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MODELLING THE MORTALITY OF MEMBERS OF GROUP SCHEMES IN SOUTH AFRICA

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ABSTRACT
In this paper, the methodology underlying the graduation of the mortality of members of group schemes in South Africa underwritten by life insurance companies under group life-insurance arrangements is described and the results are presented. A multivariate parametric curve was fitted to the data for the working ages 25 to 65 and comparisons are made with the mortality rates from the SA85–90 ultimate rates for insured lives and the ASSA2008 AIDS and demographic model. The results show that the mortality of members of group schemes is lower than that of the general population, mortality decreasing with increasing salary, as would be expected. For males it was found that there were differences in mortality rates by industry for a given salary band, whereas for females these differences only occurred in the lower salary bands. Furthermore, there is evidence of the healthy-worker effect at ages 60 and above, where the mortality rates appear to level off or even decrease as age increases. This contrasts with the mortality rates from the SA85–90 ultimate rates for insured lives and the ASSA2008 AIDS and demographic model, which increase exponentially.

KEYWORDS
Parametric graduation; mortality laws; mortality rates; force of mortality; occupational mortality; group life insurance; group schemes; ASSA2008 model

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1. INTRODUCTION

1.1 Actuaries, demographers, epidemiologists, sociologists and statisticians have long researched how behavioural, environmental, demographic and socioeconomic factors affect mortality. Early research on how mortality rates vary with age includes work by De Moivre (1725), who hypothesised that deaths were uniformly distributed by age. Since then, more complicated and realistic mortality laws have been proposed, most notably those proposed by Gompertz (1825), Makeham (1867), Perks (1932), Beard (1971) and Forfar, McCutcheon & Wilkie (1988) for adult ages only and Gompertz (1860), Thiele (1871), Heligman & Pollard (1980) and Carriere (1992) for both adults and younger lives including infants.

1.2 In South Africa, the two main problems in estimating mortality rates for the population include under-reporting of deaths and misclassification of causes (Bradshaw, Dorrington & Sitas, 1992; Dorrington, Bradshaw & Wegner, unpublished; Dorrington et al., unpublished). While official South African life tables have been produced for the white, coloured and Indian population groups, no official life tables have been produced for the African population group. As a result, only approximate mortality estimates have been obtained for the South African population as a whole (Dorrington, Bradshaw & Wegner, op. cit.; Dorrington, Moultrie & Timæus, unpublished).

1.3 In terms of the insured population in South Africa, the Actuarial Society of South Africa (ASSA) has conducted a number of investigations into the mortality experience on life-insurance, lump-sum-disability, dread-disease, disability-income, funeral and annuity products.[1] Three standard tables of the mortality of assured population in South Africa have been produced, namely SA56–62 (Mortality Standing Committee, 1974), SA72–77 (Mortality Standing Committee, 1983) and SA85–90 (Dorrington & Rosenberg, 1996), and one standard table each for male and female immediate annuitants; named SAIML98 and SAIFL98 respectively (Dorrington & Tootla, 2007).

1.4 However, very little research has been done to estimate the mortality of members of group schemes in South Africa. Whilst Lewis, Cooper-Williams & Rossouw (unpublished) and Kritzinger & van der Colff (unpublished) provide useful insights into the operation of the group life-insurance market in South Africa in terms of pricing, underwriting and the setting of free-cover limits, they do not consider the underlying mortality of members of group schemes in South Africa. However, Lewis, Cooper-Williams & Rossouw (op. cit.) highlight the need for insurance companies to develop and maintain their book rates, which are used to determine the theoretical risk rate used in pricing group schemes. The only published work on the mortality of members of group schemes in South Africa is that comparing the mortality of African members of

1 Actuarial Society, Continuous Statistical Investigation Committee, www.actuarialsociety.org.za/Societyactivities/CommitteeActivities/ContinuousStatisticalInvestigation(CSI).aspx

group schemes with the mortality of Africans in the population in general and insured lives by Dorrington, Martens & Slawski (1993).

1.5 When comparing the mortality of group-scheme members to the mortality of individual assured policyholders or to the mortality of the general population, one needs to consider the group of lives covered by a group life policy and the effects of underwriting and selection. Firstly, there is very little scope for anti-selection in group life insurance, as group life cover is compulsory for all members of a scheme. In addition, to be eligible for group life benefits there is an implicit actively-at-work selection effect and therefore one would expect the general population to have a higher mortality than that of members of group schemes due to the 'healthy-worker effect' (McMichael, 1976; Monson, 1986; Carpenter, 1987). However, only individuals with life cover greater than the free-cover limit are subjected to underwriting. On balance, one would expect the underlying mortality rates to be higher for members of group schemes than for individual assured policyholders because the lower underwriting requirements have a stronger influence than the reduced anti-selection effect and the healthy-worker effect.

1.6 In order to price group schemes accurately it is therefore important for insurance companies to have a different mortality basis for group-scheme members than those used for the general population and individual insured lives.

1.7 The purpose of this paper is to extend the research by Schriek et al. (unpublished) and Schriek et al. (2013) into the mortality experience of group-scheme members underwritten by South African life insurance companies over the period 2005–2009. This extension entails fitting a multivariate parametric model to the crude mortality rates in Schriek et al. (2013) to produce graduated rates by age, sex, industry and salary band. It was the aim of the authors for the graduation process to:

– eliminate the random sampling errors at each age and hence to produce a set of mortality rates that progress smoothly with age; and
– ensure consistency with industry experience, namely that mortality rates decrease as salary increases and mortality rates are higher for the heavier industries than the light industries.

2. DATA

2.1 The data are described in more detail in Schriek et al. (2013) but, in brief, the data used in this exercise were submitted by six South African life insurance companies—namely Sanlam, Old Mutual, Momentum, Metropolitan, Liberty and Capital Alliance—for compulsory group life insurance over the five-year period from 1 January 2005 to 31 December 2009. The data consist of information on individuals employed in the formal sector aged 20 to 69. By implication the data therefore include only lives that are in good enough health to be actively at work in the formal sector and exclude lives not covered by group schemes. Lives that are unemployed, informally employed or retired because of age or, notably, ill health are excluded. The data did not include any information on whether the optional benefit for continuation of cover during disability was included. Therefore, disabled employees are potentially still included in the data. The data include information on age, annual salary, industry grouping and sex.

2.2 The annual salary data for each of the five calendar years were adjusted for inflation, using the consumer price index, to convert them to 1 January 2010 terms and then they were grouped into five salary bands. These bands were: less than R40 000, R40 000 to R69 999, R70 000 to R124 999, R125 000 to R249 999, and R250 000 or more.

2.3 Information on the occupations of individuals was not captured by the life offices and therefore entire group schemes had to be allocated to one of five industry groupings labelled A to E. The five industry groupings were then regrouped and reclassified as light, mid or heavy. Light industries included companies operating in financial services, business administration and other services such as retail, education, healthcare and information technology. Mid industries included light manufacturing and other blue-collar work that does not involve heavy machinery. Heavy industries included companies operating in mining, transport and other heavy manufacturing.

2.4 Although data were available for ages 20 to 69, only data for ages 25 to 65 were used because of the sparseness of data outside this range when accounting for sex, industry group and salary band. To provide a summary of the relative size of the dataset for ages 25 to 65, the total central exposure and total deaths by industry group and salary band are given in Table 1 for males and in Table 2 for females. To give an indication of the overall differences by industry group and salary band, Table 3 gives the overall crude mortality rates and Table 4 shows the average salary for both males and females.

Table 1. Total central exposure and deaths for ages 25–65 by industry group and salary band: males

Salary band	Central exposure				Deaths			
	Industry group			Total central exposure	Industry group			Total deaths
	light	mid	heavy		light	mid	heavy	
1	306 986	316 651	382 636	1 006 273	3 396	3 972	6 044	13 411
2	318 776	321 503	494 707	1 134 987	3 362	3 862	8 305	15 529
3	353 552	223 267	312 682	889 501	2 242	2 026	3 992	8 260
4	392 156	156 985	246 162	795 303	1 542	840	1 598	3 981
5	376 302	108 628	85 394	570 325	1 052	344	348	1 745
Total	1 747 772	1 127 034	1 521 581	4 396 388	11 593	11 044	20 288	42 925

Table 2. Total central exposure and deaths for ages 25–65 by industry group and salary band: females

Salary band	Central exposure				Deaths			
	Industry group			Total central exposure	Industry group			Total deaths
	light	mid	heavy		light	mid	heavy	
1	306 814	209 957	103 996	620 767	2 596	2 157	1 302	6 055
2	286 415	100 152	63 538	450 104	1 425	629	551	2 605
3	427 093	73 303	92 521	592 917	1 034	216	312	1 561
4	391 653	56 900	73 558	522 111	748	115	184	1 047
5	155 921	22 970	15 688	194 579	226	28	17	271
Total	1 567 896	463 281	349 301	2 380 479	6 028	3 145	2 367	11 540

Table 3. Crude mortality rates for ages 25–65 by industry group, salary band and sex

Sex	Male crude mortality rate				Female crude mortality rate			
Salary band	Industry group			Total	Industry group			Total
	light	mid	heavy		light	mid	heavy	
1	0,01106	0,01254	0,01580	0,01333	0,00846	0,01027	0,01252	0,00975
2	0,01055	0,01201	0,01679	0,01368	0,00497	0,00628	0,00867	0,00579
3	0,00634	0,00908	0,01277	0,00929	0,00242	0,00294	0,00337	0,00263
4	0,00393	0,00535	0,00649	0,00501	0,00191	0,00202	0,00251	0,00201
5	0,00280	0,00317	0,00408	0,00306	0,00145	0,00123	0,00111	0,00139
Total	0,00663	0,00980	0,01333	0,00976	0,00384	0,00679	0,00678	0,00485

Table 4. Average salary for ages 25–65 by industry group, salary band and sex

Sex	Average male salary				Average female salary			
Salary band	Industry group			Total	Industry group			Total
	light	mid	heavy		light	mid	heavy	
1	27 475	26 771	26 846	27 031	27 096	25 316	23 205	25 206
2	54 007	52 836	54 196	53 680	54 550	52 557	54 697	53 935
3	93 810	91 303	93 516	92 876	95 897	93 777	97 000	95 558
4	181 545	177 790	170 785	176 707	172 577	172 327	166 100	170 335
5	468 918	439 840	438 475	449 078	391 280	374 899	380 349	382 176
Total	165 151	157 708	156 764	159 874	148 280	143 775	144 270	145 442

3. METHOD

3.1 THE DISTRIBUTION OF THE NUMBER OF DEATHS

3.1.1 Let $D_{x,r}$ represent the number of deaths aged x last birthday in demographic grouping r, where a 'demographic grouping' is defined as a segment of the population grouped by demographic factors such as sex, industry and salary band. If $D_{x,r}$ has a Poisson distribution with parameter $\mu_{x+0.5,r}E^{c}_{x,r}$, the probability mass function for $D_{x,r}$ is as follows:

$$P\left(D_{x,r}=d\right)=\frac{e^{-\mu_{x+0.5,r}E^{c}_{x,r}}\left(\mu_{x+0.5,r}E^{c}_{x,r}\right)^{d}}{d!};\tag{1}$$

where $\mu_{x+0.5,r}$ is the force of mortality—which for the purposes of this paper has been termed the 'mortality rate'—and $E^{c}_{x,r}$ is the central exposure to risk (i.e. person-years of exposure) in demographic grouping r aged x last birthday. Then the crude rate of mortality and the 95 per cent confidence intervals are:

$$\hat{\mu}_{x+0.5,r}=\frac{d_{x,r}}{E^{c}_{x,r}}\tag{2}$$

$$\hat{\mu}_{x+0.5,r}\pm1.96\frac{\sqrt{d_{x,r}}}{E^{c}_{x,r}};\tag{3}$$

where $d_{x,r}$ is the observed number of deaths in demographic grouping r aged x last birthday.

3.2 TRADITIONAL METHODS OF GRADUATION

3.2.1 Crude mortality rates can be graduated using methods that are parametric or non-parametric. Examples of parametric methods often used to model the rate of mortality at adult ages include the classic mortality models proposed by Gompertz (1825) and Makeham (op. cit.), which were later generalised by Forfar, McCutcheon & Wilkie (op. cit) into the generalised Gompertz–Makeham formulae for graduation. Examples of non-parametric methods include the Whittaker–Henderson method (Joseph, 1952; Lowrie, 1982; Howard, unpublished), the use of splines (McCutcheon, 1981; Dorrington & Rosenberg, op. cit.; Farmer, 2002), non-parametric generalised linear models (Green & Silverman, 1994), kernel smoothing (Haberman, 1983; Debón, Montes & Sala, 2006) and generalised additive models (Hastie & Tibshirani, 1990). In order to reduce the disadvantages of both parametric and non-parametric methods, Thomson (1999) considers a method of enhancing the likelihood of a parametric graduation by means of non-parametric methods, and Schriek et al. (unpublished) have suggested semi-parametric methods, which combine parametric and non-parametric components into one model.

3.2.2 The traditional methods, which include those listed in ¶3.2.1, all involve modelling mortality rates as a function of age, x, that is

$$\mu_x=f_{\boldsymbol{\alpha}}(x);\tag{4}$$

where $\boldsymbol{\alpha}=\alpha_1,\alpha_2,\ldots,\alpha_p$, is a vector of parameters that normally would be estimated using maximum-likelihood techniques or by minimising the residual sum of squares. In the case of splines, this would involve maximising the penalised-log-likelihood or minimising the penalised sum of squares.

3.2.3 An alternative would be, data permitting, to include additional rating factors such as sex, duration, occupation, annual salary or geographical location into the mortality-rate model as covariates, that is:

$$\mu_x = f_{\boldsymbol{\alpha}}(x, R_1, \ldots, R_p); \tag{5}$$

where R_i is the ith rating factor. For example, Madrigal et al. (2011) consider the use of age, salary at retirement, pension amount and sex–marital-status pairs in a generalised linear model to model post-retirement mortality. One method for identifying which rating factors to include in the model would be to use stepwise techniques along with information criteria, in order to determine whether the rating factor is statistically significant. However, stepwise methods should only be used as a guide and one should always consider the relevance of the rating factor in terms of how the model will be used (ibid.).

3.3 THE METHOD USED TO GRADUATE THE DATA

3.3.1 The primary objective of graduating a set of crude mortality rates, $\hat{\mu}_{x,r}$, is to produce a set of graduated rates, $\dot{\mu}_{x,r}$, that progress smoothly with age whilst still correctly reflecting the underlying pattern in mortality (Haberman, op. cit.; Heligman & Pollard, op. cit.).

3.3.2 However, when attempting to graduate the crude mortality rates separately for particular sub-populations of the members of group schemes in South Africa, for example, males working in a light industry earning R250 000 or more per year, some limitations of the process were observed. Firstly, when the data were subdivided by sex, industry group and salary band, there was a paucity of exposure for certain sub-populations involving heavy industries. Secondly, although satisfactory results were obtained in terms of goodness of fit, it was apparent that the models fitted to individual sub-populations failed to capture certain industry and salary-band effects that were present in other sub-populations of the data. Furthermore, it was difficult to explain how salary and industry affected the mortality rates at different ages when comparing various sub-populations.

3.3.3 The authors therefore decided to use a multivariate parametric model that incorporated age, industry and salary band as explanatory variables, as shown in equation (6), which are known rating factors used to determine the book rates that are used to price group schemes (Lewis, Cooper-Williams & Rossouw, op. cit.). The form of the final multivariate graduation model used to graduate the crude rates is given in equation (7).

$$\dot{\mu}_{x,S,I} = f_{\boldsymbol{\alpha}}\left(x, S, I\right) = M\left(j, k, l, m\right) \tag{6}$$

$$\dot{\mu}_{x,S,I} = \exp\left(\begin{array}{c}\sum_{i=1}^{j}\alpha_i x^{i-1} + \sum_{i=j+1}^{j+k} g(S)\alpha_i x^{i-(j+1)} + \sum_{i=j+k+1}^{j+k+l} h(I)\alpha_i x^{i-(j+k+1)} \\ + \sum_{i=j+k+l+1}^{j+k+l+m} g(S)h(I)\alpha_i x^{i-(j+k+l+1)}\end{array}\right); \quad (7)$$

where S refers to salary band, I refers to industry group, j is a positive integer, k, l and m are non-negative integers, $g(\bullet)$ and $h(\bullet)$ are discrete functions whose output values, $g(S)$ and $h(I)$, are optimised and represent the parameters allocated to salary band S and industry I, respectively. The formula is subject to the convention that if k, l or m is zero, then the second, third or fourth summation term, respectively, is defined to be zero. The parameters α_i, $g(S)$ and $h(I)$ are determined by minimising the weighted sums of squares as follows:

$$\sum_S \sum_I \sum_x w_{x,S,I}\left(\hat{\mu}_{x,S,I} - \dot{\mu}_{x,S,I}\right)^2; \quad (8)$$

where $w_{x,S,I}$ is the weight, defined as:

$$w_{x,S,I} = \frac{1}{\text{Var}(\mu_{x,S,I})} = \frac{\left(E^c_{x,S,I}\right)^2}{d_{x,S,I}}. \quad (9)$$

3.3.4 Theoretically, the model could be extended to include sex as an explanatory variable. However, when sex was included, the results suggested that there were inherent differences in mortality due to industry and salary effects over different ages between males and females. It was therefore decided to graduate the male and female mortality rates separately. Initial investigations also considered models using only age and salary band as explanatory variables, and also age and industry. However, discussion on these has not been included as the aim was to produce rates by age, sex, industry and salary band that could be used to price group schemes in practice.

3.3.5 The primary advantage of using a model of this form is that only a single graduation is required for all salary bands and industry groups, rather than a separate graduation for each individual sub-population. In addition, the model is very flexible, allowing an extensive range of different parametric equations to be fitted. When the data in a particular sub-population are sparse, the model makes use of information from other sub-populations to infer the appropriate mortality trends and ensure consistency between graduations.

3.3.6 The performance of the models for males and females was evaluated separately using standard graduation and statistical tests, including the chi-squared test, standard-deviations test, signs test, grouping-of-signs test (also called the Steven's test), cumulative-deviations test and serial-correlations test. For specific details on these tests see, for example, Benjamin & Pollard (1993). Graphical representations of the estimated values and three information criterion measures, namely the Akaike information criterion

(AIC), Bayesian information criterion (BIC) and Hannan–Quinn information criterion were used to select the final model used for the graduation.

4. GRADUATION OF THE MORTALITY RATES OF GROUP SCHEMES IN SOUTH AFRICA

As mentioned in ¶2.4, although data were available for ages 20 to 69, only data for ages 25 to 65 were used to fit the model because of the sparseness of data outside this range when accounting for sex, industry group and salary band. After exploring various combinations of different datasets by industry and salary band groupings, two datasets were graduated. The first dataset consisted of data for males and females separately for ages 25 to 65 ignoring industry and salary band, which could be used to compare mortality rates directly against those for the population as a whole or against a standard table for insured lives. These data were then divided into the five salary bands and three industry groupings, as specified in section 2, to form the second dataset to which life insurance companies can compare their group-life-insurance experience. A decision was taken to use the same salary band and industry groupings for both males and females in order to facilitate the comparison of the results.

4.1 GRADUATION OF THE AGGREGATED MALE AND FEMALE DATA

4.1.1 The aggregated data by sex were graduated using models of the form $M(j,0,0,0)$. Results from the standard graduation and statistical tests are presented in Tables 5 and 6. When considering the chi-squared test and information criteria, the fit to the data for males is significantly improved by increasing the number of parameters from $j = 5$ to $j = 6$ with a trivial improvement from $j = 6$ to $j = 7$. A similar significant improvement is seen for females from $j = 4$ to $j = 5$ with an insignificant improvement from $j = 5$ to $j = 6$. The statistical tests showed that all the models were acceptable, except for the chi-squared test for females, which was not satisfied because of the large deviations over the age of 60.

4.1.2 It was therefore decided to accept $M(6,0,0,0)$ for the graduation of male rates and $M(5,0,0,0)$ for the graduation of female rates. The final graduated rates are shown graphically in Figure 1, along with the crude rates and 95% confidence intervals for the rate of mortality, $\mu_{x+0.5}$.

4.1.3 The graduation of the aggregate data for males provides an extremely good fit whilst the graduation of the aggregate data for females is reasonable, except for ages 59 to 63 where there is significant variation in the underlying rates at some ages. From Figure 1 it is evident that the male mortality rates are higher than those of females, the gap increasing with age. There is also evidence of a slight hump between ages 25 and 45 in both sets of rates, which could be due, in part, to AIDS deaths.

4.1.4 At ages above about 60, the rates appear to level off, or even decrease, as age increases. One explanation for this phenomenon is the possibility of some form of selection due to early retirement, resulting in the healthy-worker effect. Schriek et al. (unpublished) note that, because of retirements there is a large decrease in exposure after age 55, and an even larger decrease after age 60. It is not unreasonable to expect individuals who are chronically sick to retire before age 65, whilst individuals who are healthy and want to continue working, would continue working.

Table 5. Graduation and statistical tests for the graduated male rates (aggregated)

	M(5,0,0,0)	*M*(6,0,0,0)	*M*(7,0,0,0)
Log-likelihood	538 833	538 837	538 837
AIC	–1 077 656	–1 077 661	–1 077 659
BIC	–1 077 647	–1 077 650	–1 077 647
Chi-squared test statistic (TS)	54,75*	47,45	47,42
Critical value (CV) (at 5% significance)	51,00	49,80	48,60
Standard deviations TS	5,24	1,73	1,73
CV (at 5 % significance)	14,07	14,07	14,07
Signs TS	19	19	19
(lower bound; upper bound)	(14;27)	(14;27)	(14;27)
Grouping-of-signs TS	10	11	10
CV (at 5 % significance)	8	8	8
|Cumulative Deviations TS|	0,26	0,22	0,22
Serial correlations TS	0,12	–1,22	–1,20

* significant at 5% level of significance

Table 6. Graduation and statistical tests for the graduated female rates (aggregated)

	M(4,0,0,0)	*M*(5,0,0,0)	*M*(6,0,0,0)
Log-likelihood	115 001	115 006	115 007
AIC	–229 994	–230 002	–230 002
BIC	–229 988	–229 993	–229 991
Chi-squared test statistic (TS)	76,51*	65,69*	64,71*
Critical value (CV) (at 5% significance)	52,19	51,00	49,80
Standard deviations TS	3,68	10,71	2,90
CV (at 5 % significance)	14,07	14,07	14,07
Signs TS	19	22	20
(lower bound; upper bound)	(14;27)	(14;27)	(14;27)
Grouping-of-signs TS	10	9	10
CV (at 5 % significance)	8	8	8
|Cumulative deviations TS|	0,65	0,57	0,56
Serial correlations TS	1,14	0,36	0,32

* significant at the 5% level of significance

4.2 GRADUATION OF THE INDUSTRY AND SALARY-BAND DATA BY SEX

4.2.1 When dealing with more complicated models, it is often difficult to match perfectly the curves being fitted to the data points. Therefore, it is not surprising that all these graduations are in some way statistically significantly different from the observed rates, but none of the models appear to deviate from the data in any systematic way. The poor performance of the chi-squared goodness-of-fit test statistics may be attributed to one or more of the following reasons. Firstly, the sub-division of the aggregated data, by both industry grouping and salary band, results in sub-populations with small central exposed to risk at each age, which means that it would not be appropriate to use the normal approximation for the distribution of the standardised residuals, causing a breakdown in the assumptions used to calculate the chi-squared test statistic (Pollard, 1971). Secondly, the use of industry grouping, rather than occupation, to sub-divide the data probably results in non-homogenous sub-populations, in which case it would be problematic to find a suitable curve that fits the experience well at all ages, partly due to the different features that would predominate at different ages in different salary bands and industry groupings. Finally, given the nature of group-life-insurance data and the level of accuracy in capturing personal details of individuals, the quality of data may introduce additional bias into the underlying mortality rates, resulting in higher chi-squared test statistics. Apart from these possibilities, Forfar, McCutcheon & Wilkie (op. cit.) note that in practice, many graduations with high chi-squared test statistics still produce satisfactory gradations and therefore the high chi-squared goodness-of-fit test statistics need not necessarily be of concern. The results from the overall chi-squared goodness-of-fit test and information-criterion statistics for AIC and BIC for 108 models are given in Appendix A.

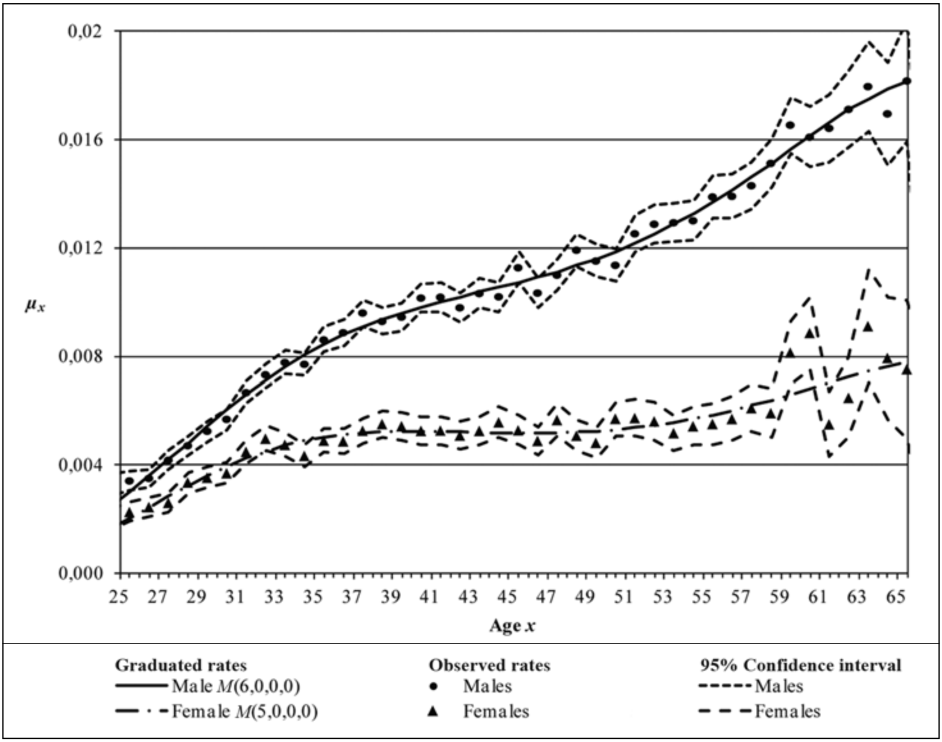

Figure 1. Observed vs. graduated rates (aggregated) with 95% confidence intervals

4.2.2 The criteria used for selecting the final model for graduating the mortality rates by industry and salary band included a combination of factors. While information criteria were used to determine if additional parameters were statistically significant, visual comparisons of the graduated rates with those of the ASSA2008 AIDS and demographic model[2] (ASSA2008) and SA85–90 were also taken into consideration to ensure that the graduated rates incorporated the appropriate trends and shapes, at the higher ages where there was evidence of the healthy-worker effect.

4.2.3 The final models selected were *M*(5,5,1,2) for the male graduation, which had the lowest BIC statistic, and *M*(5,5,2,0) for the female graduation, which had the lowest AIC and BIC statistics. The results of these graduations are shown in Figures 2 to 5 and the estimates of the parameters are given in Appendix B. Figures 2 and 4 present the graduated rates by industry for each salary band for males and females, respectively. For illustrative purposes, the graduated rates are then presented by salary band for each industry in Figures 3 and 5, in comparison with those of the population as a whole from ASSA2008, for males and females, respectively.

4.2.4 From Figures 2 to 5, it is evident that the models capture the expected salary-band and industry effects, so that the mortality rates are decreasing as salaries increase and are higher for heavy industries than for mid and light industries. An AIDS hump is more distinct in the lower salary bands (1 and 2) for both males and females, and more pronounced for heavy industries than mid and light industries. For males, there is a smaller gap in mortality rates between the lower salary bands (1 and 2) than between the higher salary bands (3, 4 and 5). For females, the grouping is at the opposite end in that there is a larger difference in the mortality rates between the lower salary bands (1 and 2) than between the mortality rates at the higher salary bands (3, 4 and 5).

4.2.5 The mortality rates for males in different industries for the same salary band are significantly higher for all salary bands except the highest salary (salary band 5) and the light and mid industries in salary band 4. For females, on the other hand, the mortality rates are only noticeably different for the lower salary bands (1 and 2) for different industries.

4.2.6 Appendix C contains the results from the individual graduation and standard-deviations, signs, cumulative-deviations, grouping-of-signs and serial-correlations tests. Individual graduated rates with 95% confidence intervals are given in Appendix D.

4.2.7 In the individual fits for females, the graduated rates adhere adequately to the mortality by age exhibited by the observed data except for salary band 2 in the heavy industries, for which mortality is underestimated. The graduated rates in salary band 5 do not satisfy the standard-deviations test, which is possibly to be expected because of the sparseness of data in salary band 5, especially for the mid and heavy industries. For salary band 2 in the mid industries there is evidence of some overall bias; however, the fit is still reasonable. For salary band 4 in the light industries, there is evidence that the graduated rates are biased below, particularly for ages 50 to 61.

2 Actuarial Society of South Africa, ASSA2008 AIDS and Demographic Model, 2011, www.actuarialsociety.org.za,03/10/2010

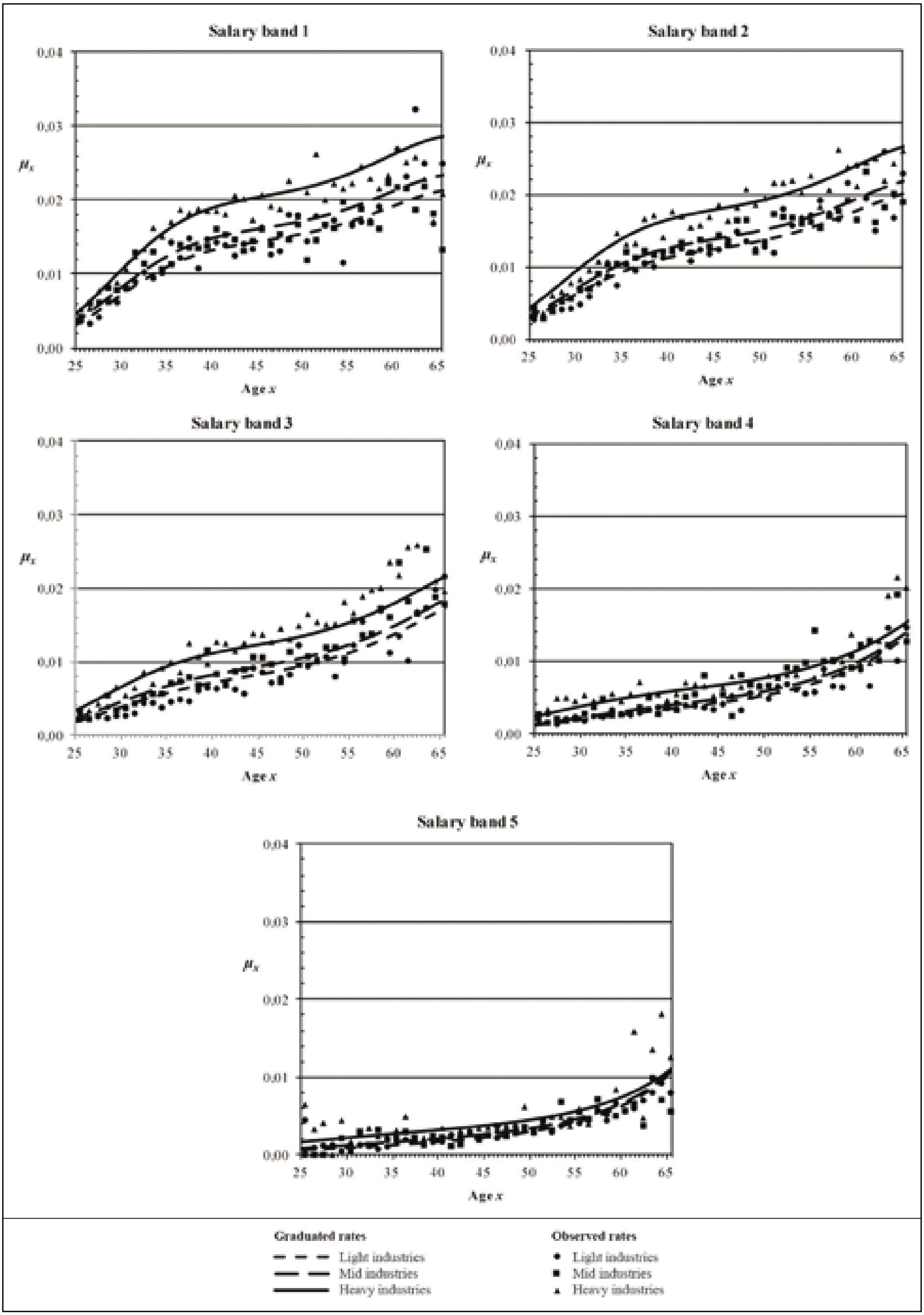

Figure 2. Observed vs. graduated rates for males grouped by salary band

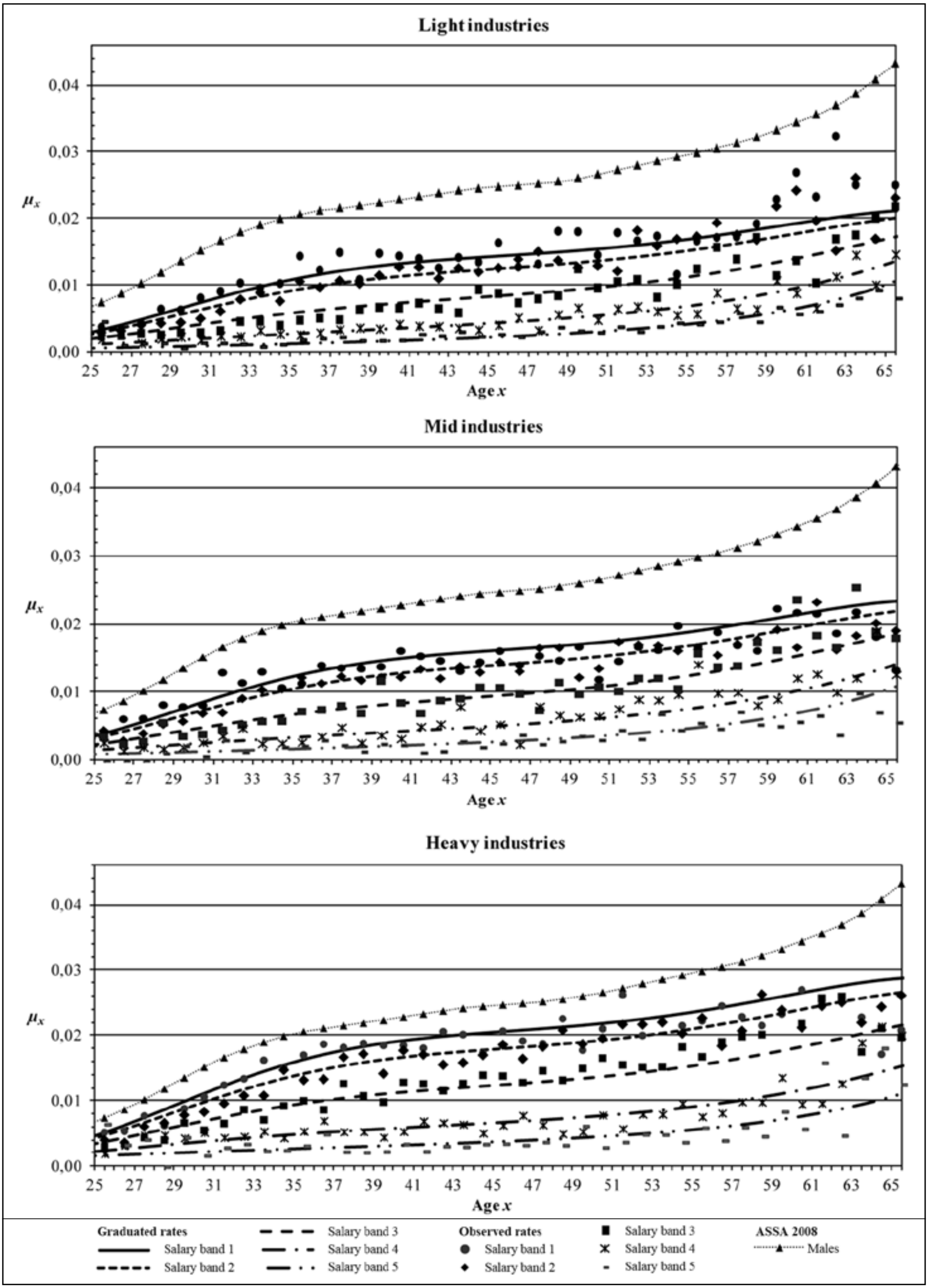

Figure 3. Observed vs. graduated rates for males grouped by industry and ASSA2008 rates

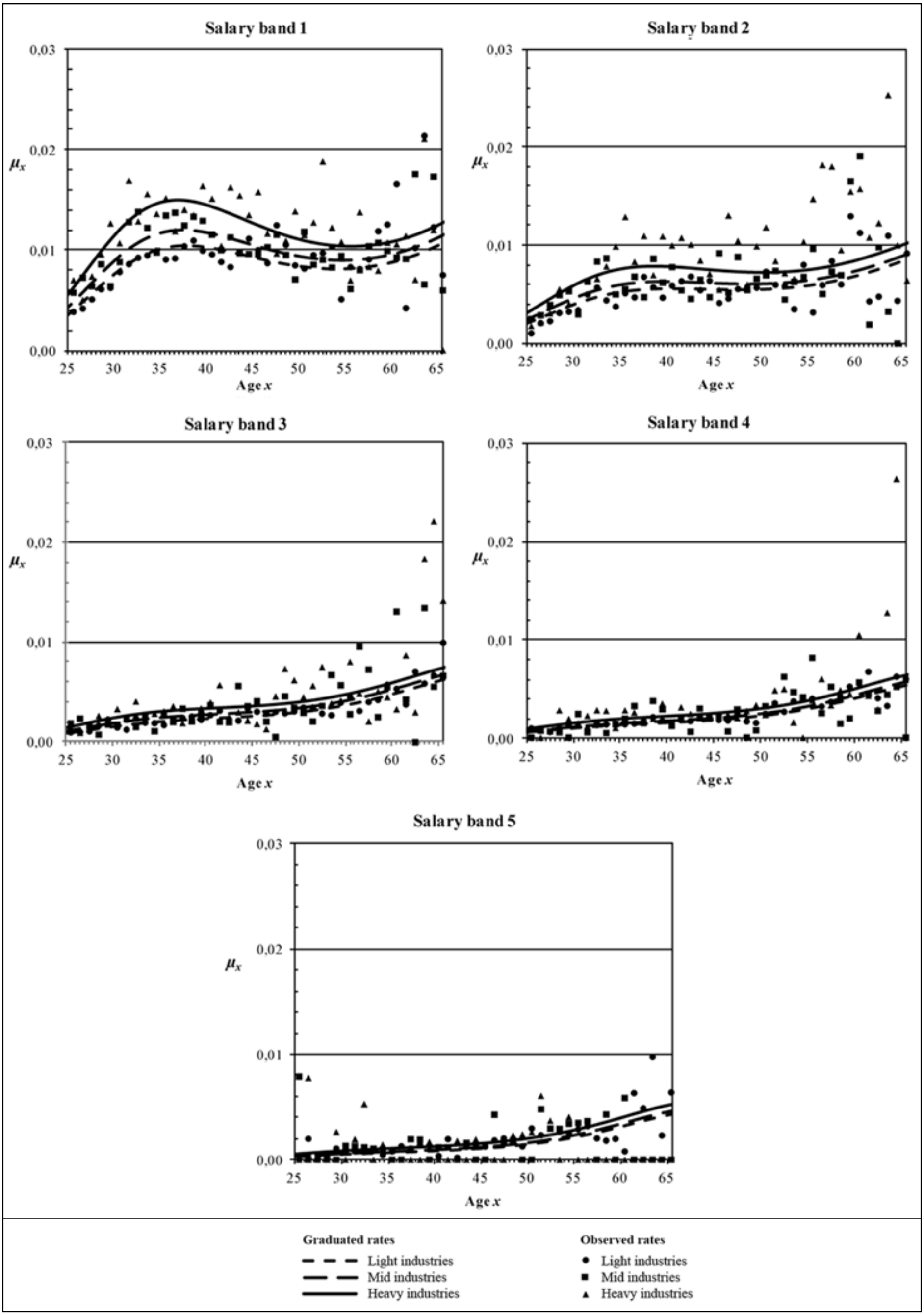

Figure 4. Observed vs. graduated rates for females grouped by salary band

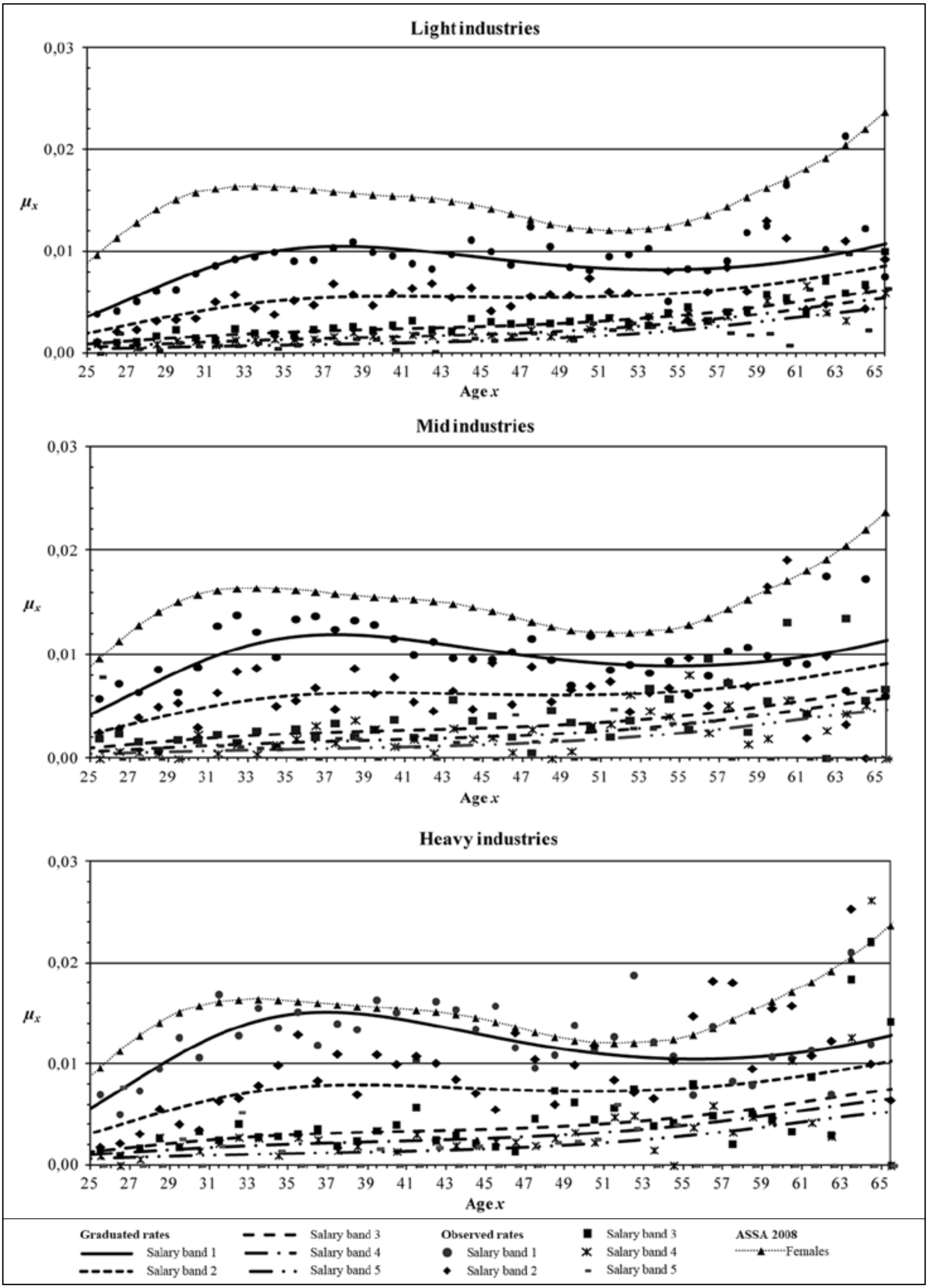

Figure 5. Observed vs. graduated rates for females grouped by industry and ASSA2008 rates

4.2.8 While the individual fits for males were adequate, there were several significant differences between the graduated and observed rates in some salary–industry combinations. For ages 40 and above, the graduated rates overestimate mortality for salary band 1 in the mid and heavy industries, whilst the graduated rates underestimate mortality for salary band 3 in the heavy industries. In addition, for salary band 3 in the light industries, the graduated rates overestimate mortality for ages 44 and below. The graduated rates for salary band 2 in the light and heavy industries still adhere reasonably to the mortality trends exhibited by the observed data; however, the rates slightly overestimate mortality below age 35. There is some evidence that the graduated rates are biased upward for salary band 4 in the heavy industries, primarily between ages 47 and 57. Furthermore, significant serial correlation was present for salary band 3 in the light and heavy industries and for salary band 4 in the heavy industries.

4.2.9 It is quite possible that the reason for the differences observed between the graduated and observed rates is that the individuals in each sub-population are not homogeneous. While industry and salary band can be used as a proxy for occupation, they are not a perfect substitute and therefore individuals allocated to a particular sub-population could be exposed to different mortality risks. This could explain why the graduated rates within a particular sub-population fit the data well over certain ages but overestimate or underestimate over other ages. Additional investigations were carried out using three and four salary bands, by aggregating the data between two salary bands, but the results showed that similar discrepancies were observed between the graduated and observed rates. It was possible to improve the overall fit for males by allowing the salary-band parameters *g(S)* to vary for each industry. However, this resulted in a model that was over-parameterised and, considering information criteria, the additional ten parameters were not statistically significant. It is also possible that part of the discrepancy between the graduated and observed rates may be due to over-graduation or the choice of the functional form of the model used to graduate the data, or both.

5. COMPARISON WITH OTHER MORTALITY TABLES

5.1 The graduated rates are compared with the ASSA2008 rates for the population as a whole and to the SA85–90 ultimate rates for insured lives, both graphically and using standardised mortality ratios (SMRs). Since SA85–90 ultimate rates are available only for males, the mortality rates for females were taken as 45% of the SA85–90 ultimate rates, following the suggestion by Dorrington & Rosenberg (op. cit.). For the purposes of comparison the rate of mortality underlying the SA85–90 ultimate rates and the rate of mortality underlying the ASSA2008 AIDS and demographic model was approximated using $\mu_{x+0.5} = -\ln(1-q_x)$ where q_x is the probability of a life aged x dying within one year.

5.2 Figures 6 and 7 show a comparison of the graduated rates aggregated by industry and salary with the rate of mortality derived from the SA85–90 ultimate rates and the ASSA2008 rates and Table 7 gives the SMRs for various age bands using either the SA85–90 ultimate rates or ASSA2008 rates as the standard population. The graduated rates for men below age 60 and for women below age 58 are as expected, the graduated rates being lower than the ASSA2008 rates by roughly 45% of these rates for males and 40% for females. However, over these ages, the graduated rates are higher than the SA85–90 ultimate rates. On the other hand, above age 60 for men and 58 for women the graduated rates are lower than the SA85–90 ultimate rates. As mentioned above, this could be due to the healthy-worker effect. Unexpectedly, the graduated rates have similar shapes to those from the ASSA2008 AIDS and demographic model, except that the ASSA2008 rates increase more rapidly with age at the older ages and in the case of females there is a far more pronounced AIDS hump between the ages of 25 and 45.

5.3 The SMRs for different salary bands and industries, and for various age bands, are shown graphically in Figures 8 and 9. Dorrington & Rosenberg (op. cit.) point out that the SA85–90 ultimate rates would probably be the last South African insured lives experience for which AIDS-related deaths would have a negligible impact. It is therefore not unexpected that when the SA85–90 ultimate rates are used as the standard, a hump shape is present in the SMRs.

5.4 As can be seen from the comparison of the graduated rates for different salary bands and industries in Figures 3 and 5, the graduated rates are lower than the ASSA2008 rates for all salary bands and industries. As a result, when the ASSA2008 rates are used

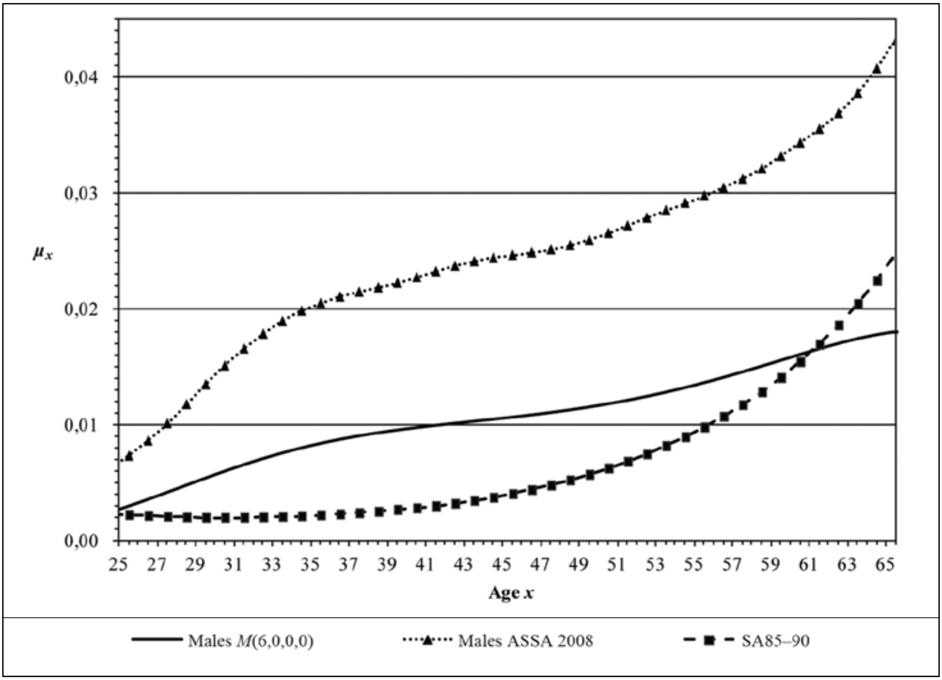

Figure 6. Aggregate graduated rates vs SA85–90 ultimate and ASSA2008 rates: male

as the standard, the SMRs in Figures 8 and 9 are all less than one. Furthermore, the levelling off of the graduated rates above age 60 is more pronounced for those in the lower salary bands (1 and 2) than for those in the higher salary bands (3, 4 and 5).

Table 7. Standardised mortality ratio comparing the graduated rates (aggregate) with the ASSA2008 rates and the SA85–90 rates

Sex	Males		Females	
Age band	SMR using ASSA2008	SMR using SA85–90	SMR using ASSA2008	SMR using SA85–90
25–29	0,41	2,01	0,23	2,98
30–34	0,40	3,39	0,28	4,75
35–39	0,42	3,68	0,33	4,69
40–44	0,43	3,10	0,35	3,55
45–49	0,44	2,29	0,39	2,40
50–54	0,45	1,66	0,45	1,63
55–59	0,47	1,25	0,43	1,18
60–64	0,46	0,94	0,38	0,89
25–65	0,44	1,54	0,35	1,67

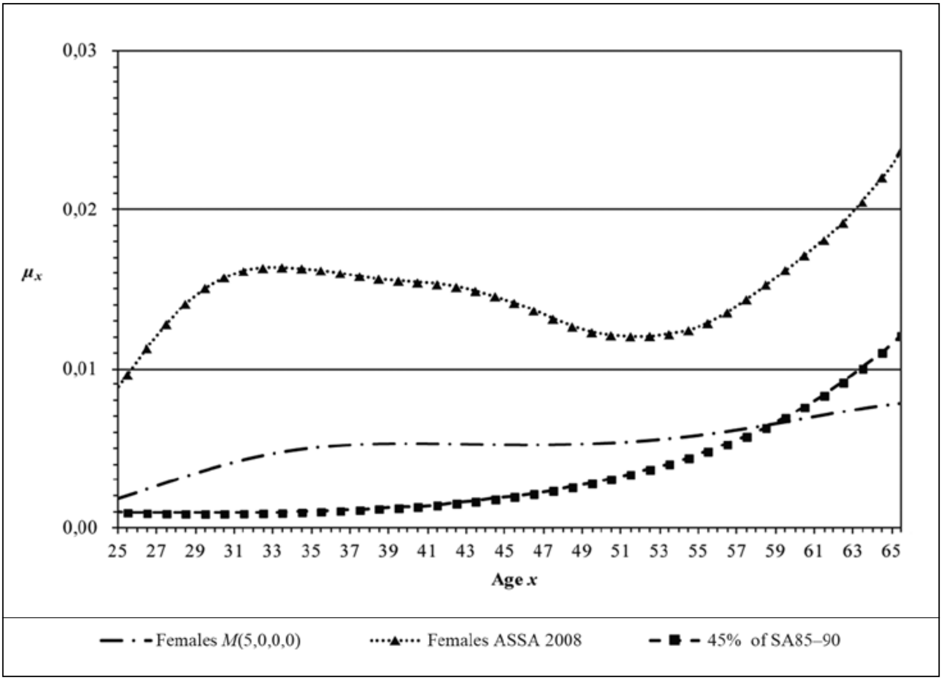

Figure 7. Aggregate graduated rates vs SA85–90 and ASSA2008 rates: female

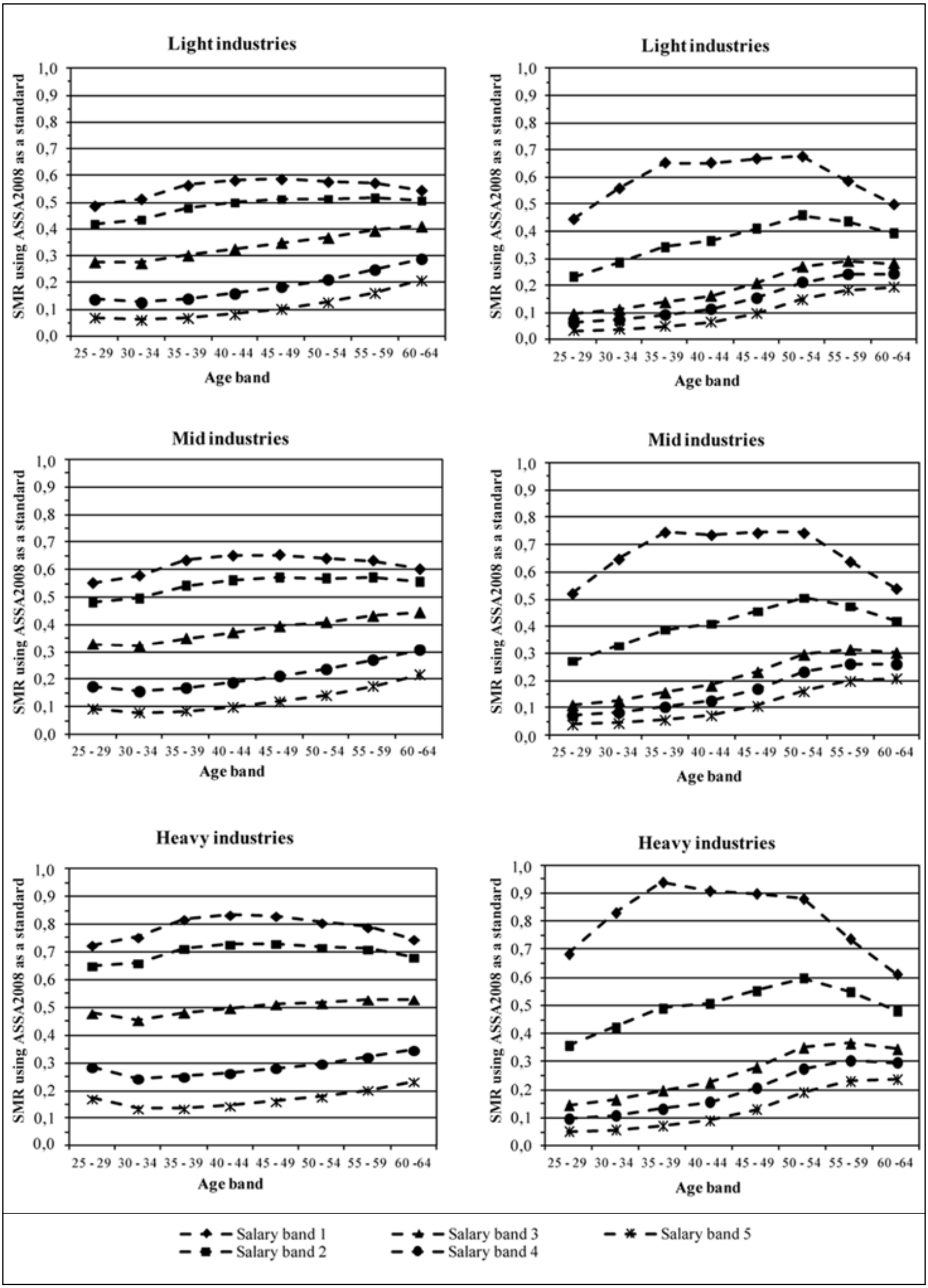

Figure 8. Standardised mortality ratios comparing the graduated rates by industry and salary band with the ASSA2008: male (left column) and female (right column)

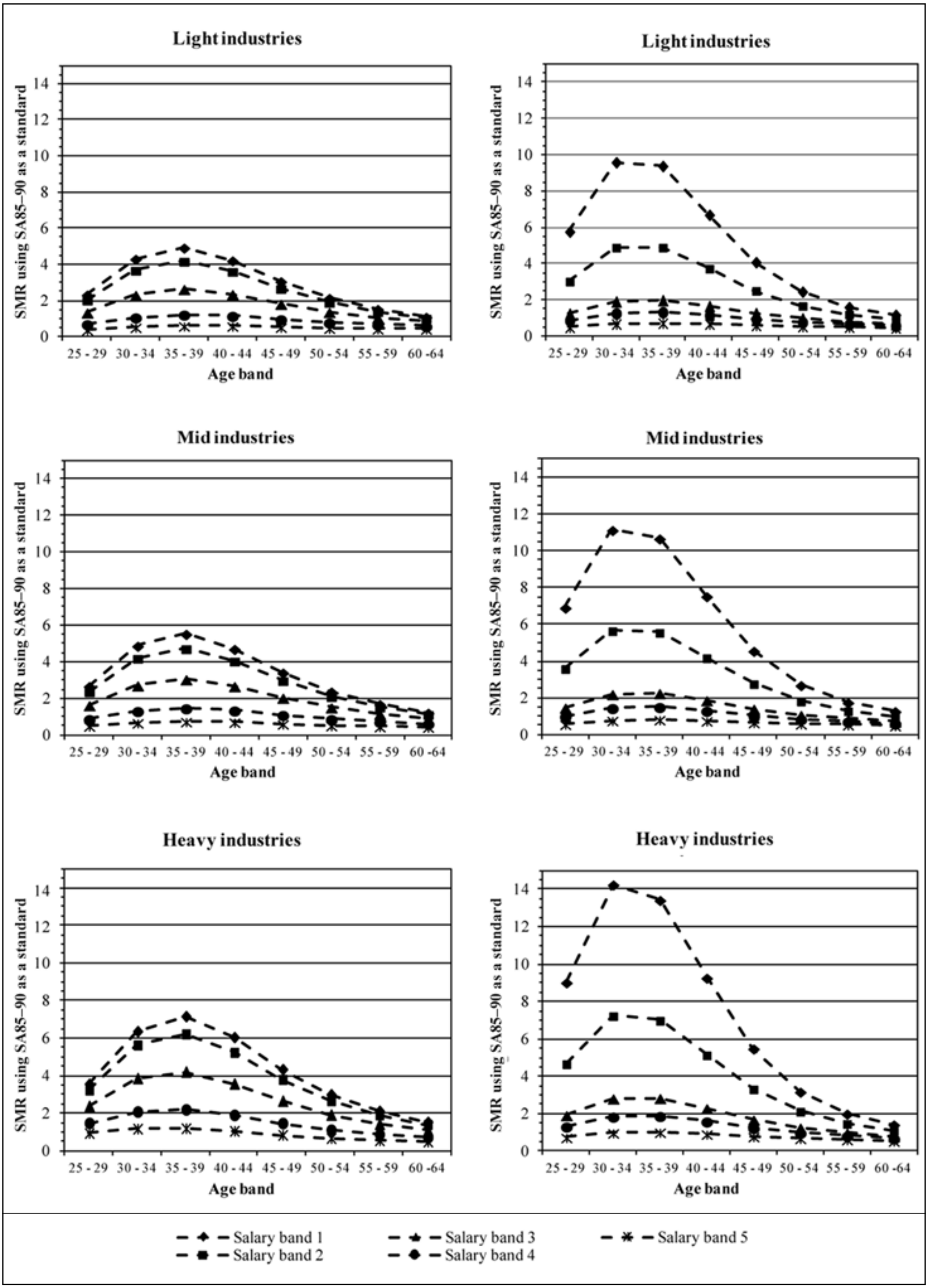

Figure 9. Standardised mortality ratios comparing the graduated rates by industry and salary band with the SA85–90 ult. rates: male (left column) and female (right column)

5.5 For males in light and mid industry groups, the graduated rates are below the SA85–90 rates. This is evidenced by the SMRs less than one in Figure 8. The SMRs for salary band 5 in heavy industries exceed one in certain age bands. It was also noted that there were SMRs less than one for other groups, such as males in salary bands 3, 4 and 5 in the older age bands and 25–29 age band. Whilst this is expected in the higher age bands because of the results from the aggregate data, as discussed in ¶5.2, it is not expected in the lower age bands..

6. SUMMARY AND AREAS FOR FURTHER RESEARCH

6.1 In this paper, the authors have presented the methodology and results of graduating the mortality of members of group schemes in South Africa over the five-year period 2005–2009 by fitting a multivariate parametric model to the crude mortality rates. A multivariate parametric model was used as it has the advantage of ensuring that the graduated rates progress smoothly with age and in addition, when the data in a particular sub-population are sparse, the appropriate mortality trends can be inferred by making use of all the data and thereby ensuring consistency between graduations. Another important advantage of the method used is that a single graduation is carried out for all salary bands and industry groups rather than performing an individual graduation for each sub-population in the dataset. However, one potential drawback of the method is the risk of over-parameterisation due to the number of parameters required to explain the effects due to industry and salary. To account for this, male and female mortality rates were modelled separately because of the inherent differences in mortality trends between them and information criterion were used to determine if additional parameters were statistically significant in order to obtain a parsimonious model.

6.2 Two sets of graduated rates for each sex, which reflect the mortality of lives in good enough health to be actively at work in the formal sector covered by group schemes for ages 25 to 65, were produced and compared with those of the population as a whole from ASSA2008 and from the SA85–90 ultimate rates for insured lives. The first set of mortality rates, referred to as the aggregated data, were age-specific mortality rates for both males and females ignoring industry and salary band, which can be used by life insurance companies for comparing their overall group life experience to that of the industry. The second set of mortality rates were age-, industry- and salary-band-specific mortality rates for both males and females, which could be used to price group schemes.

6.3 Overall the graduation of the aggregated data gave good fits except for females for ages from 59 to 63, where there were significant deviations in the crude rates at some ages. From the comparisons of the graduated rates with the mortality rates of insured lives and the population as a whole, below age 60 the rates are ranked as expected, the mortality of members of group schemes being higher than those of insured lives and lighter than the general population. Furthermore, females exhibited lighter mortality than males. However, above age 60, the graduated rates tend to level off and even fall below those of insured lives, possibly as a result of selection due to early retirement, ill-health retirement and the healthy-worker effect. Further investigations are needed in order to understand better the causes behind this selection effect and the effect it has on the pricing of group schemes.

6.4 As far as the age-, industry- and salary-band-specific mortality rates are concerned, whilst the overall fit was statistically significantly different from the observed rates, probably because of heterogeneity within industry groups, the graduated rates still adhere reasonably well to the mortality trends exhibited by the observed data. More importantly, the graduated rates are consistent with expected mortality patterns by salary and industry, so that mortality decreases with an increase in salary and increases with a movement from light to heavier industries. Furthermore, the age pattern of mortality observed in the lowest salary bands is similar to that exhibited by the general population, whereas mortality rates in the highest salary bands are similar to those exhibited by insured lives, which is reassuring.

6.5 One concern with the use of industry as a risk factor as opposed to occupation is that the contributing companies classify industries differently and the allocation of schemes to industry groups is a fairly subjective exercise. It is quite possible that, as a result of this difference, the people in a particular sub-population are not perfectly homogeneous. This could explain why the graduated rates are overestimating or underestimating mortality over certain age ranges in certain sub-populations. The way in which industry classification affects the underlying mortality rates requires further investigation. Other possible reasons for the differences between the graduated and observed rates, over certain age ranges in certain sub-populations, may be over-graduation and the choice of the functional form of the model used to graduate the data.

6.6 Whilst these mortality rates do not include the risk factors for occupation and geographic region, which are also taken into account in pricing group schemes, it is envisaged that future data collected will include fields for occupation and more accurate data collected on geographic region of employment to help improve the pricing of group schemes.

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APPENDIX A

GOODNESS-OF-FIT AND INFORMATION-CRITERION STATISTICS

Table A.1. Overall chi-squared test statistic for industry-group and salary-band data

	Males			Females		
Model	d = 4	d = 5	d = 6	d = 4	d = 5	d = 6
M(d,3,1,0)	1 079,81	1 023,99	1 015,31	863,01	836,48	823,48
M(d,4,1,0)	1 041,75	1 006,08	999,58	838,40	821,16	810,23
M(d,5,1,0)	1 038,35	992,03	979,03	846,85	812,25	793,38
M(d,3,2,0)	1 042,59	989,54	982,49	851,05	829,97	823,75
M(d,4,2,0)	1 010,10	967,15	962,84	834,99	810,18	808,44
M(d,5,2,0)	1 008,51	951,27	942,73	840,34	792,96	792,64
M(d,3,3,0)	1 035,82	996,36	980,98	849,78	822,46	823,01
M(d,4,3,0)	1 007,95	968,12	960,54	835,60	809,03	806,87
M(d,5,3,0)	1 007,81	966,52	935,84	839,99	806,66	803,79
M(d,3,4,0)	1 033,81	991,64	975,04	838,12	816,45	820,62
M(d,4,4,0)	1 003,40	967,85	964,31	824,06	806,04	806,83
M(d,5,4,0)	999,08	946,48	952,40	860,60	818,39	818,83
M(d,3,1,1)	1 066,61	1 019,85	1 011,90	864,74	841,01	839,69
M(d,4,1,1)	1 038,74	996,46	994,98	862,79	839,19	833,75
M(d,5,1,1)	1 031,18	984,62	983,51	859,09	835,16	827,04
M(d,3,2,1)	1 029,37	976,18	966,56	850,51	840,61	837,86
M(d,4,2,1)	995,34	955,38	949,30	843,28	829,14	815,96
M(d,5,2,1)	993,80	942,81	930,58	849,02	833,63	835,09
M(d,3,3,1)	1 023,31	971,30	963,31	858,15	834,37	835,99
M(d,4,3,1)	992,70	954,34	947,85	836,25	825,08	824,67
M(d,5,3,1)	991,35	937,40	927,41	847,39	828,55	825,67
M(d,3,4,1)	1 019,34	962,70	959,63	855,74	835,78	837,27
M(d,4,4,1)	991,49	956,06	948,87	844,95	815,50	819,74
M(d,5,4,1)	982,74	940,47	936,36	820,53	820,41	847,09
M(d,3,1,2)	1 025,13	968,03	961,21	863,17	844,61	843,19
M(d,4,1,2)	998,21	962,40	942,36	859,07	831,83	829,80
M(d,5,1,2)	992,84	939,39	933,80	835,32	830,35	819,47
M(d,3,2,2)	1 023,89	976,09	964,32	864,22	845,32	850,13
M(d,4,2,2)	988,54	950,08	938,22	856,99	820,58	829,88
M(d,5,2,2)	987,83	941,08	922,54	839,78	830,12	835,72
M(d,3,3,2)	1 021,49	979,87	967,53	862,28	836,56	841,31
M(d,4,3,2)	988,91	953,38	939,16	838,25	821,79	819,77
M(d,5,3,2)	971,65	947,47	930,74	828,04	832,91	822,19
M(d,3,4,2)	1 013,21	960,85	959,08	856,38	833,22	831,26
M(d,4,4,2)	987,41	947,86	944,07	831,36	820,16	823,29
M(d,5,4,2)	965,33	927,09	923,50	829,69	821,12	812,84

Table A.2. AIC and BIC statistics for male industry-group and salary-band data

	AIC			BIC		
Model	d = 4	d = 5	d = 6	d = 4	d = 5	d = 6
M(d,3,1,0)	–626 047	–626 107	–626 109	–625 976	–626 032	–626 030
M(d,4,1,0)	–626 069	–626 119	–626 117	–625 994	–626 040	–626 033
M(d,5,1,0)	–626 072	–626 121	–626 129	–625 992	–626 037	–626 041
M(d,3,2,0)	–626 075	–626 120	–626 135	–626 000	–626 041	–626 052
M(d,4,2,0)	–626 097	–626 146	–626 147	–626 018	–626 062	–626 058
M(d,5,2,0)	–626 095	–626 155	–626 153	–626 011	–626 067	–626 060
M(d,3,3,0)	–626 076	–626 130	–626 133	–625 997	–626 046	–626 045
M(d,4,3,0)	–626 097	–626 126	–626 152	–626 013	–626 038	–626 060
M(d,5,3,0)	–626 097	–626 138	–626 168	–626 008	–626 046	–626 071
M(d,3,4,0)	–626 075	–626 133	–626 139	–625 991	–626 045	–626 046
M(d,4,4,0)	–626 097	–626 148	–626 147	–626 008	–626 055	–626 050
M(d,5,4,0)	–626 090	–626 159	–626 155	–625 997	–626 062	–626 054
M(d,3,1,1)	–626 052	–626 109	–626 110	–625 977	–626 030	–626 027
M(d,4,1,1)	–626 072	–626 124	–626 120	–625 992	–626 040	–626 032
M(d,5,1,1)	–626 070	–626 124	–626 129	–625 986	–626 036	–626 037
M(d,3,2,1)	–626 083	–626 144	–626 144	–626 004	–626 060	–626 056
M(d,4,2,1)	–626 105	–626 154	–626 154	–626 021	–626 065	–626 062
M(d,5,2,1)	–626 105	–626 159	–626 168	–626 016	–626 066	–626 071
M(d,3,3,1)	–626 085	–626 150	–626 148	–626 001	–626 061	–626 055
M(d,4,3,1)	–626 105	–626 156	–626 156	–626 017	–626 064	–626 059
M(d,5,3,1)	–626 105	–626 167	–626 172	–626 012	–626 069	–626 070
M(d,3,4,1)	–626 084	–626 148	–626 147	–625 996	–626 056	–626 050
M(d,4,4,1)	–626 104	–626 153	–626 154	–626 012	–626 056	–626 052
M(d,5,4,1)	–626 109	–626 164	–626 152	–626 012	–626 062	–626 046
M(d,3,1,2)	–626 091	–626 156	–626 144	–626 012	–626 072	–626 056
M(d,4,1,2)	–626 113	–626 157	–626 170	–626 029	–626 069	–626 078
M(d,5,1,2)	–626 111	–626 175	–626 170	–626 023	–626 082	–626 073
M(d,3,2,2)	–626 090	–626 151	–626 155	–626 006	–626 062	–626 062
M(d,4,2,2)	–626 114	–626 162	–626 167	–626 026	–626 069	–626 070
M(d,5,2,2)	–626 117	–626 168	–626 167	–626 025	–626 071	–626 065
M(d,3,3,2)	–626090	–626 134	–626 152	–626 002	–626 042	–626 055
M(d,4,3,2)	–626 109	–626 161	–626 168	–626 017	–626 064	–626 067
M(d,5,3,2)	–626 124	–626 159	–626 170	–626 026	–626 057	–626 065
M(d,3,4,2)	–626 090	–626 154	–626 154	–625 997	–626 057	–626 053
M(d,4,4,2)	–626 109	–626 161	–626 166	–626 012	–626 059	–626 060
M(d,5,4,2)	–626 143	–626 176	–626 174	–626 041	–626 070	–626 064

Table A.3. AIC and BIC statistics for female industry-group and salary-band data

	AIC			BIC		
Model	d = 4	d = 5	d = 6	d = 4	d = 5	d = 6
M(d,3,1,0)	–120 125	–120 140	–120 155	–120 056	–120 066	–120 077
M(d,4,1,0)	–120 133	–120 150	–120 160	–120 060	–120 072	–120 078
M(d,5,1,0)	–120 145	–120 165	–120 168	–120 067	–120 082	–120 082
M(d,3,2,0)	–120 131	–120 154	–120 153	–120 057	–120 077	–120 071
M(d,4,2,0)	–120 141	–120 163	–120 164	–120 063	–120 081	–120 077
M(d,5,2,0)	–120 152	–120 173	–120 171	–120 069	–120 086	–120 080
M(d,3,3,0)	–120 129	–120 155	–120 151	–120 051	–120 073	–120 065
M(d,4,3,0)	–120 136	–120 162	–120 162	–120 053	–120 075	–120 071
M(d,5,3,0)	–120 151	–120 165	–120 166	–120 065	–120 074	–120 071
M(d,3,4,0)	–120 134	–120 155	–120 149	–120 052	–120 068	–120 058
M(d,4,4,0)	–120 141	–120 156	–120 153	–120 054	–120 065	–120 058
M(d,5,4,0)	–120 155	–120 162	–120 158	–120 064	–120 067	–120 058
M(d,3,1,1)	–120 124	–120 149	–120 149	–120 050	–120 072	–120 067
M(d,4,1,1)	–120 123	–120 146	–120 147	–120 046	–120 064	–120 061
M(d,5,1,1)	–120 139	–120 157	–120 153	–120 056	–120 070	–120 062
M(d,3,2,1)	–120 131	–120 153	–120 150	–120 053	–120 071	–120 063
M(d,4,2,1)	–120 131	–120 155	–120 160	–120 049	–120 069	–120 070
M(d,5,2,1)	–120 148	–120 158	–120 157	–120 062	–120 067	–120 062
M(d,3,3,1)	–120 127	–120 149	–120 147	–120 045	–120 063	–120 057
M(d,4,3,1)	–120 138	–120 156	–120 152	–120 052	–120 065	–120 057
M(d,5,3,1)	–120 143	–120 159	–120 152	–120 052	–120 063	–120 052
M(d,3,4,1)	–120 127	–120 142	–120 141	–120 040	–120 051	–120 045
M(d,4,4,1)	–120 133	–120 156	–120 151	–120 042	–120 061	–120 052
M(d,5,4,1)	–120 160	–120 161	–120 143	–120 064	–120 062	–120 039
M(d,3,1,2)	–120 124	–120 147	–120 144	–120 046	–120 064	–120 058
M(d,4,1,2)	–120 121	–120 153	–120 152	–120 039	–120 067	–120 061
M(d,5,1,2)	–120 151	–120 158	–120 156	–120 065	–120 067	–120 061
M(d,3,2,2)	–120 122	–120 147	–120 139	–120 040	–120 061	–120 048
M(d,4,2,2)	–120 129	–120 153	–120 151	–120 043	–120 062	–120 056
M(d,5,2,2)	–120 146	–120 156	–120 146	–120 055	–120 061	–120 046
M(d,3,3,2)	–120 121	–120 148	–120 143	–120 034	–120 057	–120 048
M(d,4,3,2)	–120 135	–120 154	–120 153	–120 044	–120 059	–120 053
M(d,5,3,2)	–120 140	–120 152	–120 139	–120 045	–120 053	–120 035
M(d,3,4,2)	–120 125	–120 146	–120 145	–120 034	–120 050	–120 046
M(d,4,4,2)	–120 137	–120 151	–120 146	–120 041	–120 052	–120 043
M(d,5,4,2)	–120 152	–120 156	–120 158	–120 052	–120 052	–120 050

APPENDIX B

PARAMETER ESTIMATES

In this appendix the parameter estimates of the final graduations are presented. In Table B.1 the parameters referred to are those in the following formula:

$$\mu_{x+0.5,S,I} = \exp\begin{pmatrix} \alpha_1 + \alpha_2 x + \alpha_3 x^2 + \alpha_4 x^3 + \alpha_5 x^4 + \alpha_6 x^5 + S_F\alpha_7 + S_F\alpha_8 x + S_F\alpha_9 x^2 \\ +S_F\alpha_{10}x^3 + S_F\alpha_{11}x^4 + I_F\alpha_{12} + I_F\alpha_{13}x + S_F I_F\alpha_{14} + S_F I_F\alpha_{15}x \end{pmatrix}. \quad \text{(B.1)}$$

Table B.1. Parameter estimates from the final graduations

	Male		Female	
	$M(6,0,0,0)$	$M(5,5,1,2)$	$M(5,0,0,0)$	$M(5,5,2,0)$
Parameters				
α_1	–24,84184629947	–27,05390809246	–27,71728981421	–24,01319871984
α_2	1,62285424188	1,79924832185	1,87486558620	1,56636382350
α_3	–0,04893554567	–0,05277159516	–0,05761489784	–0,04671030541
α_4	0,00065329706	0,00068410918	0,00076983311	0,00059971941
α_5	–0,00000321107	0,00000328535	–0,00000375376	–0,00000278667
α_6	$4{,}4672751\times10^{-15}$			
α_7		2,91802487546		0,50772154529
α_8		–0,29272313085		–0,05647418547
α_9		0,00904618853		0,00104199124
α_{10}		–0,00012032382		–0,00000092187
α_{11}		0,00000060381		–0,00000006131
α_{12}		0,14276792863		0,27615851385
α_{13}				–0,00299307842
α_{14}		0,10546201846		
α_{15}		–0,00199440177		
Salary band factor parameters				
$g(1)$		0,54417194058		–2,24379381534
$g(2)$		0,89149254920		–0,17133593018
$g(3)$		1,86272039256		2,71637719034
$g(4)$		3,48571773526		4,0129555080
$g(5)$		5,01003153393		5,96081632753
Industry grouping factor parameters				
$h(l)$		0,44887595531		–2,58772618449
$h(m)$		1,16788997585		–1,76734542245
$h(h)$		2,77595394385		–0,37234679591

APPENDIX C

STATISTICAL TESTS FOR GRADUATED RATES BY INDUSTRY GROUP AND SALARY BAND

Table C.1. Individual statistical tests for male graduated rates by industry group and salary band

	Light industries					Mid industries					Heavy industries				
Salary band	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
Chi-squared TS	69,87	57,25	87,89	60,70	66,40	72,17	37,37	60,22	80,11	38,81	56,23	70,85	82,83	57,86	40,83
CV (at 5% significance)	41,34	41,34	41,34	41,34	41,34	41,34	41,34	41,34	41,34	37,65	41,34	41,34	41,34	41,34	40,11
Standard deviations TS	7,98	5,24	22,41	4,85	6,02	7,98	4,46	8,76	13,44	4,07	9,54	24,76	30,61	3,29	7,59
CV (at 5 % significance)	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07
Signs TS	25	22	14	23	23	14	17	24	25	18	15	14	29	20	16
(lower bound; upper bound)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)
Grouping of signs TS	11	10	6	11	9	6	9	9	9	11	11	8	7	8	10
CV (at 5 % significance)	8	8	7	8	8	7	8	8	8	8	7	7	7	8	7
|Cumulative deviations TS|	1,71	0,76	3,79	0,62	1,30	1,40	0,85	1,36	3,63	0,09	2,19	1,24	4,66	1,03	0,74
Serial correlations TS	0,13	1,82	2,63	–0,05	1,15	1,95	–0,01	–0,17	0,55	–0,25	0,10	0,52	2,48	2,40	–1,38

Table C.2. Individual statistical tests for female graduated rates by industry and salary band

	Light industries					Mid industries					Heavy industries				
Salary band	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
Chi-squared TS	67,72	84,41	35,15	31,35	62,13	50,66	76,87	49,23	36,70	29,55	44,90	104,99	54,51	36,37	28,43
CV (at 5% significance)	42,56	42,56	42,56	42,56	41,34	42,56	41,34	41,34	37,65	18,31	41,34	42,56	42,56	38,89	5,99
Standard deviations TS	11,10	8,76	6,02	12,66	17,73	6,02	3,68	3,68	8,37	26,71	2,90	32,95	7,20	5,63	39,98
CV (at 5 % significance)	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07	14,07
Signs TS	18	20	26	29	30	22	23	24	16	22	20	27	20	18	14
(lower bound; upper bound)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)	(14;27)
Grouping of signs TS	8	10	9	10	8	12	14	14	9	9	11	6	10	11	9
CV (at 5 % significance)	7	8	8	7	7	8	8	8	8	8	8	7	8	8	7
|Cumulative deviations TS|	0,10	0,35	1,93	2,18	3,43	1,31	2,36	1,39	0,14	0,02	0,56	4,96	1,16	0,56	1,27
Serial correlations TS	1,50	0,17	–0,28	1,45	–1,49	0,23	–0,73	–0,69	0,34	–0,32	–0,26	2,63	1,63	–0,05	0,36

APPENDIX D

GRAPHICAL REPRESENTATIONS OF THE GRADUATED RATES

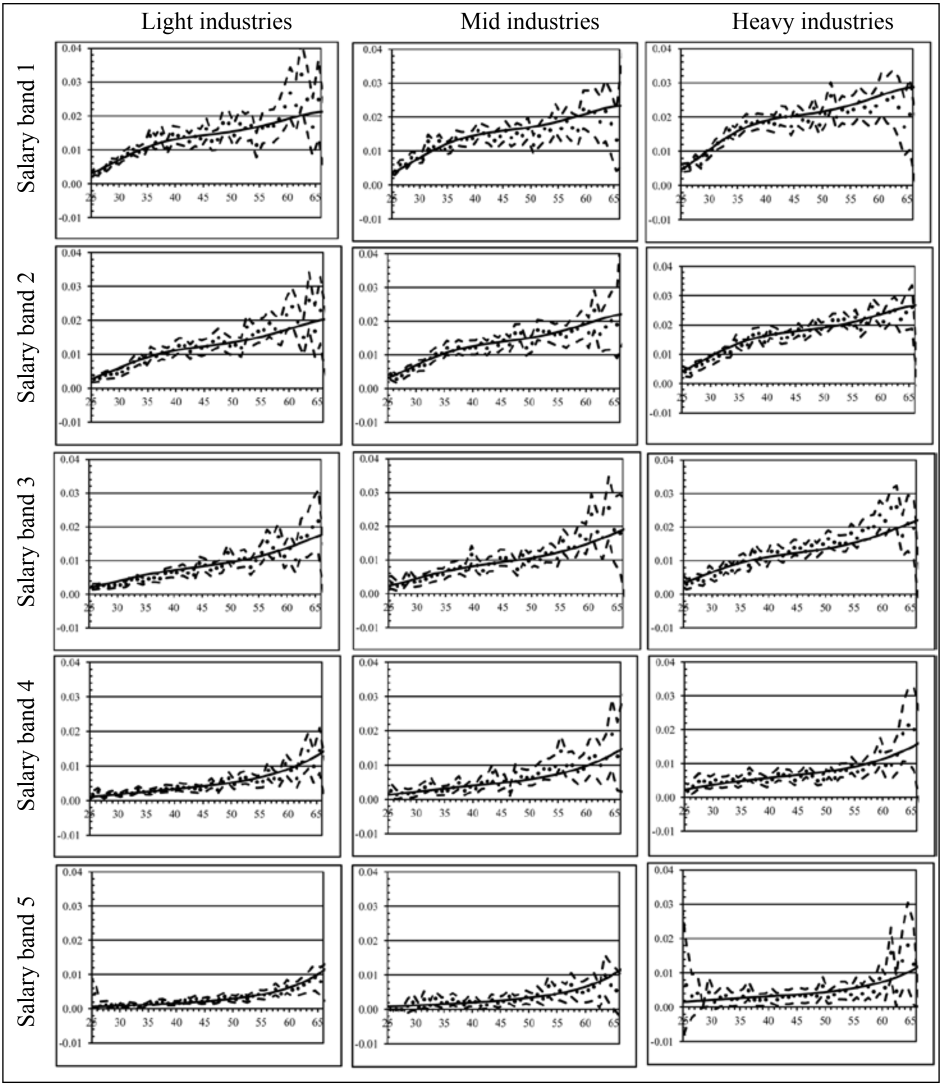

Figure D.1. Observed and graduated rates for males with 95% confidence intervals

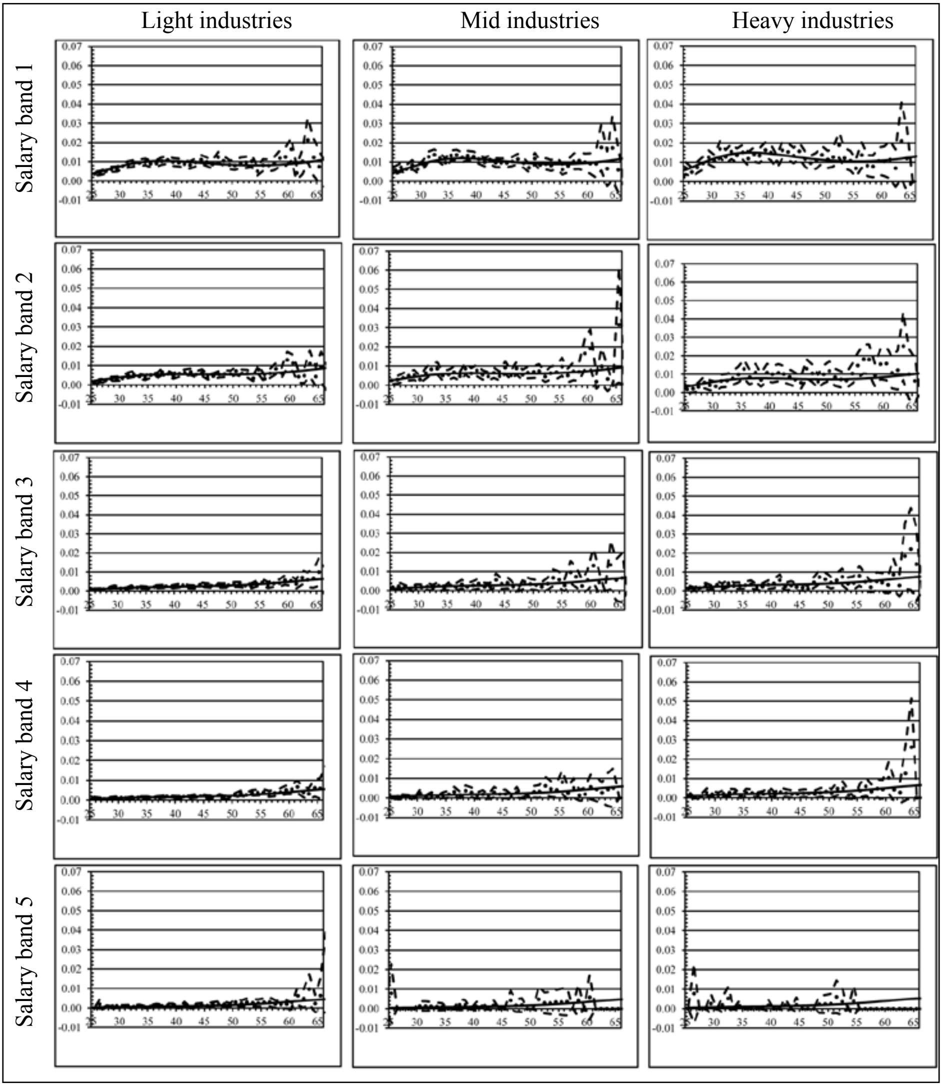

Figure D.2. Observed and graduated rates for females with 95% confidence intervals

APPENDIX E

TABLES OF THE GRADUATED RATES

This appendix presents the graduated rates by sex which have been rounded to 7 decimal places. Table E.1 shows the graduated rates by sex for all salary bands and industry groups combined. Tables E.2 to E.4 show those for males by salary band for industry groups light, mid and heavy respectively. Tables E.5 to E.7 show the corresponding rates for females.

Table E.1. Aggregated Group Life Insured Lives Table 2005–2009 (GL05-09 Aggregate)

Age x	Male $\mu_{x+0.5}$	Female $\mu_{x+0.5}$
25	0,0030138	0,0020404
26	0,0035989	0,0024338
27	0,0042080	0,0028329
28	0,0048264	0,0032247
29	0,0054400	0,0035975
30	0,0060356	0,0039409
31	0,0066022	0,0042476
32	0,0071313	0,0045125
33	0,0076171	0,0047335
34	0,0080566	0,0049106
35	0,0084492	0,0050463
36	0,0087967	0,0051442
37	0,0091023	0,0052093
38	0,0093710	0,0052469
39	0,0096082	0,0052628
40	0,0098204	0,0052625
41	0,0100138	0,0052515
42	0,0101951	0,0052345
43	0,0103704	0,0052161
44	0,0105458	0,0052002
45	0,0107268	0,0051900
46	0,0109186	0,0051885
47	0,0111257	0,0051981
48	0,0113524	0,0052210
49	0,0116021	0,0052588
50	0,0118779	0,0053130
51	0,0121823	0,0053847
52	0,0125168	0,0054746
53	0,0128825	0,0055833
54	0,0132793	0,0057111
55	0,0137061	0,0058575
56	0,0141605	0,0060220
57	0,0146386	0,0062031
58	0,0151344	0,0063987
59	0,0156401	0,0066060
60	0,0161455	0,0068207
61	0,0166378	0,0070378
62	0,0171014	0,0072506
63	0,0175181	0,0074512
64	0,0178671	0,0076300
65	0,0181257	0,0077764

Table E.2. Male Light Industry Group Life Insured Lives Table 2005–2009 (GL05-09 Male Light)

Age x	Salary band 1 $\mu_{x+0.5}$	Salary band 2 $\mu_{x+0.5}$	Salary band 3 $\mu_{x+0.5}$	Salary band 4 $\mu_{x+0.5}$	Salary band 5 $\mu_{x+0.5}$
25	0,0034277	0,0030045	0.0020783	0.0011227	0,0006296
26	0,0041787	0,0036314	0,0024522	0,0012724	0,0006871
27	0,0049822	0,0042985	0,0028448	0,0014272	0,0007467
28	0,0058198	0,0049915	0,0032493	0,0015857	0,0008084
29	0,0066720	0,0056955	0,0036588	0,0017465	0,0008720
30	0,0075195	0,0063955	0,0040667	0,0019084	0,0009377
31	0,0083443	0,0070780	0,0044672	0,0020702	0,0010053
32	0,0091309	0,0077313	0,0048551	0,0022312	0,0010750
33	0,0098670	0,0083463	0,0052266	0,0023908	0,0011469
34	0,0105441	0,0089164	0,0055791	0,0025485	0,0012210
35	0,0111568	0,0094377	0,0059108	0,0027043	0,0012975
36	0,0117034	0,0099089	0,0062214	0,0028583	0,0013768
37	0,0121851	0,0103310	0,0065115	0,0030108	0,0014590
38	0,0126057	0,0107068	0,0067823	0,0031625	0,0015446
39	0,0129707	0,0110406	0,0070360	0,0033140	0,0016341
40	0,0132870	0,0113377	0,0072753	0,0034664	0,0017278
41	0,0135624	0,0116043	0,0075032	0,0036207	0,0018264
42	0,0138053	0,0118467	0,0077231	0,0037783	0,0019305
43	0,0140238	0,0120717	0,0079384	0,0039404	0,0020410
44	0,0142261	0,0122857	0,0081528	0,0041086	0,0021586
45	0,0144200	0,0124949	0,0083698	0,0042846	0,0022845
46	0,0146125	0,0127054	0,0085930	0,0044702	0,0024197
47	0,0148102	0,0129225	0,0088259	0,0046672	0,0025655
48	0,0150189	0,0131513	0,0090720	0,0048777	0,0027234
49	0,0152435	0,0133961	0,0093344	0,0051040	0,0028951
50	0,0154884	0,0136610	0,0096163	0,0053483	0,0030826
51	0,0157571	0,0139491	0,0099207	0,0056132	0,0032879
52	0,0160520	0,0142632	0,0102503	0,0059015	0,0035138
53	0,0163748	0,0146052	0,0106076	0,0062161	0,0037630
54	0,0167262	0,0149762	0,0109949	0,0065602	0,0040390
55	0,0171057	0,0153766	0,0114141	0,0069371	0,0043457
56	0,0175115	0,0158055	0,0118667	0,0073505	0,0046876
57	0,0179403	0,0162609	0,0123534	0,0078043	0,0050699
58	0,0183872	0,0167396	0,0128746	0,0083027	0,0054990
59	0,0188454	0,0172364	0,0134296	0,0088501	0,0059819
60	0,0193061	0,0177446	0,0140166	0,0094511	0,0065272
61	0,0197580	0,0182553	0,0146325	0,0101106	0,0071449
62	0,0201877	0,0187572	0,0152727	0,0108335	0,0078467
63	0,0205790	0,0192370	0,0159309	0,0116247	0,0086466
64	0,0209135	0,0196784	0,0165985	0,0124893	0,0095613
65	0,0211706	0,0200632	0,0172647	0,0134318	0,0106105

Table E.3. Male Mid Industry Group Life Insured Lives Table 2005–2009 (GL05-09 Male Mid)

Age x	Salary band 1 $\mu_{x+0.5}$	Salary band 2 $\mu_{x+0.5}$	Salary band 3 $\mu_{x+0.5}$	Salary band 4 $\mu_{x+0.5}$	Salary band 5 $\mu_{x+0.5}$
25	0,0038803	0,0034479	0,0024778	0,0014265	0,0008493
26	0,0047268	0,0041619	0,0029157	0,0016087	0,0009202
27	0,0056312	0,0049202	0,0033735	0,0017954	0,0009929
28	0,0065728	0,0057062	0,0038428	0,0019849	0,0010672
29	0,0075294	0,0065026	0,0043156	0,0021752	0,0011431
30	0,0084792	0,0072925	0,0047840	0,0023650	0,0012203
31	0,0094019	0,0080605	0,0052410	0,0025528	0,0012990
32	0,0102802	0,0087933	0,0056809	0,0027376	0,0013791
33	0,0111003	0,0094806	0,0060994	0,0029187	0,0014607
34	0,0118527	0,0101152	0,0064933	0,0030957	0,0015439
35	0,0125317	0,0106929	0,0068610	0,0032686	0,0016290
36	0,0131354	0,0112125	0,0072023	0,0034375	0,0017161
37	0,0136655	0,0116751	0,0075180	0,0036029	0,0018056
38	0,0141261	0,0120844	0,0078098	0,0037655	0,0018979
39	0,0145237	0,0124452	0,0080803	0,0039263	0,0019934
40	0,0148663	0,0127638	0,0083329	0,0040864	0,0020926
41	0,0151626	0,0130472	0,0085710	0,0042470	0,0021962
42	0,0154221	0,0133028	0,0087986	0,0044097	0,0023048
43	0,0156540	0,0135381	0,0090198	0,0045759	0,0024193
44	0,0158675	0,0137604	0,0092386	0,0047475	0,0025404
45	0,0160712	0,0139769	0,0094592	0,0049262	0,0026693
46	0,0162730	0,0141942	0,0096856	0,0051139	0,0028070
47	0,0164803	0,0144183	0,0099216	0,0053127	0,0029549
48	0,0166995	0,0146548	0,0101710	0,0055246	0,0031143
49	0,0169360	0,0149086	0,0104373	0,0057521	0,0032870
50	0,0171947	0,0151839	0,0107238	0,0059974	0,0034747
51	0,0174793	0,0154844	0,0110337	0,0062631	0,0036797
52	0,0177925	0,0158128	0,0113699	0,0065519	0,0039043
53	0,0181362	0,0161713	0,0117349	0,0068668	0,0041513
54	0,0185110	0,0165609	0,0121309	0,0072107	0,0044238
55	0,0189162	0,0169819	0,0125598	0,0075869	0,0047256
56	0,0193498	0,0174333	0,0130229	0,0079990	0,0050609
57	0,0198082	0,0179127	0,0135210	0,0084505	0,0054346
58	0,0202858	0,0184164	0,0140539	0,0089453	0,0058523
59	0,0207751	0,0189388	0,0146205	0,0094875	0,0063207
60	0,0212663	0,0194722	0,0152188	0,0100814	0,0068475
61	0,0217472	0,0200070	0,0158452	0,0107310	0,0074418
62	0,0222028	0,0205309	0,0164944	0,0114410	0,0081143
63	0,0226155	0,0210291	0,0171593	0,0122154	0,0088775
64	0,0229651	0,0214842	0,0178307	0,0130584	0,0097463
65	0,0232293	0,0218764	0,0184969	0,0139739	0,0107384

Table E.4. Male Heavy Industry Group Life Insured Lives Table 2005–2009 (GL05-09 Male Heavy)

Age x	Salary band 1 $\mu_{x+0.5}$	Salary band 2 $\mu_{x+0.5}$	Salary band 3 $\mu_{x+0.5}$	Salary band 4 $\mu_{x+0.5}$	Salary band 5 $\mu_{x+0.5}$
25	0,0051206	0,0046909	0,0036711	0,0024373	0,0016590
26	0,0062268	0,0056462	0,0042942	0,0027180	0,0017688
27	0,0074053	0,0066559	0,0049388	0,0029998	0,0018781
28	0,0086285	0,0076971	0,0055925	0,0032794	0,0019865
29	0,0098670	0,0087463	0,0062431	0,0035540	0,0020937
30	0,0110923	0,0097808	0,0068795	0,0038211	0,0021996
31	0,0122779	0,0107798	0,0074918	0,0040786	0,0023041
32	0,0134014	0,0117263	0,0080723	0,0043253	0,0024071
33	0,0144454	0,0126068	0,0086153	0,0045602	0,0025089
34	0,0153976	0,0134122	0,0091170	0,0047830	0,0026097
35	0,0162512	0,0141378	0,0095760	0,0049939	0,0027095
36	0,0170045	0,0147824	0,0099925	0,0051936	0,0028090
37	0,0176598	0,0153484	0,0103682	0,0053830	0,0029084
38	0,0182232	0,0158410	0,0107065	0,0055634	0,0030083
39	0,0187035	0,0162674	0,0110115	0,0057365	0,0031093
40	0,0191113	0,0166362	0,0112880	0,0059039	0,0032120
41	0,0194583	0,0169571	0,0115414	0,0060678	0,0033173
42	0,0197567	0,0172399	0,0117773	0,0062302	0,0034259
43	0,0200189	0,0174948	0,0120014	0,0063932	0,0035387
44	0,0202565	0,0177314	0,0122194	0,0065592	0,0036566
45	0,0204807	0,0179589	0,0124367	0,0067304	0,0037809
46	0,0207018	0,0181860	0,0126585	0,0069092	0,0039126
47	0,0209290	0,0184204	0,0128897	0,0070980	0,0040530
48	0,0211703	0,0186691	0,0131350	0,0072991	0,0042036
49	0,0214328	0,0189381	0,0133986	0,0075151	0,0043659
50	0,0217222	0,0192328	0,0136845	0,0077485	0,0045418
51	0,0220432	0,0195574	0,0139961	0,0080018	0,0047330
52	0,0223991	0,0199152	0,0143366	0,0082778	0,0049418
53	0,0227920	0,0203085	0,0147086	0,0085791	0,0051707
54	0,0232224	0,0207385	0,0151144	0,0089087	0,0054224
55	0,0236893	0,0212049	0,0155557	0,0092693	0,0057000
56	0,0241901	0,0217064	0,0160332	0,0096641	0,0060071
57	0,0247199	0,0222397	0,0165472	0,0100961	0,0063478
58	0,0252718	0,0227998	0,0170969	0,0105685	0,0067268
59	0,0258363	0,0233795	0,0176803	0,0110845	0,0071494
60	0,0264011	0,0239695	0,0182942	0,0116473	0,0076218
61	0,0269510	0,0245575	0,0189337	0,0122600	0,0081513
62	0,0274676	0,0251285	0,0195921	0,0129258	0,0087462
63	0,0279294	0,0256648	0,0202605	0,0136473	0,0094163
64	0,0283117	0,0261454	0,0209278	0,0144270	0,0101731
65	0,0285875	0,0265466	0,0215804	0,0152667	0,0110299

Table E.5. Female Light Industry Group Life Insured Lives Table 2005–2009 (GL05-09 Female Light)

Age x	Salary band 1 $\mu_{x+0.5}$	Salary band 2 $\mu_{x+0.5}$	Salary band 3 $\mu_{x+0.5}$	Salary band 4 $\mu_{x+0.5}$	Salary band 5 $\mu_{x+0.5}$
25	0,0039608	0,0021446	0,0009122	0,0006215	0,0003491
26	0,0047637	0,0025345	0,0010520	0,0007088	0,0003917
27	0,0055894	0,0029306	0,0011919	0,0007958	0,0004338
28	0,0064100	0,0033215	0,0013289	0,0008808	0,0004748
29	0,0071982	0,0036965	0,0014605	0,0009626	0,0005145
30	0,0079292	0,0040461	0,0015845	0,0010401	0,0005527
31	0,0085825	0,0043628	0,0016995	0,0011130	0,0005893
32	0,0091430	0,0046414	0,0018046	0,0011808	0,0006244
33	0,0096016	0,0048790	0,0018996	0,0012437	0,0006582
34	0,0099547	0,0050746	0,0019846	0,0013019	0,0006911
35	0,0102042	0,0052295	0,0020603	0,0013561	0,0007235
36	0,0103560	0,0053461	0,0021278	0,0014070	0,0007558
37	0,0104194	0,0054284	0,0021883	0,0014553	0,0007885
38	0,0104059	0,0054808	0,0022433	0,0015021	0,0008222
39	0,0103280	0,0055084	0,0022943	0,0015483	0,0008576
40	0,0101989	0,0055163	0,0023429	0,0015950	0,0008952
41	0,0100310	0,0055095	0,0023906	0,0016433	0,0009357
42	0,0098363	0,0054929	0,0024392	0,0016941	0,0009798
43	0,0096255	0,0054710	0,0024900	0,0017486	0,0010282
44	0,0094080	0,0054478	0,0025445	0,0018078	0,0010818
45	0,0091920	0,0054269	0,0026041	0,0018727	0,0011412
46	0,0089843	0,0054115	0,0026702	0,0019445	0,0012075
47	0,0087906	0,0054046	0,0027442	0,0020242	0,0012814
48	0,0086156	0,0054086	0,0028272	0,0021128	0,0013640
49	0,0084630	0,0054257	0,0029204	0,0022114	0,0014561
50	0,0083359	0,0054579	0,0030251	0,0023210	0,0015589
51	0,0082368	0,0055069	0,0031424	0,0024427	0,0016731
52	0,0081677	0,0055742	0,0032734	0,0025775	0,0017999
53	0,0081303	0,0056613	0,0034189	0,0027262	0,0019400
54	0,0081263	0,0057695	0,0035799	0,0028894	0,0020941
55	0,0081569	0,0059000	0,0037570	0,0030678	0,0022626
56	0,0082235	0,0060537	0,0039505	0,0032615	0,0024456
57	0,0083274	0,0062315	0,0041605	0,0034703	0,0026425
58	0,0084700	0,0064340	0,0043864	0,0036933	0,0028523
59	0,0086524	0,0066615	0,0046273	0,0039290	0,0030729
60	0,0088759	0,0069138	0,0048814	0,0041751	0,0033014
61	0,0091415	0,0071903	0,0051458	0,0044281	0,0035336
62	0,0094502	0,0074896	0,0054168	0,0046834	0,0037640
63	0,0098026	0,0078094	0,0056893	0,0049351	0,0039858
64	0,0101986	0,0081465	0,0059569	0,0051758	0,0041906
65	0,0106376	0,0084962	0,0062117	0,0053969	0,0043691

Table E.6. Female Mid Industry Group Life Insured Lives Table 2005–2009 (GL05-09 Female Mid)

Age x	Salary band 1 $\mu_{x+0.5}$	Salary band 2 $\mu_{x+0.5}$	Salary band 3 $\mu_{x+0.5}$	Salary band 4 $\mu_{x+0.5}$	Salary band 5 $\mu_{x+0.5}$
25	0,0046664	0,0025267	0,0010747	0,0007322	0,0004113
26	0,0055986	0,0029786	0,0012364	0,0008331	0,0004604
27	0,0065528	0,0034357	0,0013973	0,0009330	0,0005085
28	0,0074965	0,0038845	0,0015542	0,0010301	0,0005553
29	0,0083976	0,0043124	0,0017039	0,0011229	0,0006002
30	0,0092277	0,0047087	0,0018440	0,0012105	0,0006432
31	0,0099635	0,0050649	0,0019730	0,0012921	0,0006841
32	0,0105883	0,0053751	0,0020899	0,0013675	0,0007231
33	0,0110920	0,0056363	0,0021944	0,0014367	0,0007604
34	0,0114718	0,0058480	0,0022870	0,0015004	0,0007965
35	0,0117304	0,0060116	0,0023685	0,0015590	0,0008317
36	0,0118757	0,0061307	0,0024400	0,0016134	0,0008667
37	0,0119191	0,0062097	0,0025033	0,0016648	0,0009020
38	0,0118745	0,0062543	0,0025599	0,0017141	0,0009383
39	0,0117567	0,0062704	0,0026116	0,0017625	0,0009762
40	0,0115812	0,0062640	0,0026604	0,0018112	0,0010165
41	0,0113627	0,0062410	0,0027080	0,0018614	0,0010599
42	0,0111148	0,0062069	0,0027562	0,0019143	0,0011071
43	0,0108500	0,0061669	0,0028067	0,0019710	0,0011590
44	0,0105788	0,0061257	0,0028611	0,0020328	0,0012164
45	0,0103105	0,0060872	0,0029210	0,0021006	0,0012801
46	0,0100528	0,0060552	0,0029878	0,0021758	0,0013511
47	0,0098120	0,0060326	0,0030630	0,0022594	0,0014303
48	0,0095930	0,0060223	0,0031479	0,0023525	0,0015187
49	0,0094000	0,0060265	0,0032438	0,0024562	0,0016174
50	0,0092362	0,0060473	0,0033519	0,0025717	0,0017272
51	0,0091039	0,0060866	0,0034733	0,0026999	0,0018493
52	0,0090054	0,0061459	0,0036091	0,0028419	0,0019845
53	0,89423	0,0062267	0,0037604	0,0029984	0,0021338
54	0,0089159	0,0063301	0,0039278	0,0031702	0,0022976
55	0,0089275	0,0064574	0,0041119	0,0033577	0,0024764
56	0,0089784	0,0066094	0,0043131	0,0035609	0,0026701
57	0,0090696	0,0067868	0,0045312	0,0037795	0,0028780
58	0,0092022	0,0069902	0,0047656	0,0040125	0,0030988
59	0,0093773	0,0072196	0,0050150	0,0042582	0,0033303
60	0,0095959	0,0074747	0,0052774	0,0045138	0,0035692
61	0,0098589	0,0077545	0,0055496	0,0047756	0,0038109
62	0,0101668	0,0080575	0,0058276	0,0050386	0,0040494
63	0,0105200	0,0083810	0,0061057	0,0052963	0,0042775
64	0,0109182	0,0087213	0,0063772	0,0055410	0,0044863
65	0,0113602	0,0090734	0,0066337	0,0057635	0,0046659

Table E.7. Female Heavy Industry Group Life Insured Lives Table 2005–2009 (GL05-09 Female Heavy)

Age x	Salary band 1 $\mu_{x+0.5}$	Salary band 2 $\mu_{x+0.5}$	Salary band 3 $\mu_{x+0.5}$	Salary band 4 $\mu_{x+0.5}$	Salary band 5 $\mu_{x+0.5}$
25	0,0061667	0,0033390	0,0014203	0,0009676	0,0005436
26	0,0073678	0,0039199	0,0016270	0,0010963	0,0006058
27	0,0085876	0,0045026	0,0018312	0,0012227	0,0006664
28	0,0097833	0,0050695	0,0020283	0,0013443	0,0007247
29	0,0109137	0,0056045	0,0022144	0,0014594	0,0007801
30	0,0119425	0,0060940	0,0023865	0,0015666	0,0008324
31	0,0128411	0,0065276	0,0025428	0,0016652	0,0008817
32	0,0135894	0,0068986	0,0026822	0,0017551	0,0009280
33	0,0141766	0,0072037	0,0028047	0,0018363	0,0009719
34	0,0146009	0,0074431	0,0029108	0,0019096	0,0010137
35	0,0148678	0,0076195	0,0030019	0,0019759	0,0010541
36	0,0149893	0,0077380	0,0030798	0,0020364	0,0010939
37	0,0149814	0,0078051	0,0031464	0,0020925	0,0011337
38	0,0148631	0,0078284	0,0032042	0,0021455	0,0011744
39	0,0146544	0,0078159	0,0032553	0,0021969	0,0012168
40	0,0143755	0,0077753	0,0033023	0,0022482	0,0012618
41	0,0140455	0,0077145	0,0033474	0,0023009	0,0013101
42	0,0136819	0,0076404	0,0033928	0,0023564	0,0013628
43	0,0133001	0,0075596	0,0034405	0,0024161	0,0014207
44	0,0129137	0,0074777	0,0034926	0,0024814	0,0014849
45	0,0125338	0,0073998	0,0035509	0,0025536	0,0015561
46	0,0121696	0,0073302	0,0036170	0,0026339	0,0016356
47	0,0118286	0,0072724	0,0036925	0,0027237	0,0017243
48	0,0115164	0,0072297	0,0037791	0,0028241	0,0018232
49	0,0112377	0,0072046	0,0038779	0,0029364	0,0019335
50	0,0109958	0,0071995	0,0039904	0,0030616	0,0020563
51	0,0107932	0,0072160	0,0041178	0,0032009	0,0021924
52	0,0106319	0,0072560	0,0042610	0,0033551	0,0023430
53	0,0105134	0,0073207	0,0044211	0,0035252	0,0025087
54	0,0104387	0,0074113	0,0045986	0,0037117	0,0026901
55	0,0104088	0,0075288	0,0047942	0,0039148	0,0028873
56	0,0104244	0,0076739	0,0050078	0,0041344	0,0031001
57	0,0104864	0,0078471	0,0052391	0,0043700	0,0033276
58	0,0105954	0,0080485	0,0054872	0,0046201	0,0035680
59	0,0107521	0,0082780	0,0057503	0,0048825	0,0038186
60	0,0109569	0,0085348	0,0060259	0,0051540	0,0040754
61	0,0112102	0,0088174	0,0063103	0,0054302	0,0043332
62	0,0115122	0,0091238	0,0065987	0,0057053	0,0045853
63	0,0118626	0,0094505	0,0068849	0,0059722	0,0048233
64	0,0122602	0,0097933	0,0071611	0,0062221	0,0050377
65	0,0127034	0,0101462	0,0074180	0,0064449	0,0052176