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Mortality risks, reinsurance and risk-based supervision

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ABSTRACT
Under risk-based supervision, mortality risks are generally considered proportional to the number of insured lives (N). This assumption is, however, incorrect for volatility mortality risks (this being the key justification for life insurance), as this risk is proportional to $\sqrt{N}$. The main benefits of reinsurance are consequently not properly reflected in the risk-based capital requirements under risk-based supervision Pillar 1. Similar findings apply to unexpired risks, also called 'premium risks', in non-life insurance. In this article, volatility risks shall therefore be thoroughly considered in the formulation and assessment of the insurer's reinsurance policy, i.e., under risk-based supervision Pillar 2.

KEYWORDS
Risk-based supervision; mortality risks; volatility; minimum capital requirements; normal power approximation; reinsurance

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1. INTRODUCTION

1.1 Many governments around the world have expressed their intention to adopt the Insurance Core Principles (ICPs) of the International Association of Insurance Supervisors (IAIS)—see IAIS (2018) for their latest version. However, only a few of these, excluding the member countries of the European Union, have already finalised and implemented the necessary changes to their national insurance law(s) and regulations. Adjusting these regulations presents a major challenge, as adopting the ICPs implies adopting the principles of risk-based supervision (RBS), including the development and implementation of a model that will allow for the calculation of RBC requirements.

1.2 The key principles of such a risk-based capital (RBC) model are reflected in the Solvency II requirements that were introduced in the European Union with effect from 1 January 2016 (European Commission, 2009, 2015). Simultaneous to the development of Solvency II by the European Insurance and Occupational Pensions Authority, but lagging a few years behind, the IAIS started developing 'ComFrame'. This includes International Capital Standard Version 1.0 for Extended Field Testing (ICS Version 1.0), which was released recently (IAIS, 2017). It is expected that ICS Version 2.0, when supported by the local insurance supervisors, will be prescribed for all 'Internationally Active Insurance Groups' (as defined by the IAIS) from 2020 or 2021 onwards.

1.3 Any RBC model needs to allow for the calculation of proper capital requirements for the key quantifiable risks that a (re)insurance company is exposed to. Furthermore, any RBC model should provide incentives for proper risk management by generating lower capital requirements as a result of having this in place. For insurance, it is important to reward the company in this way for having a proper reinsurance policy.

1.4 This article considers mortality risks in life insurance in more detail. We consider the way in which these risks can be mitigated by means of reinsurance and, most importantly, how they are generally dealt with in the common RBC models (including Solvency II). Similar conclusions can be applied to premium risks as part of non-life underwriting risks (however their distributions are more complex).

1.5 Section 2 briefly discusses the different sub-risks of mortality risks, while Sections 3 to 5 discuss how these risks are/can be modelled in a RBC model, how they depend on the size of the insurance portfolio (number of insured lives/policies), how their levels compare with each other as well as with the minimum capital requirement, and how (quota share) reinsurance would result in a lower capital requirement. Sections 6 and 7 illustrate and further discuss our findings by applying the formulae to a fictitious portfolio of one-year renewable micro-term assurance policies with a variable size and reinsurance retention level. We conclude with some remarks about other types of underwriting risks (Section 8) and some key conclusions (Section 9).

2. THE DIFFERENT TYPES OF MORTALITY RISKS

2.1 IAA's Global Framework for Insurer Solvency Assessment (IAA, 2004) distinguishes the following four types of mortality sub-risks:
— level uncertainty,
— trend uncertainty,
— volatility risk, and
— catastrophe risk.

Sub-risks 1 and 2 are generally combined as both refer to uncertainty about the expected future mortality, assuming mutual independence of the mortality of the individual insured lives.

2.2 The common way to calculate a capital requirement for level/trend mortality risks is to stress the best estimate mortality rates used in the valuation of the (life) insurance liabilities corresponding to an agreed level of risk-aversion like a '1-in-200-years event', and to define the capital requirement equal to its effect on the amount of net assets in the economic balance sheet (EBS). However, in the European Union, Solvency II also allows for an alternative, simplified approach that comprises multiplying the technical provision in the International Financial Reporting Standards (IFRS) financial statements by a certain factor that has to be calculated on the basis of certain portfolio-specific characteristics; see European Commission (2015, article 91). Either way, the correct method and the simplified approach both result in a capital requirement that is proportional to the size of the portfolio.

2.3 Volatility risk refers to uncertainty of actual mortality around the expected mortality (when given), again assuming mutual independence. Catastrophic risk also considers uncertainty of expected future mortality, but assumes full correlation between the mortality of individuals where this may be caused by, for example, an epidemic which would affect multiple individuals at the same time.

2.4 Sub-risks 1 and 2 are generally covered jointly by the capital requirement for mortality risk as included in Solvency II and several other (draft) RBC frameworks. Sub-risk 4 is often covered separately, but sub-risk 3 is generally fully ignored under RBS.

2.5 We consider the latter rather surprising. Volatility mortality risk is the key type of life insurance (underwriting) risk that forms the motivation and justification for life insurance, as this is the only sub-risk which is subject to the law of large numbers and therefore coverable by a (re)insurer who is able to pool many similar policies with this type of risk. A possible reason for the ignorance of volatility risks in RBC frameworks is the difficulty in distinguishing volatility risk, which is only incidental per year, from the more structural level/trend risk which affects mortality in the longer run. Yet, particularly for smaller portfolios, volatility risks can be significant, and should therefore not be ignored. This will be discussed further in the following paragraphs.

2.6 Literature available on the different types of mortality sub-risks is limited. The Dutch insurance supervisor (de Nederlandsche Bank) originally proposed that insurance companies should also recognise volatility mortality risks, but they have subsequently dropped this proposal in order to align with Solvency II. Interestingly, Dutch pension funds are required to consider these risks in the valuation of their pension liabilities. We refer to Van Broekhoven (2002, 2012) for a further discussion of the different mortality sub-risks. However, this literature does not discuss the relative sizes of level/trend and volatility risks for a given portfolio on N life insurance policies. This is the key subject of the next paragraphs.

3. NOTATION

3.1 We focus on the distribution of the total amount of death benefits that must potentially be paid in the first year after the valuation date, considering a portfolio of N mutually independent insured lives. To simplify the notation, we refer to average mortality rates and sums assured as defined below:

q_x = average annual mortality rate of the individual policyholders;
SA = average sum assured (death benefit) and
B = uncertain amount of death benefits in the next year, net of quota share reinsurance with a retention rate of $100f\%$, with

$$E(\underline{B}) = q_x \cdot SA \cdot N \cdot f \text{ and } \sigma(\underline{B}) = \sqrt{\{q_x \cdot (1-q_x)\}} \cdot SA \cdot \sqrt{N} \cdot f.$$

3.2 Furthermore, we define:

— $RC^{mort}{}_{pp}$ = the average capital requirement for level/trend mortality risk per 1 monetary value of SA; we assume that this amount follows from the RBC framework, and
— RC^{mort} = $\mathrm{RC}^{mort}{}_{pp} \cdot N \cdot SA \cdot f$ = the total capital requirement for level/trend mortality risk.

3.3 We refrain from using actual mortality rates $q_x^{(i)}$ and actual sums assured $SA^{(i)}$ ($i = 1, \ldots, N$), because that would make the formulae unnecessarily complex. However, recognising variability in mortality rates and sums assured would basically increase $\sigma(\underline{B})$ and therefore volatility risk; hence, this would make the key message of this article even more relevant.

3.4 Other types of reinsurance, in particular excess-of-loss and surplus reinsurance, are ignored because they are less relevant for smaller portfolios. Instead, these types primarily serve for reducing volatility due to deaths of individual policyholders with higher sums assured and higher death benefits at portfolio level caused by events affecting more policyholders at the same time.

4. ASSUMING A NORMAL DISTRIBUTION FOR FUTURE DEATH BENEFITS

4.1 Now, let us first assume that $\underline{B}$ is N[E($\underline{B}$), $\sigma(\underline{B})$] distributed, with N[•] being the normal distribution (in ¶4.2 we make an alternative assumption more suitable for smaller portfolios).

4.2 As mentioned before, it is generally difficult to assess whether an extreme observed amount of death benefits is due to an error made in the choice for the expected level of the mortality rate q_x or due to regular random volatility around this level. Here we suppose that we are very certain of q_x, meaning that an extreme outcome is a consequence of regular volatility. This raises the question: what is the minimum number of insured lives, N, required in order to ensure the (fixed) capital requirement RC^{mort} (which is now redundant for covering level/trend uncertainty) is sufficient to cover this volatility risk.

4.3 This minimum number for N follows from requiring $\mathrm{VaR}^{\mathrm{N}}[\alpha] = \mathrm{N}[\alpha] \cdot \sigma(\underline{B}) < RC^{mort}$ with $\mathrm{VaR}^{\mathrm{N}}[\alpha]$ equal to the Value at Risk at confidence level $100\alpha\%$:, i.e.,

$$N > \{\mathrm{N}[\alpha] \,/\, RC^{mort}{}_{pp}\}^2 \cdot q_x \cdot (1-q_x).$$

We call this minimum number $N^{(1,A)}$.

4.4 Volatility risk is relatively high for low numbers of insured lives. Following the formula for $\sigma(\underline{B})$, it is less than proportional to N because it is proportional to $\sqrt{N}$. On the other hand, level/trend mortality risk is fully proportional to N (see the formula for RC^{mort}). Consequently, there is a minimum number of insured lives, $N^{(1,A)}$, for which volatility risk is lower than level/trend risk. This minimum does not depend on the possible level of quota share reinsurance, i.e. the retention rate f, because both $\mathrm{VaR}^{\mathrm{N}}[\alpha]$ and $RC^{mort}{}_{pp}$ net of reinsurance are proportional to f. Moreover, it does not depend on the level of the sum assured (SA).

4.5 When the RBC framework includes a minimum capital requirement RC^{min}, and we ignore the capital requirements for risks other than mortality risks, then the minimum requirement for N would follow from

$$\mathrm{VaR}^{\mathrm{N}}[\alpha] = \mathrm{N}[\alpha] \cdot \sqrt{\{q_x \cdot (1-q_x)\}} \cdot SA \cdot \sqrt{N} \cdot f < \max[RC^{mort}{}_{pp} \cdot SA \cdot N \cdot f\,;\, RC^{min}].$$

Therefore, N should at least be equal to $N^{(1,B)} = RC^{min} \,/\, (RC^{mort}{}_{pp} \cdot SA \cdot f\,)$ when

$$RC^{min} > RC^{mort}{}_{pp} \cdot SA \cdot N \cdot f.$$

4.6 For $N < N^{(1,B)}$ the level/trend mortality risk is fully covered by the minimum capital requirement RC^{min}. However, when $N^{(1,B)} < N < N^{(1,A)}$, RC^{min} is insufficient for covering level/trend mortality risk and the (higher) capital requirement for this sub-risk is insufficient for covering volatility mortality risk. This is the case when $\mathrm{VaR}^{\mathrm{N}}[\alpha] > RC^{min}$, i.e., when

$$N > \{RC^{min} \,/\, [\mathrm{N}[\alpha] \cdot \sqrt{\{q_x \cdot (1-q_x)\}} \cdot SA \cdot f\,]\}^2 = N^{(1,C)}.$$

Note that $N^{(1,C)} <= N^{(1,B)} <= N^{(1,A)}$.

4.7 These findings are equally applicable for multi-year and one-year renewable insurance contracts. The RBC regime accounts for this difference in the way the capital requirement for level/trend mortality risk must be calculated. For volatility mortality risk we focus on the next year only, meaning that it does not matter whether the policy continues after that or not.

4.8 Summary

— When $N < N^{(1,C)}$ there is no problem at all because both level/trend and volatility mortality risk are sufficiently covered by RC^{min}.
— When $N > N^{(1,A)}$ there is no problem either, because level/trend mortality risk is covered by RC^{mort} ($> RC^{min}$) and volatility mortality risk is lower than level/trend mortality risk.
— However, when $N^{(1,C)} < N < N^{(1,B)}$ level/trend mortality risk can still be covered by RC^{min}, but more capital would be required for covering volatility mortality risk.
— Furthermore, when $N^{(1,B)} < N < N^{(1,A)}$ there isn't a problem with level/trend mortality risk because it is covered by RC^{mort}. However, there would still be more capital needed for covering volatility mortality risk.

5. USING A NORMAL POWER APPROXIMATION FOR SMALLER PORTFOLIOS

5.1 It is a well-known fact that the distribution of $\underline{B}$ is poorly approximated by the Normal distribution when the number of insured lives is relatively low. To address this, several alternative distributions, e.g., the shifted Gamma distribution and the Poisson distribution, have been proposed in literature; see Kaas et al. (2008: 33–34). We propose to use the normal power (NP) approximation; here VaR[α] in P($\underline{B}$ – E($\underline{B}$) > VaR[α]) = α is defined as

$$\mathrm{VaR}^{\mathrm{NP}}[\alpha] = \mathrm{VaR}^{\mathrm{N}}[\alpha] + \sigma(\underline{B}) \cdot \gamma(\underline{B}) \cdot (\mathrm{N}[\alpha]^2 - 1)/6 \cdot f.$$

with $\mathrm{VaR}^{\mathrm{N}}[\alpha]$ and $\sigma(\underline{B})$ defined as before and $\gamma(\underline{B})$ equal to the skewness of the distribution.

It can be shown (see Appendix) that the latter parameter $\gamma(\underline{B})$ for the sum of N identically Bernouilli (q_x) distributed variables is equal to

$$\gamma(\underline{B}) = (1-3q_x+2q_x^2) / \{\sqrt{q_x} \cdot (1-q_x) \cdot \sqrt{(1-q_x)} \cdot \sqrt{N}\}.$$

5.2 The NP adjustment of $\mathrm{VaR}^{\mathrm{N}}[\alpha]$, i.e. the term $\sigma(\underline{B}) \cdot \gamma(\underline{B}) \cdot (\mathrm{N}[\alpha]^2 - 1)/6 \cdot f$, is positive when $q_x < 0{,}5$. Hence, $\mathrm{VaR}^{\mathrm{NP}}[\alpha] > \mathrm{VaR}^{\mathrm{N}}[\alpha]$. This is the logical consequence of the fact that the distribution of $\underline{B}$ is skewed to the right for smaller levels of N.

5.3 After inserting the formulas for $\sigma(\underline{B})$ and $\gamma(\underline{B})$ into the formula for $\mathrm{VaR}^{\mathrm{NP}}[\alpha]$ we get

$$\mathrm{VaR}^{\mathrm{NP}}[\alpha] = \mathrm{VaR}^{\mathrm{N}}[\alpha] + SA \cdot (1-3q_x+2q_x^2) / (1-q_x) \cdot (\mathrm{N}[\alpha]^2 - 1)/6 \cdot f.$$

Consequently, $\mathrm{VaR}^{\mathrm{NP}}[\alpha] = RC^{mort} \cdot f$ implies for determining $N^{(2,A)}$ that

$$\mathrm{N}[\alpha] \cdot \sqrt{\{q_x \cdot (1-q_x)\}} \cdot \sqrt{N^{(2,A)}} + (1-3q_x+2q_x^2) / (1-q_x) \cdot (\mathrm{N}[\alpha]^2-1)/6 = RC^{mort}_{pp} \cdot N^{(2,A)}.$$

5.4 This equation, which is a second-order polynomial in $\sqrt{N^{(2,A)}}$, can be solved for $N^{(2,A)}$ by calculating its two null points and ignoring the smallest one that is only due to the use of the NP approximation. Note that $N^{(2,A)}$, like $N^{(1,A)}$, does not depend on the parameters f and SA.

5.5 $N^{(2,B)}$ does not differ from $N^{(1,B)}$ because it is not related to $\mathrm{VaR}^{\mathrm{NP}}[\alpha]$: $N^{(2,B)} = N^{(1,B)}$.

5.6 However, $N^{(2,C)}$ differs from $N^{(1,C)}$ because it is derived from $\mathrm{VaR}^{\mathrm{NP}}[\alpha] = RC^{min}$, i.e., by solving $N^{(2,C)}$ from

$$[\mathrm{N}[\alpha] \cdot \sqrt{\{q_x \cdot (1-q_x)\}} \cdot SA \cdot \sqrt{N^{(2,C)}} + SA \cdot (1-3q_x+2q_x^2) / (1-q_x) \cdot (\mathrm{N}[\alpha]^2-1)/6\,] \cdot f = RC^{min}.$$

This will be the case for

$$N^{(2,C)} = \{[RC^{min} - f \cdot SA \cdot (1-3q_x+2q_x^2) / (1-q_x) \cdot (\mathrm{N}[\alpha]^2-1)/6] / [\mathrm{N}[\alpha] \cdot \sqrt{\{q_x \cdot (1-q_x)\}} \cdot SA \cdot f]\}^2.$$

6. EXAMPLE: ONE-YEAR RENEWABLE MICRO-TERM INSURANCE

6.1 It has been suggested in Uganda that 'micro-insurance organisations', defined as insurers that only sell micro-insurance products, do not need reinsurance cover because the sums assured of these policies are relatively low. In particular, it has been suggested that exempting them from the minimum requirements imposed on regular insurers regarding their retrocessions with local reinsurers would stimulate the establishment of such organisations, and, consequently, insurance penetration. This section discusses the fairness of this suggestion by making specific assumptions for the parameters used above, and considering the consequences of these for such a portfolio under different levels of the retention rate f under quota share reinsurance.

6.2 We assume in ¶¶6.6 and 6.7 that:

— α = 0,5%, so N[α] = 2,82,
— q_x = 0,005,
— SA = UGX 5M (=5M Ugandan shillings; UGX 100M equals approximately ZAR 380 000 or US$ 27 500),
— RC^{mort}_{pp} = 10% of 0,005 times UGX 1 = UGX 0,0005, i.e., we assume a 10% level/trend shock for calculating the capital requirement for level/trend mortality risk,
— RC^{mort} = $RC^{mort}_{pp} \cdot SA \cdot N \cdot f$ = UGX 2 500 $\cdot N \cdot f$, and
— $RC^{min\ MR}$ = UGX 100M (for level/trend mortality risk only, with another UGX 100M for other risks, net of diversification and tax benefits, so RC^{min} = UGX 200M).

6.3 The parameters and amounts applied in this section are used to illustrate some key principles regarding mortality risks and reinsurance. More precise calculations can be made by setting the parameters equal to the ones that can be derived from the local regulatory requirements, and by using amounts that are in line with the actual portfolio held.

6.4 For instance, Financial Soundness Standard for Insurers (FSI) 4.2 in South Africa prescribes a 15% shock for mortality rates, instead of 10% mentioned above, for calculating the capital requirement for level/trend mortality risks for regular life insurers. This would result in 50% higher amounts for RC^{mort} and RC^{mort}_{pp}. Furthermore, RC^{min} in South Africa, called the Minimum Capital Requirement (MCR), is not a fixed amount but a percentage of the 'Solvency Capital Requirement' (SCR).

6.5 However, micro-insurers in South Africa, surprisingly, do not need to calculate a SCR that is the aggregate of RBC requirements for individual types of (sub) risks (see Prudential Standard Financial Soundness of Micro-insurers (FSM) 1). We therefore recommend that micro-insurers in South Africa, for this type of analysis, apply the FSI 4.2 requirements as prescribed for regular insurers.

6.6 The two null points of the equation for $N^{(2,A)}$ are now, following the assumptions of ¶6.2, approximately, $N^{(2,A)} = 32$ and $N^{(2,A)} = 160\,000$, but the first null point should be ignored because it is only caused by the NP approximation.

6.7 Furthermore, we find $N^{(2,B)} = N^{(1,B)} = RC^{minMR} / (RC^{mort}_{pp} \cdot SA \cdot f) = 40\,000 / f$. The lower the retention rate f is, the higher $N^{(2,B)}$ is. For $f = 25\%$: $N^{(2,A)} = N^{(2,B)} = 160\,000$.

6.8 Finally, again approximately, $N^{(2,C)} = 10\,110 / f^2 - 1\,160 / f + 33$, whereby the second and third term are a consequence of the NP approximation. It can be shown that $N^{(2,C)} < N^{(2,B)}$ when f, again, is larger than, approximately, 25%.

7. GRAPHICAL PRESENTATIONS

7.1 Figures 1 and 2 below show the locations of $N^{(2,A)}$, $N^{(2,B)}$ and $N^{(2,C)}$ for $f = 100\%$ and $f = 50\%$ respectively, while Figure 3 shows that for $f = 25\%$ these numbers coincide. The approximate numbers are as follows.

TABLE 1. Critical values for portfolio sizes

f	100%	50%	25%
$N^{(2,A)}$	160 000	160 000	160 000
$N^{(2,B)}$	40 000	80 000	160 000
$N^{(2,C)}$	9 000	38 000	160 000

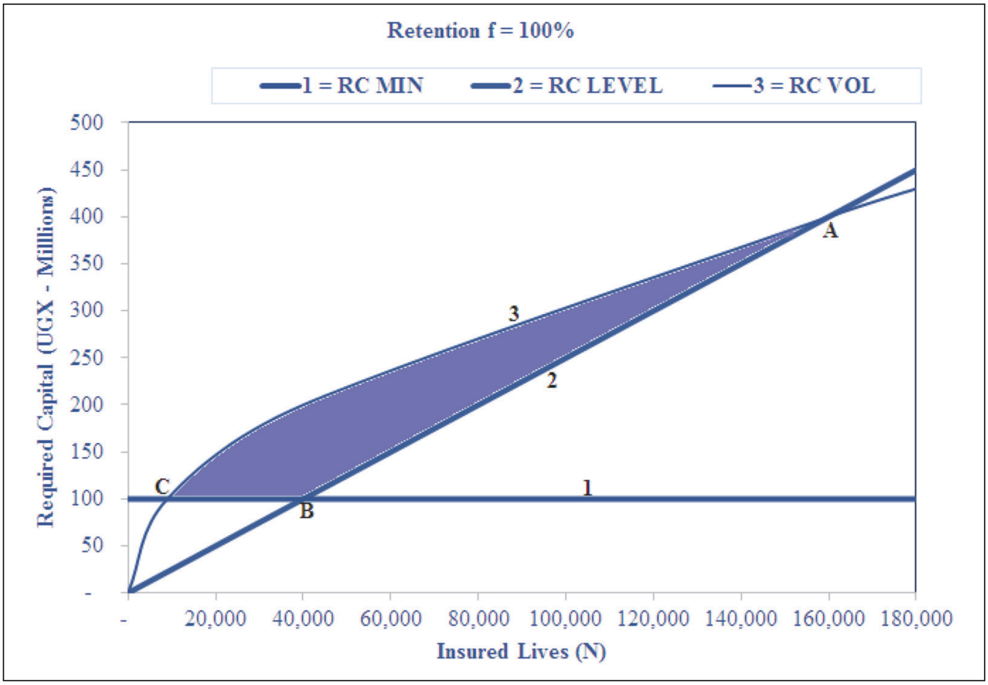

FIGURE 1. Capital requirements for non-catastrophic mortality risks, retention $f = 100\%$

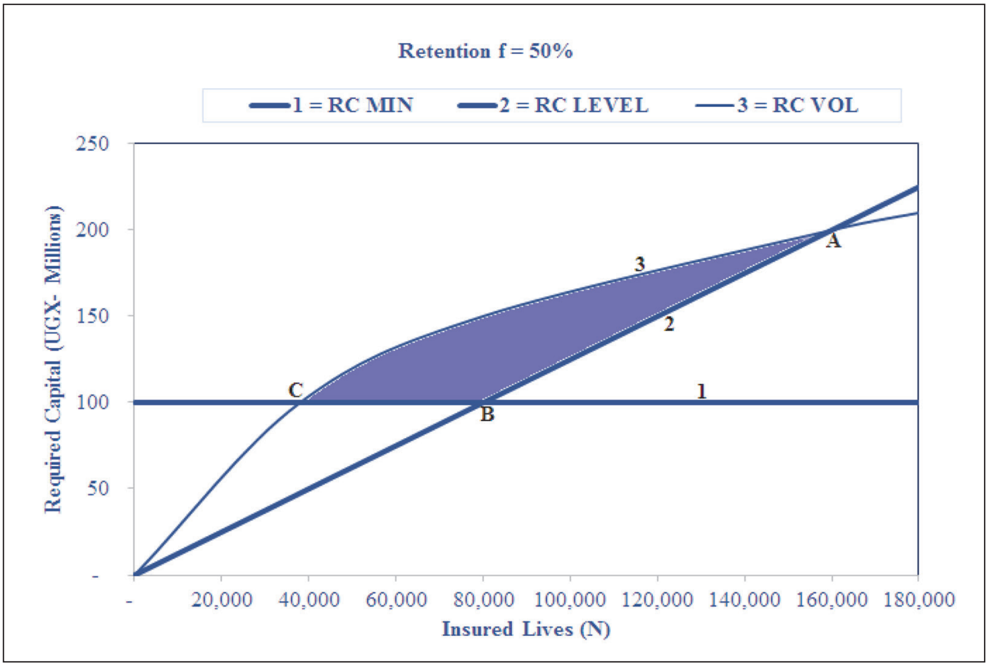

FIGURE 2. Capital requirements for non-catastrophic mortality risks, retention $f = 50\%$

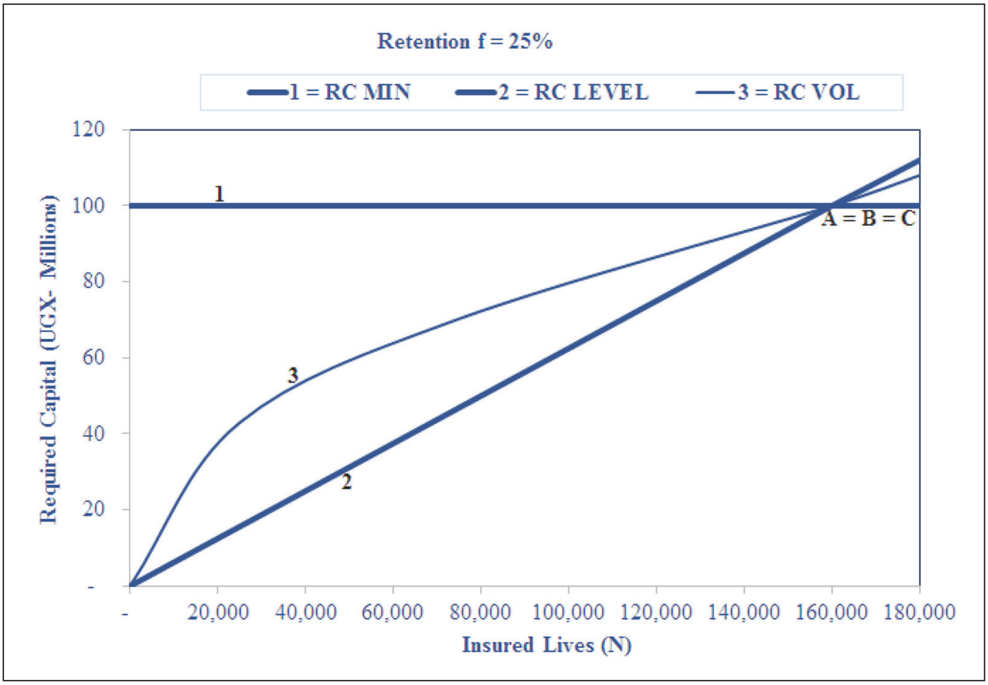

FIGURE 3. Capital requirements for non-catastrophic mortality risks, retention *f* =25%

7.2 The shaded areas in the graphs represent the areas where the volatility mortality risk exceeds the capital that is held, i.e., $RC = \max[RC^{minMR}; RC^{mort}]$. These areas are called the 'danger zones' for the corresponding levels of the retention rate *f*, as, for the corresponding numbers of insured lives, the fact that volatility mortality risks are higher than the capital that has to be held is ignored.

7.3 The numbers for $N^{(2,A)}$ and $N^{(2,C)}$ in Table 1, i.e., the intersections A and C in the graphs, are sensitive to the chosen level of α (while the locations of B and C are sensitive for the level of RC^{minMR}). Increasing α from 0,5% to the more common level of 5% lowers N[α] from 2,82 to 1,96 and therefore lowers $N^{(1,A)}$ by about 50% but, more importantly, increases $N^{(1,C)}$ by about 100%. The effect on $N^{(2,C)}$ is higher, while the effect on the much higher $N^{(2,A)}$ is only marginal. Hence, the lower α, the more stretched the danger zone around $N^{(1,B)}$ (= $N^{(2,B)}$) will be.

7.4 The successive figures clearly reveal that the size of the danger zone decreases when the retention rate is lower, i.e., when more reinsurance is taken up. The explanation is that a lower retention rate will shift $N^{(2,C)}$ and $N^{(2,B)}$ to the right towards $N^{(2,A)}$, which is fixed, i.e., independent of the level of the retention rate. Actually, taking up more reinsurance is more effective for decreasing volatility mortality risk than it is for decreasing level/trend mortality risk: $N^{(2,C)} < N^{(2,B)}$, but both converge to $N^{(2,A)}$ when *f* decreases to $f = 25\%$.

7.5 There are two other ways to decrease the size of the danger zone:
— by increasing the 'overall' minimum capital requirement, and/or
— by increasing the minimum capital requirement for level/trend mortality risk per policy ($RC^{mort}{}_{pp}$) through imposing a higher stress factor for calculating this requirement (e.g., a 15% shock of mortality rates, instead of 10% as assumed above).

7.6 Option A should (generally) be considered too blunt, as it would increase the barrier for new insurers to enter the insurance market. We even recommend licensing and setting minimum capital requirements per product group (life) and class of business (non-life); micro-insurance should then be considered a separate product group/class of business. These minimum capital requirements must always be sufficient to cover the expected administration costs of winding up the corresponding policies when necessary. These costs are generally relatively low. However, such low minimum capital requirements should only be allowed when the company is adequately reinsured, i.e., as long as $N < N^{(2,C)}$ for the retention rate chosen. The insurance supervisor should be authorised to impose 'capital add-ons' for companies with solvency problems, e.g., when the company is insufficiently reinsured.

7.7 Option B is also rather blunt; however, it recognises that volatility mortality risks are hard to distinguish from level/trend mortality risks, implying, under common RBC frameworks, that they should also be covered by the capital requirement for level/trend mortality risks. The prescribed stress level for calculating this capital requirement should therefore definitely not be too low.

7.8 Nevertheless, we believe that insurance supervisors, under the common RBC frameworks that, so far, ignore volatility mortality risks, should stimulate, and maybe even be authorised to force companies to lower their retention rate, i.e. to take up more reinsurance, when the portfolio size is still small but higher than $N^{(2,C)}$. Insurance builds on the benefits of pooling, i.e., the law of large numbers, because this reduces relative volatility. Reinsurance is just a way to benefit more from this 'law'. Hence, reinsurance is an effective risk management tool, particularly for mitigating volatility mortality risks.

7.9 The suggestion that micro-insurance organisations can be exempted from mandatory retrocession requirements with local reinsurers is therefore very much in conflict with the interests of their policyholders (who are particularly vulnerable because of their low incomes). It has also been suggested that their minimum capital requirement could be much lower than that for regular insurers. On the contrary, when their portfolios grow significantly, the minimum retrocession requirements should definitely be set much higher in order to serve the policyholders' interests because of the volatility risks.

8. OTHER TYPES OF UNDERWRITING RISKS

8.1 Catastrophic mortality risks, when covered by a local insurer without reinsurance, are proportional to the portfolio size (N). However, when covered by a globally operating, geographically well diversified (re)insurer this risk will be more proportional to $\sqrt{N}$.

8.2 The principles in this article apply equally to other types of underwriting risks in insurance, in particular to lapse/surrender risks in life and premium risks in non-life insurance, when the RBC requirement for these risks is defined as a fixed factor times the technical provision and unearned premium reserve, respectively. The reason for this is that such a capital requirement, like the common RBC requirement for (level/trend) mortality risks in life insurance, primarily covers uncertainty about the level (/trend) of the expected future underwriting losses, not the uncertainty about the level of the actual losses around their expected levels, i.e., the random volatility of the actual losses.

8.3 Even expense risks, also considered as an underwriting sub-risk, can be split between
- level uncertainty: what level of expenses should be allocated to individual policies,
- trend uncertainty: what will the future expense inflation be, corrected for the effects of the company's cost savings programmes including expected future IT benefits,
- volatility risks: how large can the 'random' elements in the annual expenses be, and, finally,
- catastrophic risks like the ones that should be addressed in the company's business continuity plan(s).

8.4 The law of large numbers, strictly speaking, does not apply to expense risks. However, start-up companies generally suffer from relatively high expenses per policy due to a lack of economies of scale. Therefore, as long as the company's portfolio size is still relatively small, it may also be better to also consider expense risks proportional to the square root of the number of policies ($\sqrt{N}$), instead of proportional to the number itself (N).

8.5 Reinsurance is less appropriate for covering expense risks. A better way to do this is by means of minimum requirements for the company's *working* capital during their start-up phase, i.e., on top of the minimum capital requirement but decreasing gradually in time down to nil when the portfolio size has grown sufficiently that economies of scale have been achieved to cover the expenses from the premium tariffs. Expenses that are still proportional to $\sqrt{N}$ should be covered by certain (decreasing) 'capital add-ons' during start-up phases.

8.6 Expense losses during start-up phases can also be covered, at least partially, by means of acquisition commissions received from reinsurers. This is basically another reason for taking up more reinsurance when the portfolio size is still small. This could even be a more sound way than raising and amortising deferred acquisition costs in the IFRS balance sheet (always to be ignored as an asset in the EBS under RBS).

9. CONCLUSIONS

9.1 Under compliance-based (insurance) supervision (CBS) there is poor recognition of specific (sub) risk types, e.g., level/trend, volatility and catastrophic risks as sub-risks of mortality risks. At most, capital requirements under such a regime are proportional to the size of the portfolio. However, volatility risk, which can also be a significant sub-risk under lapse/surrender risks in life and premium risks in non-life portfolios, is proportional to the square root of the number of policies.

9.2 Reinsurance is particularly helpful for mitigating volatility risks. CBS frameworks therefore do not, or only poorly, recognise reinsurance as an effective tool for risk mitigation.

9.3 Under RBS, there should be no less than three critical values for the size of the insured portfolio:
— the number of policies above which the volatility risks are higher than the minimum capital requirement; this number, notated $N^{(2,C)}$ in the previous paragraphs, is proportional to the square root of the number of policies ($\sqrt{N}$);
— the number of policies above which the level/trend risks are covered by the corresponding capital requirement. This level, notated $N^{(2,B)} > N^{(2,C)}$, is proportional to N; and
— the number of policies above which the capital requirement for level/trend risks exceeds the capital that is needed to cover volatility risks; this level is called $N^{(2,A)} > N^{(2,B)} > N^{(2,C)}$.

9.4 Both $N^{(2,C)}$ and $N^{(2,B)}$, but not $N^{(2,A)}$, are a decreasing function of the retention rate under quota share reinsurance. Hence, taking up more reinsurance makes the numbers $N^{(2,C)}$ and $N^{(2,B)}$ higher and therefore less critical. Insurance companies, and also the insurance supervisors applying RBS, should therefore recognise that reinsurance is a powerful risk management tool, particularly when the portfolio size is (still) relatively small (but higher than the critical level $N^{(2,C)}$) and volatility risks are still relatively large.

9.5 Unfortunately, volatility risks are not properly reflected in the existing RBC frameworks under Pillar 1 of RBS, and therefore also not in (most of the) ones that are still under development. It is therefore important for both insurers and insurance supervisors to consider them critically in the development and assessment of the adequacy of the company's reinsurance policy, respectively, i.e. under Pillar 2 of RBS.

9.6 This conclusion is particularly relevant for insurance companies that operate in less mature insurance markets, because their portfolios are generally relatively small due to a low level of insurance penetration.

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APPENDIX A
The Skewness of the Sum of *N* Independent Bernouilli Processes

The skewness parameter γ for the distribution of a random variable B is defined as

$$\gamma(\underline{B}) = \mathrm{E}\{\underline{B} - \mathrm{E}(\underline{B})\}^3 / [\mathrm{E}\{\underline{B} - \mathrm{E}(\underline{B})\}^2]^{3/2}.$$

When $\underline{B}$ reflects the outcome of the sum of N independent Bernouilli (q_x) processes, the numerator equals $\mathrm{E}\{\underline{B} - \mathrm{E}(\underline{B})\}^3 = N \cdot SA^3 \cdot q_x \cdot (1 - 3q_x + 2q_x{}^2)$ and the denominator equals $[\mathrm{E}\{\underline{B} - \mathrm{E}(\underline{B})\}^2]^{3/2} = [N \cdot SA^2 \cdot q_x \cdot (1 - q_x)]^{3/2}$.

As a result, $\gamma(\underline{B}) = (1 - 3q_x + 2q_x{}^2) / \{\sqrt{q_x} \cdot (1 - q_x) \cdot \sqrt{(1 - q_x)} \cdot \sqrt{N}\}$.