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A history and forecast of the South African life tables No. E1 to No. E9

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ABSTRACT
During the 20th century, government statistical departments in South Africa produced nine complete life tables for the white population group, seven complete life tables for the coloured population group, six complete life tables for the Asian population group and no complete life tables for the African population group. As of 2020, the South African Life Tables 1979/1981 for the white population group are still used in capitalising pensions in terms of the Compensation for Occupational Injuries and Diseases Act No. 130 of 1993. Similarly, the South African Life Tables 1984/1986 for the white population group are still used extensively in damages calculations by South African actuaries. This paper provides a historical record of the nine complete life tables for the white population group and provides a forecast of those tables to 2020. The aim of the forecast is to provide an estimate for current white population mortality rates which could then serve as a non-racial mortality basis for damages claims. It also aims to complement the work of demographers in their development of new population tables based on more sophisticated demographic techniques.

KEYWORDS
South African Life Tables; damages; mortality forecasting; Lee–Carter method; limited data

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1. INTRODUCTION

1.1 Historically there have been substantial differences in mortality between population groups in South Africa. This has been due to numerous factors the most important of which is the restriction of economic development under apartheid, poor living conditions in rural areas and urban squatter camps and insufficient health services for large segments of the population (Mostert et al., 1998).

1.2 Determining mortality rates is requisite in personal injury litigation for the purposes of calculating the lump sum damages of a claimant. Actuaries are utilised as expert witnesses for both the claimant and the defending party (which in a South African context is usually the State in the form of the Road Accident Fund or the Department of Health) to provide impartial expert evidence as to the present value of the future loss most likely to be incurred by the claimant. The starting point for calculating the present value is the application of a mortality table.

1.3 There is a conundrum faced by actuaries in having no alternative but to effectively apply race-based life tables to claimants in a post-democratic non-racial context. In post-democracy litigation the trend is for judges to approve the application of the South African Life Tables 1984/1986 for the white population group to all claimants (that is, including those previously classified as non-white) so as to remove racial bias. The following recorded decisions reflect the evolution of this principle.

1.4 In 2010, the Supreme Court of Appeal approved the application of the South African Life Table 1984/1986 for the white population group to an Indian claimant in *Singh and Another* v *Ebrahim*:[1]

> As with most things in this matter, the appropriate life tables to be applied to the assessment of Nico's life expectancy were also in issue. The high court applied the SA white male tables. The appellant contends for the application of the Koch life tables which adds between 2 to 4 years to the various scenarios calculated by Strauss. Koch's attempt to remove race from the SA life tables is obviously attractive, but the evidence of the assumptions made to compile his life tables does not, in this case, succeed to illustrate their reliability. Although the 1984/1986 SA life tables are out of date, they are still the best available. In the circumstances it seems eminently reasonable to have used the white male tables to exclude any racial component from the calculation. Consequently the dispute about whether the appellant agreed to the application of the SA life tables only to the actuarial calculation or also to the assessment of life expectancy is irrelevant.

1.5 The actuary called to give oral evidence by the appellant had published a series of life tables that represented an attempt to classify claimants on the basis of income instead of race. He had, since the 1990s, produced a series of six life tables representing various combinations

1 *Singh and Another* v *Ebrahim* (413/09) [2010] ZASCA 145

of the South African Life Tables 1984/1986 for the white population group and coloured population group.[2] Whilst the principle of moving to income-based tables is applauded, there was a lack of evidence before the Court as to the various income classifications on which his assumptions were based.

1.6 Later, in the 2016 decision of *AD and Another* v *MEC for Health and Social Development, Western Cape Provincial Government*[3] the Western Cape High Court referred to the above passage from Singh:

> This passage embodies a decision of legal policy by which I am bound in the absence of new data. The conclusion, based on this policy, would be that although IDT is a coloured child I should, to exclude any racial component, use the 1985 life table for white males (i.e. K2, not K3–K6). Apart from the reason given by Snyders JA, this may be justified on the basis that at least the legal impediments to equality which existed in 1985 were removed with our transition to a democratic country, more than 15 years before IDT was born.

1.7 Following that, in 2017, a full bench held in *PM (obo TM)* v *MEC for Health, Gauteng*[4] that:

> The unanimous decision of the Supreme Court of Appeal in Singh (supra), paras 65 and 199, was that the South African 1984/1986 life tables, although out of date, were still the best available and they were thus applied to the assessment of the life expectancy of the child in that case. Therefore I propose to apply the 1984/1986 life tables to the assessment of TM's life expectancy, which results in a further downward adjustment of Prof Strauss' estimation, and to follow Rogers J in Du Toit (supra), paras 176–197, in making a further downward qualitative adjustment based on the more recent information and data that became available after the 2010 census and Statistics South Africa (SSA) in 2015, producing male and female life expectancies at birth for persons not at risk of HIV/AIDS.

1.8 In both the *AD and Another*, and *PM (obo TM)* matters above, reference is made to new data. An attempt was made to bring the available South African Life Tables into a more current context by overlaying the life expectancy at birth produced by Statistics South Africa for the not at risk of HIV/AIDS population. A criticism of this route is that the Statistics South

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2 Life Table 1 represented 80% of the SALT1984/1986 mortality rates for the white population group, Life Table 2 represented 100% of the SALT1984/1986 mortality rates for the white population group, Life Table 3 represented two-thirds of the SALT1984/1986 mortality rates for the white population group plus one-third of the SALT1984/1986 coloured population group, Life Table 4 represented one-third of the SALT1984/1986 mortality rates for the white population group plus two-thirds of the SALT1984/1986 coloured population group, Life Table 5 represented 100% of the SALT1984/1986 mortality rates for the coloured population group and Life Table 6 represented 120% of the SALT1984/1986 mortality rates for the coloured population group.

3 *AD and Another* v *MEC for Health and Social Development, Western Cape Provincial Government* (27428/10) [2016] ZAWCHC 116

4 *PM obo TM v MEC for Health, Gauteng Provincial Government* (A5093/2014) [2017] ZAGPJHC 346

Africa estimate is not based on a mortality study and is designed as an input that conforms to the demographic mortality profile of the country. It is erroneous to make a comparison between a complete population life table and a demographic estimate of life expectancy at birth which is not based on census data. The new data to which the judiciary refers is a call for revised mortality rates.

1.9 The paper unfolds as follows: first, Section 2 provides a literature review and Section 3 provides an overview of the data. Section 4 provides a brief overview of forecasting methods. Section 5 describes the Lee–Carter method for populations with limited data. Section 6 provides results after applying that method to the South African Life Tables and an interim life table projected to 2020. The paper concludes in Section 7.

2. LITERATURE REVIEW

2.1 Lowther (2011) notes that discrimination by race and gender has been integral to actuarial calculations in the past. In the absence of availability of life tables which do not bear the weight of a historic backlog, compensation awards become tainted by the competing principles of a fair settlement value and historical unfair discrimination. With respect to the selection of an appropriate mortality table in the legal context, the actuary needs to consider whether or not past injustices should influence current rates and more contemporaneously, whether or not the rates should include or exclude the impact of HIV/AIDS.

2.2 It is significant to note that the South African Life Tables 1984/1986 pre-date the HIV/AIDS pandemic in South Africa. In addition, there are starkly contrasting rates of HIV/AIDS across the economic divide and the effect on life expectancy similarly varies depending on economically driven factors such as access to health care.

2.3 In the ordinary course of a damages claim, there is no specific requirement for the HIV/AIDS status of the claimant to be disclosed. The HIV status of a claimant may be brought into evidence by way of medico-legal reports placed before the court. In such instances, presiding officers have been found to consider the HIV status with strict regard to the specific facts applicable to the individual claimant. This personalised approach has created various possibilities in dealing with HIV-positive claimants.

2.4 In *Njoko* v *Minister of Safety and Security and Another*[5] the Court did not make explicit provision for a reduction in life expectancy of an HIV-positive claimant, but instead opted for an increased deduction for general contingencies to take into account the possibility of a reduced life expectancy:

> I accept the evidence of Professor Venter that the effect of his HIV-positive status would not have a negative effect on the plaintiff's life expectancy and future loss of earnings.

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5 *Njoko* v *Minister of Safety and Security and Another* (2011 (5) SA 512 (KZP)) [2011] ZAKZPHC 25

> Based on the normal contingencies applied by Koch, and taking into account the possibility of a reduced life expectancy, I set the contingencies which should be applied to the plaintiff's future loss of earnings at 23%.

2.5 By contrast, in *Seme* v *Road Accident Fund*[6] the Court allowed for an explicit reduction in the plaintiff's life expectancy:

> It is safe to say that the plaintiff was diagnosed with HIV in 1995. Thirteen years later he is still in a generally healthy state without the use of retrovirals. I therefore find that life expectancy of a person diagnosed with HIV is halfway between 11 years and normal. I would add the *caveat,* however, that every case is *sui generis* and must be judged on its own particular circumstances.

2.6 Koning & Van der Merwe (unpublished) constructed population mortality tables for the South African population as a whole based on 2011 census data and vital registration data on all causes of deaths including HIV/AIDS. The purpose of the study was to provide an up-to-date tool for quantum of damages assessments. Technical criticisms raised about the paper included the lack of adjustment for the under-registration of deaths and concerns around the lack of smoothness of the graduated rates.

2.7 The landscape of the effect of HIV/AIDS on the South African population and, in particular, on life expectancy has altered since 2010. There have been significant developments with respect to the roll-out and use of antiretroviral medication. In 2010 the National Department of Health expanded treatment to all children under one year, all pregnant women regardless of CD4 count and all TB-HIV co-infected patients with a CD4 count of <350 cells/μl. Further, in 2012 a decision was made to increase the initiation threshold to 350 cells/μl for all adults and to expand access thresholds for children (Simelela & Venter 2014). On the latter ground alone, the population mortality estimates of Koning & Van der Merwe centred at 2011 would be inappropriate for current use given that a significantly greater proportion of the HIV-positive population is not only receiving antiretroviral medication, but is receiving it earlier on in the progression of the virus than was the case in 2011. In particular, Strydom et al. (2016) find that life expectancy at birth has improved meaningfully with the help of the aggressive roll-out of antiretrovirals.

2.8 For HIV-negative claimants, Schneider & Kelly (unpublished) present an actuarial approach to assessing the impact of HIV/AIDS on the calculation of damages claims. The proposed model includes risk loadings that differ by age, gender, calendar year, province, urban/rural and recent HIV test results. These techniques could be applied to an HIV-negative claimant who is perceived to be at risk of becoming HIV-positive. Similarly, this approach was developed 20 years ago and as such does not take into account the subsequent impact of antiretroviral medication.

6 *Seme* v *Road Accident Fund* (13917/04) [2008] ZAKZHC 47

2.9 It is important to bear in mind that in the exercise of applying a mortality table to a litigant, the actuary does not have the array of personal information that is ordinarily available to an underwriter. More often than not, the usual factors which would load mortality rates in an insurance setting such as the HIV status of an insured are not before the court as evidence. Should a defendant argue for a reduction in life expectancy on the basis of a suspicion of HIV positivity on the part of the claimant, the onus is on the defendant to adduce evidence of this. Such evidence can be simply rebutted by the disclosure of the claimant's status as negative. Should the claimant indeed be positive, then the approach as discussed in the case law above would apply. As such there is little basis for loading mortality rates to generally take into account HIV/AIDS in order to account for the general infection rates in the population. In highly specific circumstances the actuary may encounter a situation where the defendant raises a higher probability of the claimant contracting HIV/AIDS post settlement of the damages claim. Such a scenario could evolve where the claimant engages in high risk activity such as a sex worker. This probability can be catered for by the application of a higher general contingency deduction, as one would do for any other loading factor that is present in the claimant.

2.10 Koch (2020) notes that the South African Life Tables 1984/1986 remain the best available tables for persons who do not test positive for HIV/AIDS. The author notes that in South Africa, damages claims are assessed on the assumption that the claimant is not HIV positive.

2.11 Dorrington & Richman (unpublished) note that when valuing cash flows outside of the insurance and retirement context, actuaries may need to rely upon population mortality rates. This is in alignment with the approach of Martin et al. (unpublished) who advocate for the use of the English Life Tables for damages assessments in the United Kingdom on the basis that it is thought to provide a fair reflection of the population group for plaintiffs.

2.12 Cooper-Williams et al. (unpublished) analysed mortality improvement using the various South African Life Tables for the white population group. The authors note that by so doing, the impact of HIV/AIDS is minimised to a large extent and the use of these tables may act as a rough proxy for individuals currently purchasing insurance and annuity products.

3. DATA

3.1 The South African Life Tables No. 1 (1920–1922) were obtained from the National Library of South Africa. The published tables contained values of q_x from age 0 to age 105 inclusive for males, and values of q_x from age 0 to age 107 inclusive for females (Population Census, 1921).

3.2 The South African Life Tables No. 2 (1925–1927) were obtained from the Institute and Faculty of Actuaries Library and checked against the tables contained in the Human Mortality Database that were provided by Dr V Kannisto. The published tables contained values of q_x from age 0 to age 109 inclusive for males, and from age 0 to age 109 inclusive for females (Fourth Census of the Population of the Union of South Africa, enumerated 4 May 1926).

3.3 The South African Life Tables No. 3 (1935–1937) were obtained from the National Library of South Africa. The published tables contained values of q_x from age 0 to age 103 inclusive for males, and values of q_x from age 0 to age 105 inclusive for females (Sixth Census of the Population of the Union of South Africa, enumerated 5 May 1936).

3.4 The South African Life Tables No. 4 (1945–1947) were obtained from the National Library of South Africa and checked against the tables contained in the Human Mortality Database that were provided by Dr V Kannisto. The published tables contained values of q_x from age 0 to age 104 inclusive for males, and values of q_x from age 0 to age 106 inclusive for females (Eighth Census of the Population of the Union of South Africa, 7 May 1946).

3.5 The South African Life Tables No. 5 (1950–1952) were obtained from the National Library of South Africa. The published tables contained values of q_x from age 0 to age 104 inclusive for males, and values of q_x from age 0 to age 107 inclusive for females (Population Census, 8 May 1951).

3.6 The South African Life Tables No. 6 (1959–1961) were obtained from the National Library of South Africa after a legal application in terms of the Promotion for Access to Information Act. The published tables contained values of q_x from age 0 to age 89 inclusive for males, and values of q_x from age 0 to age 89 inclusive for females (Population Census, 6 September 1960).

3.7 The South African Life Tables No. 7 (1969–1971) were obtained from the National Library of South Africa. The published tables contained values of q_x from age 0 to age 89 inclusive for males, and values of q_x from age 0 to age 89 inclusive for females (South African Life Tables 1969–71). The tables were extended from age 90 to age 100 by the then Secretary for Statistics. Correspondence dating back to 1977 was obtained from Actuary Mr GW van der Linde in this regard.

3.8 The South African Life Tables No. 8 (1979–1981) were obtained from the National Library of South Africa. The published tables contained values of q_x from age 0 to age 80 inclusive for males, and values of q_x from age 0 to age 80 inclusive for females (South African Life Tables 1979–81). These tables are still in use by the Department of Labour in calculating the capitalised value of pensions in terms of the Compensation for Occupational Injuries and Diseases Act, 1993. The factors derived from the white population group are used to calculate the capitalised value of pensions for all population groups. Up to the early 1990s, the method used was discriminatory in that the capitalised value of pensions for other than the white population group were calculated by using an age rating of five years. Values of q_x from age 81 to age 100 were obtained from the late Actuary Mr CA Scott, who was involved in the development of the capitalisation factors for the Department of Labour.

3.9 The South African Life Tables No. 9 (1984–1986) were obtained from the University of Stellenbosch Library. The published tables contained values of q_x from age 0 to age 89 inclusive for males, and values of q_x from age 0 to age 89 inclusive for females (South African Life Tables 1984–86).

3.10 A historical record of the South African Life Tables for males and females is provided in Appendix A and Appendix B.

4. MORTALITY FORECASTING METHODS

4.1 Booth & Tickle (2008) provide a comprehensive review of mortality forecasting methods. Three broad methods are discussed namely parametrisation functions, Lee–Carter variants and extensions, and generalised linear modelling.

4.2 Forecasting with parametrisation functions such as the Heligman–Pollard model involves repeated fitting to annual data, thereby obtaining a time series of each model parameter. Annual data is not available in the South African context. The parameters of the Heligman–Pollard model are difficult to interpret and the forecasting results are sensitive to the past period that is chosen to start the extrapolation (Keyfitz, 1991). Given that only nine complete life tables were available over the period from 1921 to 1985, the parametrisation method was considered inappropriate. In addition, various mortality curves were fitted to the data using the R package Mortality Laws (Pascariu, 2020) and the best fitting mortality curve varies over time.

4.3 Lee–Carter variants and extensions represent a principal components approach to mortality forecasting. The principal components approach estimates the age pattern of mortality from the data. Advantages of the Lee–Carter model include simplicity, parameters can be easily interpreted and the method involves minimal subjective judgment. A further advantage in the South African context is the extension of the model to forecast mortality for populations with limited data as discussed in detail in Section 4. Some shortcomings of the Lee–Carter model are that forecast rates lack across-age smoothness and become increasingly jagged over time. In Bengtsson & Keilman (2019), Tuljapurkar shows that we can expect a reasonable performance from Lee–Carter forecasts for as far as 40 years into the future.

4.4 Generalised linear models were not considered as researchers have found that two- or higher degree polynomials in time are required to obtain a satisfactory fit, but lead to implausible forecasts (Booth & Tickle, 2008).

5. Lee–Carter METHOD FOR POPULATIONS WITH LIMITED DATA

5.1 The Lee–Carter method (Lee & Carter, 1992) was originally developed using data at single year intervals. In particular, the method forecast mortality using United States data at single year intervals from 1933 to 1987 inclusive.

5.2 The Lee–Carter Method first applies the singular-value decomposition (SVD) on $\left\{\log\left[m(x,t)\right]-a(x)\right\}$, where $m(x,t)$ is the death rate for age x at time t, to obtain:

$$\log\left[m(x,t)\right]=a(x)+b(x)k(t)+\varepsilon(x,t) \qquad (1)$$

5.3 The $k(t)$ are modelled using standard time series methods. In most applications, it has been found that a random walk with drift provides a good fit. The model for $k(t)$ is expressed as follows:

$$k(t)=k(t-1)+c+e(t)\sigma,\ e(t)\sim N(0,1),\ E\left(e(s)e(t)\right)=0 \qquad (2)$$

5.4 The Lee–Carter Method was extended to forecast mortality for populations with limited data by Li et al. (2002).

5.5 In particular, let mortality data be collected at times $u(0),u(1),\ldots,u(T)$. In the case of South Africa, $u(0)=1921$, $u(1)=1926$, $u(2)=1936$, $u(3)=1946$, $u(4)=1951$, $u(5)=1960$, $u(6)=1970$, $u(7)=1980$, and $u(8)=1985$.

5.6 Parameters $a(x)$ are calculated as:

$$a(x)=\sum_{t=0}^{T}\frac{\log\left[m\left(x,u(t)\right)\right]}{T} \qquad (3)$$

5.7 Applying SVD on $\left\{\log\left[m\left(x,u(t)\right)\right]-a(x)\right\}$, $b(x)$ and $k\left(u(0)\right),k\left(u(1)\right),\ldots,k\left(u(T)\right)$ are obtained. SVD was performed using SVD macros in excel (Lipkovich & Smith, 2002).

5.8 For $k\left(u(t)\right)$, however, (2) becomes

$$k\left(u(t)\right)-k\left(u(t-1)\right)=c\left[u(t)-u(t-1)\right]+\sigma\left[e\left(u(t-1)+1\right)+\ldots+e\left(u(t)\right)\right] \qquad (4)$$

5.9 For different t, $\left[k\left(u(t)\right)-k\left(u(t-1)\right)\right]$ are no longer identically distributed.

5.10 An unbiased estimate of c is obtained by:

$$c=\frac{\sum_{t=1}^{T}\left[k\left(u(t)\right)-k\left(u(t-1)\right)\right]}{\sum_{t=1}^{T}\left[u(t)-u(t-1)\right]}=\frac{k\left(u(T)\right)-k\left(u(0)\right)}{u(T)-u(0)} \qquad (5)$$

5.11 The derivation of the standard error of $e(u(t))$, *see*, is somewhat complicated, and is derived as:

$$see^2 = \frac{\sum_{t=1}^{T}\left[\left[k(u(t))-k(u(t-1))\right]-c\left[u(t)-u(t-1)\right]\right]^2}{u(T)-u(0)-\frac{\sum_{t=1}^{T}\left[u(t)-u(t-1)\right]^2}{u(T)-u(0)}} \quad (6)$$

5.12 The standard error in estimating *c*, *sec*, is estimated as follows:

$$sec^2 = \frac{var\left\{\sum_{t=1}^{T}\left[e(u(t-1)+1)+\ldots+e(u(t))\right]\right\}}{\left[u(T)-u(0)\right]^2} = \frac{\sigma^2}{u(T)-u(0)} \approx \frac{see^2}{u(T)-u(0)} \quad (7)$$

5.13 Once the values of *c* and *see* have been determined, forecasting is carried out by:

$$k(t) = k(T) + \left[c + sec.e(T)\right](t-T) + see\sum_{s=T+1}^{t} e(s) \quad (8)$$

5.14 Finally:

$$\log\left[m(x,t)\right] = \log\left[m(x,T)\right] + b(x)\left[k(t)-k(T)\right] \quad (9)$$

6. RESULTS

6.1 Fitted values of $a(x)$ and $b(x)$ are shown in Appendix C.

6.2 Values of $k(t)$ from 1921 to 1985 are shown in Table 1 below.

TABLE 1. Fitted values of $k(t)$

$u(t)$	$k(u(t))$ male	$k(u(t))$ female
1921	30.636671	53.814598
1926	25.886915	44.409418
1936	20.527914	37.605683
1946	2.130471	9.116399
1951	–4.481167	–5.510732
1960	–10.888190	–22.965064
1970	–16.479784	–30.934813
1980	–20.486869	–37.525275
1985	–26.845960	–48.010214

6.3 The forecasted value for $k(2020)$ is obtained by performing 1000 stochastically simulated trajectories. The mean forecasted value of $k(2020)$ is –58.28 for males (95% confidence interval –35.45 to –81.49). The mean forecasted value of $k(2020)$ is –104.27 for females (95% confidence interval –65.06 to –145.34).

6.4 A sample of the trajectory of $k(2020)$ for males is shown in Figure 1.

6.5 Fitted values of $q(x)$ for 2020 are shown in Appendix D.

6.6 Fitted values of $q(x)$ for 2020 for females compared to prior years are shown in Figure 2 (overleaf).

6.7 The forecasted life expectancy for females has been compared to the South African Annuitant Standard Mortality Tables (Dorrington & Tootla, 2007) and to pensioner mortality rates for female pensioners in receipt of pensions in excess of R30,000 a year for 2005–2010 (Actuarial Society of South Africa Continuous Statistical Investigation Committee, 2017) in Table 2.

TABLE 2. Life expectancy for females at selected ages

Age	SALT 1984/1986	SALT 1984/1986 forecast to 2020	SAIFL98	CSI pensioners 2005–2010
40	37.80	40.30	41.81	–
50	28.74	30.93	32.48	31.44
60	20.48	22.32	23.68	23.82
70	13.37	14.78	15.77	16.27

6.8 As noted by Martin et al. (unpublished), annuitant and assurance tables cover a subset of the population and in general, mortality rates in these groups are lower than for the

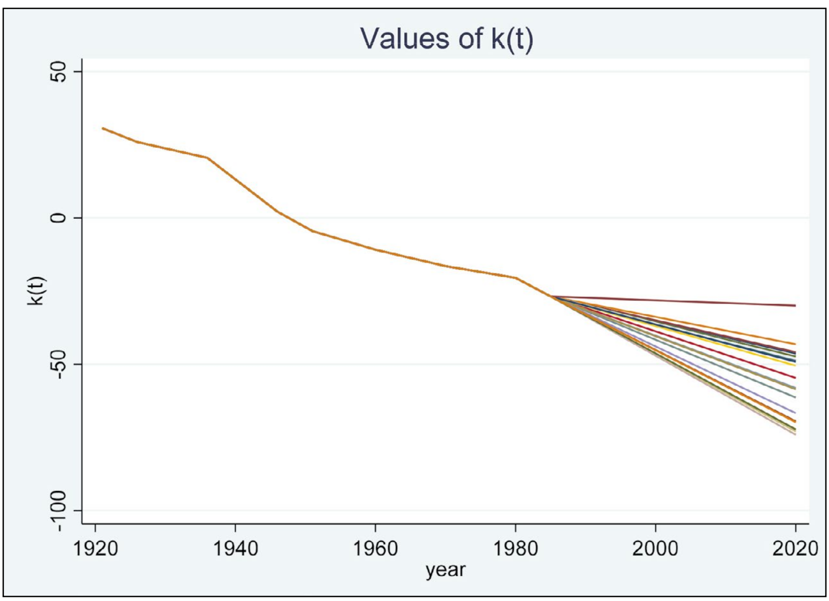

FIGURE 1. Sample trajectory of $k(2020)$ for males

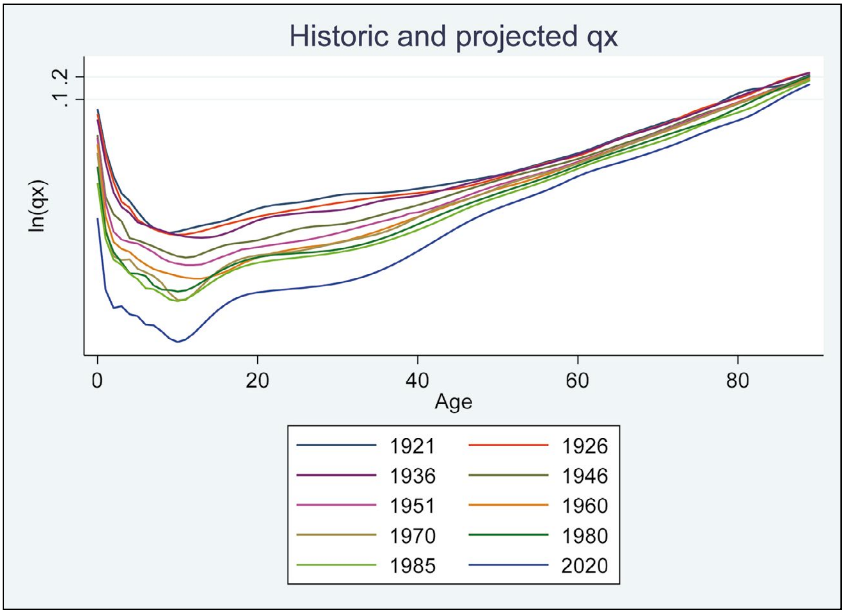

FIGURE 2. Historic and fitted mortality rates for females

population as whole. This is borne out by the forecasted rates and comparison of life expectancies in Table 2.

7. CONCLUSION

7.1 A historical record of the nine complete life tables for the white population group is provided in Appendices A and B to this paper. No such record exists at either the National Library of South Africa, Statistics South Africa or with the Actuarial Society of South Africa.

7.2 Despite government statistical departments not having produced official complete South African Life Tables in the last 35 years, the method applied in this paper produces useful forecasts for 2020.

7.3 Life expectancy at birth for white males in 2020 is estimated at 70.52 years (95% confidence interval 69.08 years to 71.55 years). The life expectancy at birth according to the South African Life Tables 1984/1986 for white males was 68.37 years. An improvement of 2.15 years is therefore estimated over the past 35 years.

7.4 Life expectancy at birth for white females in 2020 is estimated at 79.56 years (95% confidence interval 77.12 years to 81.61 years). The life expectancy at birth according to the South African Life Tables 1984/1986 for white females was 75.84 years. An improvement of 3.72 years is therefore estimated over the past 35 years.

7.5 It is recommended that the Actuarial Society of South Africa commission research into an appropriate set of population mortality tables for damages work subject to specifications such as removing racial bias and the impact of HIV/AIDS. The attempt by Koch to develop population mortality tables that are more aligned to an economic context could be advanced to establish a free, fair and equal basis on which mortality rates may be measured. The ultimate goal would be to afford claimants a balance of their right to a fair settlement and the foundational principles of the Bill of Rights. The forecasts in this paper can then be compared to the new population tables based on more sophisticated demographic techniques.

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APPENDIX A
South African Life Tables for White Males *q*(*x*)

Age	1920–1922	1925–1927	1935–1937	1945–1947	1950–1952	1959–1961	1969–1971	1979–1981	1984–1986
0	0.087844	0.074438	0.066410	0.041280	0.037890	0.032420	0.024668	0.015923	0.010598
1	0.021636	0.018703	0.014640	0.005990	0.004610	0.003140	0.001969	0.001737	0.001583
2	0.008912	0.007360	0.006200	0.002800	0.002280	0.001490	0.001202	0.001244	0.000921
3	0.005734	0.004705	0.004360	0.002090	0.001880	0.001050	0.000960	0.000854	0.000830
4	0.003438	0.003463	0.002840	0.001650	0.001420	0.000940	0.000887	0.000746	0.000714
5	0.002943	0.002920	0.002380	0.001570	0.001300	0.000820	0.000812	0.000727	0.000494
6	0.002575	0.002510	0.002130	0.001460	0.001170	0.000720	0.000665	0.000633	0.000436
7	0.002314	0.002210	0.001920	0.001310	0.001030	0.000630	0.000528	0.000612	0.000368
8	0.002140	0.002000	0.001750	0.001160	0.000810	0.000570	0.000397	0.000458	0.000301
9	0.002038	0.001864	0.001620	0.001030	0.000700	0.000530	0.000278	0.000250	0.000221
10	0.001997	0.001786	0.001540	0.000930	0.000620	0.000510	0.000200	0.000173	0.000192
11	0.002005	0.001758	0.001500	0.000880	0.000570	0.000510	0.000248	0.000214	0.000231
12	0.002056	0.001773	0.001510	0.000920	0.000580	0.000510	0.000397	0.000352	0.000331
13	0.002147	0.001827	0.001570	0.001010	0.000670	0.000540	0.000623	0.000564	0.000480
14	0.002277	0.001920	0.001680	0.001130	0.000860	0.000630	0.000901	0.000829	0.000669
15	0.002446	0.002055	0.001850	0.001280	0.001110	0.000820	0.001207	0.001126	0.000888
16	0.002661	0.002240	0.002070	0.001420	0.001390	0.001100	0.001519	0.001435	0.001126
17	0.002928	0.002483	0.002350	0.001560	0.001650	0.001470	0.001816	0.001736	0.001373
18	0.003264	0.002800	0.002720	0.001720	0.001860	0.001850	0.002080	0.002012	0.001615
19	0.003619	0.003138	0.003110	0.001880	0.001970	0.002200	0.002297	0.002250	0.001842
20	0.003942	0.003445	0.003460	0.002020	0.002010	0.002440	0.002458	0.002439	0.002040
21	0.004183	0.003668	0.003680	0.002100	0.002020	0.002540	0.002559	0.002572	0.002199
22	0.004317	0.003781	0.003750	0.002150	0.002030	0.002530	0.002603	0.002649	0.002310
23	0.004375	0.003819	0.003700	0.002150	0.002030	0.002470	0.002596	0.002674	0.002370
24	0.004396	0.003819	0.003600	0.002130	0.002050	0.002400	0.002552	0.002656	0.002384
25	0.004421	0.003820	0.003500	0.002110	0.002080	0.002350	0.002487	0.002607	0.002360
26	0.004487	0.003859	0.003450	0.002110	0.002120	0.002350	0.002415	0.002539	0.002313
27	0.004587	0.003916	0.003440	0.002140	0.002180	0.002380	0.002350	0.002465	0.002257
28	0.004693	0.003967	0.003440	0.002210	0.002260	0.002430	0.002282	0.002378	0.002179
29	0.004820	0.004040	0.003460	0.002300	0.002380	0.002500	0.002277	0.002332	0.002153
30	0.004981	0.004164	0.003520	0.002410	0.002480	0.002570	0.002316	0.002313	0.002158
31	0.005188	0.004370	0.003660	0.002530	0.002600	0.002660	0.002398	0.002322	0.002189
32	0.005456	0.004700	0.003840	0.002670	0.002720	0.002770	0.002522	0.002358	0.002243

Age	1920–1922	1925–1927	1935–1937	1945–1947	1950–1952	1959–1961	1969–1971	1979–1981	1984–1986
33	0.005778	0.005135	0.004100	0.002820	0.002850	0.002890	0.002685	0.002419	0.002315
34	0.006131	0.005613	0.004410	0.002990	0.002980	0.003050	0.002889	0.002507	0.002401
35	0.006495	0.006071	0.004720	0.003180	0.003110	0.003250	0.003133	0.002624	0.002504
36	0.006849	0.006444	0.005010	0.003390	0.003260	0.003450	0.003422	0.002777	0.002628
37	0.007173	0.006688	0.005260	0.003630	0.003440	0.003690	0.003758	0.002972	0.002776
38	0.007481	0.006845	0.005470	0.003900	0.003660	0.003970	0.004144	0.003213	0.002951
39	0.007802	0.006980	0.005700	0.004200	0.003930	0.004280	0.004584	0.003505	0.003156
40	0.008166	0.007159	0.006000	0.004560	0.004270	0.004620	0.005067	0.003848	0.003395
41	0.008601	0.007448	0.006430	0.004960	0.004660	0.005000	0.005584	0.004241	0.003669
42	0.009141	0.007871	0.007010	0.005420	0.005120	0.005410	0.006124	0.004684	0.003981
43	0.009768	0.008387	0.007710	0.005940	0.005640	0.005860	0.006678	0.005176	0.004334
44	0.010434	0.008959	0.008490	0.006530	0.006230	0.006350	0.007244	0.005718	0.004730
45	0.011090	0.009552	0.009300	0.007190	0.006880	0.006900	0.007834	0.006315	0.005176
46	0.011688	0.010130	0.010090	0.007910	0.007590	0.007510	0.008470	0.006977	0.005679
47	0.012172	0.010652	0.010820	0.008720	0.008360	0.008180	0.009175	0.007710	0.006247
48	0.012573	0.011144	0.011540	0.009590	0.009160	0.008920	0.009966	0.008524	0.006887
49	0.012973	0.011662	0.012270	0.010530	0.010010	0.009740	0.010869	0.009423	0.007606
50	0.013454	0.012266	0.013080	0.011520	0.010920	0.010640	0.011868	0.010403	0.008411
51	0.014100	0.013016	0.013990	0.012560	0.011900	0.011650	0.012959	0.011458	0.009310
52	0.014914	0.013929	0.015020	0.013640	0.012980	0.012770	0.014130	0.012581	0.010309
53	0.015841	0.014967	0.016160	0.014780	0.014170	0.014030	0.015368	0.013766	0.011414
54	0.016887	0.016114	0.017370	0.015990	0.015480	0.015460	0.016682	0.015011	0.012632
55	0.018055	0.017351	0.018640	0.017280	0.016920	0.017070	0.018062	0.016340	0.013965
56	0.019350	0.018660	0.019960	0.018670	0.018490	0.018880	0.019604	0.017784	0.015417
57	0.020797	0.020034	0.021270	0.020170	0.020170	0.020840	0.021277	0.019372	0.016990
58	0.022401	0.021490	0.022590	0.021790	0.021970	0.022900	0.023126	0.021132	0.018684
59	0.024134	0.023044	0.023990	0.023530	0.023890	0.025010	0.025191	0.023083	0.020505
60	0.025965	0.024710	0.025560	0.025410	0.025890	0.027110	0.027445	0.025193	0.022459
61	0.027860	0.026507	0.027390	0.027420	0.027950	0.029170	0.029878	0.027418	0.024554
62	0.029703	0.028386	0.029520	0.029570	0.030050	0.031260	0.032474	0.029714	0.026798
63	0.031515	0.030341	0.031890	0.031860	0.032160	0.033480	0.035206	0.032037	0.029198
64	0.033478	0.032457	0.034490	0.034260	0.034260	0.035930	0.038098	0.034367	0.031762
65	0.035788	0.034824	0.037290	0.036770	0.036430	0.038690	0.041162	0.036782	0.034489
66	0.038653	0.037537	0.040280	0.039380	0.038720	0.041840	0.044460	0.039385	0.037376
67	0.042276	0.040566	0.043340	0.042090	0.041230	0.045330	0.048044	0.042279	0.040422
68	0.046578	0.043868	0.046510	0.044990	0.044020	0.049100	0.051953	0.045566	0.043625
69	0.051343	0.047542	0.049950	0.048170	0.047150	0.053060	0.056256	0.049314	0.046999

Age	1920–1922	1925–1927	1935–1937	1945–1947	1950–1952	1959–1961	1969–1971	1979–1981	1984–1986
70	0.056328	0.051694	0.053870	0.051740	0.050630	0.057160	0.060860	0.053459	0.050625
71	0.061254	0.056446	0.058450	0.055810	0.054450	0.061330	0.065699	0.057899	0.054603
72	0.065934	0.062061	0.063970	0.060440	0.058600	0.065610	0.070696	0.062537	0.059031
73	0.070542	0.068558	0.070400	0.065590	0.063060	0.070030	0.075756	0.067272	0.064006
74	0.075336	0.075677	0.077470	0.071200	0.067850	0.074640	0.080899	0.072057	0.069565
75	0.080607	0.083100	0.084880	0.077190	0.073120	0.079480	0.086210	0.077052	0.075486
76	0.086680	0.090491	0.092260	0.083480	0.079040	0.084610	0.091892	0.082469	0.081489
77	0.093779	0.097454	0.099190	0.090070	0.085760	0.090190	0.098126	0.088521	0.087289
78	0.101793	0.104203	0.105870	0.097200	0.093470	0.096390	0.105072	0.095418	0.092603
79	0.110565	0.111236	0.112990	0.105170	0.102250	0.103390	0.112924	0.103327	0.097338
80	0.119900	0.119125	0.120950	0.114300	0.111900	0.111340	0.121460	0.112217	0.102145
81	0.129567	0.128527	0.130840	0.124870	0.122120	0.120400	0.130498	0.122091	0.107868
82	0.139641	0.140290	0.143170	0.137050	0.132650	0.130520	0.139828	0.132952	0.115351
83	0.150336	0.154302	0.157740	0.150390	0.143190	0.141650	0.148692	0.144802	0.127648
84	0.161586	0.169733	0.174230	0.164280	0.153610	0.153710	0.158150	0.157643	0.140499
85	0.173309	0.185526	0.192200	0.178110	0.164360	0.166640	0.167798	0.171478	0.155581
86	0.185404	0.200358	0.211070	0.191300	0.176030	0.180390	0.177725	0.186309	0.172552
87	0.197188	0.214000	0.231070	0.207280	0.189240	0.194950	0.188085	0.202140	0.190823
88	0.208664	0.227147	0.252820	0.224290	0.204570	0.210340	0.199081	0.218972	0.209632
89	0.220872	0.239675	0.276130	0.242340	0.222510	0.226580	0.210933	0.236809	0.228143

APPENDIX B
South African Life Tables for White Females $q(x)$

Age	1920–1922	1925–1927	1935–1937	1945–1947	1950–1952	1959–1961	1969–1971	1979–1981	1984–1986
0	0.073884	0.062761	0.053480	0.032910	0.030050	0.024930	0.018855	0.012352	0.007434
1	0.020757	0.018381	0.014020	0.004830	0.003970	0.002640	0.001779	0.001631	0.001311
2	0.009150	0.007700	0.005550	0.002810	0.001640	0.001190	0.000761	0.000867	0.000686
3	0.005347	0.004139	0.003520	0.002250	0.001280	0.000960	0.000679	0.000629	0.000579
4	0.004251	0.003435	0.002920	0.001340	0.001180	0.000870	0.000690	0.000447	0.000442
5	0.003035	0.002368	0.002190	0.001250	0.001130	0.000700	0.000514	0.000443	0.000372
6	0.002262	0.002026	0.002010	0.001140	0.000990	0.000590	0.000462	0.000411	0.000283
7	0.001823	0.001776	0.001850	0.001030	0.000860	0.000510	0.000410	0.000307	0.000273
8	0.001623	0.001609	0.001690	0.000920	0.000720	0.000470	0.000334	0.000271	0.000236
9	0.001583	0.001514	0.001570	0.000830	0.000640	0.000440	0.000241	0.000266	0.000199
10	0.001640	0.001481	0.001470	0.000760	0.000600	0.000410	0.000195	0.000254	0.000188
11	0.001743	0.001501	0.001400	0.000730	0.000580	0.000390	0.000197	0.000262	0.000200
12	0.001860	0.001564	0.001360	0.000750	0.000580	0.000380	0.000236	0.000292	0.000230
13	0.001973	0.001660	0.001350	0.000820	0.000590	0.000380	0.000302	0.000338	0.000273
14	0.002077	0.001778	0.001370	0.000910	0.000640	0.000400	0.000385	0.000396	0.000326
15	0.002186	0.001911	0.001430	0.001000	0.000710	0.000430	0.000475	0.000460	0.000383
16	0.002326	0.002046	0.001540	0.001080	0.000780	0.000470	0.000562	0.000525	0.000440
17	0.002540	0.002176	0.001690	0.001130	0.000850	0.000530	0.000641	0.000589	0.000495
18	0.002801	0.002314	0.001890	0.001160	0.000940	0.000600	0.000705	0.000646	0.000544
19	0.003078	0.002456	0.002110	0.001200	0.000970	0.000670	0.000753	0.000696	0.000586
20	0.003341	0.002599	0.002330	0.001250	0.001010	0.000730	0.000787	0.000737	0.000621
21	0.003561	0.002737	0.002500	0.001330	0.001050	0.000790	0.000809	0.000769	0.000649
22	0.003722	0.002868	0.002630	0.001430	0.001090	0.000840	0.000826	0.000793	0.000672
23	0.003842	0.002996	0.002720	0.001540	0.001130	0.000890	0.000843	0.000811	0.000692
24	0.003946	0.003123	0.002800	0.001660	0.001170	0.000930	0.000866	0.000825	0.000711
25	0.004058	0.003253	0.002870	0.001770	0.001210	0.000970	0.000897	0.000838	0.000730
26	0.004200	0.003388	0.002950	0.001860	0.001260	0.001010	0.000937	0.000851	0.000751
27	0.004391	0.003531	0.003010	0.001920	0.001320	0.001040	0.000983	0.000866	0.000774
28	0.004616	0.003679	0.003050	0.001960	0.001390	0.001070	0.001039	0.000885	0.000800
29	0.004847	0.003829	0.003100	0.001990	0.001470	0.001110	0.001095	0.000909	0.000828
30	0.005058	0.003981	0.003170	0.002040	0.001560	0.001170	0.001153	0.000941	0.000859
31	0.005221	0.004133	0.003280	0.002110	0.001670	0.001230	0.001215	0.000982	0.000896
32	0.005315	0.004286	0.003460	0.002210	0.001790	0.001320	0.001280	0.001033	0.000939

Age	1920–1922	1925–1927	1935–1937	1945–1947	1950–1952	1959–1961	1969–1971	1979–1981	1984–1986
33	0.005358	0.004441	0.003680	0.002340	0.001910	0.001420	0.001349	0.001095	0.000992
34	0.005382	0.004596	0.003930	0.002500	0.002040	0.001530	0.001427	0.001172	0.001054
35	0.005420	0.004748	0.004180	0.002680	0.002180	0.001660	0.001520	0.001265	0.001128
36	0.005506	0.004897	0.004400	0.002860	0.002320	0.001810	0.001640	0.001376	0.001215
37	0.005641	0.005034	0.004560	0.003060	0.002470	0.001980	0.001797	0.001508	0.001316
38	0.005803	0.005161	0.004690	0.003260	0.002620	0.002170	0.002002	0.001664	0.001433
39	0.005990	0.005289	0.004820	0.003480	0.002890	0.002370	0.002263	0.001846	0.001567
40	0.006201	0.005427	0.004980	0.003730	0.002970	0.002590	0.002564	0.002053	0.001723
41	0.006431	0.005585	0.005210	0.004000	0.003170	0.002830	0.002889	0.002284	0.001904
42	0.006682	0.005744	0.005520	0.004320	0.003420	0.003090	0.003219	0.002538	0.002116
43	0.006955	0.005895	0.005880	0.004670	0.003720	0.003370	0.003537	0.002814	0.002363
44	0.007251	0.006071	0.006280	0.005060	0.004070	0.003690	0.003834	0.003110	0.002647
45	0.007571	0.006301	0.006710	0.005480	0.004480	0.004030	0.004126	0.003428	0.002962
46	0.007914	0.006617	0.007170	0.005930	0.004920	0.004410	0.004439	0.003766	0.003303
47	0.008261	0.007019	0.007640	0.006420	0.005400	0.004820	0.004800	0.004127	0.003660
48	0.008611	0.007489	0.008140	0.006940	0.005900	0.005250	0.005232	0.004510	0.004027
49	0.008995	0.008026	0.008680	0.007480	0.006410	0.005690	0.005757	0.004916	0.004399
50	0.009449	0.008630	0.009240	0.008050	0.006930	0.006130	0.006351	0.005347	0.004783
51	0.010005	0.009303	0.009840	0.008640	0.007460	0.006580	0.006989	0.005809	0.005189
52	0.010670	0.010065	0.010450	0.009260	0.008010	0.007040	0.007643	0.006303	0.005626
53	0.011423	0.010920	0.011070	0.009900	0.008580	0.007550	0.008284	0.006834	0.006105
54	0.012260	0.011837	0.011730	0.010570	0.009170	0.008120	0.008900	0.007406	0.006637
55	0.013174	0.012787	0.012490	0.011270	0.009800	0.008780	0.009510	0.008037	0.007232
56	0.014160	0.013738	0.013370	0.012010	0.010500	0.009550	0.010152	0.008746	0.007904
57	0.015165	0.014584	0.014360	0.012790	0.011290	0.010440	0.010861	0.009551	0.008666
58	0.016195	0.015344	0.015420	0.013640	0.012190	0.011440	0.011669	0.010471	0.009531
59	0.017333	0.016177	0.016610	0.014590	0.013220	0.012560	0.012616	0.011520	0.010505
60	0.018664	0.017250	0.017980	0.015680	0.014380	0.013790	0.013704	0.012685	0.011578
61	0.020281	0.018735	0.019580	0.016920	0.015670	0.015160	0.014940	0.013952	0.012732
62	0.022232	0.020757	0.021470	0.018360	0.017080	0.016650	0.016333	0.015301	0.013951
63	0.024476	0.023229	0.023630	0.019980	0.018620	0.018260	0.017881	0.016717	0.015218
64	0.026966	0.026003	0.026000	0.021770	0.020280	0.020000	0.019608	0.018192	0.016522
65	0.029653	0.028921	0.028520	0.023720	0.022120	0.021880	0.021513	0.019754	0.017880
66	0.032485	0.031808	0.031130	0.025820	0.024150	0.023890	0.023616	0.021440	0.019313
67	0.035458	0.034480	0.033730	0.028080	0.026440	0.026070	0.025932	0.023287	0.020843
68	0.038623	0.037048	0.036370	0.030570	0.029020	0.028440	0.028469	0.025332	0.022493
69	0.042006	0.039782	0.039200	0.033390	0.031920	0.031030	0.031269	0.027603	0.024296

Age	1920–1922	1925–1927	1935–1937	1945–1947	1950–1952	1959–1961	1969–1971	1979–1981	1984–1986
70	0.045639	0.042973	0.042410	0.036630	0.035180	0.033880	0.034328	0.030083	0.026323
71	0.049554	0.046941	0.046180	0.040390	0.038790	0.037030	0.037679	0.032745	0.028659
72	0.053516	0.052020	0.050600	0.044740	0.042780	0.040540	0.041345	0.035563	0.031388
73	0.057512	0.058108	0.055570	0.049560	0.047150	0.044520	0.045336	0.038509	0.034590
74	0.061936	0.064864	0.061120	0.054730	0.051920	0.049040	0.049721	0.041607	0.038317
75	0.067230	0.071889	0.067250	0.060110	0.057140	0.054190	0.054515	0.045070	0.042479
76	0.073899	0.078710	0.073950	0.065560	0.062850	0.060040	0.059791	0.049164	0.046954
77	0.082968	0.084798	0.081370	0.071030	0.069100	0.066540	0.065609	0.054154	0.051618
78	0.094460	0.090404	0.089610	0.076770	0.075920	0.073640	0.072008	0.060303	0.056348
79	0.107274	0.096236	0.098600	0.083080	0.083360	0.081270	0.079091	0.067772	0.061122
80	0.119987	0.103093	0.108290	0.090280	0.091420	0.089380	0.086765	0.076305	0.066309
81	0.130816	0.111894	0.118570	0.098710	0.100080	0.097940	0.094991	0.085921	0.072383
82	0.138029	0.123765	0.129550	0.108570	0.109330	0.106980	0.103724	0.096640	0.079814
83	0.142420	0.138554	0.141390	0.119690	0.119170	0.116590	0.112571	0.108479	0.090151
84	0.145961	0.155205	0.154080	0.131770	0.129600	0.126840	0.122252	0.121460	0.101378
85	0.150788	0.172325	0.167580	0.144530	0.140690	0.137810	0.132580	0.135600	0.114367
86	0.159223	0.188116	0.181850	0.157700	0.152530	0.149550	0.143678	0.150920	0.129009
87	0.173835	0.201892	0.197000	0.172400	0.165200	0.162140	0.155707	0.167438	0.145073
88	0.193920	0.214544	0.213160	0.188230	0.178800	0.175610	0.168848	0.185174	0.162243
89	0.216674	0.226419	0.230310	0.205290	0.193400	0.190010	0.183293	0.204146	0.180166

APPENDIX C
Fitted values of *a*(*x*) and *b*(*x*) for 1921–1985

Age	*a*(*x*) male	*b*(*x*) male	*a*(*x*) female	*b*(*x*) female
0	–3.313330	0.032923	–3.557514	0.019523
1	–5.254911	0.048935	–5.365562	0.028311
2	–5.952547	0.040515	–6.141921	0.026680
3	–6.262072	0.036563	–6.465115	0.022728
4	–6.511167	0.030313	–6.675890	0.022343
5	–6.651362	0.030549	–6.876618	0.020346
6	–6.781020	0.030803	–7.030478	0.020086
7	–6.907900	0.031397	–7.171393	0.020005
8	–7.075720	0.034489	–7.299502	0.020501
9	–7.276038	0.040849	–7.406559	0.021661
10	–7.411044	0.044959	–7.472180	0.022575
11	–7.362789	0.041217	–7.471551	0.022450
12	–7.204017	0.033843	–7.415038	0.021472
13	–7.014943	0.026802	–7.328003	0.020164
14	–6.817240	0.020948	–7.222523	0.018769
15	–6.621371	0.016345	–7.116249	0.017591
16	–6.441872	0.012997	–7.015516	0.016765
17	–6.280785	0.010855	–6.920424	0.016264
18	–6.139748	0.010012	–6.830878	0.016094
19	–6.027024	0.009860	–6.758064	0.016186
20	–5.945654	0.010044	–6.695439	0.016409
21	–5.897092	0.010201	–6.642032	0.016612
22	–5.874755	0.010201	–6.597659	0.016780
23	–5.872732	0.010150	–6.558800	0.016882
24	–5.880367	0.010180	–6.522964	0.016976
25	–5.890704	0.010409	–6.488441	0.017039
26	–5.895848	0.010940	–6.452838	0.017103
27	–5.895011	0.011629	–6.418410	0.017167
28	–5.892367	0.012437	–6.383425	0.017180
29	–5.877115	0.012915	–6.347018	0.017156
30	–5.854026	0.013356	–6.306581	0.017062
31	–5.819363	0.013853	–6.263042	0.016909
32	–5.774770	0.014438	–6.213687	0.016671

Age	$a(x)$ male	$b(x)$ male	$a(x)$ female	$b(x)$ female
33	–5.721920	0.015118	–6.161734	0.016354
34	–5.663658	0.015691	–6.106666	0.015974
35	–5.603449	0.015983	–6.048159	0.015499
36	–5.543733	0.015916	–5.987419	0.014923
37	–5.483866	0.015400	–5.923896	0.014230
38	–5.422838	0.014542	–5.858577	0.013429
39	–5.358445	0.013540	–5.786195	0.012556
40	–5.288251	0.012578	–5.718931	0.011715
41	–5.212137	0.011820	–5.645498	0.010913
42	–5.129944	0.011327	–5.569356	0.010168
43	–5.044737	0.010986	–5.493042	0.009466
44	–4.958759	0.010670	–5.416009	0.008816
45	–4.873710	0.010257	–5.338799	0.008241
46	–4.790988	0.009650	–5.260971	0.007737
47	–4.711154	0.008784	–5.183101	0.007281
48	–4.633511	0.007749	–5.105997	0.006864
49	–4.555958	0.006656	–5.029578	0.006501
50	–4.477354	0.005647	–4.954164	0.006203
51	–4.396595	0.004811	–4.879404	0.005980
52	–4.314076	0.004151	–4.805415	0.005826
53	–4.230917	0.003593	–4.732230	0.005710
54	–4.147604	0.003094	–4.659696	0.005614
55	–4.064511	0.002634	–4.586609	0.005515
56	–3.981243	0.002168	–4.512068	0.005383
57	–3.898856	0.001704	–4.436986	0.005155
58	–3.817216	0.001248	–4.361072	0.004835
59	–3.736291	0.000807	–4.281911	0.004504
60	–3.656594	0.000420	–4.198714	0.004248
61	–3.578383	0.000106	–4.110892	0.004127
62	–3.502201	–0.000180	–4.018871	0.004171
63	–3.428168	–0.000448	–3.925343	0.004319
64	–3.355161	–0.000645	–3.832529	0.004504
65	–3.281736	–0.000742	–3.741435	0.004666
66	–3.206629	–0.000721	–3.652906	0.004762
67	–3.129836	–0.000606	–3.566980	0.004760
68	–3.051722	–0.000449	–3.482226	0.004695
69	–2.972348	–0.000280	–3.396680	0.004609

Age	$a(x)$ male	$b(x)$ male	$a(x)$ female	$b(x)$ female
70	–2.892141	–0.000099	–3.308913	0.004543
71	–2.811588	0.000092	–3.217933	0.004526
72	–2.730447	0.000319	–3.123924	0.004545
73	–2.649253	0.000592	–3.028542	0.004560
74	–2.568888	0.000873	–2.932344	0.004565
75	–2.489859	0.001141	–2.835740	0.004561
76	–2.412309	0.001373	–2.738944	0.004545
77	–2.336391	0.001536	–2.641425	0.004555
78	–2.260954	0.001655	–2.543098	0.004589
79	–2.184279	0.001781	–2.444760	0.004606
80	–2.105427	0.001917	–2.347224	0.004578
81	–2.022847	0.002083	–2.250502	0.004460
82	–1.935791	0.002297	–2.154397	0.004229
83	–1.844004	0.002374	–2.057525	0.003858
84	–1.753378	0.002508	–1.962312	0.003480
85	–1.664216	0.002537	–1.867709	0.003098
86	–1.578021	0.002419	–1.773470	0.002756
87	–1.492782	0.002178	–1.677445	0.002531
88	–1.408846	0.001892	–1.580593	0.002405
89	–1.325803	0.001626	–1.485034	0.002294

APPENDIX D
Fitted values of $q(x)$ for males and females in 2020

x	$q(x)$ male	$q(x)$ female	x	$q(x)$ male	$q(x)$ female
0	0.003778	0.002485	33	0.001440	0.000395
1	0.000340	0.000267	34	0.001467	0.000429
2	0.000258	0.000153	35	0.001516	0.000472
3	0.000263	0.000161	36	0.001594	0.000525
4	0.000275	0.000126	37	0.001712	0.000591
5	0.000189	0.000118	38	0.001869	0.000673
6	0.000166	0.000091	39	0.002063	0.000774
7	0.000137	0.000089	40	0.002287	0.000892
8	0.000102	0.000074	41	0.002532	0.001031
9	0.000061	0.000059	42	0.002790	0.001195
10	0.000047	0.000053	43	0.003070	0.001388
11	0.000063	0.000057	44	0.003384	0.001613
12	0.000114	0.000069	45	0.003752	0.001864
13	0.000207	0.000088	46	0.004196	0.002139
14	0.000346	0.000113	47	0.004743	0.002431
15	0.000531	0.000142	48	0.005402	0.002739
16	0.000748	0.000171	49	0.006174	0.003054
17	0.000976	0.000198	50	0.007048	0.003376
18	0.001179	0.000220	51	0.008008	0.003709
19	0.001351	0.000236	52	0.009054	0.004057
20	0.001488	0.000247	53	0.010201	0.004432
21	0.001596	0.000255	54	0.011468	0.004844
22	0.001677	0.000262	55	0.012862	0.005308
23	0.001723	0.000268	56	0.014408	0.005845
24	0.001732	0.000274	57	0.016111	0.006492
25	0.001702	0.000280	58	0.017972	0.007269
26	0.001640	0.000287	59	0.019996	0.008163
27	0.001566	0.000295	60	0.022168	0.009128
28	0.001474	0.000304	61	0.024473	0.010107
29	0.001435	0.000315	62	0.026948	0.011049
30	0.001419	0.000329	63	0.029606	0.011955
31	0.001417	0.000346	64	0.032402	0.012848
32	0.001425	0.000368	65	0.035288	0.013781

x	$q(x)$ male	$q(x)$ female
66	0.038217	0.014808
67	0.041183	0.015986
68	0.044231	0.017318
69	0.047405	0.018800
70	0.050778	0.020447
71	0.054451	0.022289
72	0.058460	0.024393
73	0.062863	0.026869
74	0.067746	0.029769
75	0.072926	0.033026
76	0.078184	0.036556
77	0.083350	0.040189
78	0.088120	0.043812
79	0.092291	0.047506
80	0.096470	0.051649
81	0.101391	0.056788
82	0.107766	0.063466
83	0.119037	0.073223
84	0.130577	0.084134
85	0.144563	0.096996
86	0.160996	0.111559
87	0.179406	0.127100
88	0.198821	0.143264
89	0.218115	0.160209

