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SYMMETRIC NUCLEAR MATTER PROPERTIES IN THE LINEAR WALECKA MODEL VIA A RELATIVISTIC MEAN FIELD APPROXIMATION AT ZERO TEMPERATURE

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ABSTRACT

Theproperties of nuclear matter at zero temperature were considered. The equations of state (EOS) of nuclear matter were studied in the linear-Walecka model at different parameterizations. At normal nucleon density, there is a strong correlation among the different parameter sets, however the linear Walecka model gives values of nucleon effective mass M_0^* and nuclear incompressibility (K) at variance to the experimental values.The calculated values of the saturation density ranges from (0.142-0.148) fm^{-3}, compressional modulus (533.53-543.95)MeV for the LWM .The results of the numerical computations were compared with the empirical analysis of the giant isoscalar monopole resonance data.

Keywords: Symmetric nuclear matter, Lagrangian density,Linear-Walecka model, relativistic mean field theory, equation of state

INTRODUCTION

A hypothetical nuclear system with equal number of protons and neutrons uniformly distributed with no coloumb interaction becomes a nuclear matter (Passamani *et al*., 2007 and Krane, 1988). It is an idealized system of nucleons that exists in several phases and structures depending on temperature and density. A great effort has been dedicated to the study of nuclear media at extreme conditions of pressure and temperature as in the case of relativistic heavy-ion reaction and the study of neutron star properties (Passamani *et al*., 2007).The linear-Walecka model is a renormalized quantum field theory(QFT) also known as $(\sigma - \omega)$ model for the description of nuclear matter and finite nuclei properties. It is based on locally Lorentz invariant fields of four fundamental particles such as the nucleons and two mesons (da-Silva, 2013).This model is otherwise called Quantun Hadrodynamics I (QHD1) which is a relativistic description of nuclear matter properties based on the work of Walecka (1974). The model describes the nucleus as a system of Dirac nucleons which interacts via the exchange of mesons and photon fields.That is to say that this model typically includes scalar (sigma) and vector (omega) meson fields. The scalar meson field contributes to the effective nucleon mass and the binding energy of nuclear matter, while the vector meson field contributes mainly to the repulsive core of the nuclear force. It is based on a phenomenological treatment of the hadronic degrees of freedom, which is a renormalizable relativistic quantum field theory (QFT). This early version (QHD1) considers the scalar (σ) meson field and a vector (ω) meson field coupled to the baryonic (nucleon) fields. It is a mean field treatment of the relativistic quantum hadrodynamics which has been found to be a very successful framework for the description of bulk and single particle properties of nuclear matter and finite nuclei (Gambhir and Ring, 1989; Abhijit and Ghosh, 2018; Dhiman, et al. 2007; Francesco, 2017; Ilona, 2007*).*The (σ-ω) model treats nucleons as interacting particles in a mean field generated by the exchange of these meson fields. This mean field approximation simplifies the description of nuclear interactions compared to more the complex quantum chromodynamics (QCD) approaches. The nucleons moving through this mean field experiences an effective mass which is different from their free-mass due to interaction with mesons. This effective mass can affect properties such as nucleon dispersion and nuclear binding energies (Francisco, 2017; Patrigani, 2016 ;Schmitt, 2010*;*Chin and Walecka, 2008*).*The linear-Walecka model provides an equation of state (EoS) for symmetric nuclear matter, relating the pressure, energy density and other thermodynamic quantities to the density of the nucleons. This is crucial for understanding the stability and structure of nuclear matter under different conditions (Von-Maco,2018). This model aims to reproduce the saturation properties of nuclear matter, such as the equilibrium density and binding energy per nucleon which are essential for understanding the stability of atomic nuclei and the conditions in neutron stars.The saturation density being the density at which the system is at staticequilibrium that is, the pressure is zero.Thus this paper paper seeks to present the basic nuclear matter (observables) at zero temperature and its applications to astrophysical sites.

MATERIALS AND METHOD

Theoretical Framework

LagrangianDensity for the $\sigma - \omega$Model

The model is governed by a Lagrangian density. And for the original linear Walecka model, also known as the $(\sigma - \omega)$ model, the Lagrangian density is given by:

$$\mathcal{L} = \mathcal{L}_{nucl} + \mathcal{L}_{\sigma} + \mathcal{L}_{\omega} + \mathcal{L}_{int.} \tag{1}$$

Where:

$$\mathcal{L}_{nucl} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi \tag{2}$$

$$\mathcal{L}_{\sigma} = \frac{1}{2}\left[\left(\partial_{\mu}\sigma\right)\left(\partial^{\mu}\sigma\right) - m_{\sigma}^{2}\sigma^{2}\right] \tag{3}$$

$$\mathcal{L}_{\omega} = -\frac{1}{4}\omega^{\mu v}\omega_{\mu v} + \frac{1}{2}m_{\omega}^{2}\omega^{\mu}\omega_{\mu} \tag{4}$$

$$\mathcal{L}_{int} = g_{\sigma}\sigma\bar{\psi}\psi - g_{\omega}\omega_{\mu}\bar{\psi}\gamma^{\mu}\psi \tag{5}$$

Where:

ψ is the nucleon field

σ is the scalar meson field

ω is the vector meson field

m is the nucleon mass

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m_σ and m_ω are the respective meson masses

g_σ and g_ω are the respective scalar and vector coupling constants and

$\omega^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu$

So that the explicit Lagrangian density is given by:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + \frac{1}{2}\left[(\partial_\mu \sigma)(\partial^\mu \sigma) - m_\sigma^2 \sigma^2\right] - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2 \omega^\mu \omega_\mu + g_\sigma \sigma \bar{\psi}\psi - g_\omega \omega_\mu \bar{\psi}\gamma^\mu \psi \quad (6)$$

Derivation of the Equations of motion of the various fields

The equations of motion of the three fields are obtained using the Euler-Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) = 0 \quad (7)$$

Eqs. (2) – (5) are substituted in turn into eq. (7) to obtain the following equations of motion for each field as:

$$\sigma - field \qquad (\Box + m_\sigma^2)\sigma(x) = g_\sigma \bar{\psi}(x)\, \psi(x) \quad (8)$$

$$\omega - field \qquad (\Box + m_\omega^2)\omega_\mu(x) = g_\omega \bar{\psi}(x)\gamma_\mu \psi(x) \quad (9)$$

$$\psi - field \qquad \left[i\gamma^\mu \left(\partial_\mu + ig_\omega \omega_\mu(x)\right) - \left(m - g_\sigma \sigma(x)\right)\right]\psi(x) = 0 \quad (10)$$

The Relativistic Mean-Field Approximation

We now assume that the ground state of nuclear matter is composed of static, uniform matter. This means that both the currents and the meson fields are independent of x^μ. Solutions of the equations of motion can be found in the relativistic mean field approximation for which meson fields are separated into classical mean field values and quantum fluctuations which are not included in the ground state. We then replace the meson field operators with their ground state expectation values to obtain the following:

$$(x) \longrightarrow \langle \Phi | \sigma | \Phi \rangle = \langle \sigma \rangle = \sigma_0 \quad (11a)$$

$$\omega_\mu(x) \longrightarrow \langle \Phi | \omega | \Phi \rangle = \langle \omega_\mu \rangle = \delta_{\mu 0}\omega_0 \quad (11b)$$

From eqns(11a) and (11b), eqs. (8) – (10) become:

$$m_\sigma^2 \langle \sigma \rangle = g_\sigma \langle \bar{\psi}\, \psi \rangle \quad (12)$$

$$m_\omega^2 \langle \omega_\mu \rangle = g_\omega \langle \bar{\psi}\gamma_\mu\, \psi \rangle \quad (13)$$

$$\left[i\gamma^\mu \left(\partial_\mu + ig_\omega \langle \omega_\mu \rangle\right) - (m - g_\sigma \langle \sigma \rangle)\right]\psi(x) = 0 \quad (14)$$

Note that equation (12) is analogous to the equation of motion of a free scalar field, the Klein-Gordon (K.G) equation with the baryon scalar density $\bar{\psi}(x)\psi(x)$ as the source term.

The eqn. (13) is analogous to the Proca equation with the coupling of the vector field to the conserved baryon current as their source term.

Eqn. (14) is the Dirac equation with the modified mass due to the scalar field.

Determination of the equation of state

The equation of state (EoS) in the linear-Walecka model can be obtained from the expression of the energy-momentum tensor in terms of the pressure and energy densities.

By definition, the energy-momentum tensor is given by (Francesco, 2017 and Diener, 2008) :

$$T_{\mu\upsilon} = \eta_{\mu\upsilon}\mathcal{L} - \frac{\partial L}{\partial(\partial^{\mu}\phi_i)}\partial_v\phi_i \qquad (15)$$

The energy density (ε) and pressure density (P) for the expectation values in the rest frame are on the diagonal of the matrix form.

$$T_{\mu\upsilon} = T^{\mu\upsilon} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix} \qquad (16)$$

From the energy-momentum tensor, the energy and pressure densities are given respectively by:

$$\varepsilon = -\langle\mathcal{L}\rangle + \langle\bar{\psi}\gamma_0 p_0 \psi\rangle \qquad (17)$$

$$P = \langle\mathcal{L}\rangle + \frac{1}{3}\langle\bar{\psi}\gamma_i p_i \psi\rangle \qquad (18)$$

Where $\langle\mathcal{L}\rangle$ is given by:

$$\langle\mathcal{L}\rangle = -\frac{1}{2}m_{\sigma}^2\langle\sigma\rangle^2 + \frac{1}{2}m_{\omega}^2\langle\omega^{\mu}\rangle\langle\omega_{\mu}\rangle \qquad (19)$$

For the linear Walecka model, we present the final result of the energy density by substituting the expectation values into equation (17) and the Lagrangian equation (19) obtaining

$$\varepsilon = \frac{1}{2}m_{\sigma}^2\langle\sigma\rangle^2 - \frac{1}{2}m_{\omega}^2\langle\omega_0\rangle^2 + \frac{2}{\pi^2}\int \partial p\, p^2\sqrt{p^2 + (m - g_{\sigma}\langle\sigma\rangle)^2} \qquad (20)$$

Similarly, for the pressure density (P) of the EOS for nuclear matter, we substitute the expectation values into equation (18))and (19) obtaining:

$$P = \frac{1}{2}m_{\sigma}^2\langle\sigma\rangle^2 - \frac{1}{2}m_{\omega}^2\langle\omega_0\rangle^2 + \frac{2}{3\pi^2}\int \partial p \frac{p^4}{\sqrt{p^2 + (m - g_{\sigma}\langle\sigma\rangle)^2}} \qquad (21)$$

Recalling that the nucleon effective mass expression $m^* = m - g_{\sigma}\langle\sigma\rangle$ (22) .

Therefore, eqns (20) and (21) modify as follows:

$$\varepsilon = \frac{1}{2}m_{\sigma}^2\langle\sigma\rangle^2 + \frac{1}{2}m_{\omega}^2\langle\omega_0\rangle^2 + \frac{2}{\pi^2}\int_0^{p_F} dpp^2\sqrt{p^2 + m^{*2}} \qquad (23)$$

and

$$P = -\frac{1}{2}m_{\sigma}^2\langle\sigma\rangle^2 + \frac{1}{2}m_{\omega}^2\langle\omega_0\rangle^2 + \frac{2}{3\pi^2}\int_0^{p_F} dp\,\frac{p^4}{p^2 + m^{*2}} \qquad (24)$$

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NUMERICAL RESULTS

Table 1:The model parameter sets. Nucleon mass is taken as 939MeV

	FSUGarnet	IOPB-1	G3	NL3
m_σ/M	0.529	0.533	0.559	0.541
m_ω/M	0.812	0.812	0.820	0.812
m_δ/M	0.833	0.833	0.833	0.833
$g_\sigma/4\pi$	0.0	0.0	1.043	0.0
$g_\omega/4\pi$	0.837	0.827	0.782	0.813
$g_\rho/4\pi$	1.091	1.062	0.923	1.024
$k_3(fm^{-1})$	1.105	0.885	0.962	0.712
k_4	1.368	1.496	2.606	1.465
ς_0	-1.397	-2.932	1.694	-5.688
η_1	4.410	3.103	1.010	0.0
η_2	0.0	0.0	0.424	0.0
η_ρ	0.0	0.0	0.114	0.0
Λ_ω	0.0	0.0	0.645	0.0
	0.043	0.024	0.038	0.0

Table 2: Calculated Nuclear Matter Observables for LWM at zerotemperature

	FSUGarnet	IOPB-1	G3	NL3
$\rho_{0(fm^{-3})}$	0.142	0.146	0.145	0.148
M^*/M	0.545	0.573	0.696	0.600
$\varepsilon_0(MeV)$	-15.71	-15.32	-16.63	-16.24
$p_F^0(fm^{-1})$	1.31	1.33	1.30	1.30
$K_\infty(MeV)$	540.4	533.55	543.95	536.36

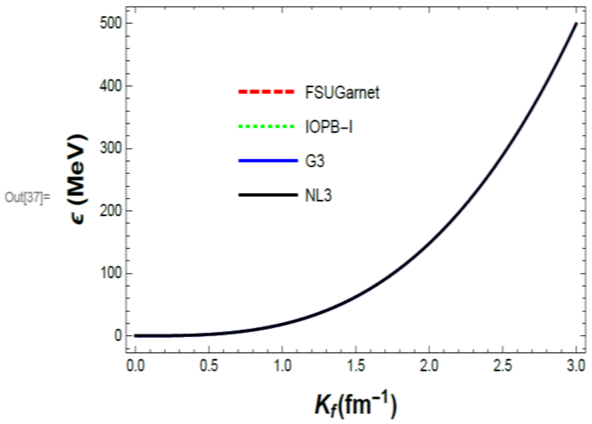

Figure 1Relationship between energy density and Fermi-wavelength for a variety of parameter sets

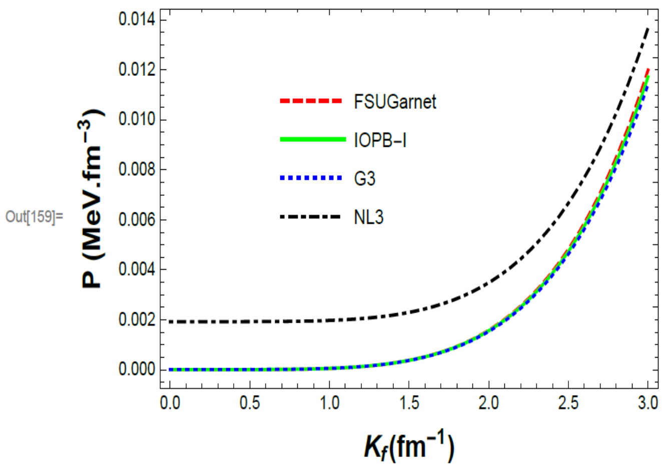

Figure 2: Relationship between pressure density and Fermi-wavelength for a variety of parameter sets

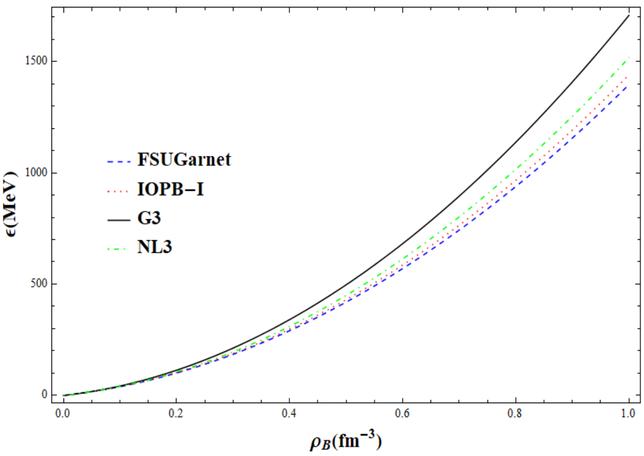

Figure 3: Energy density as a function of baryon density for symmetric nuclear matter at zero temperature (LWM)

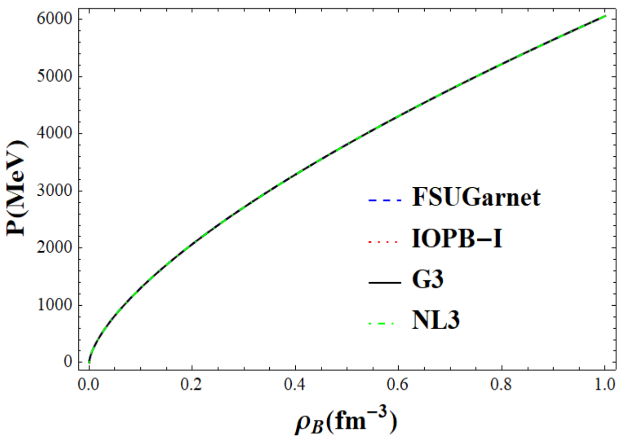

Figure 4: Pressure density as a function of baryon density for symmetric nuclear matter at T= 0(LWM)

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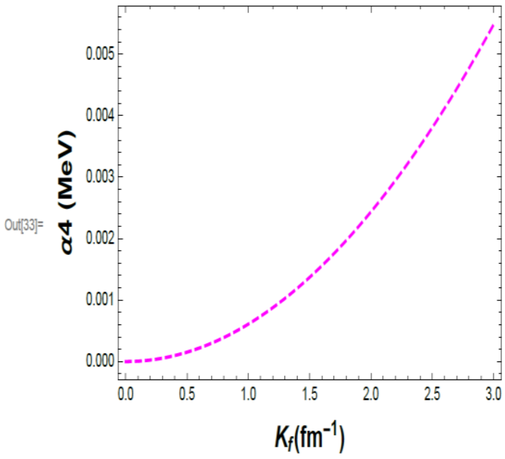

Figure 5: Relationship between nuclear Symmetry energy coefficients Fermi-wavelength for Symmetric nuclear matter at Zero temperature

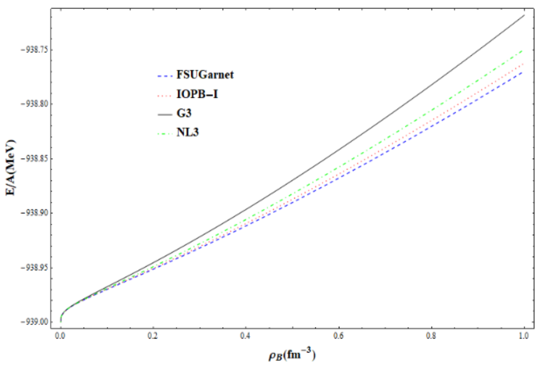

Figure .6: Binding energy as a function of baryon density for the FSUGarnet, IOPB-1, G3 and NL3 parameter sets in the LWM at T = 0

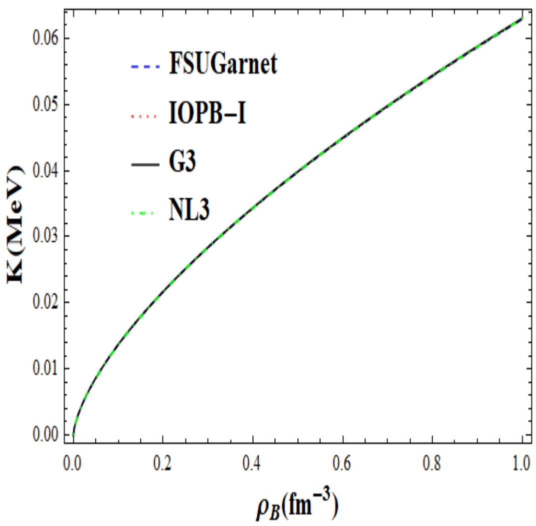

Fig. 7 : Nuclear matter compression modulus against baryon density for different parameter sets at T = 0

DISCUSSION

Properties of nuclear matter were calculated using the following parameter sets FSUGarnet, IOPB-1, G3 and NL3. The energy and pressure densities as a function of the baryon densities (ρ_B) are the equation of state (EoS).The sensitivity of the EoS were validated by plotting the energy per particle, otherwise known as the binding energy per nucleon as a function of the densities as well as Fermi-wavelength under different parameterization so as to observe the behaviour of the models.

Fig.(1) is a plot of energy density against fermi momentum which illustrates that energy of nuclear matter increases with the density and their momentum distribution. It is clear that the plots under different parameterization showed similar overlap. At lower fermi momentum, nuclear matter behaves more as a collection of loosely interacting nucleons and at higher fermi -momentum, the nucleons are squeeed closer together, leading to stronger interactions (Fig. 2). Also Fig .2 (from eqn.24) shows the variation of pressure density with fermi-wavelength.

The FSUGarnet, IOPB-1, and G3 set depict similar trends at higher fermi-wavelength while the NL3 set diverges. This is due to the stiffness of NL3 at high density observatiions due to absence of the self-cross coupling constant (see Table.1).These discrepancies at higher fermi-momentum emphasis that more information is needed for a deeper understanding of dense matter at high densities. The above results are in good agreement with (Sumiyoshi, 2019 and Dienner, 2008) and QHD1 parameterizations. The relationship between energy density and nucleon density shows a characteristic curve (Fig.3).At lower baryon densities, energy density increases slowlyand nucleons are closedly packed together. As nucleon density increases, energy density increases more steeply , indicating a reflection of increase in kinetic energy of nucleons and the rate of their interactions. Fig.6 shows the relationship between the binding energy per baryon and baryon density among the different parameter sets.It is quite obvious that all the parameter sets FSUGarnet, IOPB-1, G3 and NL3 are in agreement with high density nuclear matter observation. The G3 parameter set has a sudden divergence from the other parameter sets. However, the FSUGarnet, IOPB-1 and NL3 are in good agreement with experimental data for symmetric nuclear matter (SNM) but gives stiff EOS whereas only NL3 excellently satisfies the experimental data (Gil *et al*.,2023 and Chung*et al*.,2008). Notice that at lower nucleon density, binding energy per baryon increases due to the effects of the strong nuclear force that tends to dominate over the coulomb repulsion of the nucleons. As density increases, binding energy initially continue to increase due to the strong nucleon-nucleon interactions that results to the nuclear force. However, beyond a certain density, nuclear matter binding energy starts to decrease because of an onset of repulsive core interactions among the nucleons (Aper *et al*.,2018 ; Fiase and Gboarun, 2011). The calculated values of saturation density ranges from (0.142-0.148) fm^{-3} , nucleon effective mass (0.545-0.696) MeV, binding energy per nucleon (-15.32 to -15.71) MeV, compression modulus (533.55-543.95) MeV, and fermi-wavelength (1.30-1.33) fm^{-1} for the linear Walecka model (LWM) in table.2.

The relativistic mean field approximation method have been employed successfully to obtain the ground state properties of nuclear matter in the sigma –omega model. Also the parameter sets FSUGarnet, IOPB-1, G3 and NL3 have been used for the first time in the linear Walecka model and the results are in agreement with other experimental findings. The linear Walecka formalism was discussed in relation with the various applications both at low and high density nuclear matter. In essence, the linear Walecka model is important because it provides a foundational framework for understanding nuclear interactions, predicting nuclear properties, and exploring the behaviorof matter at extreme conditions. Its versatility and applications across different

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nuclear scales will make it indispensable in our contemporary nuclear and astrophysical research.

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