S.-Afr.Tydskr.Veek.1988,18(4)

Mathematical programming model for the optimization of nutritional strategy for a dairy cow

F.J. Kleyn and R.M. Gous*

Department of Animal Science and Poultry Science, University of Natal, P.O. Box 375, Pietermaritzburg 3200, Republic of South Africa

Received 26 June 1987; accepted 22 October 1987

The use of a mathematical programming model for determining optimal nutritional strategy for a dairy cow is described. Mixed Integer Programming (MIP) may be used to fit curvilinear functions, such as the changes in the nutrient requirements of the cow, into a standard mathematical programme. The model determines the optimum level of production and the corresponding feed that should be offered to the cow, under a particular production system. The model showed that the nutritional strategy applied is dependent upon the availability, price and quality of farm-produced roughage or purchased concentrates. Under certain conditions a change in the milk or purchased concentrate price may result in a revised nutritional strategy.

'n Wiskundige programmeringsmodel vir die bepaling van 'n optimale voedingsstrategie vir 'n suiwelkoei word voorgestel. Gemengde-geheelprogrammering is gebruik om kurvilineêre funksies, soos veranderinge in die koei se voedingbehoeftes, in 'n standaard wiskundige program in te pas. Hierdie model beraam die optimale peil van produksie waarvolgens gevoer moet word, binne 'n spesifieke produksiestelsel. Toepassing van die model het getoon dat verskillende voedingstrategieë aangewend moet word afhangende van die beskikbaarheid, koste, en kwaliteit van tuisverboude ruvoer sowel as aangekoopte kragvoere. Onder sekere omstandighede kan 'n verandering in die prys van melk of aangekoopte kragvoer tot 'n verandering in die optimale voedingsstrategie lei.

Keywords: Dairy cows, Mixed Integer Programming, nutritional, optimal, strategy

* To whom correspondence should be addressed

Introduction

Feed costs form the major part of the input costs of any milk production unit, and any attempts to maximize profitability through improving nutritional strategy are well justified. Nutritional strategy is defined here as the acquisition or production of the correct amounts of the correct feed ingredients and the allocation of those feed ingredients to a cow in such a manner as to maximize profitability of the feeding operation.

The amount of a feed ingredient which should be purchased or produced for a dairy production enterprise is determined by both its availability and relative economic value, which depend not only on price and nutrient content, but also on the type of diet in which it is to be fed, the limitations or bounds which are imposed on its use and the availability, price and nutrient content of alternative feed ingredients.

The manner in which these feed ingredients are allocated to a cow or group of cows depends upon a combination of economic and biological factors. Williams, Olenacu, Bratton & Milligan (1987) showed that most of the effect of management variables on the production cost of milk is indirect and mediated through milk production per cow. Thus the economic goal of a dairy enterprise lies in obtaining an optimal milk output per animal. Within the practical range of lactation which each animal can attain, there exists a level at which feed ingredients (the input) should be allocated to the cow in order to optimize the difference between the value of the milk sold (the output) and the cost of the inputs. The point at which this occurs will not necessarily coincide with the point at which the cow is producing at her maximum genetic potential (Street, 1975). The optimum feeding level is sensitive to both the cost of the inputs and the price realized by the output produced. In general, gross efficiency of milk production is greater in high-producing cows because a lower proportion of total costs is fixed.

There are biological aspects of dairying which have to be borne in mind when making input/output-type decisions as described above. Perhaps the most important of these is the carry-over effect within a lactation and from one lactation to the next, with regard to the level at which the cow is fed, and the resulting level of production and state of fertility. Despite the fact that a cut in the level of feeding may result in an apparent short-term improvement in the efficiency of milk production, the long-term effect on the potential of a cow must always be considered (Broster, Clements & Broster, 1985). This is particularly important during the first lactation, where the animal has a high nutrient requirement for growth.

Another feature of the dairy cow lies in the ability of the animal to increase feed intake together with a rise in production level, although this is not adequate to meet her total nutrient requirements. To overcome this problem, the nutrient density of the feed must be increased, which demands that better quality feed ingredients be offered to the cow. In addition, the proportion of crude fibre in the diet must be held constant (NRC, 1978). Alternatively, the total roughage component of the diet should not fall below 25% (Crabtree, 1985), in order to maintain proper rumen fermentation and milk-fat levels.

Linear Programming (LP) is a tool that may be used to produce least cost formulations for dairy cows, and enables the producer and the nutritionist to make decisions regarding nutritional strategy. Although this methodology has made a significant contribution to dairy feeding as a whole (Bath & Bennett, 1980; Crabtree, 1985), it is beset by many of the problems inherent in using any of the methods of formulating dairy feeds. These include the stochastic nature of the nutrient requirements of animals as well as the nutrient concentration of feed ingredients that are not taken into consideration (Black & Hlubik, 1980). Of more importance, however, is that LP is an over-simplified approach from an economic point of view (Whittemore, 1981). Meeting a fixed nutrient specification at least-cost does not necessarily imply that the economic objective, which is profit maximization, has been reached. Any approach which optimizes nutritional strategy must simultaneously consider the availability and cost of feed ingredients as well as the profit which will be realized from the sale of the product.

Profit maximization models have been developed for milk production (Bath & Bennett, 1980; Klein, Hironaka, Heller & Freeze, 1986; Hulme, Kellaway & Booth, 1986) in an attempt to consider simultaneously the costs of the input and the value of the output. The non-linear nature of factors such as nutrient density of the feed and changing milk prices, limits the usefulness of standard LP techniques. Only by carrying out repeated iterations of such a model is it possible, at least partially, to optimize nutritional strategy for a specific production situation.

The primary objective of this study was to develop a mathematical programming model, making use of Mixed Integer Programming (MIP), for the optimization of the nutritional strategy of any particular dairy cow. The model chooses the 'best' level of milk yield for an animal, in the light of the prevailing feed ingredient costs and milk price structure, so as to maximize profit. Secondly, the model was used to investigate various aspects of milk production in order to quantify some of the trends which occur in practice.

Description of the model

MIP is a subset of LP in which certain of the variables can be solved only as integers. Using an upper bound of 1 and a lower bound of 0 for these integer variables, it is possible to introduce an either/or capability into the model. Applying this methodology to a curvilinear function which has been broken up into linear segments, allows the model to select a point along a curve which will result in an optimal solution. Each 'block' of integer variables represents a particular function, and this can then be incorporated into a standard LP model. In this manner a range of possible milk outputs has been allowed for, and it has been included into a model which maximizes profit with regard to feed formulation, similar to that proposed by Dent & Casey (1967).

A suitable mathematical programme can evaluate biological data from an economic perspective. Thus a properly constructed model in this context will have two clearly defined components, namely economic and biological.

The economic component of the model may best be described in terms of its objective function:

Maximize profit $= pq - \Sigma \, Cj \, Xj$, where
p = price per kg of milk;
q = quantity sold;
Cj = cost per kg of the jth feed;
Xj = quantity of the jth feed.

The main economic variables of the model are the relative costs of the various feed ingredients, and the price realized by the sale of the milk. The costs of the various feed ingredients used in a study of this nature are critical. It is relatively easy to establish the cost of purchased feed ingredients, but it is difficult to accurately determine the costs of producing feed ingredients on the farm itself. The appropriate price to use, is one based on alternative use and value and not on the historic costs of production (Crabtree, 1983). Milk prices vary from one enterprise to another as determined by milk quality, buyer and factors such as the imposition of quotas in times of surplus production. The structure of the model, however, easily facilitates the inclusion of non-linear pricing structures.

The biological component of the model comprises the cow itself and the nutrient content of the feeds that are offered. In addition, a possible range of milk production between 10 and 45 *l* has been allowed for. In this study the 'Cedara' model (Jones & Stewart, 1987) was used to determine the nutrient requirements and dry matter intake of any cow under consideration at the various levels of milk production, although any preferred source of nutrient requirements could be used.

The nutrient composition of feed ingredients used in a model such as this is critical, and in practice specific values to the enterprise should be determined. For the purpose of this generalized model, however, standard values have been used (Bredon, Stewart & Dugmore, 1987). A facility exists within the model to constrain the consumption of each feed ingredient for reasons of palatability, availability or any other practical consideration.

The computational strategy employed by the model is best illustrated by referring to the simplified MIP model represented in Table 1.

Computer methods

In this study, the Functional Mathematical Programming System (FMPS) was used in a Sperry Univac Series 1100 computer. In particular, the Mixed Integer programming (MIP) operating module was used. Data input for the model was generated using the standard data format and input procedure for FMPS (Sperry Corporation, 1981). Sunset Software Technology of California has recently launched a product called XA (eXtended Application) which enables the model to be run on an IBM PC or compatible computer using the same data format as FMPS (Byer, 1987). The results obtained when using this package were identical to those

Table 1 Skeleton matrix for MIP model

Constraints	Feed ingredients: Dairy meal 20	Rye grass	Mineral suppl.	Milk output (Integer block): 15 *l*[d]	20 *l*	25 *l*	30 *l*	35 *l*	RHS
Dry matter	[a] −880	−1000	−880	17222	18133	18802	19300	19667	> 0
ME ruminant	[b] −11,4	−9,9		[e] 148,0	170,7	193,4	216,2	238,9	< 0
Digestible protein	−164,6	−159	1110	1370	1630	1890	2150	< 0	
Calcium	−10	−3,5	−200	77,1	90,8	104,5	118,2	131,9	< 0
Phosphorus	−6	−2,7	−120	56,4	65	73,6	82,2	90,8	< 0
Fibre	−35	−220		2239	2357	2444	2509	2557	< 0
Maximum concentrate	[c] 1								< 12,5
Maximum rye grass		1							< 6
Maximum milk output				[g] 15	20	25	30	35	< 45
Milk output restriction				[f] 1	1	1	1	1	= 1
Objective	[h] −40	−8,5	−50	675	900	1125	1350	1570	Maximize

[a] Total amount of dry matter which can be consumed on a daily basis must be entered as an independent variable. Increased amounts are allowed for at increased levels of production (Jones & Stewart, 1987).

[b] Nutrient values of each feed ingredient (Bredon *et al.*, 1987) used in the MIP, are entered in the same manner as in the profit maximization model described by Dent & Casey (1967).

[c] The necessity of placing an upper limit on the intakes of certain feed ingredients in the diet is well established. In this example an upper inclusion of concentrate has been set at 12,5 kg/cow/d while only 6 kg/cow/d of rye grass is available.

[d] An integer 'block', as described in text, is used to include different levels of milk output into the matrix of the MIP model.

[e] The nutrient requirement for each level of milk output is included in each variable of the integer 'block' (Jones & Stewart, 1987).

[f] An integer variable solves only as 1 or 0. Thus, by constraining the milk output equality to a value of 1, only a single level of milk output will be selected.

[g] It is possible to set an upper limit on the level of milk output.

[h] All prices in the model are quoted in cents.

Table 2 Prevailing feed ingredient costs used in this analysis

Feed ingredient	Cost (R/t)
Dairy meal 20% protein	400,00
Dairy meal 15% protein	371,00
Dairy meal 12% protein	354,60
Mineral supplement	500,00
Eragrostis hay	140,00
Maize silage	35,00
Rye grass pasture	85,00[a]

[a] Cost on a dry-matter basis.

Table 3 The model solution showing the standard solution as well as effect of increased rye grass availability

Variable		Rye grass availability (kg/cow/d): 0	2	4	6[a]	8
Dairy meal 20	(kg)	2,58	–	–	–	–
Dairy meal 15	(kg)	9,72	9,88	1,57	–	–
Dairy meal 12	(kg)	–	1,75	9,76	11,34	11,35
Eragrostis hay	(kg)	2,32	–	–	–	–
Maize silage	(kg)	18,00	18,00	14,97	9,23	3,49
Rye grass	(kg)	–	2,00	4,00	6,00	8,00
Optimum milk	(*l*)	31,00	31,00	31,00	31,00	31,00
Margin	(c/d)	836	876	905	910	913

[a] This is the standard situation used for comparative purposes.

obtained when using the Sperry Univac, although the PC was substantially slower.

Using the model

To demonstrate the effectiveness of the model, certain aspects of milk production were investigated. An initial standard situation was decided upon, using a 600 kg cow with a body condition score of 2,5 (Mulvany, 1977). It was assumed that the cow was genetically capable of producing milk in the range 10 to 45 *l*/d, with a milk-fat content of 3,5 g/kg. The feed ingredient prices used are given in Table 2, and an average milk price of 45 c/*l* was used.

An upper limit on concentrate intake was set at 12,5 kg/d, the maximum availability of maize silage was set at 18 kg on an as-is basis, whereas 6 kg of rye grass on a dry-matter basis was offered. This standard situation and its solution (Table 3) was then used as a basis for comparison in the exercises which follow.

Availability of farm-grown feed ingredients

The amount of farm-produced feed ingredient was varied so as to investigate the importance of this commodity. The availability of rye grass was increased from 0 to 8 kg/cow/d on a dry-matter basis. The profitability of an enterprise appears to increase as good quality farm-grown fodder becomes more available (Table 3). The manner in which the cost effectiveness of the material diminishes as it becomes more freely available is of interest. This would suggest that an even distribution of rye grass available to a herd of cows would result in profits being increased, rather than restricting this material for the use of selected animals only. The decreased inclusion of concentrate together with the decrease in the protein content of that concentrate as rye grass availability was increased, may also decide the level of overall profitability.

Concentrate availability and price

The amount of concentrate which should be fed to a lactating cow is a topic which is heatedly debated. By changing the amount of concentrate available to the animal from 0 through to 12,5 kg/cow/day, the economic consequences of 'feeding off grass' were examined. In all examples a mineral supplement was offered. It can be seen (Table 4) that the optimum level of milk production increased steadily as more concentrate was made available. More important, however, was that the margin per cow increased over this range. Concentrates allowed the correct balance of protein to energy to be restored to the complete diet. Furthermore, it enabled the high-producing cow to more easily meet her requirement for all nutrients.

In an allied example, the effect on feeding strategy of changing the concentrate price was examined. To simulate these changes, the standard costs (Table 2) were altered by either adding or subtracting increments of R100,00. As the cost of concentrates declined (Table 4), the optimum level of milk production remained constant while the overall margin increased. It is of interest to note that even when concentrate prices were increased dramatically, the change in optimum production level was small.

Table 4 The effect of differing concentrate availability and price on optimum levels of production and margin over feed cost

Concentrate			
Availability (kg/d)	Cost	Milk production (*l*/d)	Margin over feed (c/d)
0	Standard	16	541
4	Standard	24	784
8	Standard	28	881
12,5	Standard	31	910
12,5	− R100	31	1023
12,5	− R200	31	1137
12,5	+ R100	29	808
12,5	+ R200	28	733

The milk price

In the final exercise of this study the model was run using a range of milk prices. From Table 5 it can be seen that at relatively low milk prices a reduction in the optimum level of milk output occurred but that beyond a certain point (40 c/*l*) no changes occurred in terms of the feeding strategy.

Table 5 The effect of a change in milk price on the optimum nutritional strategy

Milk price (c/*l*)	Optimum milk output (*l*/d)	Margin over feed (c/d)
25	28	321
35	29	604
40	31	755
45	31	910
85	31	1530

Conclusion

The MIP model as demonstrated here would be of little value in determining day to day feeding regimes for dairy cows as it only considers the cow during a single day of her lactation. It is useful, however, in that it can be used to test the sensitivity of the optimal nutritional strategy for a cow to changes in the price and availability of feed ingredients. In addition, the model can be used to examine the effect of any changes in the milk price structure. Clearly, the results obtained from a single cow, or several representative cows from a herd, could be extrapolated to determine the 'best' nutritional strategy for an enterprise.

The model served to illustrate the importance of producing as much farm-grown material as possible, as this has a marked effect on the profitability of any milk production operation. It also demonstrated that the exclusion of concentrates from a feeding regime, even if they are costly, is unwise as comparatively small amounts of concentrate have dramatic effects on the profit of any enterprise.

The level of milk production for which feed should be offered may be reduced under certain circumstances. In many cases the level of production at which this is economically justified, is above the genetic potential of the cow, and will have little or no bearing on the average producer.

Each specific milk production enterprise has a unique set of circumstances, and the optimum nutritional strategy is affected by many factors. It is believed that the model presented here could serve as a tool in the determination of these optima under a wide range of conditions.

Acknowledgements

The authors express their appreciation to Dr P.R. Barrowman and Mr N.M.C. Schoeman for valuable comment during the preparation of this manuscript.

References

BATH, D.L. & BENNETT, L.F., 1980. Development of a dairy feeding model for maximizing income above feed costs with access to remote computer terminals. *J. Dairy Sci.*, 63, 1379—1389.

BLACK, J.R. & HLUBIK, J., 1980. Basics of computerized linear programs for ration formulation. *J. Dairy Sci.*, 63, 1366—1378.

BREDON, R.M., STEWART, P.G. & DUGMORE, T.J., 1987. Nutritive value and chemical composition of commonly used South African farm feeds. Natal Region, Department of Agriculture and Water Supply.

BROSTER, W.H., CLEMENTS, A.J. & BROSTER, V. J., 1985. Multiple lactation feeding of the dairy cow. *World Review of Animal Production,* 21, 62—69.

BYER, J.R., 1987. Personal communication. Sunset Software Technology, 1613 Chelsea Road, Suite 153, San Marino, California 91108.

CRABTREE, J.R., 1983. Interactive formulation system for cattle diets. Agricultural Systems, 8, 291—308.

CRABTREE, J.R., 1985. Computer diet formulation and feed evaluation. *Outlook on Agriculture*, 14, 104—108.

DENT, J.B. & CASEY, H., 1967. Linear programming and animal nutrition. Crosby Lockwood, London.

HULME, D.J., KELLAWAY, R.C. & BOOTH, P.J., 1986. The CAMDAIRY model for formulating and analysing dairy cow rations. *Agric. Syst.*, 22, 81—108.

JONES, R.I. & STEWART, P.G., 1987. The Cedara Program. Cedara Agricultural Research Institute.

KLEIN, K.K., HIRONAKA, R., HELLER, C.H. & FREEZE, B.S., 1986. Profit-maximizing linear program model for dairy cattle. *J. Dairy Sci.,* 69, 1070—1080.

MULVANY, P., 1977. Dairy cow condition scoring. Paper No. 4468. NIRD, Shinfield, Reading RG2 9AT, UK.

NRC, 1978. Nutrient requirements for dairy cattle. National Academy of Sciences, Washington, DC.

SPERRY CORPORATION, 1981. Sperry Univac Series 1100: Functional Mathematical Programming System (FMPS Level 9R1), Programmer Reference.

STREET, P.R., 1975. The application of business management methods to the nutrition of livestock. In: Economic factors affecting egg production. Eds. Freeman, B.M. & Boorman, K.N., British Poultry Science, Edinburgh, pp. 55—70.

WHITTEMORE, C.T., 1981. Animal production response prediction. In: Computers in animal production. Eds. Hillyer, G.M. *et al.*, Thames Ditton, BSAP, pp.47—63.

WILLIAMS, C.B., OLENACU, P.A., BRATTON, C.A. & MILLIGAN, R.A., 1987. Effects of business and dairy herd management practices on the variable cost of producing milk. *J. Dairy Sci.* 70, 1701—1709.