

BOOK REVIEW

Risk Modelling in General Insurance: From Principles to Practice

by Roger J. Gray and Susan M. Pitts. Cambridge University Press, 2012

The field of actuarial science has seen many changes over the past few years. We live in a world whose complexity becomes more apparent every day. From the well-known, such as natural catastrophes, to the esoteric, such as satellite launch coverage or cyber-terrorism, actuaries are called upon more often to provide their principals with reasonable estimates of potential profit and loss exposure. The main tool of the actuary in all these cases is the ability to create a mathematical model of real-world events and, knowing the model's limitations, to use the model to develop a range of reasonable estimates of results. In this light, it is a welcome event to have a new text on the basics of risk modelling for general insurance. The text is structured so that each section adds more complexity and utility to the one before, and each is followed by a set of questions. There are six expository sections, a seventh section containing case studies that tie the subjects taught throughout the book together. The book concludes with two appendices—one discussing utility theory and Jensen's inequality, the other containing answers to selected questions.

Two beneficial features of the text appear at its outset. In chapter 1, the authors begin with a description of the notation that will be used throughout the book, which also serves as a quickly-accessible glossary. The authors also have made liberal use of the free and open-source statistical package R throughout the book, allowing the reader to have immediate hands-on feedback on the material in question.

Chapter 2 begins the topic of risk modelling. The authors start with classic frequency distributions: Poisson, negative binomial, and binomial, both describing the distributions and showing how to use R to generate random variates and summary statistics. A minor quibble would be that, in their definition of a Poisson process on page 15, they list the three requirements as independence, stationarity and Poisson distribution, where most texts (e.g. DeGroot, 1986: 255) define the third condition as that of magnitude—the probability of more than one event in a small interval is negligible in comparison with the probability of one event in the same interval—and that the probability distribution being Poisson is an outcome, not a requirement. The authors next discuss various severity distributions, some thin-tailed reference distributions and other heavier-tailed distributions commonly

used in actuarial risk modelling such as the Pareto and the lognormal, among others, once again demonstrating both the mathematics and the practical use of R through many examples. As an aside, despite being an American myself, this was the very first time I saw reference to an “American” Pareto. Interestingly, shortly after reviewing this chapter, I found the name explained in the most recent edition of *Contingencies*, the magazine of the American Academy of Actuaries.¹ I commend the authors for their clarification, as it is unquestionably true that there are many flavours of the Pareto, often referred to with confusing or contradictory names. For example, the generalised Pareto is either a two- or three-parameter distribution depending on whether the discussion is centred in the world of extreme-value theory (McNeil, 1997) or introductory actuarial science (Klugman, Panjer & Willmot, 1998:56).

Chapter 2 continues with an introduction to Bayesian mixtures of distributions and how they can underlie well-known distributions. The chapter concludes with an introduction to the fitting of distributions to data, with emphasis on maximum-likelihood estimation and various goodness-of-fit tests. I am somewhat surprised that the authors do not include any information criteria, such as the Akaike information criterion, in their goodness-of-fit measures.

Chapter 3 introduces aggregate loss modelling, and starts with the simple compound risk model. The authors continue by demonstrating how moments alone cannot always determine compound models, and introduce convolutions of distributions, both directly and using moment-generating functions. Discussion of aggregate loss distributions continues through the use of finite mixtures and compound distributions. The authors continue with two numerical methods for fitting compound distributions, the Panjer recursion algorithm and a fast Fourier transform technique, and various approximations to the compound model, showing plenty of R code, which the reader can follow. The authors conclude their explanation of the compound model by discussing statistical properties of the estimates of the parameters of compound distributions, including the calculation of the variance of both moment and distributional statistics either direct or through the use of bootstrapping.

The authors then switch their attention to the individual risk model, briefly discussing its properties and methods for its estimation. Chapter 4 builds on the previous chapter by discussing a primary direct insurer’s need to calculate premiums given existing loss models. The chapter begins by introducing six basic ‘premium principles’ and some desirable properties of any such principle. After a brief explanation of minimum and maximum premiums in the context of utility theory, the authors tackle a mainstay of actuarial analysis—credibility theory. This section begins with a discussion of Bayesian estimation, including some of the classic prior–posterior pairs, such as the beta–binomial and the Poisson–gamma. The authors next explain Bayesian credibility using the classic prior–posterior pairs, and conclude the explanation of credibility with the empirical Bayesian credibility frameworks of Bühlmann and Bühlmann–Straub. The chapter finishes with “recipes” in R for calculating Bühlmann–Straub credibility.

1 Halliwell, LJ (2012). Probability and the 80–20 Rule. *Contingencies* 24(6), 66–69

Chapter 5 introduces the reinsurer and its effects on a primary company's results. The chapter begins with excess-of-loss reinsurance and its effect on individual and aggregate claims. It continues with the effects of quota-share (proportional) reinsurance and policyholder deductibles, and excess-of-loss retention levels. The remainder of the chapter is devoted to the optimisation of the reinsurance agreement. The authors demonstrate different optimisation criteria, including the maximisation of expected utility (which always has the drawback of quantifying the utility function itself), the minimisation of the variance of various loss-based statistics, and minimising the variance of the net profit.

The text becomes heaviest in chapter 6, where the authors discuss ruin theory. Building on the construction of aggregate loss models, they develop the classical risk model introducing premium inflow, reserves, safety loading, and the definition of the probability of survival and ruin, subject to a number of simplifying assumptions. The chapter then takes a deeper dive into the underlying principles of ruin theory, including a number of proofs.

The authors prove various results about the probabilities of ruin and survival, and then use Laplace transforms to develop a compound geometric representation of those probabilities. After some discussion of the asymptotic behaviour of the probability of ruin as a function of the initial reserves, the authors conclude the chapter with numerical methods, complete with associated R code, for calculating estimated probabilities of ruin and associated descriptive statistics.

The text concludes in chapter 7 with three case studies that tie the material together. These examples are comprehensive, with step-by-step descriptions and the R code necessary for the calculations. This serves as a good way for the student to see a more complete view of general-insurance risk and profit modelling.

On the whole, the writing style of the authors is clear, especially for a technical work. The layout is clean, with formulae typeset with plenty of surrounding white-space which makes it much easier for the reader. While much of the work does not lend itself to graphics, they are present when necessary and serve to help explain the material. The authors have done a very good job of creating a general-insurance risk modelling text that can serve as both an introduction for the interested student and as a general reference for the experienced actuary.

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References

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