

NUMERICAL SOLUTION OF FRACTIONAL ORDER MATHEMATICAL MODEL ON THE TRANSMISSION DYNAMICS OF EBOLA VIRUS DISEASE USING THE LAPLACE-ADOMIAN DECOMPOSITION METHOD

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ABSTRACT

Ebola Virus Disease (EVD) has been a significant public health concern due to its high mortality rate and potential for widespread outbreaks. This study focuses on the transmission dynamics of EVD, utilizing a fractional-order mathematical model. The model incorporates hospitalization and treatment as critical control measures against the disease. The Laplace-Adomian Decomposition method is employed to approximate series solutions, providing insights into the dynamics of EVD under varying conditions. Numerical simulations conducted using MATLAB highlight that effective treatment and high vaccination coverage are pivotal in controlling the spread of Ebola. Recommendations stemming from the study emphasized the importance of comprehensive vaccination campaigns, strengthened healthcare infrastructure, and community engagement in disease control efforts. Ultimately, this study serves as a valuable tool in understanding and managing Ebola outbreaks, contributing to global health strategies aimed at preventing and mitigating infectious diseases.

Keywords: Epidemiology, Vaccination, Hospitalization, Computer Simulation, Laplace-Adomian Decomposition method, Mathematical modelling

INTRODUCTION

Ebola virus disease (EVD), first recognized in 1976 during simultaneous outbreaks in Nzara, Sudan, and Yambuku, Democratic Republic of Congo (DRC), has since garnered global

attention due to its high mortality rates and potential for large-scale outbreaks [1]. The causative agent, Ebola virus, belongs to the family Filoviridae and genus Ebolavirus, with five identified species: Zaire ebolavirus, Sudan ebolavirus, Tai Forest (formerly Ivory

Coast) ebolavirus, Bundibugyoebolavirus, and Reston ebolavirus [1,3]. Ebola virus is transmitted through direct contact with bodily fluids of infected individuals or animals, such as blood, saliva, urine, or semen, and can also spread via contaminated surfaces and materials [1]. The virus primarily infects humans through mucosal surfaces, breaks in the skin, or parenterally. Initial symptoms of EVD include sudden onset of fever, fatigue, muscle pain, headache, and sore throat, progressing to vomiting, diarrhea, impaired kidney and liver function, and in severe cases, internal and external bleeding [1,4]. As of recent developments, a recombinant vesicular stomatitis virus-based vaccine, rVSV-ZEBOV, has shown efficacy in preventing EVD during outbreaks. This vaccine, although still under evaluation, represents a significant advancement in controlling Ebola virus transmission [1,3]. Treatment primarily involves supportive care, including fluid and electrolyte management, and addressing specific symptoms such as bleeding and infections. Experimental treatments, such as monoclonal antibodies and antiviral drugs, are also being explored [3]. Preventing EVD outbreaks relies heavily on early detection, isolation of cases, contact tracing, and implementation of infection prevention and control measures in healthcare settings [1]. Public health measures include community engagement, safe burial practices, and education on personal protective equipment (PPE) use and hygiene practices [3,4]. Surveillance and rapid response teams are crucial in containing outbreaks and preventing further transmission within and across borders.

Laplace Adomian Decomposition Method:

The Laplace Adomian Decomposition Method (LADM) is a powerful analytical technique used to solve nonlinear differential equations. It combines the classical Laplace transform method with Adomian decomposition, offering a systematic approach to approximate solutions for a wide range of differential equations encountered in science and engineering [2,7]. The LADM begins by

applying the Laplace transform to the nonlinear differential equation, transforming it into a simpler algebraic equation involving transformed functions. The nonlinearity is then decomposed using the Adomian polynomials, which are series solutions representing the nonlinearity of the equation. This decomposition simplifies the problem into a series of linear or simpler nonlinear equations, which can be solved iteratively to obtain successive approximations of the solution [2,8]. One of the significant advantages of LADM is its ability to handle nonlinear differential equations without linearization or discretization, which are common requirements in numerical methods. This method provides analytical solutions in closed-form or series form, allowing for a deeper insight into the behavior of the system and facilitating the study of the influence of parameters on the solutions [2]. Moreover, LADM is computationally efficient and can be implemented straightforwardly using symbolic computation software like Mathematica or Maple, making it accessible for researchers and practitioners in various fields. It is particularly useful in problems where numerical methods may encounter convergence issues or when exact solutions are desired for theoretical analysis or validation purposes [7,8]. The Laplace Adomian Decomposition Method has been successfully applied to various disciplines, including physics, engineering, biology, and finance, where nonlinear differential equations are prevalent. Its versatility and robustness continue to attract researchers seeking efficient analytical tools for modeling and analysis. Ongoing research focuses on refining the method's applicability to more complex nonlinear systems and extending its capabilities to handle higher-order differential equations and coupled systems [2,8]. Several authors have studied approximate solution of differential equations using Laplace-Adomian Decomposition Method (LADM). [2] focused on the combined Laplace Transform-Adomian Decomposition Method (LADM), demonstrating its efficacy in solving nonlinear

ordinary and partial differential equations. By applying the Laplace transform to eliminate the time variable and subsequently decomposing the nonlinear terms using Adomian polynomials, Wazwaz simplifies complex equations into iterative linear forms. This methodological approach provides accurate analytical solutions without the computational demands of traditional numerical techniques, making it a valuable tool in mathematical modeling across various scientific disciplines. [9] applied the Laplace-Adomian Decomposition Method (LADM) to solve nonlinear differential equations commonly encountered in engineering applications. Focusing on second-order nonlinear differential equations, Naeem illustrates the step-by-step application of LADM, highlighting its utility in providing accurate approximate solutions for engineering design and analysis. By avoiding the complexities of numerical methods, LADM offers a systematic approach to modeling nonlinear dynamics in engineering systems, enhancing understanding and facilitating practical applications. [10] explored the broad applicability of the Laplace-Adomian Decomposition Method (LADM) across various disciplines including mathematical physics and engineering. By analyzing its theoretical foundations and practical implementations, Ji demonstrates LADM's effectiveness in solving diverse nonlinear differential equations. Through computational examples and theoretical analyses, Ji showcases how LADM provides insights into the qualitative behaviors of solutions, thereby contributing to advancements in theoretical understanding and practical applications in scientific research. Some useful studies include [5, 6, 11].

MATERIALS AND METHODS

Model Formulation.

The total population (N), is subdivided into seven compartments including: Susceptible individuals (S), Vaccinated individuals (V),

exposed individuals (E), infected individuals (I), hospitalized individuals (H), dead and unburied individuals (D), and recovered individuals (R). Let Λ be the recruitment rate. Susceptible individuals become vaccinated at a rate α_1 . Let α_2 denote the rate at which vaccinated individuals become exposed due to vaccine failure, λ be the rate at which susceptible individuals become exposed, and ω_1 represent the rate at which exposed individuals become infected. Here, ω_2 represents the hospitalization rate of infected individuals, ψ_1 and ψ_2 , denote the death rates of infected and hospitalized populations, respectively. ψ_3 is the recovery rate of hospitalized individuals, μ is the natural death rate. The death rates associated with infection and hospitalization are ε_1 and ε_2 and ψ_4 , is the burial rate of deceased individuals.

Susceptible (S): This compartment represents individuals who are susceptible to contracting Ebola virus disease. Susceptible individuals have not been infected with the virus and can become infected upon exposure to infected individuals or contaminated materials [1].

Vaccinated (V): The vaccinated compartment consists of individuals who have received a vaccine against Ebola virus disease. Vaccination reduces the susceptibility of individuals to infection and can contribute to herd immunity, thereby helping to control the spread of the virus within the population [7].

Exposed (E): Individuals in the exposed compartment have been infected with the Ebola virus but have not yet developed symptoms. During the incubation period, these individuals are not infectious but can later transition to the infected compartment [8].

Infected (I): Infected individuals are those who have developed symptoms of Ebola virus disease and are capable of transmitting the virus to others. This compartment represents individuals who are actively contributing to

the spread of the disease within the population [1].

Hospitalized (H): The hospitalized compartment includes individuals who have developed severe symptoms of Ebola virus disease and require medical care. Hospitalization is necessary for managing complications and providing supportive treatment to improve patient outcomes [3].

Deceased (D): Individuals in the deceased compartment have succumbed to Ebola virus disease. This compartment represents the

unfortunate outcome of severe cases of the disease and underscores the importance of timely medical intervention and public health measures to prevent fatalities [3].

Recovered (R): Recovered individuals have successfully cleared the Ebola virus from their system and have developed immunity to subsequent infections. This compartment reflects the resilience of the human immune system and the potential for individuals to overcome the disease with proper medical care and support [1].

Variables and Parameters Interpretation

Table 1. Variables and Parameters Used.

Variables	Interpretation
$N(t)$	Total human population
S	Susceptible population
V	Vaccinated population
E	Exposed individuals
I	Infected individuals
H	Hospitalized Individuals
D	Deceased population
R	Recovered individuals
Parameters	Descriptions
Λ	Recruitment rate
α_1	Vaccination rate
λ	Force of infection
α_2	Rate of exposure due to vaccine failure
α_3	Infectionrate due to vaccine failure
μ	Natural death rate
ω_1	Rate of infection of exposed humans
ε_1	Disease induced death rate associated with I compartment
ε_2	Disease induced death rate associated with H compartment
ω_2	Hospitalized rate of infected individuals
ψ_1	Death rate of infected individuals
ψ_2	Death rate of hospitalized individuals
ψ_3	Recovery rate of hospitalized individuals
ψ_4	Rate of burial for the deceased population

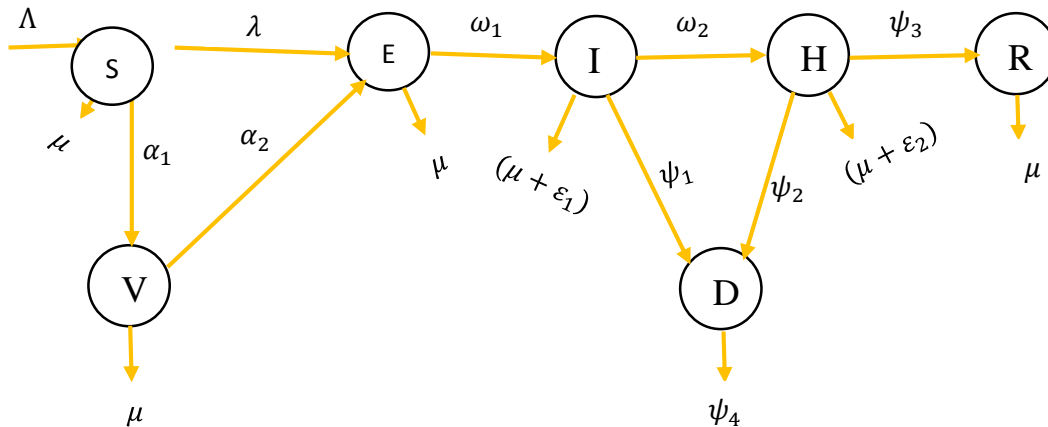


Figure 1. Schematic Diagram for the model.

Model Equations

$$\left. \begin{aligned} \frac{dS}{dt} &= \Lambda - (\lambda + \alpha_1 + \mu)S \\ \frac{dV}{dt} &= \alpha_1 S - (\alpha_2 + \mu)V \\ \frac{dE}{dt} &= \lambda S + \alpha_2 V - (\omega_1 + \mu)E \\ \frac{dI}{dt} &= \omega_1 E - (\varepsilon_1 + \psi_1 + \omega_2 + \mu)I \\ \frac{dH}{dt} &= \omega_2 I - (\psi_2 + \psi_3 + \varepsilon_2 + \mu)H \\ \frac{dD}{dt} &= \psi_1 I + \psi_2 H - \psi_4 D \\ \frac{dR}{dt} &= \psi_3 H - \mu R \end{aligned} \right\} (1)$$

$$\lambda = \frac{\beta(I + H + D)}{N}$$

Fractional Order of the Ebola Model

The Caputo derivative is measured as a differential operator in our model. We present in this segment some well-known definitions and effects that we shall be using throughout this research.

Definition 1. [7,8], The Caputo fractional order derivative of a function (f) on the interval $[0, T]$ is defined by:

$$[{}^C D_0^\beta f(t)] = \frac{1}{\Gamma(n - \beta)} \int_0^t (t - s)^{n - \beta - 1} f^{(n)}(s) ds, \tag{2}$$

Where $n = [\beta] + 1$ and $[\beta]$ represents the integer part of β . In particular, for $0 < \beta < 1$, the Caputo derivative becomes:

$$[{}^c D_0^\beta f(t)] = \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{f(s)}{(t-s)^\beta} ds, \quad (3)$$

Definition 2. [7], Laplace transform of Caputo derivatives is defined as

$$\mathcal{L}[{}^c D^\beta q(t)] = S^\beta h(S) - \sum_{k=0}^n S^{\beta-i-1} y^k(0), \quad n-1 < \beta < n, \quad n \in N, \quad (4)$$

For arbitrary $c_i \in R, i = 0, 1, 2, \dots, n-1, n = [\beta] + 1$ and $[\beta]$ represents the non-integer part of β .

Lemma 1 [7,8]. The following results hold for fractional differentiation equations

$$I^\beta [{}^c D^\beta h](t) = h(t) + \sum_{i=0}^{n-1} \frac{h^{(i)}(0)}{i!} t^i, \quad (5)$$

For arbitrary $\beta > 0, i = 0, 1, 2, \dots, n-1$, where $n = [\beta] + 1$ and $[\beta]$ represents the integer part of β .

Introducing fractional-order into the model, we now present a new model described by the following introducing fractional order derivative into the model we present new mathematical model describe by set of fractional difference of order β [7,8], for $0 < \beta < 1$

$$\left. \begin{aligned} D^\beta(S) &= \Lambda - (\lambda + \alpha_1 + \mu)S, \\ D^\beta(V) &= \alpha_1 S - (\alpha_2 + \mu)V, \\ D^\beta(E) &= \lambda S + \alpha_2 V - (\omega_1 + \mu)E, \\ D^\beta(I) &= \omega_1 E - (\varepsilon_1 + \psi_1 + \omega_2 + \mu)I, \\ D^\beta(H) &= \omega_2 I - (\psi_2 + \psi_3 + \varepsilon_2 + \mu)H, \\ D^\beta(D) &= \psi_1 I + \psi_2 H - \psi_4 D, \\ D^\beta(R) &= \psi_3 H - \mu R. \end{aligned} \right\} \quad (6)$$

The Laplace-Adomian Decomposition Method (LADM) Implementation

We considered the general procedure of this method with the initial conditions. Applying Laplace transforms to both sides of the equation (1), and then we have:

$$\left. \begin{aligned}
 S^\beta \mathcal{L}(S) - S^{\beta-1}S(0) &= \mathcal{L} \left[\Lambda - \frac{\beta(I+H+D)S}{N} - (\alpha_1 + \mu)S \right] \\
 S^\beta \mathcal{L}(V) - S^{\beta-1}V(0) &= \mathcal{L} [\alpha_1 S - (\alpha_2 + \mu)V] \\
 S^\beta \mathcal{L}(E) - S^{\beta-1}E(0) &= \mathcal{L} \left[\frac{\beta(I+H+D)S}{N} + \alpha_2 V - (\omega_1 + \mu)E \right] \\
 S^\beta \mathcal{L}(I) - S^{\beta-1}I(0) &= \mathcal{L} [\omega_1 E - (\varepsilon_1 + \psi_1 + \omega_2 + \mu)I] \\
 S^\beta \mathcal{L}(H) - S^{\beta-1}H(0) &= \mathcal{L} [\omega_2 I - (\psi_2 + \psi_3 + \varepsilon_2 + \mu)H] \\
 S^\beta \mathcal{L}(D) - S^{\beta-1}D(0) &= \mathcal{L} [\psi_1 I + \psi_2 H - \psi_4 D] \\
 S^\beta \mathcal{L}(R) - S^{\beta-1}R(0) &= \mathcal{L} [\psi_3 H - \mu R]
 \end{aligned} \right\} (7)$$

With initial conditions

$$S(0) = n_1, V_1(0) = n_2, V(0) = n_3, E(0) = n_4, I(0) = n_5, D(0) = n_6, R(0) = n_7$$

Dividing eqn. (7) by (S^β) we have:

$$\left. \begin{aligned}
 \mathcal{L}(S) &= \frac{n_1}{S} + \frac{1}{S^\beta} \mathcal{L} \left[\Lambda \rho - (\beta E + \alpha_1 + \mu)S \right] \\
 \mathcal{L}(V) &= \frac{n_2}{S} + \frac{1}{S^\beta} \mathcal{L} [\alpha_1 S - (\alpha_2 + \mu)V] \\
 \mathcal{L}(E) &= \frac{n_3}{S} + \frac{1}{S^\beta} \mathcal{L} \left[\frac{\beta(I+H+D)S}{N} + \alpha_2 V - (\omega_1 + \mu)E \right] \\
 \mathcal{L}(I) &= \frac{n_4}{S} + \frac{1}{S^\beta} \mathcal{L} [\omega_1 E - (\varepsilon_1 + \psi_1 + \omega_2 + \mu)I] \\
 \mathcal{L}(H) &= \frac{n_5}{S} + \frac{1}{S^\beta} \mathcal{L} [\omega_2 I - (\psi_2 + \psi_3 + \varepsilon_2 + \mu)H] \\
 \mathcal{L}(D) &= \frac{n_6}{S} + \frac{1}{S^\beta} \mathcal{L} [\psi_1 I + \psi_2 H - \psi_4 D] \\
 \mathcal{L}(R) &= \frac{n_7}{S} + \frac{1}{S^\beta} \mathcal{L} [\psi_3 H - \mu R]
 \end{aligned} \right\}$$

(8)

Decomposing the non-linear term of equation (6) whereby we assume the solution of $S(t), V_1(t), V_2(t), E(t), I(t), R(t)$ are in the form of infinite series given by:

$$\begin{aligned}
S(t) &= \sum_{n=0}^{\infty} S(n), & V(t) &= \sum_{n=0}^{\infty} V(n), & E(t) &= \sum_{n=0}^{\infty} E(n), \\
I(t) &= \sum_{n=0}^{\infty} I(n), & H(t) &= \sum_{n=0}^{\infty} H(n), & D(t) &= \sum_{n=0}^{\infty} D(n), & R(t) &= \sum_{n=0}^{\infty} R(n),
\end{aligned} \tag{9}$$

We have three (3) non-linear terms. The non-linear term in equation (6) are decomposed by Adomian polynomial as follows:

$$I(t)S(t) = \sum_{n=0}^{\infty} A(n), \quad H(t)S(t) = \sum_{n=0}^{\infty} B(n), \quad D(t)S(t) = \sum_{n=0}^{\infty} C(n) \tag{10}$$

Where $A(n), B(n), C(n)$ are Adomian polynomials given by

$$\begin{aligned}
A(n) &= \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n \lambda^k I(k) \sum_{k=0}^n \lambda^k S(k) \right]_{\lambda=0} \\
B(n) &= \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n \lambda^k H(k) \sum_{k=0}^n \lambda^k S(k) \right]_{\lambda=0} \\
C(n) &= \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^n \lambda^k D(k) \sum_{k=0}^n \lambda^k S(k) \right]_{\lambda=0}
\end{aligned} \tag{11}$$

The polynomials are given by

$$\begin{aligned}
A(0) &= I(0)S(0), \\
A(1) &= I(0)S(1) + I(1)S(0), \\
A(2) &= I(0)S(2) + I(1)S(1) + I(2)S(0).
\end{aligned}$$

$$\begin{aligned}
B(0) &= H(0)S(0), \\
B(1) &= H(0)S(1) + H(1)S(0), \\
B(2) &= H(0)S(2) + H(1)S(1) + H(2)S(0).
\end{aligned} \tag{12}$$

$$\begin{aligned}
C(0) &= D(0)S(0), \\
C(1) &= D(0)S(1) + D(1)S(0), \\
C(2) &= D(0)S(2) + D(1)S(1) + D(2)S(0).
\end{aligned}$$

Substituting equation (9), (10) into equation (8) we obtained:

$$\left. \begin{aligned}
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} S(n) \right\} &= \frac{n_1}{S} + \frac{1}{S\beta} \mathcal{L} \left[\Lambda - \frac{\beta \left(\sum_{n=0}^{\infty} A(n) + \sum_{n=0}^{\infty} B(n) + \sum_{n=0}^{\infty} C(n) \right)}{N} - (\alpha_1 + \mu) \sum_{n=0}^{\infty} S(n) \right] \\
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} V(n) \right\} &= \frac{n_2}{S} + \frac{1}{S\beta} \mathcal{L} \left[\alpha_1 \sum_{n=0}^{\infty} S(n) - (\alpha_2 + \mu) \sum_{n=0}^{\infty} V(n) \right] \\
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} E(n) \right\} &= \frac{n_3}{S} + \frac{1}{S\beta} \mathcal{L} \left[\frac{\beta \left(\sum_{n=0}^{\infty} A(n) + \sum_{n=0}^{\infty} B(n) + \sum_{n=0}^{\infty} C(n) \right)}{N} + \alpha_2 \sum_{n=0}^{\infty} V(n) - (\omega_1 + \mu) \sum_{n=0}^{\infty} E(n) \right] \\
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} I(n) \right\} &= \frac{n_4}{S} + \frac{1}{S\beta} \mathcal{L} \left[\omega_1 \sum_{n=0}^{\infty} E(n) - (\varepsilon_1 + \psi_1 + \omega_2 + \mu) \sum_{n=0}^{\infty} I(n) \right] \\
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} H(n) \right\} &= \frac{n_5}{S} + \frac{1}{S\beta} \mathcal{L} \left[\omega_2 \sum_{n=0}^{\infty} I(n) - (\psi_2 + \psi_3 + \varepsilon_2 + \mu) \sum_{n=0}^{\infty} H(n) \right] \\
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} D(n) \right\} &= \frac{n_6}{S} + \frac{1}{S\beta} \mathcal{L} \left[\psi_1 \sum_{n=0}^{\infty} I(n) + \psi_2 \sum_{n=0}^{\infty} H(n) - \psi_4 \sum_{n=0}^{\infty} D(n) \right] \\
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} R(n) \right\} &= \frac{n_7}{S} + \frac{1}{S\beta} \mathcal{L} \left[\psi_3 \sum_{n=0}^{\infty} H(n) - \mu \sum_{n=0}^{\infty} R(n) \right]
 \end{aligned} \right\} \tag{13}$$

Evaluating the Laplace transform of the 2nd terms in the RHS of (16), we obtain

$$\left. \begin{aligned}
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} S(n) \right\} &= \frac{n_1}{S} + \left[\Lambda - \frac{\beta \left(\sum_{n=0}^{\infty} A(n) + \sum_{n=0}^{\infty} B(n) + \sum_{n=0}^{\infty} C(n) \right)}{N} - (\alpha_1 + \mu) \sum_{n=0}^{\infty} S(n) \right] \frac{1}{S\beta+1} \\
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} V(n) \right\} &= \frac{n_2}{S} + \left[\alpha_1 \sum_{n=0}^{\infty} S(n) - (\alpha_2 + \mu) \sum_{n=0}^{\infty} V(n) \right] \frac{1}{S\beta+1} \\
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} E(n) \right\} &= \frac{n_3}{S} + \left[\frac{\beta \left(\sum_{n=0}^{\infty} A(n) + \sum_{n=0}^{\infty} B(n) + \sum_{n=0}^{\infty} C(n) \right)}{N} + \alpha_2 \sum_{n=0}^{\infty} V(n) - (\omega_1 + \mu) \sum_{n=0}^{\infty} E(n) \right] \frac{1}{S\beta+1} \\
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} I(n) \right\} &= \frac{n_4}{S} + \left[\omega_1 \sum_{n=0}^{\infty} E(n) - (\varepsilon_1 + \psi_1 + \omega_2 + \mu) \sum_{n=0}^{\infty} I(n) \right] \frac{1}{S\beta+1} \\
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} H(n) \right\} &= \frac{n_5}{S} + \left[\omega_2 \sum_{n=0}^{\infty} I(n) - (\psi_2 + \psi_3 + \varepsilon_2 + \mu) \sum_{n=0}^{\infty} H(n) \right] \frac{1}{S\beta+1} \\
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} D(n) \right\} &= \frac{n_6}{S} + \left[\psi_1 \sum_{n=0}^{\infty} I(n) + \psi_2 \sum_{n=0}^{\infty} H(n) - \psi_4 \sum_{n=0}^{\infty} D(n) \right] \frac{1}{S\beta+1} \\
 \mathcal{L} \left\{ \sum_{n=0}^{\infty} R(n) \right\} &= \frac{n_7}{S} + \left[\psi_3 \sum_{n=0}^{\infty} H(n) - \mu \sum_{n=0}^{\infty} R(n) \right] \frac{1}{S\beta+1}
 \end{aligned} \right\} \tag{14}$$

Taking the inverse Laplace transform of both sides of (14)

$$\left. \begin{aligned}
 \sum_{n=0}^{\infty} S(n) &= n_1 + \left[\Lambda - \frac{\beta \left(\sum_{n=0}^{\infty} A(n) + \sum_{n=0}^{\infty} B(n) + \sum_{n=0}^{\infty} C(n) \right)}{N} - (\alpha_1 + \mu) \sum_{n=0}^{\infty} S(n) \right] \frac{t^\beta}{\Gamma(\beta+1)} \\
 \sum_{n=0}^{\infty} V(n) &= n_2 + \left[\alpha_1 \sum_{n=0}^{\infty} S(n) - (\alpha_2 + \mu) \sum_{n=0}^{\infty} V(n) \right] \frac{t^\beta}{\Gamma(\beta+1)} \\
 \sum_{n=0}^{\infty} E(n) &= n_3 + \left[\frac{\beta \left(\sum_{n=0}^{\infty} A(n) + \sum_{n=0}^{\infty} B(n) + \sum_{n=0}^{\infty} C(n) \right)}{N} + \alpha_2 \sum_{n=0}^{\infty} V(n) - (\omega_1 + \mu) \sum_{n=0}^{\infty} E(n) \right] \frac{t^\beta}{\Gamma(\beta+1)} \\
 \sum_{n=0}^{\infty} I(n) &= n_4 + \left[\omega_1 \sum_{n=0}^{\infty} E(n) - (\varepsilon_1 + \psi_1 + \omega_2 + \mu) \sum_{n=0}^{\infty} I(n) \right] \frac{t^\beta}{\Gamma(\beta+1)} \\
 \sum_{n=0}^{\infty} H(n) &= n_5 + \left[\omega_2 \sum_{n=0}^{\infty} I(n) - (\psi_2 + \psi_3 + \varepsilon_2 + \mu) \sum_{n=0}^{\infty} H(n) \right] \frac{t^\beta}{\Gamma(\beta+1)} \\
 \sum_{n=0}^{\infty} D(n) &= n_6 + \left[\psi_1 \sum_{n=0}^{\infty} I(n) + \psi_2 \sum_{n=0}^{\infty} H(n) - \psi_4 \sum_{n=0}^{\infty} D(n) \right] \frac{t^\beta}{\Gamma(\beta+1)} \\
 \sum_{n=0}^{\infty} R(n) &= n_7 + \left[\psi_3 \sum_{n=0}^{\infty} H(n) - \mu \sum_{n=0}^{\infty} R(n) \right] \frac{t^\beta}{\Gamma(\beta+1)}
 \end{aligned} \right\} \quad (15)$$

When $n = 0$ we obtain,

$$S(0) = n_1, \quad V(0) = n_2, \quad E(0) = n_3, \quad I(0) = n_4, \quad H(0) = n_5, \quad D(0) = n_6, \quad R(0) = n_6 \quad (16)$$

When $n = 1$, we obtain,

$$\left. \begin{aligned}
 S(1) &= \left[\Lambda - \frac{\beta(A(0) + B(0) + C(0))}{N} - (\alpha_1 + \mu)S(0) \right] \frac{t^\beta}{\Gamma(\beta+1)} \\
 V(1) &= \left[\alpha_1 S(0) - (\alpha_2 + \mu)V(0) \right] \frac{t^\beta}{\Gamma(\beta+1)} \\
 E(1) &= \left[\frac{\beta(A(0) + B(0) + C(0))}{N} + \alpha_2 V(0) - (\omega_1 + \mu)E(0) \right] \frac{1}{S^{\beta+1}} \\
 I(1) &= \left[\omega_1 E(0) - (\varepsilon_1 + \psi_1 + \omega_2 + \mu)I(0) \right] \frac{1}{S^{\beta+1}} \\
 H(1) &= \left[\omega_2 I(0) - (\psi_2 + \psi_3 + \varepsilon_2 + \mu)H(0) \right] \frac{1}{S^{\beta+1}} \\
 D(1) &= \left[\psi_1 I(0) + \psi_2 H(0) - \psi_4 D(0) \right] \frac{1}{S^{\beta+1}} \\
 R(1) &= \left[\psi_3 H(0) - \mu R(0) \right] \frac{1}{S^{\beta+1}}
 \end{aligned} \right\} \quad (17)$$

When $n = 2$, we obtain,

$$\left. \begin{aligned}
 S(2) &= \left[\Lambda - \frac{\beta(A(1)+B(1)+C(1))}{N} - (\alpha_1 + \mu)S(1) \right] \frac{t^\beta}{\Gamma(\beta+1)} \\
 V(2) &= \left[\alpha_1 S(1) - (\alpha_2 + \mu)V(1) \right] \frac{t^\beta}{\Gamma(\beta+1)} \\
 E(2) &= \left[\frac{\beta(A(1)+B(1)+C(1))}{N} + \alpha_2 V(1) - (\omega_1 + \mu)E(1) \right] \frac{1}{S^{\beta+1}} \\
 I(2) &= \left[\omega_1 E(1) - (\varepsilon_1 + \psi_1 + \omega_2 + \mu)I(1) \right] \frac{1}{S^{\beta+1}} \\
 H(2) &= \left[\omega_2 I(1) - (\psi_2 + \psi_3 + \varepsilon_2 + \mu)H(1) \right] \frac{1}{S^{\beta+1}} \\
 D(2) &= \left[\psi_1 I(1) + \psi_2 H(1) - \psi_4 D(1) \right] \frac{1}{S^{\beta+1}} \\
 R(2) &= \left[\psi_3 H(1) - \mu R(1) \right] \frac{1}{S^{\beta+1}} \\
 \vdots &= \vdots
 \end{aligned} \right\} \quad (18)$$

When $n = n+1$, we obtain,

$$\left. \begin{aligned}
 S(n+1) &= \left[\Lambda - \frac{\beta(A(n)+B(n)+C(n))}{N} - (\alpha_1 + \mu)S(n) \right] \frac{t^\beta}{\Gamma(\beta+1)} \\
 V(n+1) &= \left[\alpha_1 S(n) - (\alpha_2 + \mu)V(n) \right] \frac{t^\beta}{\Gamma(\beta+1)} \\
 E(n+1) &= \left[\frac{\beta(A(n)+B(n)+C(n))}{N} + \alpha_2 V(n) - (\omega_1 + \mu)E(n) \right] \frac{1}{S^{\beta+1}} \\
 I(n+1) &= \left[\omega_1 E(n) - (\varepsilon_1 + \psi_1 + \omega_2 + \mu)I(n) \right] \frac{1}{S^{\beta+1}} \\
 H(n+1) &= \left[\omega_2 I(n) - (\psi_2 + \psi_3 + \varepsilon_2 + \mu)H(n) \right] \frac{1}{S^{\beta+1}} \\
 D(n+1) &= \left[\psi_1 I(n) + \psi_2 H(n) - \psi_4 D(n) \right] \frac{1}{S^{\beta+1}} \\
 R(n+1) &= \left[\psi_3 H(n) - \mu R(n) \right] \frac{1}{S^{\beta+1}}
 \end{aligned} \right\} \quad (19)$$

The series solution of each compartment can be expressed as:

$$S(t) = S(0) + S(1) + S(2) + \dots$$

$$V(t) = V(0) + V(1) + V(2) + \dots$$

$$E(t) = E(0) + E(1) + E(2) + \dots$$

$$\begin{aligned}
I(t) &= I(0) + I(1) + I(2) + \dots \\
H(t) &= H(0) + H(1) + H(2) + \dots \\
D(t) &= D(0) + D(1) + D(2) + \dots \\
R(t) &= R(0) + R(1) + R(2) + \dots
\end{aligned} \tag{20}$$

Numerical Solution of Laplace Adomian Decomposition Method (LADM)

In this section, we will see the numerical solution of the model. Using the initial conditions, the Laplace Adomian Decomposition Method (LADM) gives us an approximate solution in terms of an infinite series presented as [7,8]:

$$\left. \begin{aligned}
S(t) &= 4500000 - 867487.92 \frac{t^\beta}{\Gamma(\beta+1)} + 164482 \frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
V(t) &= 2500000 - 1020250 \frac{t^\beta}{\Gamma(\beta+1)} + 578180.69 \frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
E(t) &= 2000000 + 1627618 \frac{t^\beta}{\Gamma(\beta+1)} - 888367.08 \frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
I(t) &= 1500000 + 14270 \frac{t^\beta}{\Gamma(\beta+1)} + 146372.79 \frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
H(t) &= 1000000 - 260030 \frac{t^\beta}{\Gamma(\beta+1)} + 108047.10 \frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
D(t) &= 200000 + 342300 \frac{t^\beta}{\Gamma(\beta+1)} - 89392.57 \frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots \\
R(t) &= 950000 - 5235 \frac{t^\beta}{\Gamma(\beta+1)} - 742.35 \frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots
\end{aligned} \right\} \tag{21}$$

For $\beta = 1$, the series solution of (1) model becomes,

$$\left. \begin{aligned}
 S(t) &= 4500000 - 867487.92t + 82241.35t^2 + \dots \\
 V(t) &= 2500000 - 1020250 \frac{t^\beta}{\Gamma(\beta+1)} + 289090.35t^2 + \dots \\
 E(t) &= 2000000 + 1627618 \frac{t^\beta}{\Gamma(\beta+1)} - 444183.54t^2 + \dots \\
 I(t) &= 1500000 + 14270 \frac{t^\beta}{\Gamma(\beta+1)} + 73186.39t^2 + \dots \\
 H(t) &= 1000000 - 260030 \frac{t^\beta}{\Gamma(\beta+1)} + 54023.55t^2 + \dots \\
 D(t) &= 200000 + 342300 \frac{t^\beta}{\Gamma(\beta+1)} - 44696.28t^2 + \dots \\
 R(t) &= 950000 - 5235 \frac{t^\beta}{\Gamma(\beta+1)} - 371.17t^2 + \dots
 \end{aligned} \right\} \quad (22)$$

Numerical Simulation

Numerical simulation is pivotal in mathematical epidemiology, enabling researchers to model and analyze the dynamics of disease spread within populations [13]. These models typically involve systems of differential equations that capture factors such as infection rates, recovery rates, and population movement patterns [12]. By numerically solving these equations over time, simulations provide predictions on how diseases may evolve under various conditions. Numerical simulation also supports parameter estimation and sensitivity analysis in epidemiological modeling. Parameter estimation involves fitting model predictions to real-world data to determine parameters like transmission rates or initial conditions [12]. Sensitivity analysis assesses how changes in parameters affect model outcomes, providing insights into the robustness of predictions [13].

Furthermore, simulations allow researchers to explore different scenarios and intervention strategies. For instance, simulations can assess the impact of vaccination campaigns, social distancing measures, or changes in healthcare capacity on disease spread dynamics [12]. This capability is crucial for informing public health policies and interventions during disease outbreaks. Numerical simulation plays a vital role in advancing our understanding of disease dynamics, aiding in the development of effective public health strategies, and supporting preparedness efforts for future outbreaks.

Table 1 Parameter table of values

Parameter	Value	Source
Λ	0.202	Assumed
μ	0.03	[14]
α_1	0.25	[14]
λ	0.001	Assumed
ω_1	0.1	[16]
α_2	0.001	Assumed
ε_1	0.5	Assumed
ε_2	0.15	[16]
ω_2	0.80	[17]

ψ_1	0.01	[17]
ψ_2	0.02	Assumed
ψ_3	0.982	[14]
ψ_4	0.0025	Assumed
β	0.027	Assumed

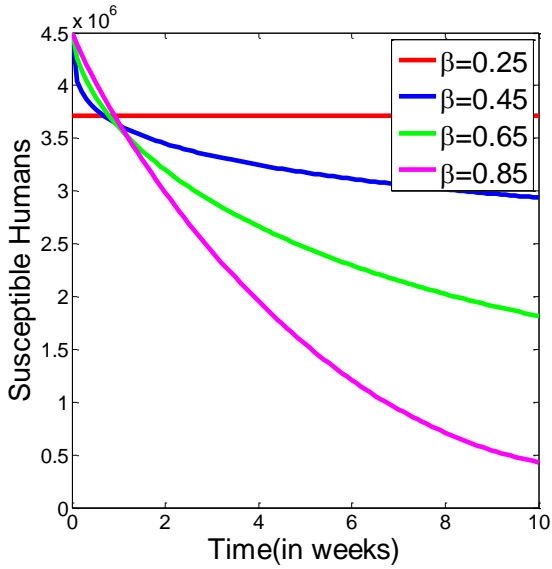


Figure 2a. Effect of varying β on susceptible Population

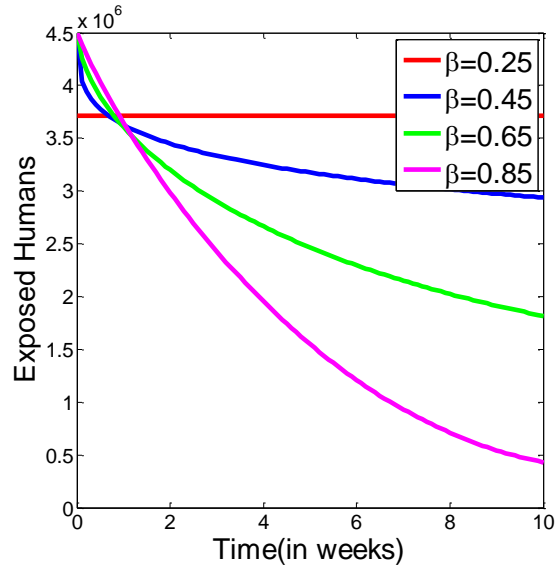


Figure 2b. Effect of varying β on exposed Population

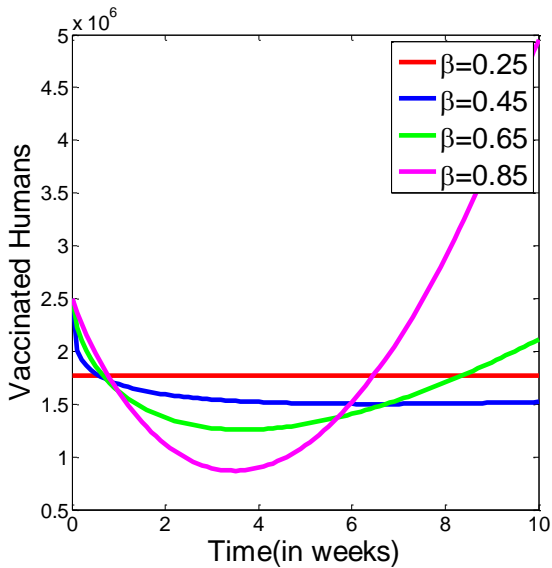


Figure 2c. Effect of varying β on vaccinated Population

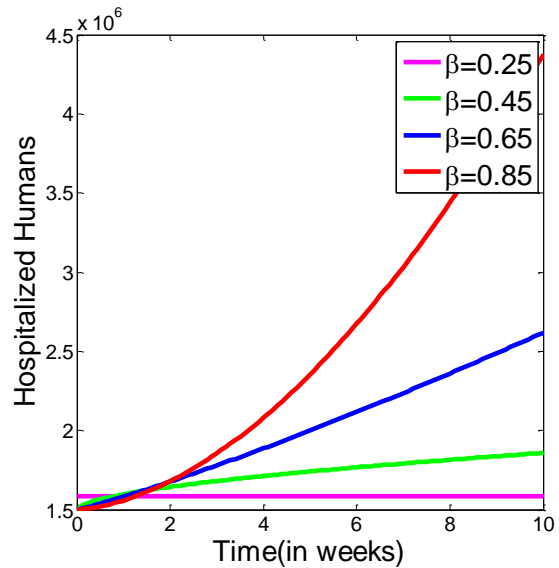


Figure 2d. Effect of varying β on hospitalized Population

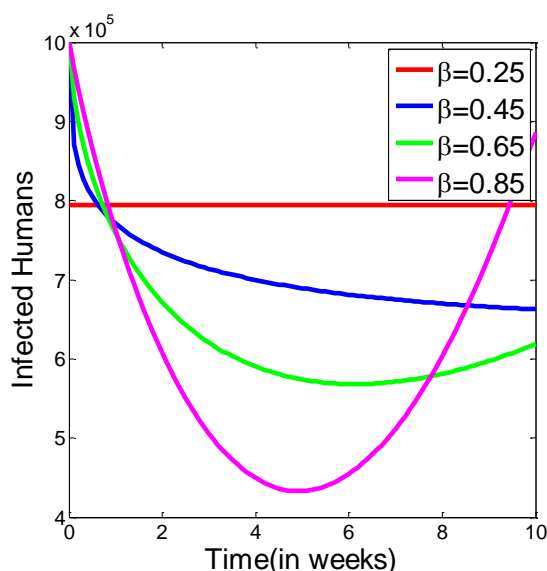


Figure 2e. Effect of varying β on infected Population

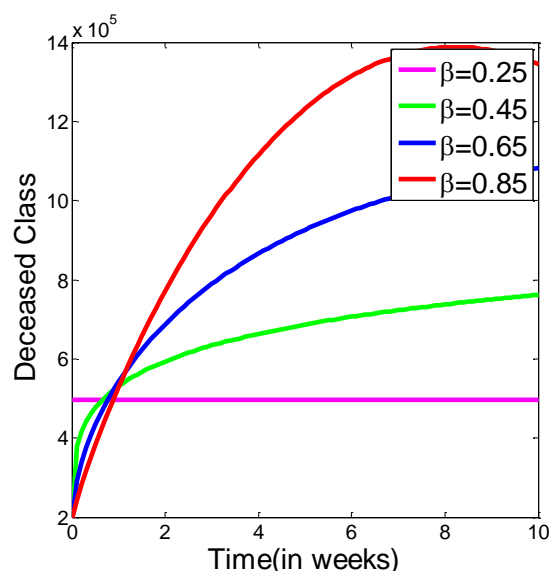


Figure 2f. Effect of varying β on deceased Population

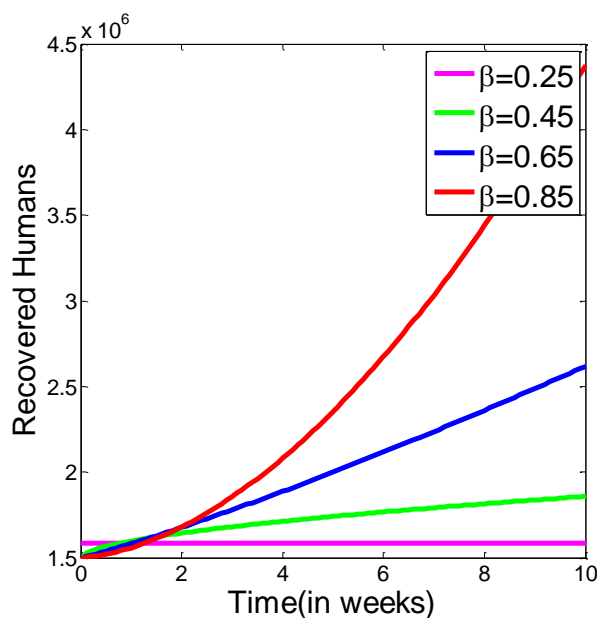


Figure 2g. Effect of varying β on recovered Population

RESULTS AND DISCUSSION

The study evaluated the effectiveness of the Laplace-Adomian Decomposition Method (LADM) for solving fractional-order models of Ebola Virus Disease (EVD). The convergence analysis confirmed that LADM provided a uniformly convergent series solution, validating its accuracy for approximating complex disease models. The method demonstrated several benefits,

including rapid convergence, reduced computational complexity, and the capability to manage various nonlinearities. This made LADM a powerful tool for analyzing disease dynamics and performing parameter sensitivity analyses. The figures presented provided detailed insights into the effectiveness of strategies for controlling Ebola Virus Disease (EVD). Figure 2a illustrated that the number of susceptible individuals declined over time, indicating that

vaccination and preventive measures were successful in reducing the at-risk population. Figure 2b demonstrated that while the number of exposed individuals remained relatively stable initially, it eventually decreased as high vaccination rates took effect, suggesting effective control over the transition from exposure to symptomatic disease.

Figure 2d depicted an increase in the number of hospitalized individuals, which aligned with enhanced treatment efforts for severe cases. This rise in hospitalizations was associated with a decrease in the number of infected individuals, as shown in Figure 2e, reflecting the positive impact of medical treatment on disease control. Figure 2f confirmed high recovery rates, indicating that treatment measures were effective in managing the disease. Meanwhile, figure 2g revealed an initial increase in the deceased population followed by a decline, which suggested that mortality rates were eventually reduced due to successful disease management. Overall, these figures highlighted the effectiveness of vaccination, treatment, and healthcare interventions in controlling and mitigating the spread of EVD.

Convergence Analysis for the Laplace-Adomian Decomposition Method (LADM).

The solution of (1) is expressed in the forms of infinite series which converged uniformly to its exact solution. To verify the convergence of the series (22), we employ the method used in [7,8]. For sufficient conditions of convergence of the LADM, we present the following theorem:

Theorem 1

Let X be a Banach space and $T : X \rightarrow X$ be a constructive nonlinear operator such that for $(x), (x') \in X$, $\|T(x) - T(x')\|, 0 < k < 1$. Then, T has a unique point x such that $Tx = x$, where $x = (S, V, E, I, H, D, R)$. The series given can be written by applying the Adomian decomposition method as follows [7,8]:

$$\begin{aligned} x_n &= Tx_{n-1}, x_{n-1}, \\ &= \sum_{i=1}^{n-1} x_i, \quad n = 1, 2, 3, \dots \end{aligned}$$

And we assume that $x_0 \in B_r(x)$, where $B_r(x) = \{x \in X : \|x' - x\| < r\}$; then, we have as follows:

- (i) $x_n \in B_r(x)$
- (ii) $\lim_{n \rightarrow \infty} x_n = x$

Proof

For condition (i), invoking mathematical induction,

For $n=1$, we have as follows:

$$\|x_0 - x\| = \|T(x_0) - T(x)\| \leq \|x_0 - x\|.$$

If this is true for $m-1$, then

$$\|x_0 - x\| \leq k^{m-1} \|x_0 - x\|.$$

This gives the following:

$$\|x_m - x\| = \|T(x_{m-1}) - T(x)\| \leq k \|x_{m-1} - x\| \leq k^m \|x_0 - x\|.$$

Therefore,

$$\|x_m - x\| \leq k^n \|x_0 - x\| \leq k^n r < r.$$

This directly implies that $x_n \in B_r(x)$.

Also, for (ii), we have that since $\|x_m - x\| \leq k^n \|x_0 - x\|$ and $\lim_{n \rightarrow \infty} k^n = 0$, we can write $\lim_{n \rightarrow \infty} x_n = x$.

Benefits of Using the Laplace-Adomian Decomposition Method (LADM) to Obtain Series Solutions for Disease Models, Such as an Ebola Model:

- **Analytical Insight:** LADM provides an analytical series solution that offers a deep understanding of disease dynamics, revealing insights into how different

parameters influence the spread and control of diseases [13].

- **Rapid Convergence:** The series solution obtained through LADM typically converges rapidly, even for nonlinear problems like disease models, ensuring that truncated series provide accurate approximations of solutions [12].
- **Reduced Computational Complexity:** LADM reduces the computational complexity of solving partial differential equations (PDEs) or nonlinear ordinary differential equations (ODEs) compared to traditional numerical methods, making it efficient for modeling complex disease dynamics [12].
- **Parameter Sensitivity Analysis:** With the series solution from LADM, researchers can easily perform sensitivity analyses on model parameters, assessing how changes in parameters affect disease dynamics and intervention strategies [20].
- **Validation and Comparison:** The series solution derived from LADM serves as a reliable benchmark for validating results obtained through other numerical or simulation methods, ensuring the robustness and accuracy of disease models [22].
- **Flexibility in Model Complexity:** LADM can handle a wide range of nonlinearities and complexities in disease models, making it adaptable for studying various real-world scenarios and dynamic interactions [19].
- **Insights into Long-Term Dynamics:** The series solution obtained through LADM provides insights into long-term behaviors and asymptotic stability of disease models, which is crucial for predicting outcomes over extended periods [18].

CONCLUSION

Figure 2a shows the graph of Susceptible individuals over time. It is observed that the number of susceptible individuals decreases to zero over time, indicating effective disease control. In Figure 2b, the number of exposed

individuals remains almost constant initially but later decreases as time progresses, likely due to effective high vaccination rates (refer to Figure 2c), which suggests that the disease can be controlled within the population. In Figure 2d, the number of hospitalized individuals increases, leading to a decrease in the number of infected individuals in Figure 2e and a high recovery rate in Figure 2f, implying effective treatment measures. Consequently, the deceased population initially increases but later decreases, as illustrated in Figure 2g.

In conclusion, the use of the Laplace Adomian decomposition method to model Ebola Virus disease incorporating vaccination and hospitalization as control measures demonstrates significant efficacy in disease control. Numerical simulations underscore the critical role of high treatment and effective vaccination in mitigating Ebola transmission and reducing its impact on population health. Implementing comprehensive vaccination campaigns, strengthening healthcare infrastructure, and fostering community engagement are essential steps in enhancing preparedness and response strategies against future Ebola outbreaks.

Findings from the Study

- **Effective Disease Control:** High treatment and vaccination rates lead to a significant reduction in Ebola transmission within the population.
- **Impact of Vaccination:** Effective vaccination strategies result in a stable and subsequently decreasing number of exposed individuals over time.
- **Hospitalization Effects:** Increased hospitalization rates correspond to a decrease in the number of infected individuals, reflecting successful treatment interventions.
- **Recovery Rate:** The model shows a high recovery rate among infected individuals due to prompt hospitalization and treatment.
- **Mortality Trends:** Initially rising mortality rates stabilize and decrease with sustained

control measures, indicating successful disease management.

Recommendations:

- Enhanced Vaccination Campaigns: Effort should be taken to implement comprehensive vaccination programs to ensure high coverage and effectiveness against Ebola outbreaks.
- Improvement in Hospital Infrastructure: Government should Strengthen healthcare facilities to enhance the capacity for prompt diagnosis, treatment, and isolation of Ebola cases.
- Training and Education: Government should provide continuous training for healthcare workers on Ebola prevention, early detection, and management protocols.
- Community Engagement: Effort should be taken to foster community participation and awareness to promote vaccination acceptance and adherence to treatment protocols.
- Global Collaboration: Effort should be taken foster international collaboration and resource-sharing to enhance preparedness and response capabilities against Ebola outbreaks.

Conflict of interest

The authors declare that they have no competing interest

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Availability of data.

The data used in this study are referenced and presented in table 2 above.

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