SOUTH AFRICAN ACTUARIAL JOURNAL
SAAJ **22** (2022) 1–28
https://dx.doi.org/10.4314/saaj.v22i1.1

Suitability of the 2.5% net discount rate for quantum of damage calculations in South Africa

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Submission date 20 September 2021
Acceptance date 13 June 2022

ABSTRACT
This study is on the 2.5% real discount rate used to calculate the lump sum payment in the event of a compensation claim. The aptness of this 2.5% real discount rate is assessed through a statistical analysis of government bonds and inflation data over the past 59 years. The investigation yields evidence that the discount rate over the last 59 years may be mean reverting and would be a good approximation to use in the future. Also, an interesting relationship was found between the introduction of inflation targeting in the year 2000 and the stationarity of the series. It must be considered that the timing and duration of variation from the 2.5% cannot be predicted, but reversion to the series mean seems to always occur. The practice of using the 2.5% in the South African context is also compared to other practices globally. Finally, circumstances for the departure from the 2.5% discount rate are investigated, with some suggestions.

KEYWORDS
Discount rate, risk discount rate (RDR), net discount rate, liability claims, damages, loss of income (LOI), loss of support (LOS)

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1. INTRODUCTION

1.1 In the event of a claim for the loss of future income, the commonly used practice worldwide is to award a single lump sum to the claimant as compensation for the future loss. This award represents the present value of the differences between future income with and without the claim event occurring, such as an accident or death. Future income is assumed to increase at inflation, and then discounted to the present at the risk discount rate. The resultant real discount rate applied to the lost future income is therefore of critical importance, since it will have a great impact on the size of the loss calculated. This discount rate is often based on assumptions about the characteristics of the claimant and the nature of the loss.

1.2 The current South African industry real default liability discount rate of 2.5% is based on the difference between the implied risk discount rate of the backing assets of a prudent investment portfolio, assumed to be South African government bonds, and the inflation rate. This research sets out to do a comprehensive statistical analysis of the difference between the two variables over a period of 59 years, using monthly data. The purpose is to determine if government bonds consistently exceeded CPI inflation by an average of 2.5% over the longer term, and then to investigate the periods and triggers of such periods where this was not the case. The aim is to validate the 2.5% assumption currently in use and to assess and explain any major fluctuations from it.

1.3 This paper also investigates how the socio-economic position of individuals will impact their expected risk preference and expected investment return on their likely investment portfolio, from which their risk discount rate would be derived. This is done to determine what factors should be considered when departing from the 2.5% base discount rate for each individual. The practices in other countries on this matter are considered and the reasons behind them investigated. These are compared to the South African setting to see whether adjustments should be required to ensure a more appropriate compensation is awarded to each claimant.

2. LITERATURE STUDY

The core concepts of this study are defined below, followed by the factors that influenced the South African inflation rate and government bond yields over the recent past of 59 years. An explanation is given of the expected effect of the socio-economic position of individuals on their investment preferences. Finally, the socio-economic landscape of South Africa is discussed and the practices used in various countries to set the discount rate are explained and considered as a South African alternative.

2.1 Socio-economic position and Gini coefficient

2.1.1 SOCIO-ECONOMIC POSITION

Socio-economic position refers to the "social and economic factors that influence what positions individuals or groups hold within the structure of a society" (Galobardes et al., 2006: 1).

2.1.2 GINI COEFFICIENT

> The Gini coefficient can be interpreted as the percentage of inequality represented in the population with respect to the given resource. (Catalano et al., 2009: 1).

This percentage of inequality is represented in a coefficient that ranges from 0 to 1, with 0 indicating perfect equality and 1 the exact opposite. The Gini coefficient is usually used to quantify the level of income inequality in a population (Catalano et al., 2009).

2.2 Historical South African government bond yields and inflation

Since the discount rate is based on the difference between inflation and government bond yields, the economic factors that caused the variation of these two values in the 59 years from 1960 up to the start of 2019 in the South African context are explained. The analysis is performed over the past 59 year since the data source with the longest uninterrupted available data points was 59 years.

2.2.1 KEY EVENTS THAT AFFECTED THE SOUTH AFRICAN ECONOMY AND ITS EFFECT ON GOVERNMENT BOND YIELDS AND INFLATION FROM 1970 TO 1994

2.2.1.1 The South African GDP leading up to 1974 grew at an average of 4.9 percent per year. In the years 1974 to 1987, the average GDP growth slowed down to 1.8 percent per year. One explanation of the slowed economic growth (Levy, 1999) was the OPEC nations applying an oil embargo which started in 1973 at the same time as a worldwide oil crisis.

2.2.1.2 Two other noteworthy factors beyond the oil shocks affected the South African economy at this time. The first was the labour shortage that South Africa faced in the early 1970s. The second was an increased outflow of foreign capital from the South African economy which averaged 2.3% of the GDP from 1976–1980 (Levy, 1999). This outflow could be ascribed to the political unrest caused by the Soweto riots in 1976 which drew international attention. The dire economic position extended into the 1980s when the South African economy fell into a recession between 1984 and 1985 and a state of emergency was declared in 1985 (Hefti & Staehelin-Witt, 1986: 3).

2.2.1.3 Shortly after former South African president, PW Botha, delivered his infamous “Rubicon” speech in 1985, Chase Manhattan Bank declared that it would not be rolling over its short-term loans with the South African government. Other lenders soon followed, initiating a liquidity crisis (Giliomee, 1985: 38). At this time the appeal of investing into South Africa diminished due the increased financial risk caused by political unrest and economic instability (Levy, 1999). This drastic decrease in investment in the South African economy saw government bond yields rise in order to retain some investors.

2.2.1.4 The significant increase in the South African inflation rate from 1970 to 1980 can be ascribed to oil price shocks of the time (Nowak & Ricci, 2005: 190). However, the South African inflation remained high despite its trading partners experiencing disinflation from 1980 to 1990. This was due to a loose monetary policy adopted to stimulate economic activity which came at the cost of increased inflation (Nowak & Ricci, 2005).

2.2.2 Key events that affected government bond yields and inflation from 1994 until 2019

2.2.2.1 During the mid-1990s, the South African Reserve Bank (SARB) introduced inflation targeting informally by targeting the money supply as the primary way of combating inflation (Nowak & Ricci, 2005). The SARB formalised the inflation-targeting regime in 2000. This caused "a substantial decline in the inflation differential with trading partner countries" (Nowak & Ricci, 2005).

2.2.2.2 After 1994, the government improved fiscal transparency and predictability and monetary policy has become more transparent and accountable (Nowak & Ricci, 2005: 195). These improvements led to an increase in investor confidence. This can be observed in the steady improvement of South Africa's sovereign debt rating by major credit ratings agencies from 1994 until 2008.[1]

2.2.2.3 However, in 2012 South African sovereign debt was gradually downgraded until Standard & Poor's downgraded South Africa's sovereign debt to sub-investment grade (or junk status) in April 2017 with Fitch to follow soon thereafter.[2] Slowed economic growth and a political agenda which overshadowed policy-making led to the downgrade.[3]

2.2.2.4 In the post-1994 era the largest variations in the discount rate were mainly caused by the rapid changes in inflation. Government bond yields caused some variation but were mainly dominated by the effects of the inflation-targeting strategies.

2.3 Effect of socioeconomic position on expected individual investment preferences

2.3.1 When setting the lump sum compensation amount, Koch (1993) notes that the future payments compensated for should be reduced for the advantage of being received earlier. The extent of this advantage and hence reduction required could however be differently contested by each claimant. As a result, the court would aim to objectivise this advantage by observing that "the claimant should mitigate his damages by profitably investing the compensation money" (Koch, 1993). The resulting expected investment return could serve as a measure of the advantage of early receipt of the compensation payments and their resulting reduction. According to Koch (1993), the personal investment idiosyncrasies of a claimant should not be considered in this regard, but rather the investment decisions adopted by "the reasonable man who is neither too cautious nor too reckless".

2.3.2 However, the South African context (see section 2.4) creates a socio-economic setting where the definition of a "reasonable man" and his generally accepted investment decisions may differ significantly between groups of individuals. As a result, the factors affecting the investment preferences of a claimant are discussed here as they could influence the investment decisions made and the resulting expected investment return earned by the claimant.

1 https://businesstech.co.za/news/finance/213641/one-graph-showing-south-africas-credit-rating-history-1994-2017/

2 Fitch downgrades SA to junk status. https://ewn.co.za/2017/04/07/breaking-fitch-downgrades-south-africa-to-junk-status

3 Donnelley, L (2017). https://mg.co.za/article/2017-11-25-global-credit-ratings-agency-has-downgraded-south-africa-to-junk-status/

2.3.3 It is important to understand the needs and socio-economic characteristics of the claimant as this will most likely influence the investment returns earned by the claimant on the awarded amount (Edwards, 1975: 342). An allowance for the net rate of return that the claimant can earn on an awarded lump sum must be set according to the return earned on an investment portfolio with an appropriate level of risk. Fairgrieve & Gauci (2016) describe this "risk" as the liquidity preference of the involved individual. They explicitly distinguish the difference in liquidity preference of individuals based on two socio-economic characteristics: age and level of wealth. Several other characteristics also influence the liquidity preference and risk aversion of individuals.

2.3.4 Impoverished individuals, especially in developing countries, have been found to be more risk averse than wealthier individuals and "were more likely to choose smaller and earlier monetary rewards over larger, delayed ones" (Haushofer & Fehr, 2014: 862).

2.3.4.1 The difference in risk preference for individuals of different levels of wealth can indeed be ascribed to liquidity constraints. This is observed since poorer individuals face much higher interest credit rates and receive more constrained amounts of credit than wealthier individuals. The higher cost of capital and the expectation of future liquidity constraints will lead to safer investment decisions. These investment decisions are not necessarily made due to higher risk aversion, but rather due to the need to alleviate liquidity constraints (Haushofer & Fehr, 2014).

2.3.5 Individuals with a higher level of attained education can be considered less risk averse than individuals with lower levels of attained education (Grable, 2000: 628).

2.3.5.1 Observations indicate that a higher educational qualification enables an individual to assess the probable risks and benefits of investing more accurately. An investor with a higher educational degree will thus be less risk averse and as a result it is expected that the individual will make investment decisions with a higher expected return. (Chattopadhyay & Dasgupta, 2015: 607; 618).

2.3.6 Individuals, who can invest larger amounts, are usually less risk averse than those who invest smaller amounts of capital (Chattopadhyay & Dasgupta, 2015).

2.3.6.1 The increased level of funds available for investment will increase an individual's ability to take on more risk since the range of investment options at the investor's disposal will increase. The increased ability to take on more risk is then expected to lead to an increased willingness to take on more risk (Riley Jr & Chow, 2016: 35).

2.3.7 Individuals with a higher occupational status exhibit lower levels of risk aversion than individuals with a lower occupational status (Grable, 2000).

2.3.7.1 Individuals with a higher occupation level are expected to have a higher level of disposable income and can thus afford to take on more investment risks. Also, individuals who follow a professional occupation tend to have lower risk aversion than those engaging in non-professional occupations (Chattopadhyay & Dasgupta, 2015).

2.3.8 It is apparent from the above that income level, education, occupation, and investment size of individuals are factors that could have a significant influence on the chosen discount rate.

2.4 The current South African socio-economic landscape

2.4.1 In the South African context, distinction between individuals can be made regarding the extreme unequal distribution of two, among several, indicators of socio-economic position: Education and wealth.

2.4.1.1 The World Bank has recently described South Africa as "one of the most unequal countries in the world by any measure". According to the World Bank, South Africa was found to have a Gini coefficient of 0.63 in 2015, the highest in the world. It has also displayed levels of wealth inequality that exceeded its level of income inequality. This prominent inequality is evident when observing that 71% of net household wealth is held by the top 10% of households while 7% of the net household wealth was held by the bottom 60% households (The World Bank & Statistics SA, 2018: xv)

2.4.1.2 South Africa's education system also produces a very unequally educated society. With the exception of a wealthy minority, most South African pupils cannot read, write and preform calculations according to the appropriate grade level (Spaull, 2013: 2). This is evident in the fact that South Africa's educational inequality is "much higher than its income inequality would suggest" (Crouch et al., 2008: 2). The South African Gini coefficient for education was 0.560 in 2010 and ranked 132nd most unequal out of 146 countries that participated in the study to determine global education inequality (Ziesemer, 2016).

2.4.2 Stark differences exist in the socio-economic positions of individuals in South Africa when considering the indicators of income and education. It is also clear that the socio-economic position of an individual influences the risk preference and investment needs and hence expected investment returns earned by individuals.

2.4.3 The next section refers to the practices accepted in South Africa and other countries to determine the discount rate. The aptness of current industry practice and the possible alternatives available to ensure an appropriate discount rate is chosen are assessed.

2.5 Accepted practices to calculate the discount rate in different countries

The countries chosen as comparatives to the South African practice are Canada, Hong Kong, and the United States of America. Hong Kong and Canada were chosen since these countries have unique practices when setting the discount rate compared to other countries on which literature could be found. The possibility of considering these practices in the South African case is worth investigating. The practice of the United States is similar to South Africa's since the discount rate is set in most states by a court ruling and expert evidence provided. The practices in the United States could provide some insights into possible alterations that can be made in South African practice. Practices of other developing countries with similar socio-economic conditions as South Africa would also have been considered but very little and incomprehensive literature is available on any such countries.

2.5.1 SOUTH AFRICA

2.5.1.1 In South African courts no specific statutory laws have been put into place to govern the quantum of damages calculation. South African courts "readily admit and frequently insist on actuarial testimony" when calculating the compensation of future losses.

As a result, the court allows for a great deal of flexibility when deciding on a method of calculating the compensation and do not wish to apply rigid regulations to it (Du Plessis, 2012: 5–9).

2.5.1.2 Thus, in the South African case actuaries have a good opportunity to contextualise the applied discount rate to ensure the fairest compensation is determined.

2.5.1.3 According to Fairgrieve & Gauci (2017: 53), in the South African case, the discount rate is calculated based on the allowance for the net rate of return expected to be earned by the claimant on the lump sum awarded. This expected return accounts for the investment portfolio that will meet the specific investment needs and risk appetite of the claimant. The particular circumstances of the claimant should be taken into account to determine their specific needs.

2.5.1.4 Although this is the case, it is argued by Fairgrieve & Gauci (2017) that even the most extensive analysis of a claimant's circumstances (for our study, socio-economic context) would not yield the application of the correct calculation. The result is a similar basis applied to all claimants regardless of circumstances.

2.5.1.5 This belief is evidently applied in practice: South African actuaries generally apply a discount rate of 2.5% to 2.73% to all cases (Koch, 2011: 113). This rate is based on recommendations made in the Quantum yearbook and originates from the assumed minimum risk real rate of return earned in South Africa.

2.5.2 CANADA

2.5.2.1 According to the Report to the Civil Rules Committee on rules 53.09 and 53.10 of April 27, 2020, Ontario Canada have been using a two-tier approach for losses before and after 15 years. The 2.5% real rate is the current rate for losses after 15 years. The initial 15 years would use the real rate of interest on long-term Government of Canada real return bonds, with possible adjustments and a floor of 0%. (Feldman et al., 2020).

2.5.2.2 The Ontario Civil Rules committee is compelled to review the applied discount rate at least every four years and the most recent review has left the interest rates at 0.5% for the 15-year period that follows the start of the trail, increased from 0% since 2020 The rate used for any later period remains at 2.5% since 2000.[4]

2.5.2.3 There was also an evidence-based adjustment made to the risk discount rate, where evidence of the claimant was produced in court, in order to adjust the discount rate. However, the latest report in 2020 suggested that the two-tiered system be transformed into a single-tier system, and the ability of the judicial system to change the discount rate based on evidence should be eliminated. They also recommended that no different discount rates should be used for different kinds of damages, and specifically referred to the use of a different rate for medical damages (Feldman et al., 2020).

4 Courts of Justice Act (Ontario) RRO 1990 Regulation 194: Calculation of Awards for Future Pecuniary Damages Rules 53.09(1) and (2) Rules of Civil Procedure (2022)

2.5.3 HONG KONG

2.5.3.1 Until 2013 Hong Kong followed the accepted practice as in Wales and England to allow for a discount rate between 4%–5% as applied by the English common law principles laid down by the House of Lords (Chan & Chan, 2015: 511).

2.5.3.2 In 2013 however, the Hong Kong court gave two claimants, Li Ka Wai and Yuen Hiu Tung, an opportunity to investigate the validity of the 4.5% discount rate. As a result of the investigation the court ruled that different interest rates should apply to claimants with different financial needs, especially with reference to the time period of these needs (Bowers, 2013).

2.5.3.3 The structure of selecting the discount rate is currently set to allow for three different interest rates based on the future time period in which the claimant's future financial needs or liabilities occur (Fairgrieve & Gauci, 2017). These are:

— For the time period of 0–5 years, a rate of –0.5% per annum was accepted.
— For the time period of 5–10 years, a rate of 1% per annum was accepted.
— For the time period exceeding 10 years, a rate of 2.5% was accepted.

2.5.3.4 These rates were set according to the varying investment returns earned on portfolios set up to specifically meet the various liquidity preferences of these investors. These portfolios were made up of proportional investments in high quality bonds, blue chip stocks, Hong Kong Government Exchange Fund Notes and time deposits.

2.5.3.5 This decision was also based on the practice in the state of Ontario, Canada, as described above, but was altered to match the availability of certain investment instruments in the Hong Kongese context.

2.5.4 UNITED STATES OF AMERICA

2.5.4.1 In some states of the country the discount rate is set either by statute or prescribed by case law or the jury. In most states expert evidence is allowed to lead the discount rate set by the court. This has resulted in different decisions being made on deciding the applied discount rate. These include a rate or 0% as a result of assuming future inflation to be equal to future interest rates. Another assumption entailed the rate of interest should be based on "the best and safest investments". It has also been assumed that the claimant should be entitled to a risk-free stream of income if the lost income is assumed to be earned with certainty in the future if the damage did not occur (Whittaker, 2018).

2.5.4.2 In most states allowance is made for expert evidence to be used when setting a suitable discount rate. Currently members of the National Association of Forensic Economists are of the opinion that a rate of 1.36% is suitable to apply with respect to a 30-year loss period (Whittaker, 2018).

2.6 Approaches to determine a fair liability discount rate

2.6.1 The fair amount to award to a claimant is difficult to determine, since the goal is to put the claimant in the same financial position as if the loss event had not occurred (although in practice this is rarely the case). Several factors need to be kept in mind.

2.6.2 The first factor is the concept of the disutility of delayed payment, which Robert J. Koch explains in Chapter 8 of his thesis (Koch, 1993). It simply states that people will generally prefer a payment made now rather than a payment at a later date, even if this amount has been adjusted for inflation. Awarding a lump sum to the claimant already puts the claimant in a better position than what would have been the case if the loss event had not occurred. Therefore, this amount will need to be adjusted. Koch explains: “Future payments, however, when compensated by a present lump sum, will be reduced for the advantage of being received earlier.” (Koch, 1993).

2.6.3 However, one can also argue that the lump sum would need to be adjusted to allow for future inflation. Keeping all these factors in mind we need to find a rate that would adjust the lump sum which would reflect a fair value to all parties involved.

2.6.4 Assuming that the claimant would invest the lump sum, the expected rate of return, as Koch states: “will include what the investor requires to offset inflation and also to offset the risks attaching to the investment”. Koch goes on to explain that this expected rate of return is referred to as the nominal rate of return. To focus specifically on the pure risk element of the return, we use the real rate of return which takes inflation into account.

2.6.5 Using the real rate of return as a discount rate to adjust the present lump sum amount would ignore the individual circumstances of the claimant. However, this would introduce a consistent measure by which the lump sum would be discounted for all claimants. In chapter 8 of Koch’s thesis, he states that: “The expected return on the investment would then be the measure, for compensation purposes, of the utility of the time value of money”.

2.6.6 Deciding on which investments classes would be a fair assumption to base the real return on is a difficult task on its own and is discussed later on in this paper.

2.7 Using CPI inflation and government bond yields to calculate the real rate of return

2.7.1 To calculate the real return at a certain point in time, a proxy for inflation would be needed. In chapter 8 of Koch’s thesis, he mentions: “The consumer price index as published by the Department of Statistics, despite its many weaknesses, is generally accepted as the fair reflection of the rate of inflation”. Since the CPI is the most widely used measure of inflation, this was decided to be the most appropriate measure. See also the work by Dimson et al. (2009) which confirms this practice from a global perspective.

2.7.2 There is difficulty in choosing an appropriate investment index on which to base the nominal rate of return, since any choice made can be debated. Koch describes the yield index on fixed interest securities as a “barometer” of the investment opportunities at the time of investment. He also goes on to explain that: “It is submitted that if fairness is to be achieved the award that is made for damages should have regard to the state of the market at the time that the award is made”. Since the yield on fixed interest securities would then be a good proxy for the market conditions at the time the investment would have been made, government bond yields were chosen as the measure on which to base the statistical analysis.

3. STATISTICAL ANALYSIS OF THE 2.5% DISCOUNT RATE

3.1 Although there are different approaches in determining the liability discount rate to be used in quantum damages calculations (see sections 2.6 and 2.7), this section tests the statistical appropriateness of the 2.5% discount rate used in practise as an ex-ante real return based on a risk-free investment.

3.2 This section only looks at the statistical significance in using the real return on fixed interest securities; a later section of this paper focuses more on the socio-economic factors that one needs to consider when setting a discount rate.

3.3 Using South-African asset class results from Firer & McCleod (1999), Firer & Staunton (2002) and extended by Dimson et al. (2016) for the period 1900–2015, we observe that the annualised total return for bonds, adjusted for inflation (i.e., real returns) amounted to 1.8% p.a. on a geometric return basis and 2.3% on an arithmetic mean basis with a standard deviation of 10.5% and standard error of the mean of 1.0%.

3.4 It is important to understand that the above result considers real returns on an ex-post basis. On an ex-ante basis, which we consider below, we need to understand that we observe nominal bond returns and will be subject to realised inflation. From an analysis perspective, we need to form a view on inflation.

3.5 Mankiw & Reis (2002, 2006) describe a model of dynamic price adjustment in the economy based on the assumption that information dissemination is slow throughout the population and expectations are formed based on past data. Döpke et al. (2008) find a period of roughly 18 months for European households to update information. Kabundi et al. (2015) find that economic agents in South Africa, for example, have heterogeneous inflation expectation beliefs with anchoring i.e., expectations linked to lagged inflation, displayed by individual price setters. Their work is extended by Miyajima & Yetman (2018) who describe anchoring of inflation expectations by unions and businesses.

3.6 Following the line of reasoning above, we therefore consider nominal bond returns adjusted by the current inflation rate in the analysis below. This line of reasoning is consistent with investors assessing real asset class returns on an ex-ante basis while using the current inflation rate as an estimate of future realised inflation. Given the comments by Bauer & McCarthy (2015) and Fisher et al. (2002) this is seen as a pragmatic approach.

3.7 Time frame of the analysis

For this study both returns on government bonds and inflation data dating from 1 January 1960 to 1 December 2018, as obtained from I_Net, were used for the analysis of the discount rate. This is a period of approximately 59 years in monthly values. For visualisation purposes, Figures 1 and 2 illustrate government bonds and inflation respectively.

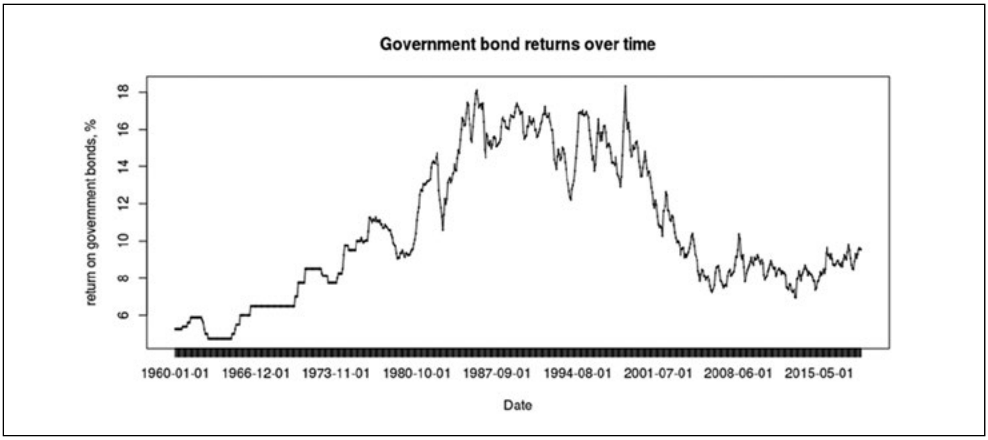

FIGURE 1. Return on government bonds over the past 59 years

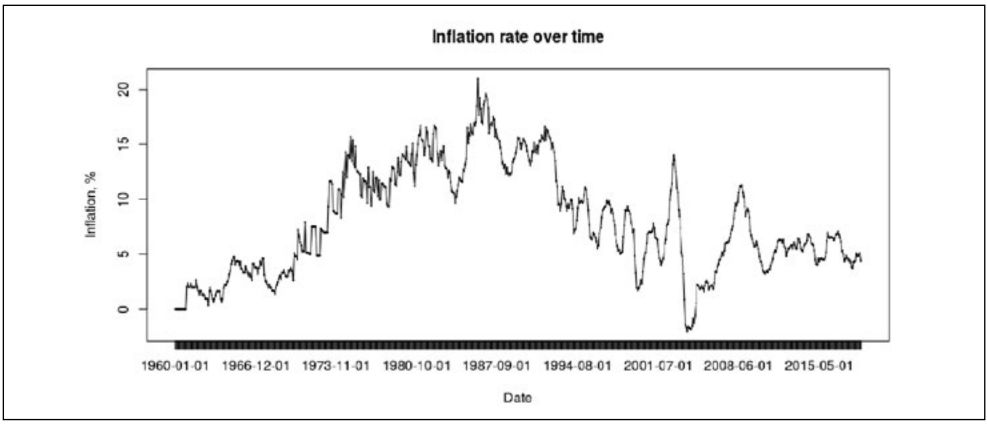

FIGURE 2. Inflation over the past 59 years

3.8 Method for calculating the discount rate

The method used for calculating the implied base discount rate is given by the formula,

$$r = \frac{(1+g)}{(1+i)} - 1,$$

where g is the return on government bonds and i the level of inflation.

3.9 Statistical analysis of implied discount rates

3.9.1 The discount rate r was calculated for each monthly entry of the government bond yields and inflation data. By plotting these rates over time and adding the 2.5% discount rate for reference, Figure 3 is obtained.

3.9.2 The mean real discount rate over the 59-year period was calculated and yielded a surprising result of 2.4972%, which for all practical purposes is equal to the 2.5%.

The significance of this result lies in the fact that the 2.5% that has been used for decades based on sound actuarial arguments, yet never statistically confirmed, was in fact remarkably accurate. From Figure 3, it is evident that from a visual perspective the 2.5% discount rate assumption in industry seems reasonable over long periods of time.

3.9.3 The geometric mean discount rate over the 59-year period was also calculated and yielded a result of 2.4452%. This is still close enough to the 2.5% rate currently in use. The long periods above and below the mean now requires attention.

3.9.4 The geometric averages over visual variation periods are calculated and shown in Figure 4.

3.9.5 The average from 1960 to 1973 was relatively stable around the 2.5% rate. The sharp decline in the average from 1973 to 1994 could be attributed to the numerous events such as labour shortages, Soweto riots, sanctions and recession that took place in the period leading up to the 1994 elections, as discussed in Section 2.2.1. Similarly, the steep incline in the average from 1994 to 2008 can be related to the factors discussed in

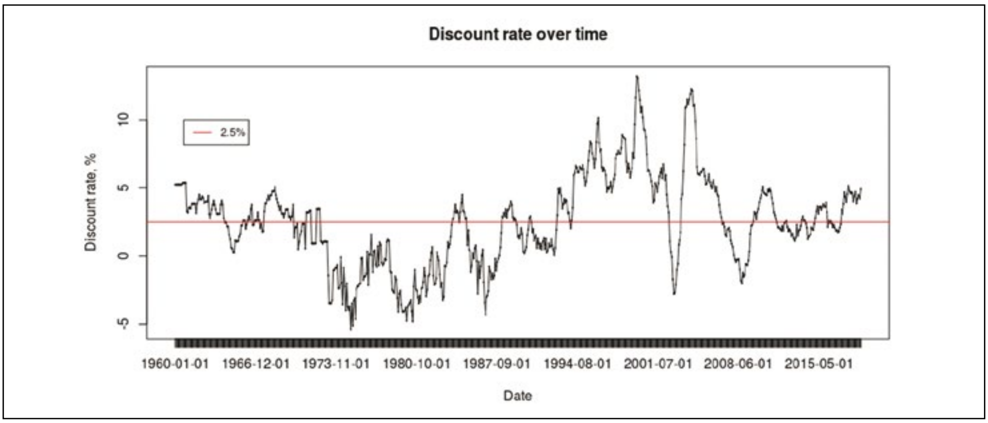

FIGURE 3. Discount rate over the past 59 years

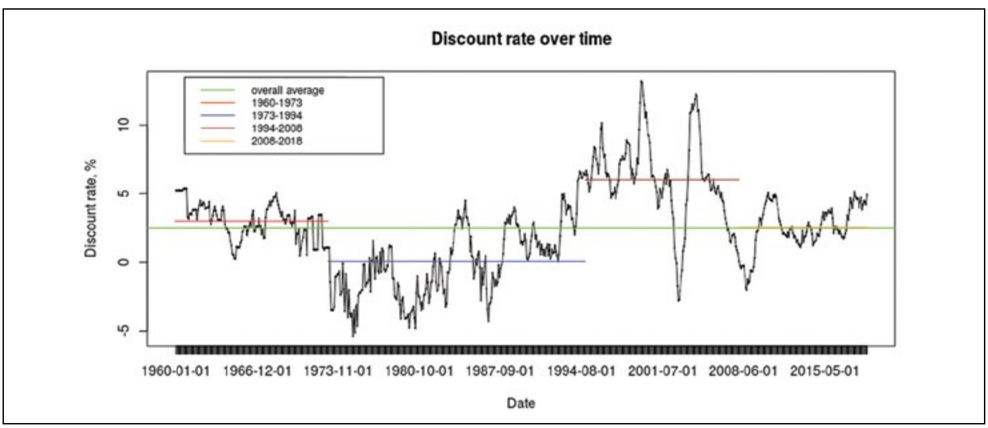

FIGURE 4. Geometric averages of the different periods

Section 2.2.2. The average stabilised around the 2.5% discount rate from 2008 onwards. Figure 5 visually illustrates all the factors that coincided with key turning points in the South African economy, some acting as trigger events for changes to the discount rate.

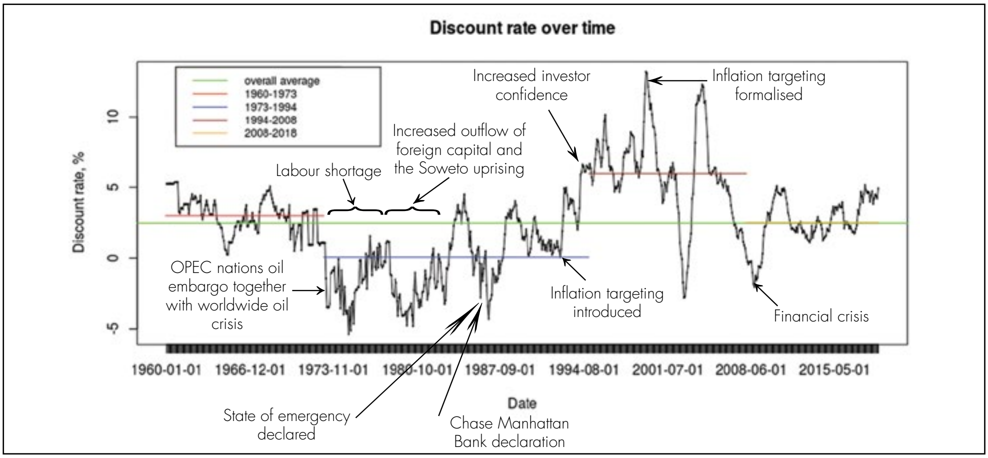

FIGURE 5. Key turning points in the South African economy

3.9.6 From the results in Figure 5, we can see that the South African economic and political landscape did not stabilise after the 1994 period. One can assume that the variation in the discount rate will continue into the future, but most of the variation seems to occur around the 2.5% rate and it seems that even after long periods of higher or lower variation, it eventually reverts to the 2.5%.

3.9.7 For the above statement to be statistically valid, the series will have to be mean reverting, meaning that the series will eventually revert to its mean after a period of deviation. To test this, the Augmented Dickey–Fuller (ADF) test and a Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test was performed on the data to test for weak stationarity (Kwiatkowski et al., 1992).

3.9.8 The ADF test has a null hypothesis that there is a unit root present, implying non-stationarity in the time series sample with an alternative hypothesis that the series is trend-stationary. The KPSS test has a null hypothesis that the time series sample is trend-stationary with an alternative hypothesis that the sample is not stationary. The Phillips-Perron (PP) test has a null hypothesis that there is a unit root present and an alternative hypothesis that the series is stationary. The ADF and PP tests' alternative hypotheses and the KPSS test null hypothesis are the similar in this context and should not be confused to have the same null and alternative hypotheses. The three tests yielded the following contradictory results:

— ADF:	p-value = 0.01	stationary
— KPSS:	p-value = 0.01	non-stationary

3.9.9 The indicated p-value for the ADF test is very small meaning that the null hypothesis cannot be accepted, and concluding that the series is stationary. The p-value for the KPSS test was also very small which meant that the null hypothesis could also not be accepted, but meaning that the series is not stationary. These two tests produced contradicting results so further investigation is needed. The next step in finding out if the series is stationary is to plot the autocorrelation functions and the partial autocorrelation function, shown in Figure 6.

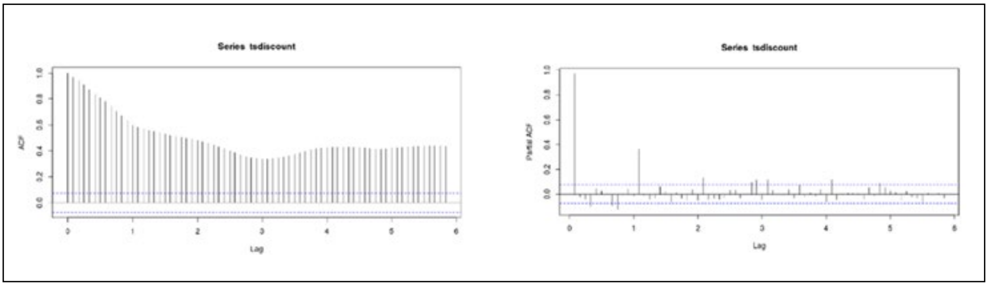

FIGURE 6. Correlograms of the series

3.9.10 The autocorrelation function has large significant lags that decrease but are still significant even at high lag values. This is a clear indicator that the series is not trend-stationary (Kestel, 2011). These are analysed using power spectra (plots of variance in time series data versus frequency. This was in contradiction with the Dickey Fuller unit root test. Although higher order autocorrelations are evident, the ADF test will remain well behaved (DeJong et al., 1992).

3.9.11 The results of the stationarity tests and the correlograms produced greater statistical evidence against the assertion of stationarity than in favour of it. Hence the possibility that the series varies around the mean cannot be confirmed statistically. However, when visually assessing the data, two proposed structural breaks (Hansen, 2001) can be observed. Figure 7 illustrates this.

3.9.12 To test if the proposed structural breaks were significant, the Chow test

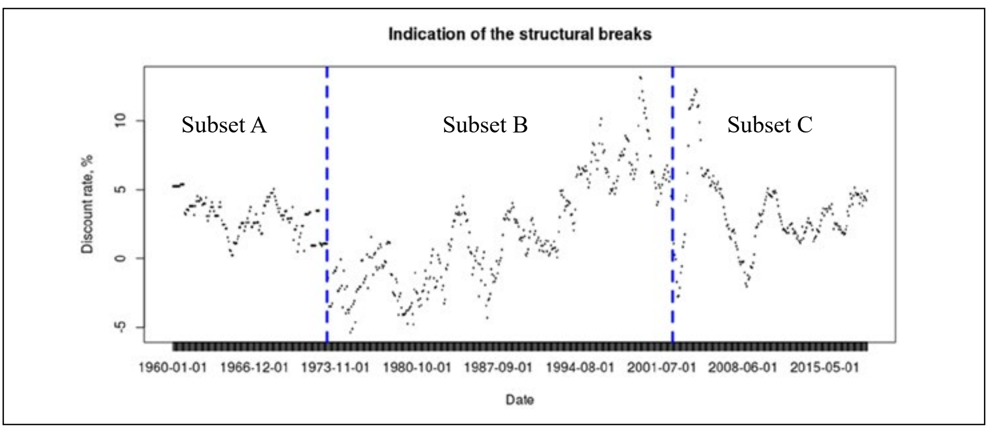

FIGURE 7. Structural breaks in the time series

(Morley, 2006) was done and Chow statistics were obtained for each of the breaks. The formula to calculate the Chow statistic is as follows:

$$F = \frac{\frac{RSS_P - RSS_A - RSS_B}{k}}{\frac{RSS_A + RSS_B}{n - 2k}}$$

where each of the above values can be defined as:

- RSS_A is defined as the residual sum of squares using only subsample A in regression model.
- RSS_B is defined as the residual sum of squares using only subsample B in regression model.
- RSS_P is defined as the residual sum of squares using the entire pooled sample.

3.9.13 To calculate the Chow statistic for the two breaks, subsets A and B were taken as the respective subsamples in the formula for the first test statistic, and the next test statistic used B and C as subsamples.

3.9.14 The Chow statistic was then compared to the critical F value to determine if the structural breaks were significant. The following results were obtained for the Chow statistics and the critical F value.

- Chow statistic for first break: 470.20
- Chow statistic for second break: 190.72
- F critical value for first break: 1.24
- F critical value for second break: 1.23

3.9.15 Both Chow statistics are significantly larger than the critical values meaning that both structural breaks are significant.

3.9.16 Next, each individual part of the breaks was tested for stationarity using the ADF and KPSS tests on all three individual parts to see if any part of the time series is mean reverting. This yielded the following if the p-values are compared to a 5% confidence level.

For subset A:

— ADF test	p-value of 0.3645	non-stationary
— KPSS test	p-value of 0.01	non-stationary

For subset B:

— ADF test	p-value of 0.07423	non-stationary
— KPSS test	p-value of 0.01	non-stationary

For subset C:

— ADF test	p-value of 0.01	stationary
— KPSS test	p-value of 0.1	stationary

3.9.17 From the results of the stationary tests it can be seen that the only stationary part is subset C. An interesting occurrence is that after inflation targeting was formalised in the year 2000, the series became stationary. This could be a coincidence, but it would be worth investigating this in a separate study. When looking at the series it is apparent that, although there is a large amount of variation, there is some mean reversion present, at least over the last 19 years, with this mean equal to 2.5%. To demonstrate this, the discount rate over the period with the mean and a two standard deviation interval is plotted in Figure 8.

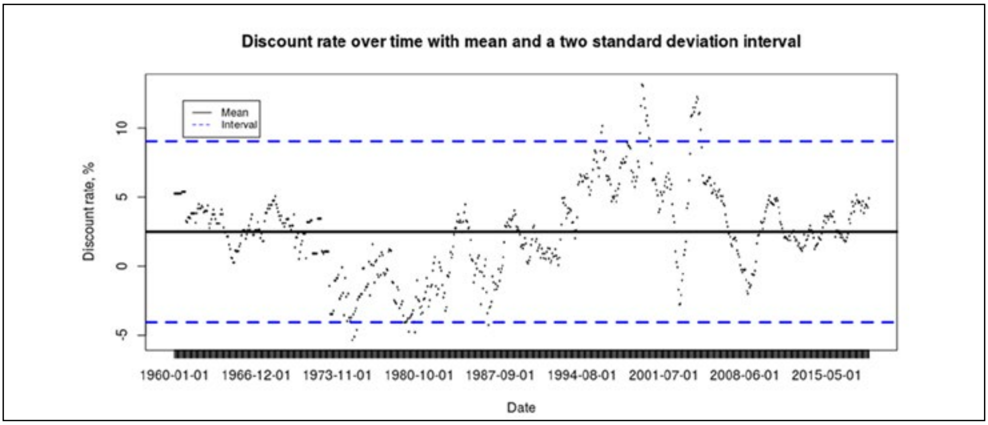

FIGURE 8. Geometric mean of the series with two standard deviation intervals

3.9.18 In Figure 8, it is shown that the discount rate only broke through the interval on a handful of occasions during the 1970–2009 period which relates to the various factors discussed in the literature study. Keeping in mind that the two standard deviations is only used with normal distributions, this provides some evidence that the series varies around the mean and that the 2.5% rate is a sensible rate to use in the future.

3.9.19 Future periods of large variation will be present in the data and structural breaks might occur that would compromise the mean reverting properties of the series, but it is important to note that the length of these large variation time periods cannot be predicted, but are likely to be temporary. The data have shown that the series will eventually revert to the mean of approximately 2.5%. More weight to the argument is produced by the period after the year 2000 when inflation targeting was introduced, and the series was found to be mean reverting. With inflation targeting not expected to change soon, it might be appropriate to apply the 2.5% discount rate going forward.

3.10 Analysis of the yield curves of South African, Hong Kong and Canadian government bonds

3.10.1 Data are analysed to determine whether setting the discount rate according to the term of the future loss, similar to Hong Kong and parts of Canada (2.5), is relevant in South Africa. This investigates whether the returns earned on investments will differ for investments with differing terms.

3.10.2 To determine this, the yields earned on government bonds with different terms are compared for each of these three countries. Clear differences between the yields of bonds with different terms would indicate that the use of different interest rates for each term is appropriate.

3.10.3 The historic yields earned on government bonds with different terms which include 5, 10, 15 and 20 years for South Africa, Hong Kong and Canada respectively were plotted on the same graph in Figure 9 to determine whether distinct differences in the yields rates could be seen.

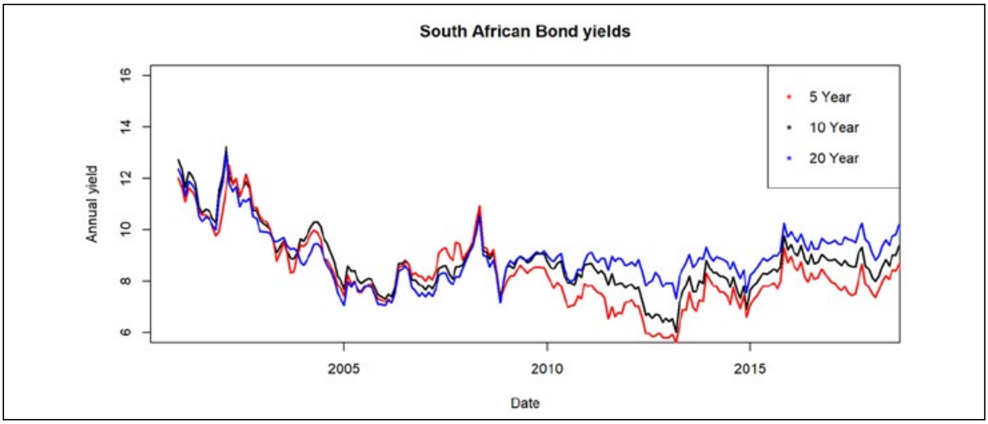

FIGURE 9. South African 5-, 10- and 20-year bond yields

3.10.4 From the plots in Figures 9 to 11, a clear and consistent difference between the yields of bonds with different terms can be seen in the case of Hong Kong and Canada compared to South Africa. In the South African case, the yields on longer term bonds do not consistently exceed that of shorter-term bonds in the time period from 2000 to 2009, indicating that a flat term structure existed in this time period. From 2009 the difference between the yields of the different South African bonds became clearer, which indicated that a steep yield curve, similar to that of Canada and Hong Kong, could be seen.

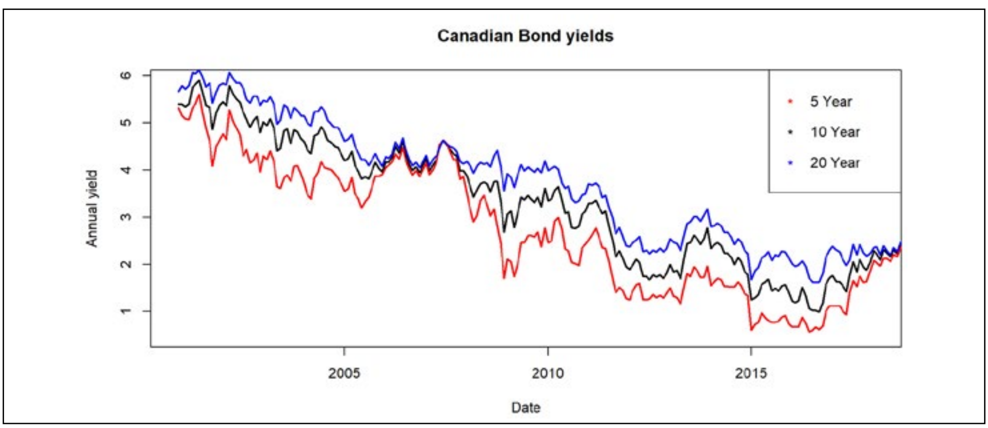

FIGURE 10. Canadian 5-, 10- and 20-year bond yields

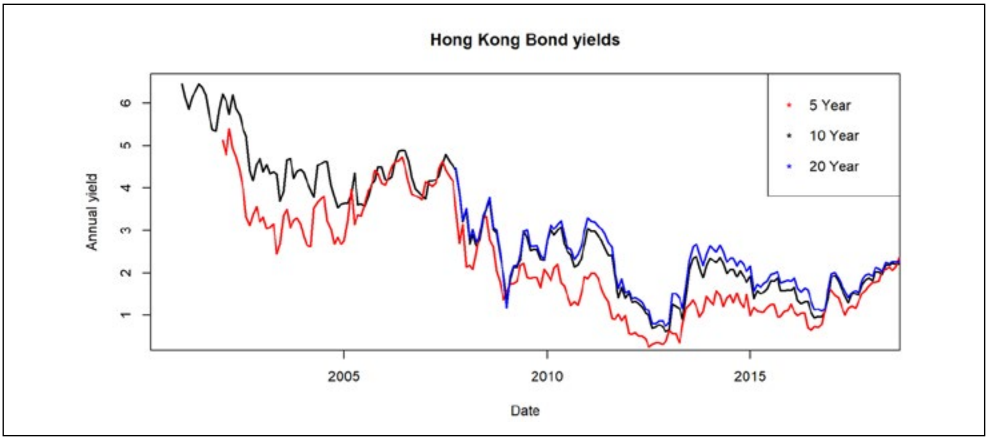

FIGURE 11. Hong Kong 5-, 10- and 15-year bond yields

3.10.5 For the period from 2009 until present, consideration should also be given to the proportional size of the differences between the yields of bonds with different terms for these three countries. To determine this proportional difference, plots for the three bonds of each country based on the following formula were made:

$$Proportional\ Difference = \frac{Yield\ on\ Bond\ A - Yield\ on\ Bond\ B}{Yield\ on\ Bond\ B}$$

3.10.6 The following plots were made in Figure 12. The proportional difference between long- and short-term bonds for each country were:

— South Africa: 20- and 5-year bonds
— Canada: 20- and 5-year bonds
— Hong Kong: 15- and 5-year bonds

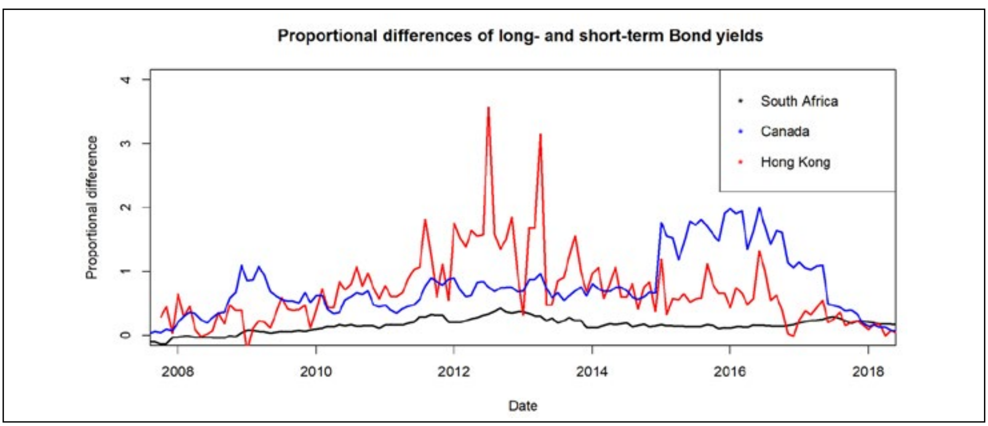

FIGURE 12. Proportional differences between long- and short-term bond yields.

The proportional difference between medium- and short-term bonds for each country were (Figure 13):
— South Africa: 10- and 5-year bonds
— Canada: 10- and 5-year bonds
— Hong Kong: 10- and 5-year bonds

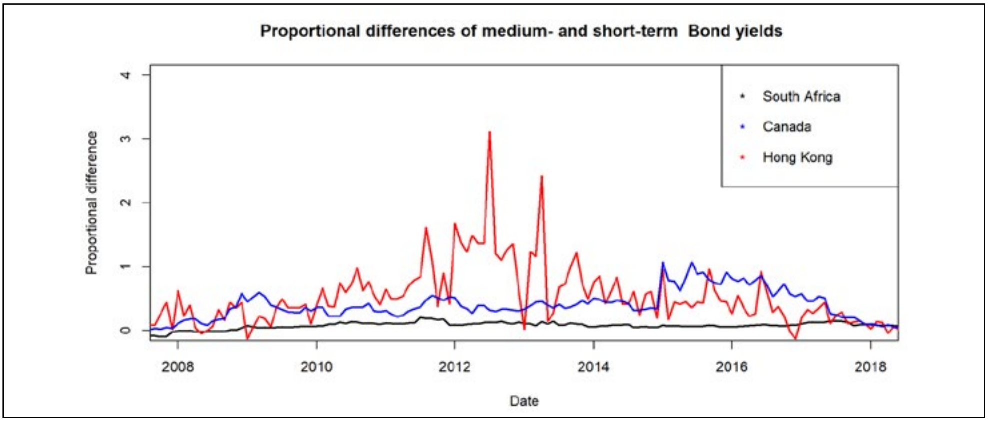

FIGURE 13. Proportional differences between medium- and short-term bond yields

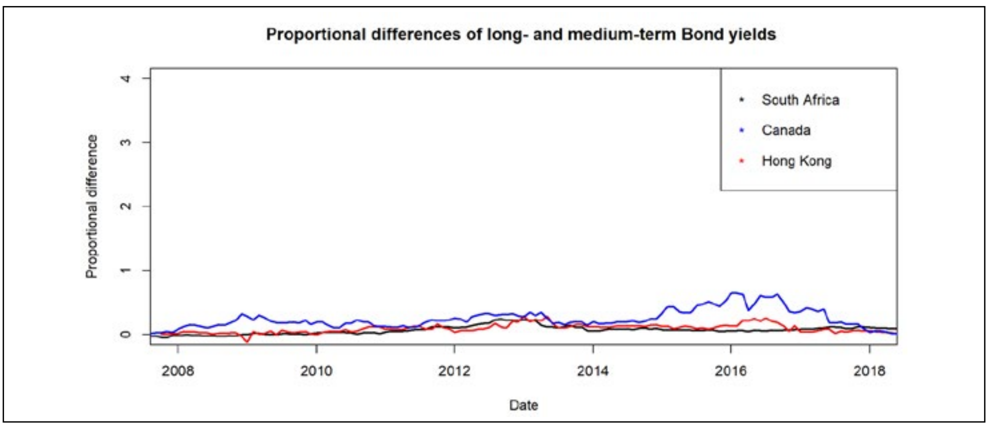

FIGURE 14. Proportional differences between long- and medium-term bond yields

The difference between long- and medium-term bonds (Figure 14) were:
— South Africa: 20- and 10-year bonds
— Canada: 20- and 10-year bonds
— Hong Kong: 15- and 10-year bonds

3.10.7 From Figures 12 to 14 a much larger proportional difference is seen between the yields on bonds from Canada and Hong Kong compared to those of South Africa when 20- and 10-year bonds are compared to 5-year bonds. When 20- and 10-year bonds are compared, the proportional difference between the yields of each country are quite similar. This indicates that although a more distinctive difference between bonds of different terms can be seen in the South African case from 2009, this difference does not have the same proportional size as in the case of Canada and Hong Kong.

3.10.8 It can be concluded that the term structure of South African bond yields differs from the term structure of Canadian and Hong Kongese bond yields based on two factors: the consistency of the gap between the yields of bonds with different terms and the proportional size of the gap between these yields. Hence the conclusion can be made that the practice of dividing the discount rate into a set of different rates based on the future time period of the loss as done in Hong Kong and Canada, will not be relevant in the South African case yet. The relevance of such a practice in future can however not be excluded. The term structure of South African bond yields can be monitored to determine if the consistent gap between the yields of bonds with different terms continues and if the proportional difference between the yields increases. If this is the case, an alteration to the practice can be considered.

4. POSSIBLE ADJUSTMENT TO THE BASE DISCOUNT RATE FROM CLAIMANT CHARACTERISTICS

4.1 The relevance of government bonds

4.1.1 From the above analysis, it can be concluded that the assumption that government bonds will exceed inflation by 2.5% in the future is the best deterministic result to use in the calculation of the quantum of damages. This could be used as a base rate from which small adjustments are possible depending on the characteristics of the claimant.

4.1.2 The assumption that all claimants can and should invest the awarded lump sum in government bonds is contestable. Although government bonds are considered a reasonably safe security, there are many issues regarding government bonds that render them irrelevant to some investors as an investment choice, and debatable if they would have similar returns on their alternatives. The issues around bonds include the size of government bonds, lack of liquidity of such an investment and lack of investment knowledge (Edwards,1975: 345).

4.1.3 Hence, a suggestion is made that investment decisions that are assumed to be made by a claimant with the awarded lump sum should reflect the ability and need of a claimant to invest in government bonds as well as alternative investment instruments. The assumed investment decision made will greatly affect the expected return earned on the lump sum and hence the discount rate applied to the lost future earnings of the claimant.

4.2 Finding an accurate discount rate

4.2.1 Although Fairgrieve & Gauci (2017) contest the usefulness of adjusting the discount rate applied according to the specific needs and characteristics of the claimant, the literature described in sections 2.3 and 2.4 suggests a need to take the socio-economic position of claimants into consideration when determining the discount rate to ensure fair compensation. This should be done since claimants with different socio-economic positions will have diverging investment needs and abilities and this should be considered when selecting the applied discount rate.

4.2.2 A lack of similar investigations done into such a practice in other countries can be attributed to the uniqueness of the South African socio-economic context, as mentioned in section 2.4.

4.2.3 From the literature study it was found that the level of income, education, occupational status and the size of investment are prominent factors affecting the expected investment returns earned by investors. These factors could thus be considered when contextualising the discount rate.

4.3 Suggested future research

4.3.1 Future research into division of claimants into segments according to their socio-economic position and hence their investment preferences is suggested, possibly dividing claimants according to the Paterson Job Grading System. This system is suggested since it provides an objective measure of skill level and income level that is widely in use, understood and accepted in the South African practice.

4.3.2 The Paterson Job Grading System is used to analytically evaluate jobs based on an employee's ability to make decisions (Diamond, 2019). This system divides jobs into six different decision-making bands according to the complexity of decisions made by the employee. The complexity of the decisions made by the employee is closely related to the skill level of the employee. Hence the skill level of the employees in each decision-making band is explicitly indicated in the system. This system is used to define pay scales and thus determines the income level of an employee.

4.3.3 The Paterson Grading System is suggested as an appropriate system to use to classify the socio-economic position of a claimant since it considers the relevant socio-economic factors identified in the literature study (2.3) that influence investor preference.

4.3.4 This proposed research could include consideration of a reduction in the net discount rate of this paper for unskilled or semi-skilled, and even an addition for the highly skilled.

5. CONCLUSION

The statistical analysis suggests that the 2.5% industry discount rate is in reality a very sound base rate, with statistical mean reversion to the 2.5% average since the year 2000, when inflation targeting was introduced. Deviations above and below the 2.5% do occur over time, but damage calculations are often done over 20–60 years of future life span, and the current research confirms the current 2.5% as the best standard over the longer term.

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APPENDIX 1

CODE

```
library("tseries")

GovBonds<- read.csv("Government Bond yield.csv")
CPI<- read.csv("ZA CPI.csv")
Inflation <- CPI

plot(GovBonds, xlab = "Date", ylab = "return on government bonds, %", main = "Government bond
returns over time")
lines(GovBonds)
plot(CPI)
Inflation[1:12,2] <- 0
for (i in 13:708) {

 Inflation[i,2]<- ((CPI[i,2]-CPI[i-12,2])/CPI[i-12,2])*100
}

plot(Inflation, xlab = "Date", ylab = "Inflation, %", main = "Inflation rate over time")
lines(Inflation)

Diff<- CPI
Diff[,2]<- GovBonds[,2]-Inflation[,2]

vecdiff<- c(Diff[,2])
vecdate<- seq(1,708,1)

summary(regression <- lm(vecdiff~vecdate))

plot(Diff, xlab = "Date", ylab = "Discount rate,%", main = "Difference between Government bonds
and Inflation" )
lines(Diff)
curve(0.0047505*x+0.9056766 , from = 1, to = 708, add = TRUE, col = "blue")
legend(x = 600, y=13, c("Best fit line"), col=c("blue"), lty=1, cex=0.8)

Discountrate <- Inflation
Discountrate[,2]<- ((1+(GovBonds[,2]/100))/(1+(Inflation[,2]/100))-1)*100

curve(0.0047505*x+0.9056766 , from = 1, to = 708, add = TRUE, col = "red")

vecdiscount<- c(Discountrate[,2])

summary(regression <- lm(vecdiscount~vecdate))
```

```
plot(Discountrate, xlab = "Date", ylab = "Discount rate, %", main = "Discount rate over time" )
lines(Discountrate)
abline(a=2.5, b=0, col = "red")
curve(0.0047505*x+0.9056766 , from = 1, to = 708, add = TRUE, col = "blue")
legend(x = 10, y=10, c("2.5%", "best fit line"), col=c("red","blue"), lty=1, cex=0.8)

tsdiscount<- ts(Discountrate$ZAFCPIALLMINMEI,frequency = 12)
pacf(tsdiscount)
acf(tsdiscount)

#looks like there is a seasonal component but it is very small as to make a difference in the actual
usage of the discount rate

#Now checking if I can see trends of high and low periods
geoaverage<- (prod(1+(Discountrate$ZAFCPIALLMINMEI)/100)^(1/708)-1)*100
OverallAverage <- geoaverage
Average1960to1973<- (prod(1+(Discountrate[0:157,2])/100)^(1/157)-1)*100
Average1973to1994<- (prod(1+(Discountrate[158:420,2])/100)^(1/262)-1)*100
Average1994to2008<- (prod(1+(Discountrate[421:577,2])/100)^(1/156)-1)*100
Average2008to2018<- (prod(1+(Discountrate[578:708,2])/100)^(1/130)-1)*100

plot(Discountrate, xlab = "Date", ylab = "Discount rate, %", main = "Discount rate over time" , type
= "l")
lines(Discountrate)
abline(a=OverallAverage, b=0, col = "green")

segments(y0 = Average1960to1973, x0= 0 , x1 = 157, y1 = Average1960to1973,col ="red")
segments(y0 = Average1973to1994, x0= 157 , x1 = 420, y1 = Average1973to1994,col ="blue")
segments(y0 = Average1994to2008, x0= 420 , x1 = 577, y1 = Average1994to2008,col ="brown")
segments(y0 = Average2008to2018, x0= 577 , x1 = 708, y1 = Average2008to2018,col ="orange")

legend(x = 10, y=13.5, c("overall average","1960-1973", "1973-1994","1994-2008","2008-2018"),
col=c("green","red","blue","brown", "orange"), lty=1, cex=0.8)

#Dickey fuller test

adf.test(diff(tsdiscount))
plot(diff(tsdiscount))

acf(tsdiscount,70)
pacf(diff(tsdiscount))
kpss.test(tsdiscount)

CPIpart <- CPI[100:200,]
```

```
#structural breaks

plot(Discountrate)

nobreak<- ts(Discountrate$ZAFCPIALLMINMEI)

sb1<- ts(Discountrate[1:158,]$ZAFCPIALLMINMEI)

plot(sb1)

sb2<- ts(Discountrate[158:510,]$ZAFCPIALLMINMEI)

plot(sb2)

sb3<- ts(Discountrate[510:708,]$ZAFCPIALLMINMEI)

plot(sb3)

rNoBreak <- lm(nobreak~vecdate)

rsb1 <- lm(sb1~vecdate[1:158])
rsb2 <- lm(sb2~vecdate[158:510])
rsb3 <- lm(sb3~vecdate[510:708])

plot(Discountrate)

curve(rsb1$coefficients[2]*x+rsb1$coefficients[1] , from = 1, to = 158, add = TRUE, col = “red”)
curve(rsb2$coefficients[2]*x+rsb2$coefficients[1] , from = 158, to = 510, add = TRUE, col = “blue”)
curve(rsb3$coefficients[2]*x+rsb3$coefficients[1] , from = 510, to = 708, add = TRUE, col = “green”)
curve(rsbpart1$coefficients[2]*x+rsbpart1$coefficients[1] , from = 1, to = 510, add = TRUE, col =
“purple”)
curve(rsbpart2$coefficients[2]*x+rsbpart2$coefficients[1] , from = 158, to = 708, add = TRUE, col =
“black”)

sbpart1<- ts(Discountrate[1:510,]$ZAFCPIALLMINMEI)
sbpart2<- ts(Discountrate[158:708,]$ZAFCPIALLMINMEI)

rsbpart1 <- lm(sbpart1~vecdate[1:510])
rsbpart2 <- lm(sbpart2~vecdate[158:708])

rss1<- sum(residuals(rsb1)^2)
rss2<- sum(residuals(rsb2)^2)
rss3<- sum(residuals(rsb3)^2)
rssp1<- sum(residuals(rsbpart1)^2)
rssp2<- sum(residuals(rsbpart2)^2)
```

```
Chowstat1<- ((rssp1-(rss1+rss2))/2)/((rss1+rss2)/(rsb1$df+rsb2$df-4))
fcrit1<-qf(.95,df1=rsb1$df,df2=rsb2$df)

Chowstat2<- ((rssp2-(rss2+rss3))/2)/((rss2+rss3)/(rsb2$df+rsb3$df-4))
fcrit2<-qf(.95,df1=rsb1$df,df2=rsb2$df)

kpss.test(sb3)
adf.test(sb3)

#plotting 2 standard deviations around the mean

upperbound<- OverallAverage+ sd(Discountrate$ZAFCPIALLMINMEI)
lowerbound<- OverallAverage- sd(Discountrate$ZAFCPIALLMINMEI)

plot(Discountrate)
curve(OverallAverage + 0*x , from = 1, to = 708, add = TRUE, col = “blue”)
curve(upperbound + 0*x, from = 1, to = 708, add = TRUE, col = “green”, type = “l”)
curve(lowerbound + 0*x , from = 1, to = 708, add = TRUE, col = “green”,type = “l” )
```