their graphs. But for Sumaiya, who was very concerned about the sharpness of the exchange it was an opportunity to teach learners about how to criticise, an important quality in a critical citizenship.

The role of the teacher in project work as participant and supervisor is to guide students, giving suggestions and advice. But it also included explicit teaching with each of the smaller groups or the class as a whole as the project progresses. In fact it was the teachers who directed the learners into engaging relevant and appropriate mathematics. There was much debate in Devan's group about what kinds of graph to draw, bar graphs or "pizza" graphs.

Sumaiya: *Yesterday you said you are going to draw the pizza graph.*
Vikesh: *No, ma'am showed us another one [bar graphs].*
Devan: *This is easier.*
Sumaiya: *Why don't you draw both and then see which is easier?*
Devan: *No ma'am, we like it.*
Sumaiya: *What are you going to say?* (Looking at text) *here they have number of pupils, what are you going to write* (pointing to the y-axis)?
Vikesh: *How much it costs us to go to school.*
Sumaiya: *And here* (pointing on the x-axis)?
Vikesh: *Stationery, school fees ...*

Once the decision to draw bar graphs was made, they discussed if they should be horizontal or vertical bar graphs or line graphs. A kind of democratic concern in the process of negotiation in attempting to take all group members' points of view into account, even within the mathematics, was observed as I interviewed some groups while they were working.

Renuka: *Where are all the members of your group?*
Vikesh: *Three are absent* (Harry, Vikesh and Loresha due to impending teacher strike).
Devan: *We can't really go on to just writing the thing onto the basic chart ma'am, they not here, because what if they disagree.*
Renuka: *What does Harry think?*
Devan: *Harry what you think?*
Harry: *Ma'am I think this one is better ma'am because ...*
Devan: *But Harry it's the same graph.*
Vikesh: *Ok, let him say.*
Harry: *I think it's ...* (inaudible)
Vikesh: *But they the same.*
Renuka: *You think it's easier*?
Harry: *Yes ma'am.*
Renuka: *But you'll have to wait until the others come before you decide.*
Devan: *Ma'am, Bernard will be ok with the decision and* (inaudible) *will be ok.* (Vikesh nods in agreement)
Renuka: *But then what about Harry's view?*
Devan: *Mam we'll listen to Harry's view. We'll do what he wants because it's basically the same thing we doing.*

Besides explicit within-group teaching by Sumaiya, she also conducted a "traditional" whole class lesson on the drawing of bar graphs in the middle of the project. This was done some time after the learners had grappled with drawing graphs themselves by referring to relevant chapters from textbooks made available to them by her. Devan, who almost never volunteered to go to the chalkboard, as well as Mohan and Bernard from his group, participated in drawing graphs on the board during this lesson. What is observed in this is the teacher's active role as participant according to the obligations of the didactical contract of teaching and learning mathematics in the project work while simultaneously creating spaces for student interest and choices to manifest, and then acting on those. "The form of the teaching becomes open to discussion. The organisation of the educational process, which controls the activities, is no longer the responsibility of the teacher alone" (Nielsen *et al.*, 1999:18); it is jointly shared, even if unequally. That is, both teachers and learners participate and jointly direct the project with full recognition of the inherent relations of inequality.

Inter-disciplinarity

The concept of inter-disciplinarity is derived from the problem-oriented nature of project work. An interdisciplinary perspective recognises the separation of disciplines but takes into account their relations and intersections towards a synthesis of knowledge to better understand, from different points of view, the same object of study (Bastos & Costa, 2000). Since the point of departure is not necessarily a narrowly defined topic in mathematics but a more general problem of social or political relevance "inter-disciplinarity implies drawing on different disciplines to the extent to which they are useful for the treatment of a specific problem" (Vithal *et al.*, 1995:211). This was a key feature of the project work experience identified by Devan, when asked by the class teacher to explain the reason for successfully drawing graphs even though he was unable to execute many of the basic traditional algorithms:

> *I think when you go out in the front and you go a little in each subject, ma'am, I think that's interesting.*

Clearly for some learners the entry into mathematics needs to be negotiated differently. But school mathematics is somewhat resistant to inter-disciplinary problems (Nielsen *et al.*, 1999) and the strongly entrenched nature of this was also initially observed in Devan. Sumaiya recounted his response to a "realistic" problem based on a table about annual rainfall for Cape Town. The problem linked mathematics to geography and was taken from a textbook not used in the school. Sumaiya used this to prepare pupils for the problem-oriented and inter-disciplinary expectations of project work, and to signal a new approach and role as facilitator and supervisor rather than teacher:

> *We were given a task to teach addition of decimal fractions. So we said instead of doing that, we will do it this way where it's a problem-centred approach. Initially the pupils looked at it and they said we didn't do things like this. And how can we? We doing maths now, we don't do geography? Devan was the one, he's got the [math] anxiety, he says, "but ma'am I don't know, there's so much stuff like ... English and I don't understand". So I said, "read it, come let's sit with it". So slowly we went through it. He's a bit of a slow worker, but even for him to do those four sums, that was like an effort. He made some progress. He added those things.*

Devan struggled to complete basic number operations and even resisted the use of technology such as calculators in earlier lessons when advised to do so by Sumaiya, claiming that using calculators was "cheating". Yet he demonstrated competence in most other areas of learning with his favourite and strong disciplinary interests being the language subjects. Not surprisingly, being given a text to read, even in mathematics, and gradually making his way through it with questions and discussion about when and how to draw a graph, facilitated a different kind of learning for him.

However, implicit in the concept of inter-disciplinarity is an inherent tension within the structures of schooling, that all disciplines are not deemed equal. This is after all the mathematics classroom and both teachers and learners are bound by the didactical contract which privileges and prioritises mathematics teaching and learning. Hence lear-

ners are allowed to go outside mathematics so long as they also remain inside it. The data showed that students do not naturally gravitate towards mathematics when dealing with their project problem. It is the teachers who insist on this. In this respect inter-disciplinarity does not serve to undermine or equalise the power and status of mathematics as a discipline and may in fact reinforce it. A discussion about how the problem of school fees could be dealt with without mathematics is, in a sense, not viable in the mathematics classroom. Indeed the amount of fees levied by schools is as much a product of our apartheid history as it is of any mathematical or financial consideration and is hugely disparate across schools. As mentioned above, Devan and some members of his group considered the R60 they paid good value for what they got in the school. They had made comparisons with their neighbouring school and also knew that predominantly "white" public schools charge several thousand rands. The point is that even though backgrounds and societal contexts are noticed and discussed by the learners they are not fully developed. For example, the potential to learn about differences in the costs incurred by parents is not explored, and neither are the indirect costs, through taxes, for school buildings, teachers' salaries, etc. raised. No mention of the constitution is made to debate ideas such as education is a basic right, that it is compulsory and yet requiring payment. This in turn also appears to limit full mathematical investigation of the problem. For instance, no attempt was made to calculate exactly how much the school in fact collects as school fees or on what basis and in what proportion these monies are apportioned to different school needs. More political questions about who makes such decisions, and any possibility for learners themselves to participate remained unexplored, in part due to the conflicting and co-operating didactical disciplinary authority and emancipatory authority (Giroux, 1997) that the teachers are called on to exercise in this approach and conception of project work (Vithal, 2003).

Inter-disciplinarity in project work does not only have to account for the relationship of mathematics to the other formal school disciplines but also has to consider the relationship between mathematics and a broader everyday reality both inside and outside school. It is here that the notion of the formatting power of mathematics — the ways in which mathematics not only produces new inventions in reality but also in a sense colonises and reorganises parts of it (Skovsmose, 1994) — needs to be interrogated. In particular a key concern is how school mathematics provides or denies opportunities to learners to both participate in this power and to be able to react to it. The former refers to the production of mathematics, mathematicians and those involved in applications of mathematics — the "formatters"; while the latter refers to the development of a "mathematical literacy" for a critical citizenship — those who must read and respond to that formatting in everyday life. When Devan lists the cost of his education in a bar graph and presents it to the class he is in some sense being inducted into this formatting power of mathematics and becoming a formatter:

> *And then we have my graph. Stationery for me is only two hundred rands because I have high lighters, and different kind of equipment that I use. My schoolwear only comes up to three hundred rands, because you buy your ties, takkies, shoes, PE clothes, you buy your school uniform. And your school fund, that's about sixty rands there* (pointing on graph). *My transport is hundred rand because I just live down the road, so I count mine yearly. And spending for me is only hundred rand a week.*

Later when he comments and questions the single graph drawn by another group investigating the same problem, he becomes a "critical reader" of a formatting done by someone else:

> *What I don't understand, they said how much their education cost, how can you have one calculation?*

These refer to two related yet quite different competences. Further, competence in one does not necessarily pre-suppose or imply competence in the other. It is, however, possible to consider becoming a "critical formatter" in which ethical, social and political concerns are actively considered in the formatting process by the formatter. After Loresha from his group presented her graph, Devan draws the attention of the class to the fees she pays.

> *Loresha has to pay hundred and twenty rand for school fund because she's got a little smaller sister and it becomes more costly for her because there's two that's schooling.*

After Harry's presentation he makes explicit a social concern

> *Harry takes a bit of money (R1200) in transport because Harry lives in Clermont and every day he comes from Clermont to [the school]. It's quite hard because Harry actually leaves at five o' clock in the morning. He has to take the combi and come to school, so it's quite costly.*

A critical formatter also expresses concern about the capacity of the citizenry to understand the formatting undertaken. This aspect influenced the learners' decision to choose bar graphs rather than pie graphs. It was argued by learners that not only were pie graphs more difficult to draw involving working with fractions, but that the majority of the class would be lost and not understand this mathematical representation. In both these instances of critical formatting and reading, the teachers focus on the correctness of the graph and honour the strict conditions of the didactical contract, to teach and learn the mathematical content. No discussion about the enormous disparities in learners' socio-economic and broader societal conditions are pursued. This is after all the mathematics classroom and with so much mathematics to teach and learn, any emancipatory authority or curriculum goals are expendable. Or perhaps the wounds of apartheid are still too fresh to engage inequalities and injustices so close to the participants' lives.

Exemplarity

"Where "participant direction" and "problem orientation" have to do with the **form** in which projects are carried out, exemplarity has to do with the **reason for problem orientation**" (Christiansen, 1999:57, emphasis in original). The exemplarity principle, developed with reference to worker education by Oscar Negt, is central to justifying an alternative project-based curriculum framework, and has been interpreted for a critical mathematics education by Skovsmose (1994). The main idea is that some larger totality or complexity can be reflected in and comprehended by focusing on some smaller part of it, a particular problem or phenomenon. In addition, learners are construed as epistemic subjects who have an active interest in the process of "coming to know" and in trying to improve their situation. An exemplary organisation of the curriculum requires that the subject matter is relevant and meaningful from the perspective of individual experience, relevant to an objective and conceptual understanding of society, and relevant to meaningful action to improve social conditions related to learners' lives (Rasmussen, 1991). In his cultural approach to a mathematics curriculum, Bishop (1988:110) alludes to a similar idea when he writes "one principle which is appropriate for this (societal) component of the curriculum is 'exemplification' rather than 'coverage'". Exemplarity draws sharp attention to the kind of problem identified for exploration and the process of problem selection involving both learners and teachers. Exemplarity makes it possible to demonstrate that "general theoretical concepts (are) not necessarily far removed from individual experience and that inter-disciplinarity could be combined with involving students more actively in the formulation of knowledge" (Rasmussen, 1991:49). For Skovsmose (1994) critical mathematics education refers to a form of practice that incorporates problem orientation, inter-disciplinarity, and exemplarity and through which a critical mathematics literacy or "mathemacy" can develop. Thus project work in mathematics has the potential to allow learners to not only learn to provide mathematical solutions, but also to identify the extent to which mathematical solutions can be both useful and problematic in solving real life problems both in the micro- and macro-scale. On the basis of the exemplarity principle it is also possible to argue for a space to provide learners with experiencing project work, even within the framework of a conventional curriculum. Theoretically, according to the exemplarity principle, one experience of doing a project could provide some insight for how mathematics as a discipline is connected to real

life problems. Devan reflected at the end of the project:

> *I learnt about how much I spend on my education. And I learnt that maths, although it does not appear very interesting, but now I'm actually starting to like maths. Before, I used to hate maths. Actually maths was my biggest danger. I used to pretend and had to copy or things like that because I used to hate doing maths. I'm starting to like it a bit and I find it quite interesting — this group project ma'am, I like it.*

As a learner who regularly failed mathematics tests with scores seldom more than 30%, for Devan the exemplarity potential was realised in relation to at least two different sets of complexities. The first set, connected to the content of the project, refers to different aspects of the problem that was investigated. It can be seen in how learners expressed surprise at how much was spent on their education, and in their broader understanding of how mathematics can be applied to this particular problem or reality. The second set refers to how learners can and do participate in class and in their groups in the process of coming to know something about the content of the project problem. These two sets of complexities make it possible to distinguish between a theoretical or intellectual exemplarity related to the content of the project problem, and a practical or lived exemplarity connected to the processes by which the project work is undertaken (Vithal, 2003). While the former is produced through engagement with the project problem itself, the epistemic object; the latter is realised through the interactions and processes of the project work experience and especially working in a group, the epistemic subjects.

> *One of my schools that I went to ma'am, they used to bring like the cleverest children in the front and the children that wasn't too clever they should just leave them in the back but now when I done this ma'am, everyone asked me questions, and ma'am, I felt like quite important in the group. I feel great doing project work and I am finally starting to like maths. Maths is quite interesting now. We are working with our budget and am enjoying it. And for the first time I am feeling important doing something. Everyone needs me in this group. Mostly people used to treat me like nothing but now I feel very happy working like this and I hope to get a good pass in maths.*

By working on a single problem of how much is spent on their education, learners have the possibility to come to know the broader system of education funding and its attendant inequalities — a societal complexity. They can also come to know how mathematics participates in distributing those funds at the level of school and state — a disciplinary totality. Both these enable an intellectual exemplarity which is expected in a mathematics class. However, an experiential exemplarity may also be realised because by working in a single instance of a diverse group setting, learners can come to know how to act and be in other or larger settings in society. Of course these can be both positive and negative. While Devan points to an affirming experience, another group member who is constantly marginalized may come to a different understanding of what it means to work on a project and within a group, and therefore also within a broader community of school or home.

An important component of exemplarity is that the learners are interested and active in coming to know different and multiple realities. In project work this may be interpreted to include the practice that having reflected and understood the problem they could engage in action to change some aspect of that reality. Not only is action made active in the learning process itself but it is also present in the outcome of the learning. Project work from a critical perspective builds in the possibility to change, to empower the participants toward better life conditions or opportunities.

This was explicitly engaged by Sumaiya in the classroom where they were questioned and encouraged to act on their findings, such as making representations to the school parent-teacher organisation or the principal. Devan was one of the few students who directed questions about possible action to the various groups. For example, he questioned the group who undertook a sports survey:

> *You have realised that the school's got equipment. You said, Naresh, that the school has lots of many equipment, like baseball. So you know that. Now what are you all going to do with that information? Are you going to take it up to the Parent-Teacher Association? Or you all not just going to leave it aside?*

In this the students realise that they have the possibility to act, to do something about what they consider to be an important problem to them but they also come to know the limits of that action as the sports group respond to Devan:

Navin: *We asked the principal, the teachers and they say there is no space for baseball.*

Naresh: *Baseball, there's no space, there's hardly any space we have here.*

The exemplarity potential can be realised in different domains and directions. By engaging the project problem of school fees and sports equipment in the classroom and school they can come to know something about the problems of unequal distribution of these resources in the community or society. That they consider acting on this problem means that they also come to know something about what possibilities they have for acting on such problems in other settings in the school and in the broader democratic society.

Intellectual and experiential exemplarity, which refer to the content and processes of project work, respectively, are realised through the activity constituting the problem. But this activity, made possible by the commitment of the learners, includes more than an engagement with the problem. It seeks to recruit the classroom and the school into the attempt to act on the understandings and experiences gained. It is in this respect that Mellin-Olsen's (1988) notion of Activity is useful. For him Activity is not only an educational concept, but also a social and political concept that includes communication, oppression, resistance and culture as a basis for mathematics education. This is demonstrated in the project work description of how Devan gains voice but also experiences silencing in the class. The notion of Activity refers and gives priority to the activity of pupils, activity that is owned by them (rather than the teacher). In this approach the learner is always deemed to "have some important knowledge which is significant for the learning process, which should thus be recognised by the curriculum maker" (*ibid.*:18). But this point must be extended to refer also to interactions within groups among learners themselves to value what each participant brings to the project. A main concern for Mellin-Olsen is the relation between Activity and critical awareness. Activity follows critical awareness but Activity is also necessary for critical awareness to develop. Schools are the sites for challenging ideologies, for politicising mathematics through Activities. Critical awareness without Activity is clearly a problem because "It is hard to see how pupils who may be in a difficult life situation can have faith in their future curriculum if their meta-concept of it tells them that school helps them to understand what is wrong, but does not stand up for them in a common attempt to challenge the difficulties." (*ibid.*: 203-204). The task for mathematics education is to offer experiences of how to apply the thinking tools of the curriculum in such a way that the tools are recognised as functional knowledge by pupils to become critically aware and to transform that awareness into social or political action. Project work appears to offer one possibility for realising such goals and exemplarity is a conceptual tool for seeing how this may be possible. Devan was the one learner in this class who seemed to come closest to realising these multiple ideals through the facilitation and efforts of Sumaiya. Even though not completely successful, since no action or Activity was observed within or following the project, the potentialities were experienced and observed.

Conclusion

Project work provides a means for meeting some of the expectations of the new curriculum reforms in South Africa. For example, each of the concepts illuminated through the project work experienced by Devan may be seen to be reflected in the critical intended outcomes of the new National Curriculum Statements for grades R-9 (Department

of Education, 2002b). The problem oriented feature of project work may make it possible to produce "learners who are able to identify and solve problems and make decisions using critical and creative thinking". The concept of participant direction suggests that learners are able to "organise and manage themselves and their activities responsibly and effectively and work(ing) effectively with others as members of a team or group". Exemplarity may be inferred from learners being able to "demonstrate an understanding of the world as a set of related systems" (Department of Education, 2002b:1). Inter-disciplinarity is expected in requiring learners to be able "to transfer mathematical knowledge and skills between learning areas and within Mathematics" (Department of Education, 2002b:5). Therefore, project work as a pedagogy provides a means for realising some of the important features of teaching and learning mathematics (such as problem solving and investigations) expected by the new South African curriculum reforms. To achieve this, clearly such "progressive pedagogies" need to be made available in teacher education programmes. However, what is also equally important is that they need to be interrogated through research to assess their viability for the broad range of South African classrooms, to point to what is possible and what is not, and to develop theory and practice that is relevant to this diversity.

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