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Grade 7 teachers' and prospective teachers' content knowledge of geometry

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The geometry content knowledge of Grade 7 teachers (n =18) and prospective teachers (n =100) was investigated, using the Van Hiele theory and acquisition scales of Gutiérrez, Jaime and Fortuny. Results indicated that both teacher and prospective teacher populations failed to reach the level of geometric thinking and degree of acquisition expected from successful teachers. The impact of teaching experience and different pre-service time frames (3 years *vs* 4 years) on the level of geometrical thought was also investigated. The conclusion was that teachers and prospective teachers do not have adequate control of the Grade 7 geometry subject-matter they have to teach. This holds implications both for pre-service and in-service teacher education as well as classroom practice.

Introduction

The education system in South Africa has been the focus point of various major research projects, which include the Monitoring Learning Achievement Project completed in November 1999 (by the Research Institute for Educational Planning at the University of the Free State) (Strauss, 1999), the Presidential Educational Initiative Project completed in November 1998 (managed by the Joint Education Trust) (Taylor & Vinjevold, 1999) and the Third International Mathematics and Science Study in 1995, as well as the repeat project, TIMSS-R, in 1999 (managed by the Human Sciences Research Council) (Howie, 2001).

A common factor identified as necessary for effective education in all these studies was the crucial role that the teacher plays in teaching and learning. A general concern raised in most of these studies, as well as in studies in other countries, such as the USA (National Research Council, 2001:4; Ball, Lubienski & Mewborn, 2001:437), was the state of the teachers' knowledge, particularly their subject content knowledge.

What the teacher knows is one of the most important variables that impact on what is done in the classroom (Turner-Bisset, 2001: 148; Fennema & Franke, 1992:147) with Ball *et al.* (2001:440) noting that the "assertion that teachers' own knowledge of mathematics is an important resource for teaching is so obvious as to be trivial". Teachers form an important link in the success of any curriculum as it is teachers who filter the curriculum through to the learners (Du Plooy, 1998:15; Graham & Fennell, 2001:326). Although Thompson (1992: 129) acknowledges that it is difficult to distinguish between beliefs and knowledge, especially since teachers frequently treat their beliefs as knowledge, Kennedy (1998:261) appeals for concrete evidence of what prospective teachers and practising teachers do in fact know about the subjects they learn or teach, as teachers' behaviour is influenced by their knowledge (Ball & Bass, 2000:86; Koehler & Grouws, 1992:118).

This article focuses on the state of teachers' and prospective teachers' (PTs) content knowledge of Grade 7 geometry, as well as the impact of different time-frame models of pre-service training.

Literature review

Teacher content knowledge

Shulman (1987:8-9) distinguishes between different kinds of knowledge that an effective teacher should possess. These include knowledge about the subject matter they are teaching (content knowledge), knowledge of specific strategies for teaching a particular subject matter (pedagogical content knowledge), and knowledge of the materials and media with which instruction and assessment are carried out (curricular knowledge).

Content knowledge involves knowledge of the substance of the field; major concepts, principles and procedures, and the relationships between these (Aubrey, 1997:7). Teachers must have an in-depth knowledge of the specific mathematics that they teach (Ball, 2000: 244) as well as the mathematics that their learners will learn in future as teachers' subject knowledge impacts on their behaviour and thus indirectly affects learner achievement (Muijs & Reynolds, 2002:12). Teachers must not only know the subject matter but must also have the ability to understand it from the perspective of the learner (Moseley, 2000:39).

Ball *et al.* (2001:441) propose that there are two approaches involving research that focus on teachers' mathematical knowledge. The first approach which focuses on characteristics of teachers assumes that knowledge of and skill with mathematical content are essential to teaching whilst indicators (e.g. courses taken or degrees earned) will represent the required knowledge (Stedman, 2001:2). In this approach teacher knowledge can be measured by these relatively straightforward indices. Monk (1994:130), however, reaches the conclusion that the number of mathematics courses taken in mathematics does make a difference in teachers' knowledge, but only up to a point. He determines that five courses in mathematics (independent of the specific content) are the threshold beyond which few (student achievement) effects accumulate. Some mathematics education researchers (e.g. Muijs & Reynolds, 2002:5; Ball, 1999:21) indicate their disapproval of this approach as it is felt that formal mathematics qualifications cannot be linked to student gains, with Ball *et al.* (2001:443) noting that "simply counting courses does not permit an examination of whether the teacher has the mathematical knowledge needed for the lesson".

The second approach, which focuses on teachers' knowledge, builds on the first approach by acknowledging the importance of the content of teachers' mathematical knowledge but also the quality of the nature of teachers' knowledge. In this approach focus falls on the understanding of specific mathematical topics (e.g. geometry), procedures (e.g. long division) and concepts (e.g. parallelism) rather than on global conceptions of mathematical knowledge (Ball *et al.*, 2001:444). This substantive knowledge of mathematics is recognized as "content knowledge'.

Van Hiele theory

A widely respected and accepted theory, the Van Hiele theory, was used to examine the content knowledge of teachers and prospective teachers, as the theory identifies a way in which a learner's level of geometric argumentation or thinking can be measured (Van Hiele, 1959:1-31).

The Van Hiele theory postulates that learners advance through progressive levels of geometrical thought from a Gestalt-like visual level through increasingly sophisticated levels of description, analysis, abstraction, and proof (Van Hiele, 1999:315; Van Hiele, 1986:39). At the first level (Recognition) learners identify and operate on shapes and other geometric configurations according to their appearance alone (Mason, 1997:39). On the second level (Descriptive / Analytic) learners are able to recognize and explicitly characterize shapes by their properties (Van Hiele, 1986:40), but cannot recognize relationships between classes of figures (Battista, 1994:89) or even redundancies (repetitions) (Spear, 1993:393). Learners at level three (Abstract / Relational) can form abstract meaningful definitions (Mason, 1997:39), distinguish between necessary and sufficient sets of conditions for a concept, classify figures hierarchically (by ordering their properties), give informal arguments to justify their classification (Battista, 1994:89), and understand and sometimes even provide logical arguments in the geometric domain (Van Hiele, 1999:316; Clements & Battista, 1992:427). At level four (Formal Deduction) learners are able to establish theorems within an axiomatic system. They recognize the differences among undefined terms, definitions, axioms, and theorems and are capable of constructing original proofs (Clements & Battista, 1992:428). At the fifth level (Rigor / Meta-mathematical) learners reason formally about mathematical systems, understand the formal aspects of deduction (Presmeg, 1991:9), establish and compare mathematical systems (Mason, 1997:40), and reason by formally manipulating geometric statements such as axioms, definitions, and theorems.

Spear (1993:393) postulates that the first three levels identifying thinking are within the capacity of elementary school learners whilst the last two levels involve mathematical thinking typically needed in high school and tertiary courses. It stands to reason that all elementary school mathematics teachers and prospective elementary school mathematics teachers should at least attain the first three Van Hiele levels. Furthermore, it seems reasonable to expect that teachers and prospective teachers who choose mathematics either at college or university level should have completed high school with mathematics as a subject, and should therefore have obtained the formal deduction level (level four).

Gutiérrez, Jaime and Fortuny (1991:237-239) theorize that the Van Hiele geometric thought levels are not discrete and present an additional method to evaluate and identify those answers (learners provide) that denote a possible transition between levels. Answers are firstly classified according to the Van Hiele levels of thinking they reflect, by using the descriptions of the levels and secondly by assigning to one of a number of types of answers (and given a numerical weight), depending on its mathematical accuracy and how complete the solution to the activity is. Determining the average of the numerical weights of answers of a specific topic (e.g. squares) leads to a classification of the degree of acquisition (see Table 1) for that specific topic (58% average = intermediate level of acquisition for squares).

As the importance, impact and expected level of teachers' and PTs' content knowledge, including geometric thinking level attainment, have been established, the state of Grade 7 teachers' and PTs' content knowledge is also explored, with some attention to different time frame models in PTs' training.

Method

Participants

The population of teachers who participated in this study was composed of all Grade 7 mathematics teachers (n = 23) teaching at ex-Model C primary schools in five towns in North West province. Of the 23 teachers invited to participate, 18 volunteered to take part in this study. The available population of PTs consisted of all final-year education students taking mathematics as a subject in any of the seven higher education institutions in North West province of South Africa (two universities and five colleges of education). One university and one college completely withdrew from the study due to administrative difficulties at these institutions. The total population consisted of 100 PTs, of which 78 students (Institutions 1 to 3) received second language instruction and followed a 3-year college curriculum. Students at these colleges followed an integrated mathematics syllabus, consisting of 70% academic mathematics content and 30% mathematics curriculum content (Nieuwoudt, 1998:214). The 22 remaining students (Institutions 4 and 5) received first language instruction. Both institutions follow a four-year curriculum, with the university students completing a three-year degree (with mathematics on a first/second/third year level) followed by a one-year higher education diploma in which a mathematics method course is taken. The college students complete a four-year teacher's diploma with students selecting the academic mathematics course for four years, and with all students taking a compulsory methodology course for four years (Nieuwoudt, 1998:214).

All PTs involved in the study are qualified to teach Grade 7 (the first grade in the senior phase), as the curricula at the colleges prepare students for either the intermediate to senior phase (Grades 4–9) or the senior to FET phase (Grades 7–12), whilst the university prepares the students for the senior to FET phase (Grades 7–12).

Research design

Field study research was done where data were gathered directly from individuals and groups in their natural environment for the purpose of studying interactions, attitudes, and characteristics of individual groups. An *ex post facto* research design (Leedy, 1997:111) was used to determine the state of teachers' and PTs' knowledge of Grade 7 geometry (including their geometric thought level attainment). In this design no direct manipulation of conditions took place while investigating a possible cause-and-effect relationship (Leedy, 1997:227).

Instrumentation

The questionnaire that teachers and PTs had to answer investigates geometry subject knowledge. The questionnaire consists of 56 items on a variety of geometric concepts and is based on the Mayberry Test (Lewin-Pegg Version as published by Lawrie, 1998) where only questions dealing with concepts relevant to Grade 7 are incorporated into the questionnaire. This questionnaire assesses concepts (such as the parallel lines) and shapes (such as square and isosceles triangle) over the first four Van Hiele levels. In selecting the relevant items, it was found that the items from the Mayberry Test were not sufficient and therefore additional items were introduced from a test developed by the Research Unit for Mathematics Education of the University of Stellenbosch (RUMEUS) (1984). Cronbach Alpha (CA) values were calculated to determine the degree of reliability of the test items. Acceptable to high CA values were found, namely: 0.77 for level 1; 0.71 for level 2; 0.68 for level 3; and 0.56 for level 4. A CA value in excess of 0.5 indicates an acceptable degree of reliability; a value in excess of 0.7 indicates a high degree of reliability (Anastasi, 1988). The Van Hiele instrument used in this research clearly yielded reliable results for the study population.

Figure 1 shows examples of questions relating to right-angled triangles on various Van Hiele levels. The answers were evaluated according to the acquisition scales of Gutiérrez *et al.* (1991:237-239), presented in Table 1.

Statistical procedures

All the computations for this article were done with SAS® (SAS Institute Inc., 1999). As the populations were not random samples, inferential statistics could not be used and statistical significance (*p* values) was not applicable (Steyn, 1999:1-2; Cohen, 1988:20-27). Effect sizes were determined to indicate the practical significance between groups by using the following formula (Steyn, 1999:3):

$$d = \left| \frac{\bar{x}_1 - \bar{x}_2}{\max \sigma} \right|$$

with $\bar{x}_1$ = mean of population group 1; $\bar{x}_2$ = mean of population group2; max σ = maximum standard deviation of the two population groups. Where $d \geq 0.2$ indicates a small effect; $d \geq 0.5$ indicates a medium effect; and $d \geq 0.8$ indicates a large effect. Only when $d \geq 0.8$, is it considered that there is a practically significant difference between groups. The results are applicable only to the study population and no generalisations will be made.

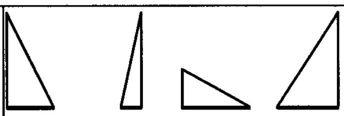
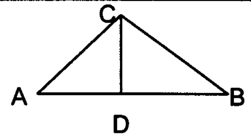
Level 1	 Are all of these triangles? *Yes/No*, Explain your answer.	Level 2	Does a right-angled triangle always have a longest side? *Yes/No* If so, which one? (Make a drawing and indicate on your drawing.)
Level 3	Circle the smallest combination of the following which guarantees a triangle to be a right angled triangle: a. *It has two acute angles.* b. *The measures of the angles add up to 180⁰.* c. *An altitude is also a side.* d. *The measures of two angles add up to 90⁰.*	Level 4	 CD is perpendicular to AB. $A\hat{C}B$ is a right angle. If you would measure $A\hat{C}D$ and $\hat{B}$, you would find that they have the same measure. Would this equality be true for all right angled triangles? Why or why not?

Figure 1 Examples of test items on various Van Hiele levels

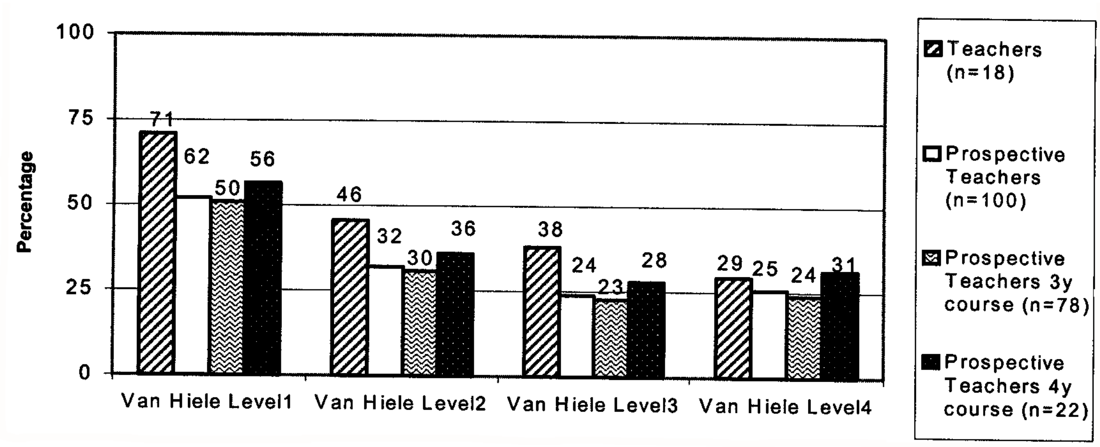


Figure 2 Degrees of acquisition of teachers and prospective teachers

Procedure

A pilot study was conducted to refine the questionnaire in five schools with five Grade 7 teachers taking part on a voluntary basis. During this piloting phase, teachers were encouraged to comment, criticize and make suggestions regarding the questionnaires. The teachers in the pilot study noted that all items in the questionnaire were relevant and were on the correct "level" for Grade 7. They unanimously agreed that all items should remain in the final questionnaire. The teachers in the pilot study were not included in the main research project.

Grade 7 teachers participating in the final study did so voluntarily after individual meetings with each of them to discuss the aim of the study. Each of the institutions participating was visited in turn where all students taking mathematics as a subject (that would qualify them to teach senior phase mathematics) were requested to complete the questionnaire. Both population groups were visited during the final quarter of 2000.

Results

In analysing the data for both Grade 7 teachers and PTs, it became clear that neither group had achieved a complete degree of acquisition (85%), even for the first level of geometric thought (see Figure 2).

The Grade 7 teachers reached only a high degree, and PTs an intermediate degree, of acquisition for the Van Hiele level 1 (Recognition). On the descriptive level (Van Hiele level 2) PTs could only manage a low degree of acquisition with teachers reaching an intermediate level. In the third Van Hiele level (Abstract), where, for example hierarchical classification of figures is expected (Battista, 1994: 89), as well as level 4 (Formal Deduction), both teachers and PTs could only reach a low degree of acquisition. The results of relevant items of the general questionnaire are included in the discussion below.

Discussion

Kanes and Nisbet (1996) noted that the level of mathematics content knowledge is an important indicator of overall teacher effectiveness, whilst Meredith (1993) admitted that it is a truism that teachers require subject knowledge, but there is no agreement on what is sufficient and what is necessary subject knowledge. Considering that the quality of instruction is a function of teachers' knowledge (National Research

Table 1 Answer type and degree of acquisition (Gutiérrez *et al.*, 1991)

Answer type and weight		Description	Degree and weight		Description
0	0%	No reply, or answers that cannot be categorized.	No degree of acquisition	0 – 15%	Learners are not in need or are not conscious of the existence of thinking methods specific to a new level
1	0%	Answers that indicate that the learner has not reached the given level but has no knowledge of the lower level either.			
2	20%	Answers that contain incorrect and incomplete explanations, reasoning processes, or results.	Low degree of acquisition	15 – 40%	Learners are aware of methods of thinking, know their importance and try to use them. These learners make some attempts to work on a higher level, but have little or no success due to their lack of experience.
3	25%	Correct but insufficiently answered to indicate that the given level of reasoning has been achieved. Answers contain very few explanations as well as inchoate reasoning processes, or very incomplete results.			
4	50%	Correct and incorrect answers that clearly show characteristics of two consecutive Van Hiele levels. Answers contain clear reasoning processes and sufficient justifications.	Internediate degree of acquisition	40 – 60%	Learners use methods of the higher level more often and with increasing accuracy but still fall back on methods of a previous level. Typical reasoning is marked by frequent jumps between the two levels.
5	75%	Answers that represent reasoning processes that are complete but incorrect, or answers that reflect correct reasoning but that still do not lead to the solution.	High degree of acquisition	60 – 85%	Characterized by progressively strengthened reasoning that indicates that a learner is using a higher level of reasoning. Learners still make some mistakes or sometimes go back to the lower level.
6	80%	Correct answers that reflect the given level of reasoning that are complete or insufficiently justified.			
7	100%	Correct, complete and sufficiently justified answers that clearly reflect a given level of reasoning.	Complete	85 – 100%	Learners have completely mastered the new level of thinking and use it without difficulties.

Table 2 Effect sizes for teachers and PTs

	All PTs (n = 100)				PTs – 3 years (n = 78)				PTs – 4 years (n = 22)			
	L1	L2	L3	L4	L1	L2	L3	L4	L1	L2	L3	L4
Teachers	1.68*	1.69*	2.40*	0.39	2.30*	2.62*	4.41*	1.16*	1.79*	1.00*	1.15*	0.13
PTs – 3 year	-	-	-	-	-	-	-	-	0.47	0.66	0.98*	0.91*

* practically significant ($d \geq 0.8$)

Council, 2001:315), the results, as demonstrated in Figure 2, do however suggest that the current level of content knowledge attainments amongst teachers and PTs may be an issue that requires further scrutiny.

The results appear to indicate that both Grade 7 teacher and PT populations failed to reach the expected level (level four) and degree (high or complete) of acquisition expected from the learners they are to teach. This could suggest that pre-service teacher education does not adequately prepare teachers for teaching geometry.

It should however be noted that the Grade 7 teachers consistently outperformed PTs on the first, second and third Van Hiele level (see Table 2 for practical significant values). If a comparison is made between Grade 7 teachers and students following a three-year course, the differences in the level of geometric thought deliver large practical significance values on level 1, level 2, level 3, as well as level 4. The difference in respect of the level of geometric thought on level 3 is especially disconcerting as this level of thought requires (hierarchical) classification of figures (by ordering their properties) — one of the focus points of the learning outcomes of the Revised National Curriculum Statements (Department of Education, 2002:64). The effect of mathematics teaching experience of teachers on the level of geometric thought seems to be an influencing factor (in this study) as it could explain why teachers' attainment is significantly higher than the PTs'. This could verify Ormond and Cole's (1996:40) statement that true expertise in teaching any topic typically requires in-depth conceptual knowledge of the subject matter plus many years of experience teaching it. Unfortunately teaching experience in this study seems only to benefit the lower geometric thought levels to a limited degree, but this thought still remains below the expected levels and degrees of acquisition.

In comparing the two models of teacher preparation, namely, the three- and four-year options, the students receiving the four-year training consistently outperformed the students following the three-year course. These differences appear to have practical significance on level 2, level 3, as well as level 4 (see Table 2), which could lead to the conclusion that the extra year of education does positively affect content knowledge of PTs. Unfortunately the levels and degrees attained (even by the students following the four year course) still remain below the expected levels and degrees of acquisition. This result brings the relevance and level of education the PTs receive into question, but also provides evidence of the fallacy of the assumption that content knowledge is not a problem for high school teachers, who, by virtue of specialized study, "know their subjects" (Ball *et al.*, 2001: 444).

Shulman (1987:5) notes that a person who presumes to teach subject matter must demonstrate knowledge of that subject matter as a prerequisite to teaching and helping learners to learn with understanding (*cf.* Ball, 1993:395). It seems reasonable to expect that teachers should be experts in the classroom (Reinke, 1997) as a depth of knowledge of the content is a prerequisite for good teaching (Von Minden, Wallis & Nardi, 1998). Quinn (1998) warns that teachers who have inadequate meaningful mathematical content knowledge often exacerbate the problems that students experience in learning mathematics, but it will also be unlikely for such teachers to be able to provide adequate explanations of concepts they do not understand (National Research Council, 2001:377).

The final and general conclusion of the study is that PTs and Grade 7 teachers are not adequately in control of the subject matter they have to teach, concurring with the findings of Stacey *et al.* (2001:222) and Reeves & Long (1998:182). This study also supports the findings of Webb *et al.* (1998:56), a South African research team, regarding the poor state of teacher knowledge of the subject matter they are to teach to learners. In the Webb *et al.* study, teachers were from previously disadvantaged communities and education systems whilst educators in this study were from previously advantaged communities and privileged education systems (both in schooling and on tertiary level). Furthermore, the findings are in agreement with Ball *et al.* (2001:444) that elementary and secondary, pre-service or experienced teachers all revealed universal weakness in the understanding of basic fundamental mathematical ideas and relationships. These correlations could imply that teachers, irrespective of educational history, could currently be teaching without the necessary subject content knowledge.

If the central goal of teacher preparation and professional development is to help teachers understand the mathematics they teach (National Research Council, 2001:398), the Grade 7 teachers' and PTs' content knowledge (as reflected in this study) needs serious attention and/or renewal of both pre- and in-service training. The National Research Council (2001:373-374), however, cautions that simply taking more of the standard college or university mathematics courses does not appear to improve this situation (see also the work of Monk, 1994). The specialized knowledge of mathematics that teachers need is different from the mathematical content contained in most college or university mathematics courses, which are primarily designed for professional use of mathematics in fields such as mathematics, science and technology (National Research Council, 2001: 375). Teachers need to make connections within and amongst their knowledge of mathematics, students and pedagogy in ways that enable them to help learners learn. The implication for teacher preparation and in-service training is that teachers need to acquire these forms of knowledge in ways that forge connections between them and are supported by the integration of mathematics, methods of teaching and psychology (*cf.* Ball, 2000:241; National Research Council, 2001: 381). The findings of this research suggest that current mathematics teacher education programmes may fail to a substantial degree in this respect. It is therefore our challenge as teacher educators to solve the question of how to create fruitful representational contexts (Ball, 1993:394) and design programmes that will result in a deep understanding of elementary mathematics (Barnett, 2001:35) from a higher standpoint, by treating elementary content in such a way that teachers gain in both pedagogical strength and knowledge of higher mathematics, including thorough knowledge of the subject matter taught in schools (*cf.* Noddings, 1999:214). We should also provide teachers and PTs with opportunities for learning subject matter that will enable them to not only know but also learn to use what they know in varied contexts of practice (*cf.* Ball, 2000:246).

Cooney (2001:9) indicates how crucial pre-service education is by noting that the role of teacher educators is to reveal and make evident the complexity of teaching and to propose alternatives for dealing with that complexity as teachers have neither the luxury, nor the resources, to experiment with or fantasize about a different school environment. With the results apparently indicating that pre-service training is not adequately preparing PTs and therefore not assisting teachers, the following recommendations could be useful for teacher educators and tertiary institutions wishing to make a meaningful contribution to the preparation of mathematics teachers.

A long-term teacher education programme is suggested (Swafford *et al.*, 1999:79) that incorporates constructive learning environments that:

1. Enable PTs to develop knowledge of mathematics that permits the teaching of mathematics from a constructive perspective (Cooney, 1994b:16) with courses provoking PTs to confront their possible "naïve notions of teaching mathematics" (Lerman, 2001: 48);
2. offer teachers and PTs an opportunity to reflect on their experience as learners of mathematics (Cooney, 2001:16; Krainer, 1999:110), but also as teachers/mentors (Fresko & Wertheim, 2001:159; Serrazina & Loureiro, 1999:57), in an environment where they experience the learning in the same way as they will be expected to work with learners (Farah-Sarkis, 1999:47);
3. Mapolelo (1999:724) suggests balancing mathematics content knowledge with pedagogical competency in mathematics teacher education. Swafford *et al.* (1999:79) add that enhancing PTs' content and pedagogical knowledge in combination with colla-

oration and reflection can serve as a catalyst for change in in-structional practice;

4. provide a context in which teachers and PTs develop expertise in identifying and analysing the constraints they face in teaching (Farah-Sarkis, 1999:45; Cooney, 1994b:19) and explore strategies and ways to deal with those constraints (Steele, 2001:170); and
5. afford contexts in which PTs and teachers can gain experience in assessing learners' understanding (Cooney, 1994b:16) and learning (Mapolelo, 1999:724) of mathematics. Franke and Kazemi (2001:104) suggest that PTs are transformed from teachers into learners by listening to learners' mathematical thinking/explanations with the benefit of PTs learning about the teaching and learning of mathematics in the context of their practice.

The advantages of following a programme of this nature could include teachers being enabled to become less textbook dependent with less emphasis given to computational tasks and a focus shifting from teaching to learning (Swafford *et al*., 1999:80). By providing opportunity to apply instructional strategies and techniques (Tirosh, Stavey & Tsamir, 2001:73), teachers acquire knowledge that enables them to reflect on their own learning and knowledge base and so generate the realization for the need to relearn forgotten knowledge and gain new knowledge.

Cooney (2001:16) theorizes that the greatest moral dimension of teacher education is the challenge which enables teachers to see knowledge acquisition as power they can use to enable learners to acquire the same kind of power. If teachers are to teach according to the visions of reform, they must be convinced of the value of reform and have exposure to similar learning environments first-hand as learners (Manouchehri, 1997). This places great responsibility on the shoulders of tertiary institutions to reform teacher education by establishing a theory for their practice and by giving attention to content and pedagogy, while training teachers to be reflective problem-solving intelligent professionals (Sullivan & Mousley, 2001:162; Cooney, 2001:16).

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