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A COMPARISON OF PROBABILITY OF RUIN AND EXPECTED DISCOUNTED UTILITY AS OBJECTIVE FUNCTIONS FOR CHOOSING A POST-RETIREMENT INVESTMENT STRATEGY

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ABSTRACT
Individuals in defined-contribution retirement funds currently have a number of options as to how to finance their post-retirement spending. The paper considers the ranking of selected annuitisation strategies by the probability of ruin and by expected discounted utility under different scenarios. 'Ruin' is defined as occurring when income falls below a given threshold, but does not relate to the extent of that deficit. If there is insufficient money to buy an inflation-linked annuity at retirement, then the minimisation of the probability of ruin tends to result in living annuities with a high equity content. This is because the objective function does not reflect the extent of shortfall of income or the investor's level of risk aversion. The authors argue that this is a limitation to using the minimisation of the probability of ruin. Expected discounted utility may be more difficult to apply in practice, because of the complexity of explaining the approach to investors and the need to estimate a greater number of parameters explicitly. The authors argue that the use of expected discounted utility is, however, likely to be more representative of most investors' perception of risk, and illustrate its use by applying an extended discounted utility model that caters for the bequest motive and different reference income levels.

KEYWORDS
Annuities; income drawdown; ruin theory; discounted utility

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1. INTRODUCTION

1.1 BACKGROUND

1.1.1 Members of defined-contribution retirement funds face a number of options as to how to finance their post-retirement spending (Emms, 2010) and may rely on themselves as opposed to financial advisors when making this decision and other decisions relating to retirement planning (Mitchell et al., 1999). The decision as to how to finance post-retirement spending involves a number of risks including that of outliving income (Milevsky & Robinson, 2000; Albrecht & Maurer, 2002), being unable to support a desired consumption level (Levitan, Dolya & Rusconi, unpublished; Emms, 2010) or choosing sub-optimal strategies (Sweeting, 2009).

1.1.2 In South Africa, the risk of inappropriate decumulation leading to old-age poverty has been cited by National Treasury as a reason for the proposed reform of the annuities market.[1] This reform may affect all retirement-fund members. Currently, pension-fund members must annuitise at least two-thirds of their accumulated wealth at retirement as opposed to taking benefits as a cash lump sum.[2] National Treasury[3] has suggested that provident-fund members, who are currently not compelled by the Income Tax Act[4] to annuitise, may have to do so in future. In this context, the term 'annuities' includes life annuities as well as income-drawdown accounts, commonly known as 'living annuities' in South Africa.

1.1.3 National Treasury has put forward two proposals on the decumulation phase. The first was made in May 2012 and proposed the introduction of "standardised products into which retirement funds can automatically place members when they retire, without requiring financial advice".[5] If the placement is to be made by trustees, the requirement that trustees must act in the "best interest of all members"[6] would apply. This may imply that trustees would need to identify the optimal placement decision for members and may be jointly and individually liable should a member be able to prove that an inappropriate decision was made negligently on the member's behalf.

1 National Treasury. Strengthening Retirement Savings: An overview of proposals announced in the 2012 Budget. Discussion Paper, 14 May 2012
2 Income Tax Act (Act 58 of 1962), as amended
3 National Treasury, supra
4 supra
5 National Treasury, supra
6 Financial Services Board. Circular PF No. 130: Good Governance of Retirement Funds. Financial Services Board, Pretoria, 2007

1.1.4 The second proposal was made in September 2012 and clarified that fund credits of less than R150 000 would be exempt from annuitisation. For larger fund credits, a maximum of one-third could be taken in cash. The balance would be used to purchase an annuity. For the fund credit less than R1 500 000 this may be a prescribed life annuity but for larger credits, the retiree would have some choice as to the annuity purchased.[7]

1.1.5 Under both proposals, it is unclear how these defaults will be chosen or what tools will be made available to stakeholders choosing life annuities or living-annuity strategies for themselves or others. These decisions relating to annuities and living annuities are termed the 'annuitisation decisions' for the purposes of this paper.

1.1.6 Statistical and mathematical models, such as ruin-theory models (Milevsky & Robinson, 2000; Albrecht & Maurer, 2002; Levitan, Dolya & Rusconi, unpublished) or discounted-utility models (Yaari, 1965; Mitchell et al., 1999) can be used to guide the annuitisation decision. Thomson (2003a; 2003b) has proposed the use of expected-utility-theory models to aid decision-making in South African defined contribution funds.

1.2 PROBLEM STATEMENT AND AIM

1.2.1 The aim of this paper is to document the findings of investigations into:

- the highest ranking annuitisation strategies under various circumstances and objective functions;
- whether different objective functions would suggest different annuitisation decisions for members; and
- the sensitivity of the preferred annuitisation decision to the parameters used.

1.2.2 As the use of probability of ruin or utility theory to determine annuitisation strategies is a practical problem, the authors have focused on the results that these two approaches would yield under scenarios that are likely to occur in retirement funds.

1.3 CONTRIBUTION TO KNOWLEDGE

1.3.1 To the authors' knowledge, the literature on the annuitisation decision itself in the South African context is limited to a 2010 conference paper by Levitan, Dolya & Rusconi (unpublished). In addition, Lodhia & Swanepoel (unpublished) compared living and life annuities in 2012, based on deterministic assumptions.

1.3.2 Furthermore, although there are several international studies on the annuitisation decision, there is a lack of literature on the comparison of the results under different classes of models in the context of annuitisation. Bayraktar & Young (2007) considered income drawdown accounts and the optimal borrowing and lending behaviour of investors under ruin theory and discounted-utility metrics and found convergence between models given a known date of death. However, this result is unhelpful given uncertain life expectancies and, in addition, their discussion ignored life annuities.

7 National Treasury. Enabling a Better Income in Retirement: Technical Discussion Paper B for Public Comment. Discussion Paper, 21 September 2012

1.3.3 This paper is expected to contribute to the understanding of the dynamics around the annuitisation decision, based on stochastic simulations and varying key parameters, at a time when reform proposals may require such understanding by policy-makers, actuaries and trustees. The emphasis on the application of these approaches, as opposed to their theoretical derivation, is expected to assist in this regard.

2. THE RISKS AND CONSIDERATIONS INVOLVED IN THE ANNUITISATION DECISION

2.1 The literature suggests that the annuitisation decision involves consideration of a number of different risks as well as consideration of the bequest motive.

2.2 Longevity risk, or the risk of outliving available funds before death, has been cited as a key risk by Blake, Cairns & Dowd (2003), Milevsky & Robinson (2000) and Albrecht & Maurer (2002). National Treasury[8] has highlighted this as a key risk of living annuities.

2.3 In the context of annuitisation, liquidity risk arises because the income stream becomes inflexible on the purchase of a life annuity (Milevsky & Robinson, 2000; Albrecht & Maurer, 2002; Sexauer, Peskin & Cassidy, 2012). The elderly require flexible incomes in order to fund sudden and possibly prolonged increases in consumption during retirement arising from health shocks (Albrecht & Maurer, 2002; Murtaugh, Spillman & Warshawsky, 2001).

2.4 Inflation risk, which refers to the risk that expenditure will rise faster than income, is considered critical to the annuitisation decision by Mitchell et al. (1999).

2.5 The bequest motive is often considered a critical assumption when modelling optimal annuitisation strategies (Yaari, 1965; Albrecht & Maurer, 2002; Sweeting, 2009). However, studies by Hurd (1987) and Shefrin & Thaler (1988) have called into question whether the bequest motive really exists. In addition, Brown (2001) suggested that self-reported bequest motives are not necessarily consistent with the annuitisation strategies that are selected.

2.6 Sweeting (2009) cited the importance of the tax and regulatory regimes in influencing the annuitisation decision.

2.7 Hence the literature suggests a number of competing considerations are relevant when the individual annuitises. Life annuities hedge longevity risk but introduce liquidity risk. Life annuities may not explicitly allow for bequests, although they may implicitly provide for bequests via the guarantee period. Inflation risk may be hedged to a greater

8 Strengthening Retirement Savings, supra

or lesser degree under different living-annuity and life-annuity products (Levitan, Dolya & Rusconi, unpublished).

3. DISCOUNTED UTILITY AND RUIN-THEORY APPROACHES TO PREFERENCES

The bulk of the literature on the optimisation of annuitisation uses utility maximisation or ruin probabilities, although Sweeting (2009) considered a risk–return trade-off. It is, however, noted that quadratic utility is a sufficient condition for analysis in Markowitz-space (Thomson, 2003b) and, hence, Sweeting's analysis is consistent with discounted utility approaches, albeit with a somewhat limiting utility function (Thomson, 2003b). The two dominant approaches are described below.

3.1 DISCOUNTED UTILITY

3.1.1 When considering different post-retirement strategies, investors need to make choices regarding the value of different benefits at different times. Samuelson (1937) proposed the expected-discounted-utility model as a means to examine choices over time, where utility measures the degree of human satisfaction offered by a specific outcome (Fishburne, 1968). The typical implementation of the model may be decomposed into the following components:

– an instantaneous utility function, which examines how the investor values different quantities of money or goods; and
– a discount factor, which accounts for the investor's preference to obtain money or goods sooner rather than later.

3.1.2 Under expected-discounted-utility models, the preferred annuitisation strategy is the strategy that maximises expected discounted utility. However, Samuelson himself stated that it was "completely arbitrary" to assume that individuals would behave so as to maximise expected discounted utility (Samuelson, 1937: 159). Thomson (2003a), however, asserts that even if expected discounted utility does not describe actual behaviour, it is valid as a normative theory. In other words, it can be used to describe ideal behaviour given certain axioms. The authors agree with this view and hence use the results of ranking strategies by expected discounted utility as a touchstone against which to measure the results of the approach based on ruin probability.

3.1.3 The six fundamental axioms described in Thomson (2003a) can be described, in the context of this research, as follows:

– Individuals can specify a preference between any two annuitisation options or income streams and bequests.
– Preferences are transitive. In other words, if a first annuitisation option is preferred to a second and that the second option is preferable to a third, the first option must also be preferable to the third option. This means that rankings are sensible.
– Where outcomes are uncertain, the overall probability of an outcome is important, not the probability of the series of events leading to that outcome.

– If an individual is indifferent between two annuitisation options, they must be indifferent between the outcomes associated with the annuitisation options.
– If offered a choice of two annuitisation options, the annuitant will favour the one with a greater probability of favourable outcomes and a lower probability of worse outcomes.
– If an individual prefers an income and bequest stream that is certain to one that is uncertain, there must be some fraction that can be applied to scale down the guaranteed income and bequest stream so that the individual is indifferent between the two options.

3.1.4 There are both practical and theoretical difficulties with discounted utility. Firstly, the practical elicitation of utility functions is in itself a non-trivial task. Thomson (2003b) raises the issue of framing bias, which is concerned with how the way questions are phrased during elicitation can influence the results. In addition, Thomson (2003b) suggests there are at least four different theoretical approaches to establishing preferences. Bayraktar & Young (2007) concur that expected-discounted-utility models are difficult to parameterise. A second practical issue is that of intelligibility. Bayraktar & Young (2007) state that the probability of ruin is objective and hence ruin-theory models may be easier for individuals to understand, while utility functions are by definition highly subjective. Furthermore, Thomson (2003a) indicates that there have been challenges to the descriptive validity of expected-utility theory.

3.1.5 In terms of theoretical difficulties, Samuelson (1937) comments that the constant discount rate is unrealistic. Frederick, Loewenstein & O'Donoghue (2002) provide a detailed discussion of the shortcomings of assuming a constant discount rate and constant instantaneous utility function. These shortcomings include:
– the assumption that there is no interdependence between the utility experienced in each period, implying that the total discounted expected utility is merely the sum of the expected utility in each period;
– the assumption that investors' preferences will not change with time and that instead one 'cardinal instantaneous utility function' applies at all time horizons;
– the assumption that the discount rate is constant, irrespective of what is being consumed, whereas investors' level of preference for consumption at different times may be different for critical medical care than for candy; and
– the assumption of a constant discount rate, which does not reflect the hyperbolic discounting observed in many studies, including Thaler (1981) and Benzion & Yagil (2002).

3.1.6 The shortcomings outlined may suggest that the expected-discounted-utility model with a constant discount factor and instantaneous utility function is inadequate to realistically capture human preferences. Frederick, Loewenstein & O'Donoghue (2002) collected the results of forty studies into the estimation of an annual discount rate. The results vary from negative values to infinite positive values.

3.1.7 Despite these theoretical and practical challenges, expected discounted utility is the most common optimisation criterion in the literature (Bayraktar & Young, 2007).

3.2 RUIN THEORY

3.2.1 Ruin theory involves the consideration of the probability of entering ruin, which could be defined in various ways. Bayraktar & Young (2007) point out the application of ruin theory both to insurers and other corporate institutions and individuals saving for retirement. Ruin theory might be used to determine optimal strategies for an individual when that individual wishes to avoid running out of funds as their primary objective (Bayraktar & Young, 2007).

3.2.2 Milevsky & Robinson (2000) considered the lifetime probability of ruin ('LPoR') which is the probability of depleting wealth fully before death and the eventual probability of ruin ('EPoR') which is the probability of the wealth ever being depleted ignoring mortality. Albrecht & Maurer (2002) considered an LPoR measure. Levitan, Dolya & Rusconi (unpublished) considered a slightly modified LPoR by considering the possibility that consumption falls below a threshold level before death.

3.2.3. The preferred strategy under ruin theory is that which minimises the probability of ruin. Bayraktar & Young (2007) have suggested that minimising the probability of running out of funds is intuitively more appealing than maximising utility, because a probability is more objective than a value calculated using a subjective utility function. However, it is worth noting that ruin theory requires stochastic simulations, which may involve subjectively parameterised models, and hence ruin theory is not strictly objective. In addition, the threshold chosen for ruin can be very subjective. The results in section 6 show clearly that the results are extremely sensitive to this threshold level.

4. PREFERRED STRATEGIES IDENTIFIED IN THE LITERATURE

4.1 EXPECTED DISCOUNTED UTILITY

4.1.1 Expected-discounted-utility models found in the literature suggest a range of results depending on the level of risk aversion of the individual and the bequest motive.

4.1.2 Yaari (1965) explored how a rational retiree would seek to maximise utility given a starting level of wealth and the constraint that the asset value at death must be non-negative. Yaari (1965) found that in the absence of a bequest motive, the rational strategy would be to annuitise fully, as opposed to investing in an income drawdown account.

4.1.3 Blake, Cairns & Dowd (2003) were able to explore the annuitisation problem more fully given advances in computing and new investment products. Blake, Cairns & Dowd (2003) considered three annuitisation options:

– a level life annuity;
– an equity-linked annuity with a level life annuity purchased at an older age; and
– an equity-linked drawdown account with a level life annuity purchased thereafter.

The equity-linked life annuity provides an investment-linked income together with mortality credits to hedge against mortality risks, whereas the equity-linked drawdown account does not provide mortality credits.

4.1.4 Blake, Cairns & Dowd (2003) found that life annuities are preferable for risk-averse pensioners and that income drawdown accounts are more suitable for pensioners with lower risk aversion. However, the level of equity exposure in the equity-linked drawdown account and equity-linked annuity may be an even more important decision than the choice between the life annuity and the drawdown account.

4.2 RUIN THEORY

4.2.1 Much of the literature on ruin-theory models in the context of annuitisation considers the optimal asset allocation in the income drawdown account as opposed to the balance between life annuities and income drawdown accounts. Milevsky & Robinson (2000) and Albrecht & Maurer (2002) considered the asset allocation problem in the Canadian and German contexts respectively.

4.2.2 Milevsky & Robinson (2000) and Albrecht & Maurer (2002) both considered the complete exhaustion of funds in an income drawdown account. Albrecht & Maurer (2002) used drawdown rates set with reference to income that could be earned under a with-profit life annuity while Milevsky & Robinson (2000) used an arbitrary drawdown rate.

4.2.3 Milevsky & Robinson (2000) found that females had much higher probabilities of ruin than males and each sex had a different optimal investment strategy, although all investors benefitted from diversification. Albrecht & Maurer (2002) similarly found that the ruin probability was minimised by holding a diversified portfolio. Higher post-retirement interest rates, and hence lower initial drawdown rates, were associated with lower exposure to growth assets (Albrecht & Maurer, 2002).

4.2.4 Levitan, Dolya & Rusconi (unpublished) used ruin theory to explore the trade-offs between life annuities and income drawdown accounts with various investment strategies. Ruin was defined as the contingency that income falls below the level required to sustain a desired level of spending and hence the drawdown rates were set according to this expenditure level (Levitan Dolya & Rusconi, unpublished). The annuitisation decision involved consideration of four strategies:

– a life annuity level in nominal terms;
– a life annuity increasing at 3% a year;
– an inflation-linked life annuity; and
– an income drawdown facility.

4.2.5 Four levels of equity exposure in the income drawdown account were tested, namely 0%, 25%, 50% and 75%, the balance of the assets being invested in conventional fixed-interest instruments.

4.2.6 Levitan, Dolya & Rusconi (unpublished) found that the results were very sensitive to the ratio of the accumulated credit to the annual income requirement.

4.3 SENSITIVITY OF ANNUITISATION PREFERENCES TO VARIOUS PARAMETERS

The literature considered thus far suggested that the optimal annuitisation decision is dependent on many parameters, such as the level of initial wealth, bequest motive, utility function, mortality and other sources of wealth. These parameters are discussed in turn in the rest of this paper.

4.3.1 Income Preferences Relative to Initial Wealth

4.3.1.1 Albrecht & Maurer (2002), Blake, Cairns & Dowd (2003) and Emms (2010) found that higher income preferences lend themselves to higher equity exposure, given that an income drawdown account is purchased at retirement.

4.3.1.2 Levitan, Dolya & Rusconi (unpublished) found that where the accumulated wealth was sufficient to secure an inflation-linked annuity, this was the most preferable strategy. However, if this was unaffordable, the income drawdown accounts provided lower ruin probabilities than the life annuities. The higher the income requirement relative to the capital available at retirement, the higher was the equity component required in the income drawdown account to be expected to meet the income requirement. This result was consistent with Albrecht & Maurer (2002), Blake, Cairns & Dowd (2003) and Emms (2010).

4.3.2 The Bequest Motive

4.3.2.1 Although low purchase rates of life annuities are often attributed to the bequest motive (Davidoff, Brown & Diamond, 2005), Yaari (1965) suggested that if annuities are available individuals can separate the bequest and consumption motives. In other words, the bequest motive can be accommodated via a cash withdrawal at retirement as opposed to influencing the annuitisation decision. Davidoff, Brown & Diamond (2005) similarly established that most mathematical models of the bequest motive fail to explain low rates of life-annuity purchases.

4.3.2.2 An important result from Blake, Cairns & Dowd (2003) was that the choice of annuity product was not sensitive to the bequest motive. However, the strength of the bequest motive may influence how much should be held in equities post-retirement and the age at which the individual eventually purchases a life annuity. Greater bequest motives suggest a later age at which a life annuity is purchased after investment in an income drawdown account, as well as a higher equity exposure in the drawdown account (Blake, Cairns & Dowd, 2003). The latter was also concluded by Emms (2010).

4.3.3 Utility Function and Relative Risk Aversion

4.3.3.1 Blake, Cairns & Dowd (2003) suggested that whether an exponential or power utility function was used was less important than the value of the relative risk aversion (RRA) parameter. They tested the annuitisation preferences under a range of RRA parameters, varying from 0,25 to 25. Investors with RRA parameters below 1,25 (described as having a ‘very strong appetite for risk’) were found to prefer equity-linked annuities with 100% equity exposure. For an investor with RRA parameters greater than 1,25 and less than 10, equity-linked annuities were still preferable to annuities but the equity exposure decreased with increasing risk aversion. For ‘more risk averse plan members’ with RRA parameters greater than 10, life annuities were found to be preferable.

4.3.3.2 These results are consistent with Sweeting (2009), who considered risk–return trade-offs, where the return criterion was the pension in excess of what could be earned on a decumulation strategy of a compulsory-purchase fixed annuity and the risk criterion was the value at risk. Sweeting (2009) found that for relatively low levels of risk aversion an income drawdown account converting to a life annuity later in retirement was preferable.

4.3.4 Mortality

Blake, Cairns & Dowd (2003), Albrecht & Maurer (2002) and Levitan, Dolya & Rusconi (unpublished) all considered only male lives. However, results from Milevsky & Robinson (2000) suggest that female lives, with lighter mortality and longer life expectancies than males in general, should have higher equity allocations than shorter-life males. Blake, Cairns & Dowd (2003) tested their results for impaired lives and found, unsurprisingly, that impaired lives may benefit from strategies that accelerate the payment of their benefits.

4.3.5 Other Wealth

Blake, Cairns & Dowd (2003) also found that neither the level of non-retirement-fund wealth nor the introduction of a fixed State pension significantly influenced the preferred annuitisation strategy. Although the former result may seem surprising, non-retirement assets may consist largely of illiquid assets such as property which would not influence income levels, under the assumption that equity release structures are not entered into.

5. METHODOLOGY

Whilst the expected-discounted-utility approach has well–documented short-comings, it is mathematically tractable, has been used widely and arguably has an intuitive appeal. It does offer an interesting basis of comparison against the ruin-theory approach, which has been used widely in recent literature on annuitisation. The authors are also of the opinion that ruin theory enjoys far greater use in South African post-retirement investment strategy planning, whereas expected discounted utility is perceived as being of limited practical use. The authors hence believe there is value in comparing the results obtained using the two approaches. A number of annuitisation strategies and scenarios were considered, which are set out in sections 5.1 and 5.2 respectively. Section 5.3 sets out how the income levels under each strategy were evaluated. The discounted-utility and ruin-theory models are set out in sections 5.4 and 5.5 respectively.

5.1 ANNUITISATION STRATEGIES

5.1.1 The attractiveness of different annuitisation strategies was assessed under the expected discounted utility and probability of ruin as objective functions. Eleven strategies were considered and are summarised in Table 1. The list of strategies is not exhaustive and can be expanded in future research.

5.1.2 For simplicity the risk that the insurer will default is ignored.

5.1.3 The life annuities all included a 75% spouse's reversion, which means that any surviving spouse will receive 75% of the prevailing income when the principal pensioner dies. This would allow the widowed spouse to meet his or her reduced variable costs of living, as well as bear the fixed costs associated with running a household that do not reduce when the principal pensioner passes away. The life annuities also included a ten-year guarantee period, so annuity payments continue for at least ten years after purchase.

Table 1. Annuitisation strategies considered

Strategy description	Abbreviation
Purchase a level annuity at retirement, where payments remain constant.	Level
Purchase a fixed-escalation annuity, where payments increase by 5% each year. The 5% figure was chosen to be slightly above the middle of the 3%–6% inflation target band used by the South African Reserve Bank	Fixed 5%
Purchase an inflation-linked annuity at retirement, where payments increase by the inflation rate each year.	IL
A living-annuity strategy annuitising into an inflation-linked annuity at age 75. Before annuitisation, the portfolio has a 100% allocation to fixed-interest instruments.	LwA 0/100
A living-annuity strategy annuitising into an inflation-linked annuity at age 75. Before annuitisation, the portfolio has a 25% allocation to equities and a 75% allocation to fixed-interest instruments.	LwA 25/75
A living-annuity strategy annuitising into an inflation-linked annuity at age 75. Before annuitisation, the portfolio has a 50% allocation to equities and a 50% allocation to fixed-interest instruments.	LwA 50/50
A living-annuity strategy annuitising into an inflation-linked annuity at age 75. Before annuitisation, the portfolio has a 75% allocation to equities and a 25% allocation to fixed-interest instruments.	LwA 75/25
A living-annuity strategy without annuitisation, the portfolio has a 100% allocation to fixed-interest instruments.	L 0/100
A living-annuity strategy without annuitisation, the portfolio has a 25% allocation to equities and a 75% allocation to fixed-interest instruments.	L 25/75
A living-annuity strategy without annuitisation, the portfolio has a 50% allocation to equities and a 50% allocation to fixed-interest instruments.	L 50/50
A living-annuity strategy without annuitisation, the portfolio has a 75% allocation to equities and a 25% allocation to fixed-interest instruments.	L 75/25

5.1.4 For simplicity, no asset classes were considered beyond local equity and fixed-interest, which represent the risky and less-risky asset classes respectively. The allocations between equity and fixed-interest were the same as were adopted by Milevsky & Robinson (2000), Blake, Cairns & Dowd (2003) and Levitan, Dolya & Rusconi (unpublished).

5.1.5 For the purposes of this research, the living annuity with subsequent purchase of a life annuity is referred to as the 'lifestage' annuity. Even though annuitisation at the age of 75 is no longer compulsory in the UK,[9] the age of 75 was used as the annuitisation age for the lifestage annuity in order to be consistent with Blake, Cairns & Dowd (2003). Further investigation of the optimal age to purchase a life annuity is left for future research.

5.2 SCENARIOS

5.2.1 A range of demographic profiles were selected, which were believed to be reasonably realistic in the South African context. The purpose was to assess whether the optimal annuity choice would be influenced significantly by changes in the demographic profile. The scenarios were selected to be reasonable and to produce a range of ruin probabilities. The base case was set so that the income required was marginally higher than that available from an inflation-linked annuity. This choice was to highlight how an inflation-linked annuity has either a 0 or 100% chance of ruin, depending on whether it offers an income below or above the required income specified. The exact choice is admittedly arbitrary, and for this reason a number of scenarios were run.

Table 2. Member scenarios

Case	Main member age	Main member gender	Spouse age	Income requirement in first year as a percentage of initial accumulated wealth	
				Necessities case	Comfort case
1	65	male	61	5,16%	6,60%
2	60	male	56	5,16%	6,60%
3	65	male	–	5,16%	6,60%
4	65	female	69	5,16%	6,60%
5	65	male	61	4,20%	5,64%
6	65	male	61	6,00%	7,44%

5.2.2 The 'comfort case' refers to drawing income levels sufficient to allow the individual to live comfortably without much budget austerity. In contrast the 'necessities case' refers to drawing income levels to just meet the costs of living (Levitan, Dolya & Rusconi, unpublished). The base-case-retirement age of 65 is consistent with both Blake, Cairns & Dowd (2003) and the modal retirement age observed from administrator data.[10]

5.2.3 The drawdown percentages ranged from 4,2% to 7,44% and were generally slightly lower than the 7,14% used by Milevsky & Robinson (2000).

5.3 GENERATING THE INCOME STREAMS FOR EACH ANNUITISATION STRATEGY

5.3.1 For the life-annuity strategies, Level, Fixed 5% and IL, the initial income levels were determined using actual annuity quotes in the market, valid from 1 July to

9 Blake D, Cannon, E & Tonks, I (2010). Ending Compulsory Annuitisation: What are the Consequences? Pensions Institute Report dated July 2010.

10 Alexander Forbes Member Watch 2011 database, extracted July 2012

7 July 2012, published on 29 June 2012. The Fixed 5% income stream was increased at a rate of 5% a year and the IL income stream was increased by stochastically generated inflation simulations.

5.3.2 For living annuities, the income drawdown during the member's lifetime is managed according to a rules-based system. The member chooses at the outset whether to draw down at the rate of income required for either comfort or necessities. This drawdown is limited by a minimum of 2,5% per annum of prevailing fund credit at point of drawdown, and a maximum of 17,5% of fund credit per annum as per the Income Tax Act. Although a drawdown rate that is a fixed proportion of the living annuity fund, as opposed to a proportion selected to meet a monetary income need, may delay ruin (Emms, 2010), it can be argued that the budgetary needs of households are in currency terms and not strictly determined by the size of the living annuity.

5.3.3 The drawdown account at the start of the year is immediately reduced by the drawdown and increased by a stochastically simulated investment return derived from using the Maitland stochastic investment model (Maitland, 2010), based on the asset allocations for the strategy. The following year the process repeats itself, except that the drawdown amount is increased by the stochastically simulated inflation rate.

5.3.4 For the purposes of this research, the Maitland stochastic investment model was parameterised with reference to bond yields as at 31 March 2012. The model was calibrated to a ten-year yield on nominal bonds of 8,3% a year and a nominal ten-year expected return on equities of 11,3% a year. The expected return on equities was based on a long-term equity risk premium of 3%, which is a rounded-off value from Hu (unpublished). The model was calibrated to give an expected inflation rate of 5,8% a year on average over ten years, based on the difference between ten-year nominal and inflation-linked bond yields and a 0,5% inflation-risk premium on nominal bonds. Hu (unpublished) had estimated the inflation-risk premium in South Africa at 1,3% in the early 2000s, but this was done at the launch of inflation-linked bonds as an asset class in South Africa, when the inflation-targeting regime was not as well established. Hu (unpublished) warned that this was a crude estimate, which was high relative to international research and may have reflected market uncertainty at the time on whether inflation would be kept under control. Hence, the authors believe a 0,5% inflation-risk premium to be justifiable given the greater maturity of the inflation-linked bond market more recently, and the entrenched policy of inflation-targeting by the South African Reserve Bank. The authors did not test the sensitivity of the results of this paper to the return assumptions underlying the asset model used, which could alter the relative attractiveness of the investment strategies considered. This is left for future research.

5.3.5 For each strategy involving a living annuity or lifestage annuity, 2500 simulations were completed. It is the authors' experience that this is a sufficient number of simulations to obtain stable results in investigations such as these. Because of the large number of cases tested, a larger number of simulations would have resulted in excessively long run times.

5.3.6 For the lifestage annuity, the annuity price at the date of purchase was estimated using the stochastically simulated inflation-linked bond yields prevailing at that time.

5.3.7 The lifetime of the pensioner is stochastically simulated from a mortality table of PA(90) rated down by three years for males and by two years for females, both with rate of mortality improvement of 1,5% a year from 2012. This was informed by the experience of a large South African pension fund administrator.

5.4 THE EXPECTED DISCOUNTED UTILITY APPROACH

5.4.1 Formulation

5.4.1.1 The expected-discounted-utility approach is based largely on the framework set out by Blake, Cairns & Dowd (2003), with some modifications for South African conditions. These modifications were:

– introducing a spouse's reversion on the death of the principal pensioner;
– calibrating the real interest rate to the South African inflation-linked bond market;
– considering South African experience for the assumption made on fund credit and income drawdown levels;
– using the Maitland model to give stochastically generated South African asset-class returns;
– using a mortality table informed by South African experience; and
– varying the denominator of the consumption utility curve to be based on the income from a level annuity, inflation-linked annuity, or the initial income required to meet necessities.

5.4.1.2 The authors used the Blake, Cairns & Dowd (2003) framework because it measures the utility of the income delivered by one strategy relative to another (the 'base' level and type of income). This may be intuitively appealing when comparing various annuitisation strategies. The authors are not aware of any other examples of the application of this approach to a South African asset model. It is shown below that changes to the base strategy affect the results obtained. An alternative type of utility function, which the authors did not explore, uses a replacement ratio or income as the argument. Further research is therefore needed with regard to an appropriate formulation of the argument of the utility function used.

5.4.1.3 In general, the expected-discounted-utility framework involves identifying the strategy with the highest discounted utility, defined as:

$$U^* = max\int_0^b U\left(x_t\right)e^{-\beta t}dt\,; \qquad (1)$$

where:

b is the end of the time period considered;
x_t is income at time t;
$U(\bullet)$ is the utility function; and
β is the constant force of discount which is independent of the instantaneous utility function.

5.4.1.4 For the purposes of this investigation, a modified power function was used for the utility function. The value function, or discounted utility function, contingent on the life being alive at time s, denoted $V(s,f)$ is:

$$V(s,f)=E\left\{\sum_{t=s}^{K}e^{-\beta t}J_1\left(P(t)\right)+k_2e^{-\beta(K+1)}J_2\left(D\left(K+1\right)\right)\mid F(s)=f\right\}; \tag{2}$$

where:

$F(s)$ is the accumulated pension wealth s years after retirement which is a function of initial wealth at retirement and past pension amounts;
K is the curtate future lifetime of the member at the date of retirement;
$J_1(\bullet)$ is the utility of consumption;
$P(t)$ is the pension in year t;
k_2 is the preference for bequests;
$J_2(\bullet)$ is the utility of bequests; and
$D(t)$ is the bequest payable at time t given that death occurred within the year ending at time t and is determined by the accumulated pension wealth at a point prior to t and all subsequent pension payments.

5.4.1.5 Constant RRA was used for the utility of consumption, i.e. the utility of the pension p, as follows:

$$J_1(p)=h_1(\gamma_1)\left(\frac{p}{P_B}\right)^{\gamma_1}; \tag{3}$$

where:

P_B is the base pension;
$1-\gamma_1$ is the RRA associated with consumption; and where:

$$h_1(\gamma_1)=\frac{1}{1-d_1^{\gamma_1}}; \tag{4}$$

where d_1 is a shape parameter for the consumption utility curve.

5.4.1.6 For the utility of bequests, $J_2(D(t))$, a function from the hyperbolic absolute risk aversion class was used, given by equation 5.

$$J_2\left(D(t)\right)=h_2(\gamma_2)\left\{\left(\frac{D(t)+d_2}{d_2}\right)^{\gamma_2}-1\right\}; \tag{5}$$

where:

$1-\gamma_2$ is the RRA associated with bequests;
d_2 is the value of assets held outside the pension fund such as a house; and $d_2>0$;

and where:

$$h_2(\gamma_2)=\left\{\left(\frac{F(0)+d_2}{d_2}\right)-1\right\}^{-1}. \tag{6}$$

Blake, Cairns & Dowd (2003), Yaari (1965) and Levitan, Dolya & Rusconi (unpublished) do not deal with reversionary annuities. For the purposes of this research, the income received from a spouse's contingent annuity at time *t* was treated as a bequest made at time *t*. In other words, annuity income to a spouse was treated as a series of bequests. Further research is required to test whether this is reasonable.

5.4.2 Calculation of Discounted Utility

For the strategies considered, the income receivable each year is stochastically simulated using transformations of the Maitland stochastic investment model. Based on the simulated income and the bequests made, the member's discounted utility is calculated under a particular simulation. The average utility over the simulations is then used as an estimate for the expected discounted utility for the strategy under consideration.

5.4.3 Parameterisation

In the parameterisation of the model, care was taken to be consistent with the literature, particularly with Blake, Cairns & Dowd (2003), in order to allow some comparability of results. The specific parameters adopted are described as follows.

5.4.3.1 Risk-aversion Parameters

The relative-risk-aversion parameter is represented by $1-\gamma_1$. A higher value represents a more risk-averse person. Blake, Cairns & Dowd (2003) adopted a range from 0,25 to 25 in order to accommodate both very risk-averse and very risk-tolerant preferences and adopted a base-case value of 3,96. For the purposes of this research the same base-case value was used. To test the sensitivity of this parameter, six equidistant point estimates were taken from this range, namely 0,3, 5,2, 10,2, 15,1, 20,1 and 25. As per Blake, Cairns & Dowd (2003), γ_1 was set equal to γ_2. No further evidence for the reasonability of these parameters is provided here. Further research is hence required on reasonable values for the RRA where the argument of the utility function is the income earned under a strategy relative to a base strategy. There is a vast literature dealing with estimating RRA where the argument of the utility function includes consumption (Mehra & Prescott, 1985), payoffs (Holt & Laury, 2002), wealth or even a replacement ratio at retirement (Thomson, 2003a; Thomson, 2003b). It would be unreasonable to assume that the RRA of an investor is similar for these different utility function arguments without further evidence. Justification is also required for the use of a particular functional form of utility if a new argument is used. Some authors, including Blake, Cairns & Dowd (2003) have, however, extended the use of utility functions to new arguments without justification.

5.4.3.2 Discount Rate

A flat real yield of 2,45% a year was used, which represented the average of the yields of the two longest-dated inflation-linked bonds issued by the South African government.[11] A real yield was chosen given the need to protect income against inflation risk. Although, more recently, real yields have declined, it is shown in section 7.2 that the results would not be materially affected. In the modelling, this rate of interest was converted into a force of interest of 2,44% a year, for application in the model. For simplicity, this real yield was used as an intertemporal discount rate. Blake, Cairns & Dowd (2003) used a nominal intertemporal discount rate chosen to be consistent with the rate used by the United Kingdom's Government Actuary to value benefits. Sensitivity testing was also performed using forces of interest of 0,1% and 10% a year.

5.4.3.3 Initial Fund Credit

The average and median retirement benefits for clients of a certain large financial services provider were approximately R1 130 000 and R480 000 in 2011.[12] A figure of R1 million was used at the outset for $F(0)$.This resulted in income levels that fell below the income-tax thresholds. This limits the application of the results to higher-income earners as Sweeting (2009) has shown that tax regimes can influence the annuitisation decision.

5.4.3.4 Bequests and Other Assets

The base-case value for the bequest motive parameter, k_2, was 5. The greater the value of this parameter, the greater the utility derived from bequests relative to utility derived from income. In sensitivity-testing values of 1 and 10 were tested. Blake, Cairns & Dowd (2003) comment that studies on the importance attached to bequests by retirees are inconclusive, but found that their results were "not over-sensitive to changes in the value of k_2". For the other assets outside the retirement fund, a value of R1 million was used for simplicity and values of R500 000 and R2 million were adopted for sensitivity testing.

5.4.3.5 Base Pension

$J_1\left(P(t)\right)$ includes a ratio of prevailing income to the base pension at inception. Three types of base pensions were used for the parameter P_B. As per Blake, Cairns & Dowd (2003), a level life annuity was adopted for the first case and the resultant discounted utility function is referred to as 'DUL' for the purposes of this paper. In the second, an inflation-linked life annuity was adopted and the resultant function is termed 'DUI'. In the third, the necessity spending level, increased by inflation, and termed 'DUN', was used as per Levitan, Dolya & Rusconi (unpublished). While the purchase of level annuities exposes pensioners to inflation risk, the DUL function that uses a level

11 Bloomberg, June 2012
12 Alexander Forbes Member Watch 2011 database, supra

annuity as a base pension is arguably a valid means of comparison given that 90%[13] of life annuities sold in South Africa are level annuities.

5.4.3.6 *Shape Parameter, d_1*

The parameter d_1 acts as a shape parameter for the instantaneous utility of consumption, but has no intuitive interpretation. By altering the parameter d_1 we may therefore consider individuals with different utility functions within the constraints of the functional forms of $J_1\left(P\left(t\right)\right)$ and $h_1(\gamma_1)$. Unfortunately one cannot make any generalisations regarding the relationship between the value chosen for d_1 and the value of utility derived. The effect of increasing or decreasing the value of d_1 depends on whether γ_1 is positive or negative and whether the ratio of pension income at time t to the base pension, P_B, is less than, equal to or greater than one. A value of 0,75 was chosen as per Blake, Cairns & Dowd (2003), who remarked that this parameter value could be chosen freely in the range between 0 and 1 to adjust the shape of the consumption utility curve. To test for sensitivity, values of 0,05 and 0,95 were also used.

5.5 The Ruin-theory Model

The ruin-theory approach involved finding the strategy with the lowest LPoR, where 'ruin' is defined as occurring when income falls below a level of income for comfort, or for necessity, increased by simulated inflation each year, as per Levitan, Dolya & Rusconi (unpublished). If the income available from the annuitisation strategy is lower than the income needed, while the pensioner is alive, the strategy is deemed to have resulted in ruin. The number of simulations resulting in ruin was divided by 2500 to give the probability of ruin.

6. RESULTS ON THE STANDARD BASIS

6.1 SCENARIO 1: BASE CASE

6.1.1 The results for the base case of a male aged 65 with a spouse aged 61 are given in Table 3. The strategies are ranked under different measurement criteria, where a lower number for the rank represents a more favoured strategy. For ease of reference, the best and worst results are highlighted. For the ruin-theory measure the probability of ruin is also shown, as calculated from the percentage of simulations giving a ruin result.

6.1.2 When the required level of income is that to meet comfort, ruin theory favours aggressive living-annuity strategies with a 75% allocation to equities, resulting in a material probability of ruin of 48%. Other aggressively managed strategies also fared well, as aggressive investment increases the probability of achieving the desired income for comfort from living annuities. By not annuitising and not being locked into fixed-interest instruments, expected returns were higher in pure living-annuity strategies, albeit with a higher risk of running out of income, possibly by a significant margin.

6.1.3 In contrast, locking into income from a fixed 5% escalation or inflation-

13 National Treasury (2012b). Enabling a Better Income in Retirement: Technical Discussion Paper B for Public Comment. Discussion Paper, 21 September 2012

linked annuity was certain to result in ruin, because the incomes from these annuities were lower than the level of desired income for comfort. Importantly, the ruin-theory criterion does not take the extent of shortfall into account and hence treats a shortfall of R1 and, say, a shortfall of R10 000 equally.

6.1.4 When the required level of income is that for necessities only, the highest-ranked strategy under ruin theory now switches to the annuity with fixed 5% escalation, as the income purchasable from the life annuity with its increases in general exceeds the level of income for necessity. The inflation risk remains, which accounts for the 20% probability of ruin. This strategy is followed by a fairly aggressively managed living annuity, albeit less aggressive than when the income threshold for comfort was used. Again, an inflation-linked strategy remained the lowest ranked, as income from it is still less than the income level for necessity.

Table 3. Results for the base case

	Comfort income: R5500 p.m.					Necessity income: R4300 p.m.				
	Ruin theory		Discounted utility			Ruin theory		Discounted utility		
	% ruin	rank	DUL	DUI	DUN	% ruin	rank	DUL	DUI	DUN
Level	83%	9	1 (best)	3	8	69%	10	1 (best)	5	10
Fixed 5%	100%	10 (worst)	4	1 (best)	6	20%	1 (best)	2	1 (best)	8
IL	100%	10 (worst)	6	2	7	100%	11 (worst)	6	4	9
LwA 0/100	74%	8	3	5	4	51%	9	4	3	7
LwA 25/75	71%	7	2	4	3	33%	7	3	2	5
LwA 50/50	60%	5	5	6	2	30%	3	5	6	3
LwA 75/25	53%	3	8	7	1 (best)	32%	6	9	7	2
L 0/100	69%	6	9	10	10	50%	8	8	9	6
L 25/75	55%	4	7	8	5	30%	3	7	8	1 (best)
L 50/50	51%	2	10	9	9	28%	2	10	10	4
L 75/25	48%	1 (best)	11 (worst)	11 (worst)	11 (worst)	30%	3	11 (worst)	11 (worst)	11 (worst)

6.1.5 The expected-discounted-utility results are strongly influenced by the choice of the base pension, and hence the choice of a level annuity, inflation-linked annuity or necessity spending adjusted for inflation. In other words, the results for DUL, DUI and DUN vary significantly. This is due to the formula used to define the utility function, and in particular the expression given in equation 3, which is the ratio of prevailing income to the base pension, being raised to an exponent linked to the degree of risk aversion. A risk-averse individual would have a low value for the exponent, and an individual with a low risk aversion would have a high value, which would compound the ratio. Hence the overall utility is very sensitive to the relative size of any shortfall to the base pension. By taking utility into account, strategies providing higher levels of income upfront are favoured. In sensitivity testing, summarised in section 7.1, it was found that the effect of variations in the utility function on the rankings of base pensions was much greater than the effect of variations in the discount rate.

6.1.6 Under DUL, level annuities emerged as most preferred, largely because the base pension under DUL is also a level annuity. There was not much difference in the interpretations between the income for comfort and necessities cases under DUL.

6.1.7 It is noteworthy that the annuity with fixed escalation at 5% emerged as the most preferred strategy under DUI, for both income for comfort and income for necessities. This suggests that the slightly higher income from the annuity with fixed escalation at 5% relative to the inflation-linked annuity made it more appealing, even though ongoing increases may not match inflation, which in 48% of simulations was above 5% a year and was in one case as high as 20,9% a year. Similarly to the DUL result, the purchase of life annuities is favoured, and more conservative asset allocations were preferred to overly aggressive ones where living-annuity strategies are considered.

6.1.8 The picture was somewhat different under DUN, which selected the most aggressive strategies out of the expected-discounted-utility measures. When income for comfort is used, the most preferred strategy was the high-equity living annuity with an inflation-linked annuity purchased at age 75. When income for necessities is used, this moves to the living annuity without annuitisation with more conservative asset allocations. The reason is that when the threshold is income for necessities, which is lower than income for comfort, less is drawn each year from the living annuity, which means less capital is depleted, making the living annuity arrangement more sustainable. Hence, annuitisation is not as highly valued as in the case of income for comfort.

6.2 SCENARIO 2: LOWER RETIREMENT AGE

6.2.1 The ruin probabilities and strategy rankings for the case of a man aged 60 with a spouse aged 56 are given in Table 4.

6.2.2 For an earlier retirement age, the ruin-theory results were mostly unchanged, except that, in the case of income for necessity, the annuity with fixed escalation at 5% was not as highly ranked. This is due to the higher cost of the life annuity at younger ages.

6.2.3 Under the DUL, level annuities emerged as the most preferred once again for both levels of income requirement. Similarly, the favoured strategy under DUI remained annuities with fixed escalation at 5%. Under DUN, the more aggressively managed lifestage annuities were generally favoured.

6.3 SCENARIO 3: SINGLE MEMBER

6.3.1 The ruin probabilities and strategy rankings for the case of a man aged 65 without a spouse are given in Table 5.

Table 4. Results for retirement age 60

	Comfort income: R5500 p.m.					Necessity income: R4300 p.m.				
	Ruin theory		Discounted utility			Ruin theory		Discounted utility		
	% ruin	rank	DUL	DUI	DUN	% ruin	rank	DUL	DUI	DUN
Level	91%	9	1 (best)	3	7	81%	10	1 (best)	4	10
Fixed 5%	100%	10 (worst)	2	1 (best)	5	55%	7	2	1 (best)	7
IL	100%	10 (worst)	5	2	6	100%	11 (worst)	7	2	8
LwA 0/100	82%	7	4	5	3	71%	9	4	5	5
LwA 25/75	75%	6	3	4	2	51%	6	3	3	3
LwA 50/50	67%	4	6	6	1 (best)	39%	4	5	6	1 (best)
LwA 75/25	60%	2	9	7	4	38%	2	10	7	4
L 0/100	82%	7	8	9	9	65%	8	8	9	6
L 25/75	71%	5	7	8	8	48%	5	6	8	2
L 50/50	62%	3	10	10	10	38%	2	9	10	9
L 75/25	56%	1 (best)	11 (worst)	11 (worst)	11 (worst)	35%	1 (best)	11 (worst)	11 (worst)	11 (worst)

Table 5. Results for the single male

	Comfort income: R5500 p.m.					Necessity income: R4300 p.m.				
	Ruin theory		Discounted utility			Ruin theory		Discounted utility		
	% ruin	rank	DUL	DUI	DUN	% ruin	rank	DUL	DUI	DUN
Level	78%	11 (worst)	1 (best)	5	8	63%	11 (worst)	1 (best)	5	10
Fixed 5%	27%	2	2	1 (best)	6	2%	2	2	1 (best)	8
IL	0%	1 (best)	5	2	7	0%	1 (best)	3	2	9
LwA 0/100	72%	10	4	4	4	10%	4	5	4	5
LwA 25/75	51%	6	3	3	2	8%	3	4	3	3
LwA 50/50	42%	3	6	6	3	13%	5	6	6	4
LwA 75/25	42%	3	7	7	5	19%	6	8	7	6
L 0/100	70%	9	9	9	9	49%	10	9	9	2
L 25/75	60%	8	8	8	1 (best)	35%	9	7	8	1 (best)
L 50/50	51%	6	10	10	10	29%	7	10	10	7
L 75/25	48%	5	11 (worst)	11 (worst)	11 (worst)	30%	8	11 (worst)	11 (worst)	11 (worst)

6.3.2 Under ruin theory, for a single male aged 65 inflation-linked annuities were the best strategy for both income for comfort and income for necessity, as both levels were affordable under the annuity, which no longer included a spouse's reversion.

The least preferred strategy was the level annuity. This was due to the effects of inflation eroding the income purchased from the annuity, which was initially higher than the income requirements.

6.3.3 Under the DUL, level annuities emerged as the most preferred once again for both levels of income requirement. Similarly, the favoured strategy under DUI remained annuities with fixed escalation at 5%. Under DUN, low-equity living annuities without annuitisation were favoured.

6.4 SCENARIO 4: FEMALE MAIN MEMBER

When the analysis was changed to a female pensioner with an older husband, the relative rankings were similar to the base case in Scenario 1, as shown in Table 6. The only noteworthy difference from the base case was for the DUN function which favoured a lifestage annuity but with a 50% equity exposure under the comfort income case.

Table 6. Results for female member

	Comfort income: R5500 p.m.					Necessity income: R4300 p.m.				
	Ruin theory		Discounted utility			Ruin theory		Discounted utility		
	% ruin	rank	DUL	DUI	DUN	% ruin	rank	DUL	DUI	DUN
Level	84%	9	1 (best)	4	8	71%	11 (worst)	1 (best)	5	10
Fixed 5%	100%	10 (worst)	4	1(best)	6	15%	2	2	1 (best)	8
IL	100%	10 (worst)	6	2	7	0%	1 (best)	3	3	9
LwA 0/100	76%	8	3	5	4	40%	9	5	4	6
LwA 25/75	69%	6	2	3	3	25%	3	4	2	5
LwA 50/50	56%	4	5	6	1 (best)	26%	4	6	6	4
LwA 75/25	51%	2	8	7	2	29%	5	9	7	3
L 0/100	70%	7	9	9	9	50%	10	8	9	2
L 25/75	62%	5	7	8	5	36%	8	7	8	1 (best)
L 50/50	51%	2	10	10	10	29%	5	10	10	7
L 75/25	48%	1 (best)	11 (worst)	11 (worst)	11 (worst)	29%	5	11 (worst)	11 (worst)	11 (worst)

6.5 SCENARIO 5: LOWER INCOME REQUIREMENT

6.5.1 The analysis then returned to the base profile of a male aged 65 and female spouse aged 61, but with the income levels lowered per Table 2. The results are shown in Table 7.

6.5.2 Under ruin theory, the absolute probabilities of ruin dropped, but the relative rankings amongst the strategies remained similar relative to the base case. With lower income requirements, conservative living annuities without annuitisation were preferred under DUN; however, the fixed 5% escalation annuity was still the highest ranked strategy under DUI. Again, level annuities were the most preferred under the DUN measure.

Table 7. Results for the lower income

	Comfort income: R5500 p.m.					Necessity income: R4300 p.m.				
	Ruin theory		Discounted utility			Ruin theory		Discounted utility		
	% ruin	rank	DUL	DUI	DUN	% ruin	rank	DUL	DUI	DUN
Level	75%	10	1 (best)	5	10	57%	11 (worst)	1 (best)	3	10
Fixed 5%	47%	6	2	1 (best)	8	3%	2	2	1 (best)	8
IL	100%	11 (worst)	6	3	9	0%	1 (best)	3	2	9
LwA 0/100	71%	9	4	4	6	11%	4	5	5	7
LwA 25/75	54%	7	3	2	4	8%	3	4	4	6
LwA 50/50	44%	4	5	6	3	12%	5	6	6	5
LwA 75/25	42%	3	9	7	2	17%	8	9	7	4
L 0/100	57%	8	8	9	7	29%	10	8	9	3
L 25/75	45%	5	7	8	1 (best)	15%	7	7	8	1 (best)
L 50/50	36%	1 (best)	10	10	5	13%	6	10	10	2
L 75/25	36%	1 (best)	11 (worst)	11 (worst)	11 (worst)	18%	9	11 (worst)	11 (worst)	11 (worst)

6.6 SCENARIO 6: HIGHER INCOME REQUIREMENT

6.6.1 The results for the same couple with higher income requirements per Table 2 are shown in Table 8.

6.6.2 Under ruin theory, when the income requirements were raised, the absolute probabilities of ruin increased, but the relative rankings again remained similar to the base case. The DUI and DUL rankings remain largely unchanged but the DUN favoured annuitisation and more equity exposure when the living annuities were set to deliver the necessity level of income.

7. SENSITIVITY TESTS ON DUL RESULTS

The sensitivity of the results from the discounted utility models has already been tested for variations in the base pension and differences in demographic profiles. The sensitivities of five other key parameters in the DUL function were also tested, namely:

- RRA, $1-\gamma_1$;
- Force of discount for consumption, β;
- Shape parameter d_1, which influences h_1 and hence consumption utility J_1;

– Assets outside retirement fund, d_2; and
– Bequest motive, k_2.

The reason for performing the analysis on the DUL function was the fact that National Treasury[14] states that 90% of life annuities sold in South Africa are level. This suggests that the income provided by a level annuity is likely to be a useful point of comparison. For presentation purposes, the relative rankings of the 11 strategies under these sensitivity tests are shown in sections 7.1 to 7.5.

Table 8. Results for the higher income

	Comfort income: R5500 p.m.					Necessity income: R4300 p.m.				
	Ruin theory		Discounted utility			Ruin theory		Discounted utility		
	% ruin	rank	DUL	DUI	DUN	% ruin	rank	DUL	DUI	DUN
Level	89%	9	1 (best)	3	7	78%	9	1 (best)	5	10
Fixed 5%	100%	10 (worst)	3	1 (best)	5	79%	10	3	1 (best)	6
IL	100%	10 (worst)	6	2	6	100%	11 (worst)	6	4	7
LwA 0/100	78%	8	4	5	4	73%	8	4	3	4
LwA 25/75	76%	6	2	4	3	58%	6	2	2	3
LwA 50/50	70%	4	5	6	2	48%	4	5	6	2
LwA 75/25	64%	3	7	7	1 (best)	44%	3	8	7	1 (best)
L 0/100	76%	6	10	10	10	62%	7	9	9	9
L 25/75	70%	4	8	8	8	50%	5	7	8	5
L 50/50	62%	2	9	9	9	41%	1 (best)	10	10	8
L 75/25	59%	1 (best)	11 (worst)	11 (worst)	11 (worst)	41%	1 (best)	11 (worst)	11 (worst)	11 (worst)

7.1 SENSITIVITY TEST ON RELATIVE RISK AVERSION

7.1.1 The rankings for various levels of the RRA parameters are shown in Table 9.

7.1.2 For annuitants with an RRA parameter of 0,3, the aggressive living annuity with annuitisation was preferred. However, once more risk aversion is introduced, above 5,2 there is no change in the rankings of the attractiveness of these strategies. The comparable results from the base case scenario are shown in Table 10.

7.2 SENSITIVITY TEST ON FORCE OF DISCOUNT

7.2.1 The rankings under various discount rates are shown in Table 11.

7.2.2 It can be seen that under both the income for comfort and necessities

14 Enabling a Better Income in Retirement, supra

scenarios, even very large differences in the force of discount has little effect on the rankings across the strategies. This is attributable to the high sensitivity of results to the ratio of income from annuitisation to the base pension level. Relatively speaking, the impact of the discount rate is much lower. Hence, although the absolute utility levels changed with a change in the discount rate, the rankings did not change much.

Table 9. Results for various RRA parameters

	Comfort income: R5500 p.m.						Necessity income: R4300 p.m.					
RRA	0,3	5,2	10,2	15,1	20,1	25	0,3	5,2	10,2	15,1	20,1	25
Level	2	1 (best)	1 (best)	1 (best)	1 (best)	1 (best)	2	1 (best)	1 (best)	1 (best)	1 (best)	1 (best)
Fixed 5%	8	4	2	2	2	2	4	2	2	2	2	2
IL	11 (worst)	5	4	4	3	3	10	5	5	4	4	4
LwA 0/100	9	2	3	3	4	4	8	4	3	3	3	3
LwA 25/75	7	3	5	5	5	5	5	3	4	5	5	5
LwA 50/50	4	6	6	6	6	6	3	6	6	6	6	6
LwA 75/25	1 (best)	8	7	7	7	7	1 (best)	9	9	9	8	8
L 0/100	10	9	9	9	9	9	11 (worst)	8	7	7	7	7
L 25/75	6	7	8	8	8	8	9	7	8	8	9	9
L 50/50	5	10	10	10	10	10	7	10	10	10	10	10
L 75/25	3	11 (worst)	11 (worst)	11 (worst)	11 (worst)	11 (worst)	6	11 (worst)	11 worst)	11 (worst)	11 (worst)	11 (worst)

Table 10. Comparative results for the base case scenario

	Comfort DUL	Necessity DUL
Level	1 (best)	1 (best)
Fixed 5%	4	2
IL	6	6
LwA 0/100	3	4
LwA 25/75	2	3
LwA 50/50	5	5
LwA 75/25	8	9
L 0/100	9	8
L 25/75	7	7
L 50/50	10	10
L 75/25	11 (worst)	11 (worst)

Table 11. Results for various forces of discount

	Comfort income: R5500 p.m.			Necessity income: R4300 p.m.		
β	0,10%	2,44%	10%	0,10%	2,44%	10%
Level	1 (best)	1 (best)	1 (best)	1 (best)	1 (best)	1 (best)
Fixed 5%	3	4	6	2	2	2
IL	6	6	8	6	6	8
LwA 0/100	4	3	3	4	4	4
LwA 25/75	2	2	2	3	3	3
LwA 50/50	5	5	5	5	5	7
LwA 75/25	7	8	10	9	9	10
L 0/100	9	9	7	8	8	6
L 25/75	8	7	4	7	7	5
L 50/50	10	10	9	10	10	9
L 75/25	11 (worst)	11 (worst)	11 (worst)	11 (worst)	11 (worst)	11 (worst)

7.3 SENSITIVITY TEST ON SHAPE PARAMETER

7.3.1 The rankings under various shape parameters are shown in Table 12.

Table 12. Results for various shape parameters

	Comfort income: R5500 p.m.			Necessity income: R4300 p.m.		
d_1	0,05	0,75	0,95	0,05	0,75	0,95
Level	11 (worst)	1 (best)	1 (best)	11 (worst)	1 (best)	1 (best)
Fixed 5%	9	4	4	9	2	2
IL	10	6	6	10	6	5
LwA 0/100	8	3	3	8	4	4
LwA 25/75	7	2	2	7	3	3
LwA 50/50	5	5	5	6	5	6
LwA 75/25	4	8	8	5	9	9
L 0/100	6	9	9	4	8	8
L 25/75	3	7	7	2	7	7
L 50/50	1 (best)	10	10	1 (best)	10	10
L 75/25	2	11 (worst)	11 (worst)	3	11 (worst)	11 (worst)

7.3.2 The range of values for the shape parameter for the consumption utility recommended by Blake, Cairns & Dowd (2003) is between 0 and 1. With the base at 0,75, higher values had virtually no effect on the rankings. However, for lower values, there was a significant effect, with those values favouring more aggressive living-annuity strategies as opposed to level annuities and more conservative lifestage-annuity strategies. As there is no intuitive interpretation for the shape parameter, it is impossible to infer an appropriate value without further research. Estimates for this parameter should be tested together with the form of the utility function against utility functions elicited from a broad sample of retirees.

7.4 SENSITIVITY TEST ON ASSETS OUTSIDE FUND

Other values tested were R500 000 and R2 million, in addition to the base of R1 million. These amounts were implicitly assumed to be non-interest bearing and had no impact on taxation. There was no effect on the rankings and no material changes in the utility levels and hence the tabulated results are not shown.

7.5 SENSITIVITY TEST ON BEQUEST MOTIVE

7.5.1 Other values tested were 1 and 10, in addition to the base of 5. The results are shown in Table 13.

Table 13. Results for various strengths of bequest motive

	Comfort income: R5500 p.m.			Necessity income: R4300 p.m.		
k_2	1	5	10	1	5	10
Level	1 (best)	1 (best)	1 (best)	1 (best)	1 (best)	1 (best)
Fixed 5%	4	4	4	2	2	2
IL	6	6	6	5	6	7
LwA 0/100	3	3	3	4	4	4
LwA 25/75	2	2	2	3	3	3
LwA 50/50	5	5	5	6	5	5
LwA 75/25	8	8	8	9	9	9
L 0/100	9	9	9	8	8	8
L 25/75	7	7	7	7	7	6
L 50/50	10	10	10	10	10	10
L 75/25	11 (worst)	11 (worst)	11 (worst)	11 (worst)	11 (worst)	11 (worst)

7.5.2 Interestingly, varying the strength of the bequest motive had only a slight impact under the income for necessities case and no effect when considering income for comfort. Although this may not be the case if mortality were significantly heavier, explicitly varying the mortality basis is left for future research.

8. DISCUSSION

8.1 SENSITIVITY OF THE RESULTS

8.1.1 The Base Pension in the Discounted Utility Approach

The sensitivity tests support the view that under the expected-discounted-utility framework, for a given base income, the rankings amongst the strategies are relatively stable against variations in the key parameters. However, varying the base income level can have a significant effect on the rankings.

8.1.2 The Bequest Motive and Other Assets

8.1.2.1 The DUL result was relatively insensitive to the bequest motive. This is consistent with the findings of Blake, Cairns & Dowd (2003), whose results were not overly sensitive to changes in the strength of the bequest motive. They stated that mis-specification of the bequest parameter is unlikely to be as serious as having the wrong equity exposure.

8.1.2.2 The size of assets outside the fund, including assets that could be bequeathed, similarly had no impact.

8.1.3 Sensitivity to Relative Risk Aversion and the Utility Function

The rankings of the various strategies given a base pension of the level life annuity changed substantially for very low RRA parameter values, which represent a low RRA. However, the preferred strategy was relatively stable for moderate and high RRA parameter values. Expected discounted utility is often criticised for being difficult to parameterise, however the results from this research suggest that unless the investor is very risk-seeking, the preferred annuitisation decision will not be overly sensitive to this parameter.

8.1.4 Sensitivity to Life Expectancy at Retirement

The scenarios using a female pensioner and using an earlier retirement age increase the term for which income is required (on average). This is very similar to reducing the initial funding level which is discussed further in ¶8.1.5.1. Under the discounted utility model using income for necessities as the base, the effect of increased life expectancy at retirement was to reduce the equity exposure in a living annuity, or to annuitise at a later stage, which would mitigate the investment and longevity risks associated with a longer time spent in retirement. The reduction in equity exposure is contrary to the preferences found in Milevsky & Robinson (2000).

8.1.5 Sensitivity to Initial Funding Level

8.1.5.1 The ruin-theory approach was extremely sensitive to the initial funding level, or the drawdown rate relative to the initial wealth at retirement. When the drawdown is low relative to initial wealth, the funding level is said to be high, and vice versa. If there was sufficient capital to purchase an inflation-linked annuity equal to the desired initial income or higher, the ruin-theory approach gave the inflation-linked life annuity the highest ranking. Otherwise a living annuity with relatively high equity allocation was selected. This is because in an underfunded position the ruin-theory model will give higher rankings to strategies that have at least some possibility of not ending in ruin (relative to the desired income level), even though the income threshold may be high, and the end result may be a much lower income level than desired. Due to the higher expected return on equities and volatility of equity returns (particularly on the upside), the probability of not achieving the desired income level is lower for higher equity exposures. Obviously, equity volatility can also lead to very adverse outcomes, which result in a material depletion of available funds. Under a ruin-theory approach, the extent of this possible downside is not factored into the measure for the attractiveness of a strategy. It is possible that most investors would be sensitive to the extent of the shortfall of income relative to the level of desired income. The authors therefore argue that the simple approach of measuring whether income exceeded a single benchmark level may be unrealistic and hence limits the usefulness of ruin theory.

8.1.5.2 The DUL framework gave the highest ranking to the level annuity, irrespective of the funding level. Similarly, the DUI gave the highest ranking to life annuities with fixed escalation at 5%.

8.1.5.3 The DUN framework was sensitive to the funding level but gave considerably more variation in the rankings. It typically favoured living annuities. Living annuities without the protection via later purchase of a life annuity were favoured if the funding level was high. Otherwise, living annuities with annuitisation later were ranked highest.

8.2 DIFFERENT RESULTS UNDER DIFFERENT OBJECTIVE FUNCTIONS

8.2.1 Overview

8.2.1.1 Browne (1995) has shown that, if there is a fixed investment term that is known in advance and insurance is ignored, the optimal investment strategy under ruin theory and under expected discounted utility with an exponential utility function will be the same. If the future lifetime is not known and discounted utility is used with constant RRA, then the best investment mix for income drawdown accounts will be similar but not identical (Bayraktar & Young, 2007). However, once life annuities are considered, there is no prior research to suggest that the annuitisation decision should be similar under ruin theory and expected discounted utility.

8.2.1.2 The results of this research show that, under most circumstances, the minimisation of the probability of ruin suggested different strategies than what expected discounted utility would suggest. Even when expected discounted utility would also suggest living annuities, the ruin-theory model ranked more aggressive investment strategies more highly than expected discounted utility. In addition, the ranking of various strategies under expected discounted utility depended on the income type used to anchor the utility function.

8.2.2 Ruin-theory Rankings

8.2.2.1 It was noted that the results suggest that ruin theory tends to select aggressive strategies, particularly for lives that have higher income needs relative to their savings. This was consistent with the literature as discussed in section 4.2.6. In this paper the bequest motive was not explicitly included in the ruin-theory approach; however, the literature would suggest that the more under-funded a retiree is relative to their income or bequest needs, the higher the equity exposure that would be suggested. Some commentators might view this as gambling.

8.2.2.2 Given the propensity of the ruin-theory objective function to select aggressive strategies, it is noteworthy that even under the highest ranked strategy, the probability of ruin often remained significant. In addition, the extent of ruin is not clearly shown. It could be argued that a retiree may be uncomfortable with these probabilities in absolute terms, and may instead be willing to accept some level of shortfall relative to required or desired levels upfront, rather than run the risk that sharply poorer investment performance would jeopardise the entire living-annuity strategy.

8.2.3 DUL Rankings

8.2.3.1 In contrast to the ruin-theory criterion, the DUL rankings amongst the 11 strategies remained surprisingly stable to changes in demographics. Inflation risk was assumed in exchange for higher initial incomes.

8.2.3.2 This result supports the theory that most people would rather revise their goals downwards than run the risk of falling materially short of these standards. In addition, because of the lack of adequate contributions in the lead-up to retirement, most people need high income levels relative to their savings at, and throughout, retirement. This is one possible reason why level annuities, which provide income over life, and provide the highest level of income out of the life annuities, are popular. A possible second driver of this effect is the interaction of the discount rate, implied discount factors and the ratio of tested strategy income to base pension in the utility-function formulation. For example, an inflation-linked annuity defers the distribution of income relative to a level annuity, creating a strongly increasing ratio over time. The trend in the ratio varies by the strategy being tested and the benchmark or denominator used in the utility function. This can then be amplified or dampened according to the discount rate and the level of risk aversion. Additionally, the formulation of the utility function uses a ratio raised to an exponent. Hence the utility result is highly sensitive to the ratio of actual income to the base level of income and the effect of the exponent on the ratio.

8.2.4 DUI Rankings

The most preferred strategy using the DUI utility function was an annuity with fixed escalation at 5%. This is near to the upper band of the South African Reserve Bank's target range of 3% to 6% for inflation. The geometric average inflation rate was 5,3% a year and, for the first ten years of simulation in 79% of simulations it fell within the range of 4% to 6% a year. This annuity also achieves some balance between relatively good future increases, and an upfront income that is not too low by forgoing full inflation-proofing. Further research is required on the sensitivity of this result to the parameterisation of the asset model with regard to expected inflation.

8.2.5 DUN Rankings

When income for comfort was the threshold, aggressive lifestage annuities were favoured. When this threshold was changed to necessities, straight living annuities without annuitisation were favoured. The annuitisation aspect is consistent with expected discounted utility in general, which prioritises the management of longevity risk. Due to the need to keep up with inflation each year, aggressive asset allocations were favoured.

8.3 RELATIVE ADVANTAGES AND DISADVANTAGES OF THE OBJECTIVE FUNCTIONS CONSIDERED

There are advantages and disadvantages to both the ruin-theory and expected-discounted-utility approaches as discussed below.

8.3.1 Ruin Theory

8.3.1.1 At face value, the probability of running out of funds is easy to understand and to relate to, and is intelligible both when used to compare strategies and to assess the merits of a strategy in isolation. However, the precise formulation of the ruin-theory model can be extremely complex and subjective and this is difficult to explain. In addition, in a country with low numeracy, the comparison of probabilities may be quite difficult for certain groups of fund members.

8.3.1.2 If there is insufficient capital to buy an inflation-linked annuity, ruin theory favours aggressive living annuities. This is because it does not measure depth of shortfall when a shortfall is experienced. This level of risk tolerance may be inconsistent with how many individuals would see this risk–reward trade-off. The tendency to increase risk within an investment strategy to reduce the probability of underperforming a single income level benchmark may not reflect how many people would want to react to address this problem.

8.3.1.3 Results under ruin theory are also very sensitive to the level of capital available to purchase an annuity at retirement. Individuals may have accumulated retirement wealth in more than one retirement vehicle. If full and accurate information is not available at the outset a very different result could be produced to what it would be if full information were available. It also means that it is difficult to use with projected investment values. In addition, ruin theory requires fairly accurate forecasting of needs in retirement. A difference of a few rands in forecast needs could change the outcome, because ruin theory is geared towards beating that level of desired or required income, without taking downside outcomes into account.

8.3.2 Discounted Utility

8.3.2.1 In contrast, expected discounted utility provides stability of the preferred solution across different demographic profiles and generally across different parameters, unless the annuitant has a low RRA or an unusual consumption utility curve. The results under expected discounted utility tend to favour strategies providing more stability around the achievement of the base outcome, as the extent of shortfalls, when they occur, are taken into account in the analyses. In addition, to the authors, the results from the discounted utility model seemed more compatible with human behaviour. In essence, when faced with higher income requirements relative to money available, ruin theory tends to favour strategies that take more risk in an attempt to meet those requirements, whereas expected discounted utility favours those that take less risk to avoid missing those requirements drastically.

8.3.2.2 On the other hand, the results under expected discounted utility are less intuitive and less easy to explain. While an individual may understand ranking of preferences, a utility level may not be meaningful. This complicates the comparisons across strategies, unless rankings are used. In addition, terms like ‘risk-seeking’ and ‘risk-averse’ may be difficult to explain. Although the elicitation of the utility functions may be challenging, Thomson (2003b) demonstrated an interactive system which he used to elicit utility functions for members of a South African defined-contribution fund.

8.3.2.3 It is important to note that the base pension chosen in the consumption utility function can affect the preferred solution, for example:

– if income from level annuities were used as the base, level annuities were the best option;
– if income from inflation-linked annuities were used, annuities with fixed escalation at 5% a year or inflation-linked annuities were preferred, but a conservative living annuity with annuitisation came up as a reasonable alternative; and
– if income-for-necessity levels were used in the denominator, moderate to aggressive living annuities with annuitisation were favoured, especially when income for comfort was required to be met.

8.4 AREAS FOR FUTURE RESEARCH

8.4.1 This paper is limited to considerations around the annuitisation decision. However, it is noted that the proportion of retirement wealth to take as a lump sum at retirement is non-trivial and requires further investigation.

8.4.2 This research considered purchasing inflation-linked annuities after holding a living annuity. Given current annuity prices, fixed-escalation and with-profit annuities would be more affordable and these options should be included in future research.

8.4.3 Further research is required into the appropriate ages for holders of living annuities to purchase life annuities. In addition, alternative drawdown patterns for living annuities could be considered such as drawing down a fixed percentage of the assets each year.

8.4.4 The extension of the work to higher-income earners, liable for income tax, is an important area for future research.

8.4.5 More work is also required on the formulation of the utility function, its parameterisation and its arguments, particularly with regard to whether income should be measured relative to a base strategy or in absolute terms. The treatment of reversionary annuity benefits under expected discounted utility requires further consideration.

8.4.6 Further research is required as to:

– the preferred strategies given health shocks in retirement;
– the impact of other demographic factors such as education and very low or very high income levels on the results;
– the effects of heavier mortality; and
– the sensitivity of the results obtained to the structure and parameterisation of the asset model used.

9. IMPLICATIONS AND CONCLUSIONS

9.1 Although Milevsky & Robinson (2000) proposed their ruin-theory model as a way of helping individuals to make decisions regarding income drawdown accounts and in particular the degree of equity exposure in the income drawdown accounts, this research indicates that the preferred annuitisation strategy under one model may be very different from that under another equally plausible model.

9.2 The chief difficulties are that each fund may not have access to all the individual's financial information and individuals may not know their own information, either because their investments are subject to market fluctuations or because they have lost track of their finances. This could make automated suggestions or member-populated models very misleading.

9.3 Given the sensitivity of the result to funding levels and retirement budgets, the separation of the annuitisation decision from general financial coaching may produce sub-optimal results. In addition, it would be difficult to find a one-size-fits-all solution that is appropriate for everyone or to suggest a preferred annuitisation strategy given the information available to the trustees of the retirement fund.

9.4 Consequently, it would be risky for trustees to put members into annuity products without advice. Trustees may not wish to take on this responsibility without legislative protection. If trustees leave the annuitisation decision with members and make a discounted utility optimisation tool available to members to guide them in their decision, Thomson (2003a) suggests that trustees may either require that the axioms be explained to members or a suitable warning be used explaining that the results may be inconsistent with the individual's approach to preferences. On-site education and training may also be required to facilitate the process (Thomson, 2003b).

9.5 The living annuity can form part of a preferred annuitisation strategy under all of the models and this suggests that it might be inappropriate to prevent people from using living annuities altogether.

9.6 In conclusion, all the expected-discounted-utility methods tended to reward certainty, which seems to reflect actual behaviour and hence may appear palatable to members as well as being consistent with the goals of National Treasury. However, the expected-discounted-utility approach produced results that are very different from those produced using ruin theory. Ruin theory does not measure the severity of any shortfalls experienced and may hence not be reflective of how investors perceive risk. Optimisation using expected discounted utility as an objective function addresses this specific shortfall. This method is, however, more complex to explain to investors and more work is required to ascertain reasonable parameters and to justify the functional form and arguments of this approach.

CONFLICT OF INTEREST DISCLOSURE

All three authors work for Alexander Forbes Financial Services, a registered financial services provider, in the Research and Product Development division. This research was performed in order to advance best practice within the organisation. Other divisions of Alexander Forbes Financial Services provide personal financial advice on retirement and annuities. Given the use of models in the arena of personal financial advice, the authors believed that their results and insights might be useful to a more general audience.

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