

PERFORMANCE EVALUATION OF TEN NUMERICAL METHODS FOR WEIBULL DISTRIBUTION PARAMETER ESTIMATION APPLIED TO NIGERIAN WIND SPEED DATA

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ABSTRACT

Utilizing wind energy necessitates a thorough understanding of wind profiles as well as a precise forecast of wind speed at a study location. In this study, ten Numerical Methods (NEMs), which include the Empirical Method of Lysen (EML), Percentile Method (PCM), Maximum Likelihood Method (MLM), Modified Maximum Likelihood Method (MMLM), Empirical Method of Justus (EMJ), Alternative Moment Method (AMM), Median and Quartiles Method (MQM), Probability Weighted Moments Based on Power Density Method (PWMBPM), Method of Mabchour (MOMAB) and Energy Variance Method (EVM) were applied to estimate the two-parameter (k and c) Weibull (Wbl) distribution in five locations (Jos, Kano, Maiduguri, Abuja, and Akure) in Nigeria. The performance of these NEMs was assessed using five different metrics and the most effective NEM was determined for each studied location. Daily wind speed data spanning 11 years for the studied locations were sourced from the Meteorological Agency in Nigeria and used in this study. The k and c parameters range from 2.91 to 5.46 and 9.95 to 10.26 (Kano); 2.31 to 4.50 and 5.63 to 6.20 (Maiduguri); 3.19 to 7.61 and 12.16 to 12.99 (Jos); 2.18 to 6.77 and 4.99 to 5.50 (Abuja), and 1.84 to 3.18 and 3.83 to 3.90 (Akure). Findings revealed that the best methods for estimating Wbl parameters for the Kano, Maiduguri, Jos, Abuja, and Akure locations were MMLM, MMLM, MQM, MQM, and EMJ, EML, and AMM, respectively, as MOMAB remained the least performing NEM for all the studied locations. The results also showed that the V_{ms} , V_{mps} , and $V_{e\max}$ varied from 3.47 m/s to 11.63 m/s, 3.40 m/s to 11.90 m/s, and 4.58 m/s to 12.59 m/s, respectively, with the most recorded for Jos. The P_{WPD} augmented from 36.45 W/m² (Akure) to 1000.06 W/m² (Jos), at a hub height of 10 m. Based on these results Jos was the best location for installing wind turbines while Kano was an excellent place for integrating the grid. Additionally, the Maiduguri location was determined to be suitable for a stand-alone application while Abuja and Akure were considered to be unsuitable for wind energy applications.

Keywords: Wind speed, Wind energy, Numerical method, Weibull distribution, Probability density function.

INTRODUCTION

The primary source for meeting global energy demand has been the utilization of fossil fuels. However, there are just a few fossil fuel deposits left, and burning them has a terrible impact on human health and the ecosystem. The possibility of wind energy source for electricity generation has gained more acceptance around the world due to its affordability, cleanliness, abundance in nature, and environmental friendliness (Okakwu *et al.*, 2021). The need to reduce the continuous use of fossil fuels due to its rapid depletion, unstable prices, and harmful effects on public health and the environment has necessitated the need for a cleaner and more sustainable energy source such as wind. Utilizing a wind turbine to transform wind into a more usable type of energy (electricity) is known as wind energy (or wind power) (Panwar *et al.*, 2011). Wind energy, one of the main renewable energy sources, can provide more than four times the annual world electricity demand (Lu *et al.*, 2009; Jung and Schindler, 2016). Numerous benefits come with using wind energy as an alternative energy source, such as lower energy costs, environmental friendliness, job opportunities, increased economic activity, etc. (Masseran, 2015; Abbasi and Abbasi, 2016; Masseran and Razali, 2016; Sulaiman *et al.*, 2020; Sohoni *et al.*, 2016).

Due to the cubic relationship of wind speed with wind power, which means a significant variation in power will occur even with a small variation in wind speed. The wind speed is the most important factor in determining wind energy, and is therefore essential to harnessing the wind energy potential of a location. Therefore, some specific statistical distribution (probability distribution function) that must provide the best fit for the wind data must be used to describe the data. Due to its flexibility, two-parameter nature, ease of calculating the parameters, and near-exact findings when compared to other probability density functions, the Weibull (Wbl) distribution

function is reportedly a more practical way of describing wind speed data for wind energy evaluation applications (Golam *et al.*, 2023). Therefore, choosing an effective method for calculating the Wbl parameters is a crucial component of evaluating the wind energy potential of a location using the Wbl distribution. A biased evaluation of the Wbl characteristics would undoubtedly result in an incorrect estimation of wind power production.

The numerical method (NEM) is the technique that is most frequently employed in the literature for evaluating the Wbl parameters in wind energy potential evaluation. Prominent NEMs deployed for estimating the Wbl parameters have been published in several researches. In a study by Werapun *et al.*, 2015, five techniques (Empirical Method of Justus (EMJ), energy pattern factor method (EPM), Maximum Likelihood Method (MLM), Modified Maximum Likelihood Method (MMLM) and graphical Method (GM)) were used to calculate the Wbl parameters from 2012 to 2014 for the wind speed data. To find the best fit, they evaluated the data using various statistics-based metrics (root mean square error (RMSE), R^2 , percent error, and Kolmogorov-Sminorv (KS)). The MLM was the best NEM, according to their findings. A comparison of NEMs, including the mean-standard deviation method (MSDM), rank regression method (RRM), power density method (PDM), and MLM was conducted in (Ahmed, 2013) to model the wind speed frequency distribution for 4 years (2001-2004). RMSE and R^2 were the metrics utilized to determine the best fits. The best NEM was found to be the RRM. In (Parajuli, 2021), the Wbl parameter for wind speed data in Jumla, Nepal was estimated using the method of moment (MOM), least square error method (LSEM), and PDM. The most effective strategy, which was MOM, was determined using the RMSE. For the purpose of calculating the Wbl parameter utilized in wind energy, the wind energy assessment of Meşelik region in Eskişehir was modeled

with two-parameter Wbl distribution between 2013 to 2014, comparison research of NEMs employing MLM, LSEM, MOM, logarithmic moment method (LMM), percentile method (PCM), and L-moment technique (LM) was conducted in (Aras *et al.*, 2020). The best approach for estimating the Wbl parameters was MLM, which was demonstrated by comparing the accuracy of their results using mean square error (MSE) and mean absolute percent error (MAPE). The Wbl parameters of the wind speed data of Jeju Island, South Korea were estimated using six NEMs (EMJ, MOM, GM, EPM, MLM, and MMLM) by Kang *et al.*, 2018. Their findings show that, while estimating the Wbl parameters, MOM performed the best while GM did the worst.

To estimate the Wbl characteristics of wind data in Zuwara, Libya, Teyabeen *et al.*, 2017, explored the use of seven NEMs (GM, MSDM, EMJ, Empirical Method of Lysen (EML), EPM, MLM, and MMLM). Using MAPE, mean absolute bias error (MABE), RMSE, and R^2 , respectively, the performance of these seven NEMs was assessed. EMJ and EML recorded the best performance whereas GM performed least according to the deployed performance metrics. In Guarienti *et al.*, 2020, an examination of the performance of six NEMs for calculating Wbl characteristics applied to Brazilian wind speed data was carried out. R^2 , RMSE, and KS were used to evaluate the approaches' accuracy. The best NEMs were determined to be the MLM and MMLM. The equivalent energy method (EEM) and seven alternative NEMs (GM, MOM, MSDM, MLM, PDM, MMLM, and STDm) were utilized to estimate the Wbl parameters for the assessment of wind speed in Bangladesh (Azad *et al.*, 2014). The two techniques found to be most effective for estimating the Wbl were MOM and MLM. To estimate the Wbl characteristics for a wind energy application in Eastern Jerusalem, Palestine, Alsamamra *et al.*, 2022, deployed five NEMs (MLM, MMLM, MOM, EPM, and EMJ). The EMJ and MOM were the most accurate, according to their findings, which were supported by

three goodness-of-fit methodologies (MAPE, RMSE, and χ^2). A study of the wind energy potential of China's onshore and offshore coastal regions was conducted by Li *et al.*, 2020. In this study, they examined four NEMs for estimating Wbl parameters: MOM, MLM, PDM, and curve fitting technique (CFM). Six distinct NEMs were used by Mohammadi *et al.*, 2016 in four different Canadian locales. The authors concluded that MLM, EMJ, and EML were superior to the other approaches. Six NEMs were examined by Khalid *et al.*, 2019 for estimating Wbl parameters of wind speed between 2016 to 2018 in Pakistan, and they concluded that the MLM is the most effective of all the NEMs.

The above literature surveys showed the estimation of Wbl parameters of wind speed using data garnered from various locations in different countries of the world which cut across developed and developing nations and intercontinental spread. In addition, various NEMs have been deployed by different authors for the estimation of Wbl parameters of wind speed in different countries. However, documentation in this regard is lacking for the continent of Africa as there is potential for wind power generation as a renewable energy source in the continent (Oyewo *et al.*, 2023). Of huge interest is Nigeria, which has the untapped wind power potential and estimating the Wbl parameters of wind speed in the country would be a giant stride in harnessing and documenting the wind energy potential in the country. As a signatory to the sustainable development goals (SDG) and other similar parties on renewable and sustainable energy development, Nigeria is committed to transitioning from a carbon-based economy to a zero-carbon economy. In the current study, the Wbl parameters of wind speed data for five different locations in Nigeria were estimated using 10 distinct NEMs (both standard and rare approaches). Five goodness-of-fit parameters were used to evaluate the effectiveness of these NEMs in estimating the Wbl parameters. The best NEM that would provide a more accurate

estimate of the Wbl parameter was achieved in an attempt to lower the level of uncertainty in forecasting wind energy at the five different locations in Nigeria.

MATERIALS AND METHODS

Study coverage area and Data source

In the current study, Kano, Maiduguri, Jos, Abuja, and Akure were chosen as the locations in Nigeria whose wind speed data were used to estimate the Wbl parameters. The choice of Nigeria was premised on its popularity, population, and need to shift from a fossil fuel-dependent nation to renewable energy to achieve sustainable development goals. The choice of these locations was based on the high wind speed published concerning these areas (Okakwu *et al.*, 2023) and the need to shift from fossil fuel dependence in Nigeria. The wind speed data for these geographical locations in Nigeria was sourced from a Meteorological Agency

in Nigeria. The sourced data were for 11 years (2002 – 2012). The geographical coordinates of the case study areas is given in Ayodele *et al.*, 2018.

Numerical methods for estimation of Weibull parameters

Various numerical techniques can be used to estimate the Wbl parameters (k and c). However, in the present work, 10 distinct NEMs (both standard (MLM, MMLM, EMJ, EML, and PCM) and rare approaches) were used to estimate the Wbl parameters of the sourced wind speed data. The choice of these NEMs was based on their performance in the estimation of Wbl parameters as related in the literature (for standard NEMs) and prediction accuracy for the rare NEMs (Bidaoui *et al.*, 2019). Five goodness of fit metrics were used to evaluate the prediction effectiveness of these NEMs.

Maximum likelihood method (MLM)

Eqs. (1) and (2) were used to compute the k and c parameters using the maximum likelihood method (MLM) (Mostafaeipour *et al.*, 2014; Belabes *et al.*, 2015; Safari *et al.*, 2022):

$$k = \left[\frac{\sum_{i=1}^n V_i^k \ln(V_i)}{\sum_{i=1}^n V_i^k} - \frac{\sum_{i=1}^n \ln(V_i)}{n} \right]^{-1} \quad (1)$$

$$c = \left[\frac{1}{n} \sum_{i=1}^n V_i^k \right]^{\frac{1}{k}} \quad (2)$$

where; n is the number of "non-zero" wind speed data points and V_i is the wind speed in time step i . The Wbl parameters in the MLM must be determined by numerical iterations.

Modified maximum likelihood method (MMLM)

Eqs. (3) and (4) were used to compute the k and c parameters using the modified maximum likelihood method (MMLM) (Mostafaeipour *et al.*, 2014; Belabes *et al.*, 2015; Safari *et al.*, 2022):

$$k = \left[\frac{\sum_{i=1}^n V_i^k \ln(V_i) f(V_i)}{\sum_{i=1}^n V_i^k f(V_i)} - \frac{\sum_{i=1}^n \ln(V_i) f(V_i)}{f(V_i \geq 0)} \right]^{-1} \quad (3)$$

$$c = \left[\frac{\sum_{i=1}^n V_i^k f(V_i)}{f(V_i \geq 0)} \right]^{\frac{1}{k}} \quad (4)$$

where; $f(V_i)$ is the Weibull frequency of the wind speed in intervals i and $f(V \geq 0)$ is the probability of the wind speed for $V_i \geq 0$, where V_i is the wind speed central to bin i . The Wbl parameters in the MMLM must be determined by numerical iterations.

Empirical method by Justus (EMJ)

The k parameter was estimated by Eq. (5) while the c parameter was determined using Eq. (6) according to the empirical technique (EMJ) suggested by Justus (Bidaoui *et al.*, 2019):

$$k = \left[\frac{\sigma}{\bar{V}} \right]^{-1.086} \quad (5)$$

$$c = \frac{\bar{V}}{\Gamma\left(1 + \frac{1}{k}\right)} \quad (6)$$

In such a situation, the standard deviation and mean wind speed were represented by σ and \bar{V} respectively (Akdag and Guler, 2018).

Empirical method by Lysen (EML)

The c parameter was determined using Eq. (7), while the k parameter was estimated by Eq. (5) in the empirical technique (EML) suggested by Lysen (Bidaoui *et al.*, 2019):

$$c = \left[0.568 + \frac{0.433}{k} \right]^{\frac{1}{k}} \quad (7)$$

Alternative moment method (AMM)

The k parameter was computed using Eq. (8) for the alternative moment method (AMM), and the c parameter was estimated by applying Eq. (6) (Akdag and Guler, 2018):

$$k = \frac{a_0 + a_1 \left[\frac{\sigma}{\bar{V}} \right] + a_2 \left[\frac{\sigma}{\bar{V}} \right]^2 + a_3 \left[\frac{\sigma}{\bar{V}} \right]^3 + a_4 \left[\frac{\sigma}{\bar{V}} \right]^4}{b_0 + b_1 \left[\frac{\sigma}{\bar{V}} \right] + b_2 \left[\frac{\sigma}{\bar{V}} \right]^2 + b_3 \left[\frac{\sigma}{\bar{V}} \right]^3 + b_4 \left[\frac{\sigma}{\bar{V}} \right]^4} \quad (8)$$

Using Eq. (6), Table 1 provides the coefficients of a_i and b_i for $i = 0, 1, 2, 3, 4$.

Table 1: Coefficients of a_i and b_i when using AMM (Akdag and Guler, 2018).

i	a	b
0	294843×10^{-5}	320694×10^{-12}
1	150722×10^{-5}	229887×10^{-5}
2	256734×10^{-5}	248525×10^{-5}

3	90316.4×10^{-5}	235103×10^{-5}
4	20899.5×10^{-5}	1.0000×10^0

Percentile method (PCM)

To use the percentile method (PCM) to evaluate the Wbl parameters, a Wbl distribution's quantile function is expressed as follows according to Bidaoui *et al.*, 2019:

$$V_p = c \left[-\ln(1-p)^{\frac{1}{k}}, 0 < p < 1 \right] \quad (9)$$

By putting $p = 1 - \exp[-1] \cong 0.6321$, we obtain the parameter c as follows:

$$c = V_{0.6321} \quad (10)$$

where $V_{0.6321}$ is the 63.21th percentile of the wind speed data.

The percentile-based estimate for k was obtained by putting c into Eq. (9) to estimate V_p which was substituted in Eq. (11).

$$k = \frac{\ln[-\ln[1-p]]}{\ln[V_p] - \ln[V_{0.6321}]}, 0 < V_p < V_{0.6321} \quad (11)$$

The optimum value of p was assumed to be 0.31 (Bidaoui *et al.*, 2019):

Where V_p , c , and p , are quantile functions of the Weibull distribution, Weibull parameter, and optimum percentile estimation respectively.

Median and quartiles method (MQM)

Using the median and quartiles method (MQM) to estimate the Wbl parameters, the k and c parameters were given by Bidaoui *et al.*, 2019:

If the median, 25th percentile, and 75th percentile were V_m , $V_{0.25}$ and $V_{0.75}$ respectively.

$$k = \frac{1.572534}{\ln \left[\frac{V_{0.75}}{V_{0.25}} \right]} \quad (12)$$

$$c = \frac{V_m}{[\ln(2)]^{\frac{1}{k}}} \quad (13)$$

Probability weighted moment based on power density method (PWMBPDM)

The k parameter was derived using Eq. (14) (Bidaoui *et al.*, 2019):

Using the probability-weighted moment based on the power density method (PWMBPDM):

$$k = \frac{\ln(2)}{\ln(\bar{C})} \quad (14)$$

where, \bar{C} was given by Eq. (15) (Bidaoui *et al.*, 2019):

$$\bar{C} = \frac{\bar{V}}{\frac{2}{n(n-1)} \sum_i^n V_i [n-i]} \quad (15)$$

where V_i is the ordered sample wind speed data and \bar{V} is the mean wind speed, with $V_1 \leq V_2 \leq \dots \leq V_n$. Eq. (16) (Bidaoui *et al.*, 2019):

provides the value for the c parameter.

$$c = \left[\frac{\bar{V}^3}{\Gamma\left(1 + \frac{3}{k}\right)} \right]^{\frac{1}{3}} \quad (16)$$

Method of Mabchour (MOMAB)

Using the method of mabchour (MOMAB), Eqs. (17) and (6) were used to determine the k and c parameters, respectively (Bidaoui *et al.*, 2019):

$$k = 1 + [0.483(\bar{V} - 2)]^{0.51} \quad (17)$$

Energy variance method (EVM)

The energy variance method expressed in Eq. (18) was used to calculate the k parameter and Eq. (6) was used to determine the c parameter (Bidaoui *et al.*, 2019):

$$k = \left[\frac{\sum_i^n V_i^2}{n\sigma^2} \right]^{\frac{1}{2}} \quad (18)$$

Numerical method accuracy assessments

Five different statistical-based estimators—root mean square error (RMSE), coefficient of determination (R^2), the chi-square error (χ^2), mean absolute percentage error (MAPE), and mean absolute bias error (MABE)—were used to assess the precision of the NEMs deployed for estimating the Wbl parameters that were presented. The estimators were calculated using Eq. (19–23) (Azad *et al.*, 2014; Alsamamra *et al.*, 2022; Okakwu *et al.*, 2019):

$$RMSE = \left[\frac{1}{n} \sum_{i=1}^n (y_i - x_{iw})^2 \right]^{\frac{1}{2}} \quad (19)$$

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - x_{iw})^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (20)$$

$$\chi^2 = \sum_{i=1}^n \frac{(y_i - x_{iw})^2}{x_{iw}} \quad (21)$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{x_{iw} - y_i}{y_i} \right| \times 100\% \quad (22)$$

$$MABE = \frac{1}{n} \sum_{i=1}^n |x_{iw} - y_i| \quad (23)$$

where y_i is the frequency of the actual data, x_{iw} is the frequency of the Wbl parameter, n is the number of intervals, and \bar{y} is the average of the measured data.

Wind speed analysis

Eqs. (24) and (25) (Sulaiman *et al.*, 2020), were used to obtain the daily mean wind speed (\bar{V}) and the standard deviation (σ) of the wind resource data:

$$\bar{V} = \frac{1}{n} \left(\sum_{i=1}^n V_i \right) \quad (24)$$

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (V_i - \bar{V})^2} \quad (25)$$

where n and V_i are the number of wind speed data points and the daily wind speed, respectively. Eqs. (26) and (27) provide the wind power density function ($f_w(v)$) and the cumulative distribution function ($f_w(V)$) (Sulaiman *et al.*, 2020):

$$f_w(v) = \left(\frac{k}{c} \right) \left(\frac{v}{c} \right)^{k-1} \exp \left[- \frac{v}{c} \right]^k \quad (26)$$

$$f_w(V) = 1 - \exp \left[- \frac{V}{c} \right]^k \quad (27)$$

where the wind speed (in m/s), the shape parameter (which has no dimensions), and the scale parameter (in m/s) are each represented by the variables v , k , and c . Eqs. (28) and (29) also provide the wind speed at the highest energy ($V_{e \max}$) and the wind speed at the most likely speed (V_{mps}) (Sulaiman *et al.*, 2020):

$$V_{mps} = c \left(\frac{k-1}{k} \right)^{\frac{1}{k}} \quad (28)$$

$$V_{e \max} = c \left(\frac{k+2}{k} \right)^{\frac{1}{k}} \quad (29)$$

Wind power estimation

The wind potential capability of a specific location per unit swept area of the blades is indicated by the wind power density (P_{WPD}). The wind power density was estimated using Equation (30) (Sulaiman *et al.*, 2020):

$$P_{WPD} = \frac{1}{2} \rho c^3 \Gamma \left(1 + \frac{3}{k} \right) \quad (30)$$

where the air density (i.e., 1.225 kg/m^3) is represented by ρ , while the P_{WPD} parameter is expressed in W/m^2 .

RESULTS AND DISCUSSION

Wind speed distribution of estimated wbl probability distribution function

Figures 1 to 5 present a comparative graphic wind speed histogram with the predicted probability density functions (PDFs) for the various NEMs at the designated study locations. These Figures make it obvious that each NEM's PDF corresponds to the location-specific observed wind speed histogram. Generally, it was observed that as k increases, the Wbl density function narrows and becomes peaked, indicating that wind speeds tend to remain within a narrow range. As the value of c rises, the peak also shifts toward the direction of higher wind speeds. As an illustration, figure 1 show that the computed value of k was higher for PWMBPM, AMM, EMJ, and EML than for other NEMs, leading to the peak character of the PDFs relative to others. The NEMs in Figure 1 have close-knit parameter c ranges, hence there was hardly any significant overlap amongst the curves. PWMBPM, AMM, EMJ, EML, and MMLM have peakier PDFs than other NEMs in Figure 2 due to the computed value of k being higher for these NEMs than for other NEMs. Except for MOM, which is higher, and MQM, which is least, the parameter c ranges of the NEMs in Figure 2 are near to one another. When compared to other NEMs in Figure 3, the computed value of k is higher for PCM, MQM, PWMBPM, AMM, EMJ, EML, and MMLM, which results in the peak character of the PDFs. The NEMs in Figure 3 have close-knit parameter c ranges, hence there was hardly any significant overlap amongst the curves. The peak character of the PDFs in Figure 4 is due to the computed value of k being higher for PCM, MQM, PWMBPM, AMM, EMJ, and EML compared to other NEMs. The curves in Figure 4 hardly overlap because the NEMs' parameter c ranges are so near to one another. AMM, EMJ, EML, MLM, and PWMBPM PDFs in Figure 5 have peaks compared to other PDFs because the computed value of k for these NEMs is higher than it is for other NEMs. There wasn't much overlap between the curves because the parameter c ranges of the NEMs in Figure 5 are near to one another.

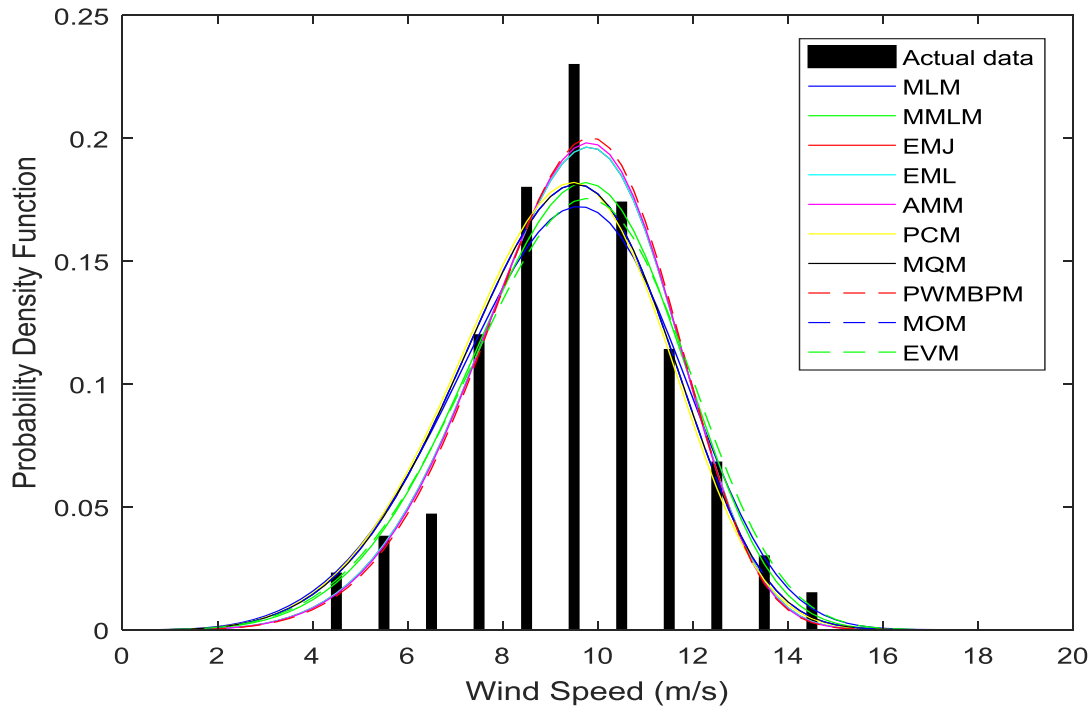


Figure 1: Comparison of the PDFs for Kano

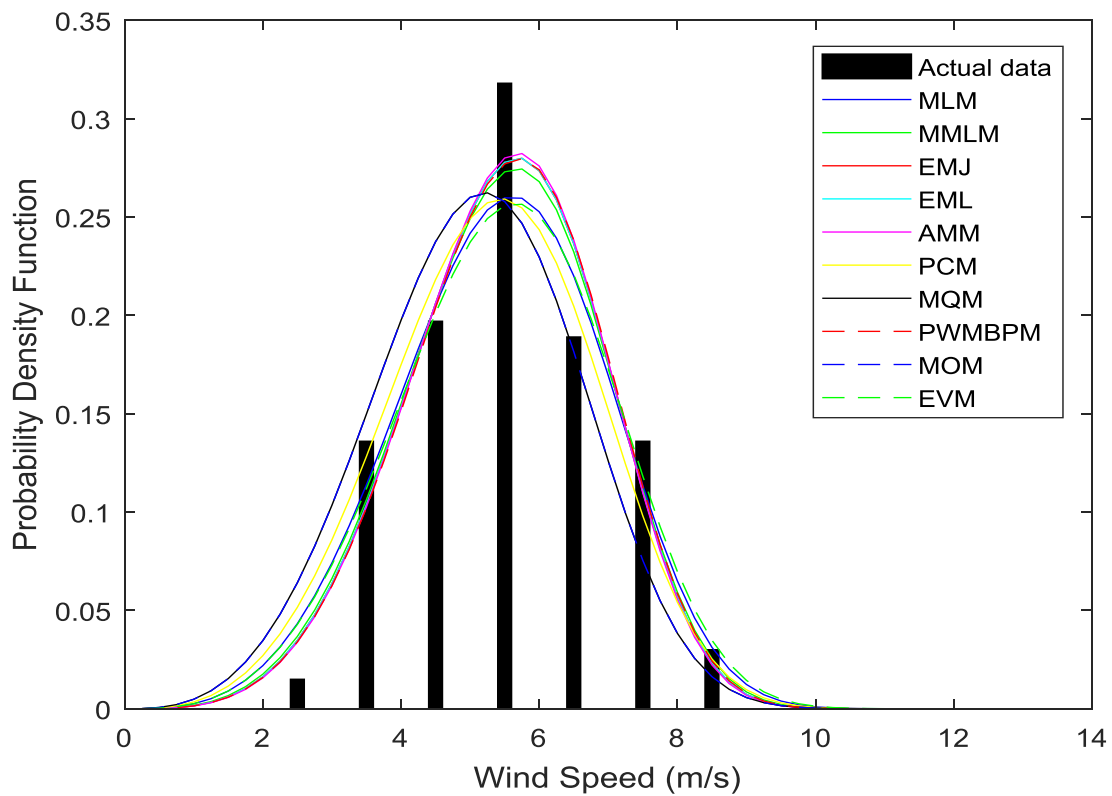


Figure 2: Comparison of the PDFs for Maiduguri.

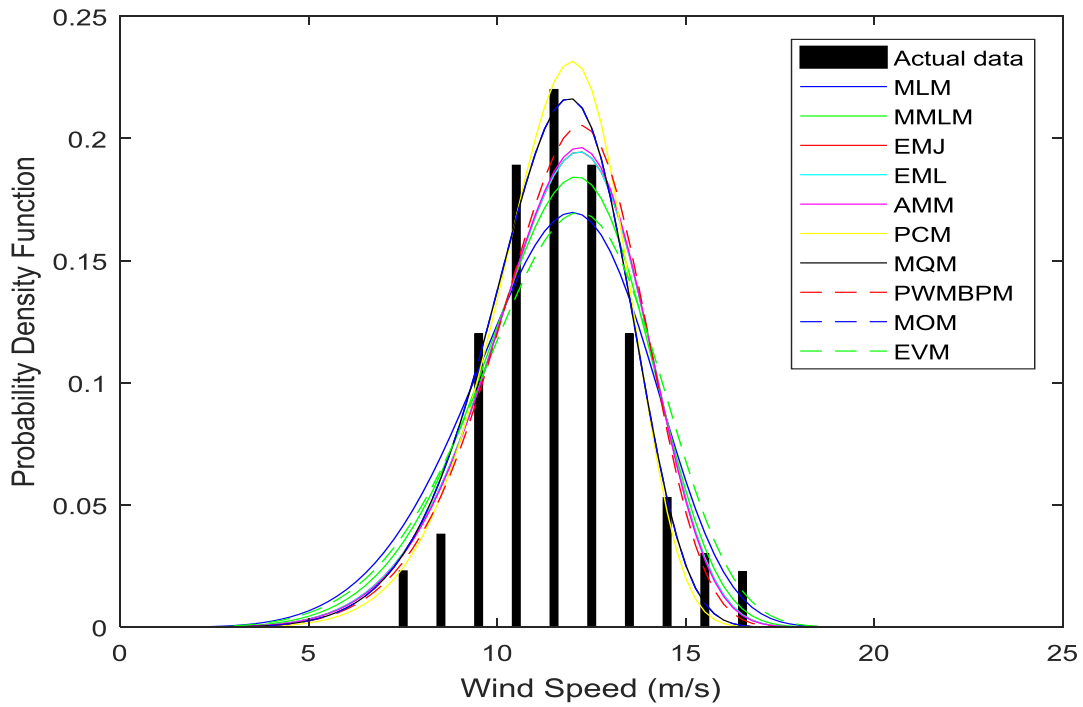


Figure 3: Comparison of the PDFs for Jos.

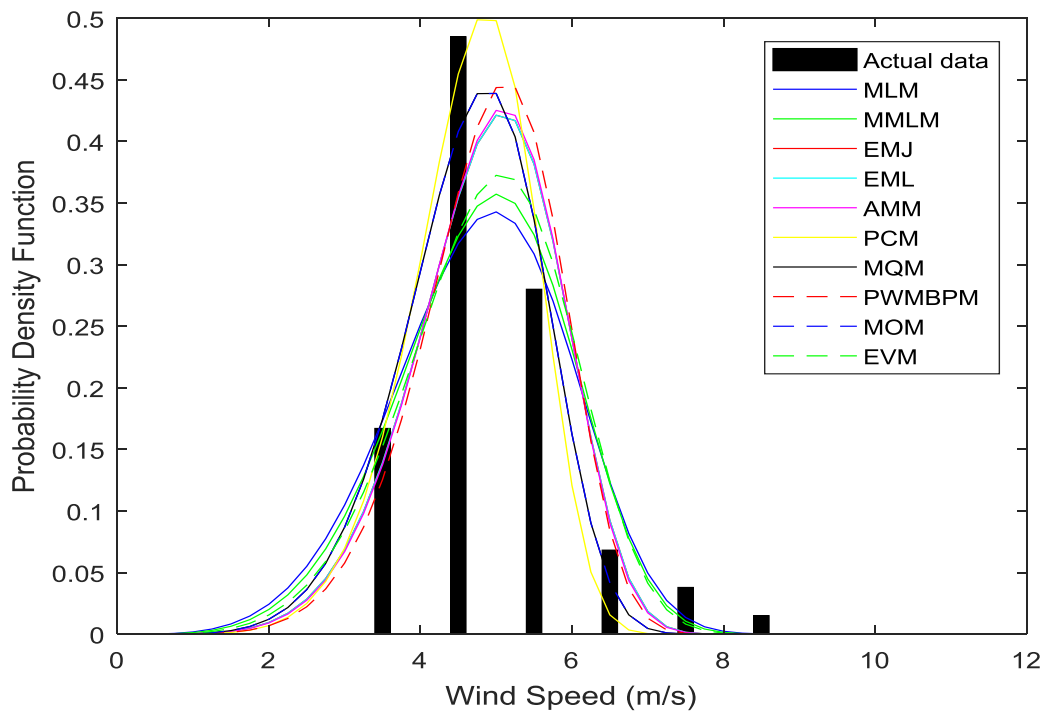


Figure 4: Comparison of the PDFs for Abuja.

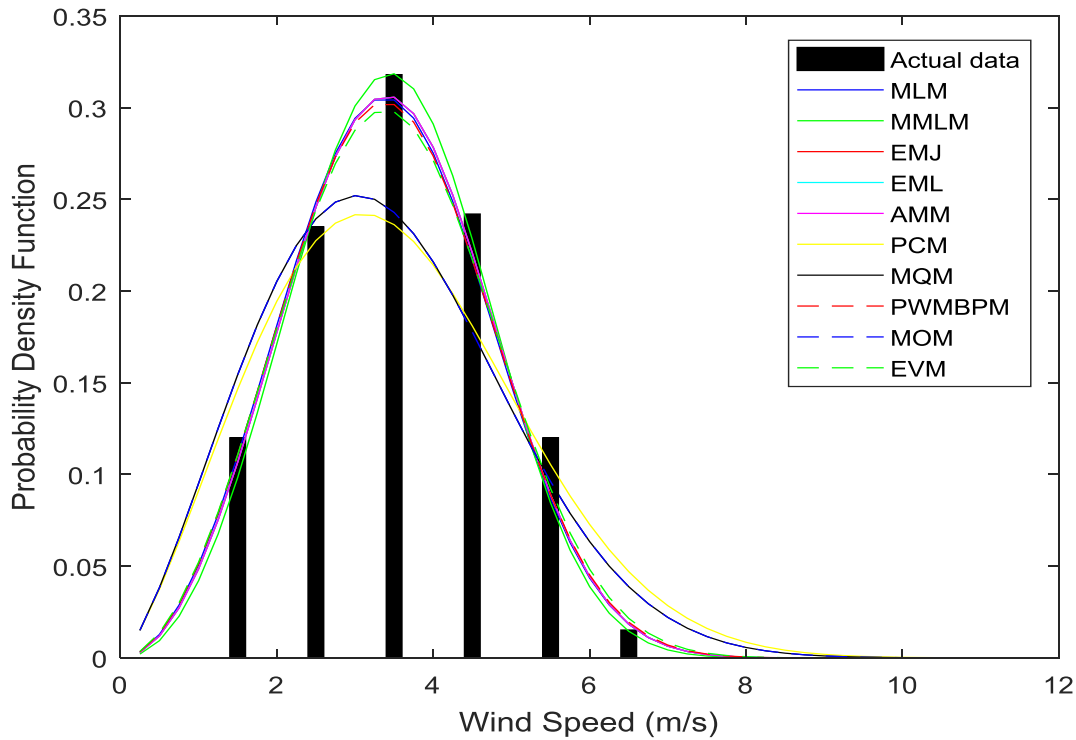


Figure 5: Comparison of the PDFs for Akure.

Table 2: Statistical comparison of numerical methods for Kano

	MLM	MMLM	EMJ	EML	AMM	PCM	MQM	PWMBP M	MOMA B	EVM
k	4.62	4.93	5.34	5.34	5.39	4.81	4.82	5.46	2.91	4.78
c	10.12	10.19	10.19	10.19	10.19	9.95	10.01	10.21	10.53	10.26
RMSE	0.0223	0.0189	0.0159	0.0159	0.0157	0.0201	0.0197	0.0158	0.0530	0.0213
Ranking	8	4	3	3	1	6	5	2	9	7
R^2	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
Ranking	8	4	3	3	1	6	5	2	9	7
χ^2	0.0456	0.0402	0.0681	0.0681	0.0753	0.0648	0.0548	0.0821	0.3227	0.0399
Ranking	3	2	6	6	7	5	4	8	9	1
MAPE	17.910	18.339	23.255	23.255	24.026	24.073	22.179	24.850	65.460	17.862
Ranking	2	3	5	5	6	7	4	8	9	1
MABE	0.0140	0.0126	0.0131	0.0131	0.0133	0.0145	0.0137	0.0138	0.0423	0.0145
Ranking	6	1	2	2	3	8	4	5	9	7

Tables 2 – 7 show the results of the calculated Wbl parameters from each of the ten NEMs (MLM, MMLM, EMJ, EML, AMM, PCM, MQM, PWMBPM, MOMAB, and EVM) and the five performance metrics (RMSE, R^2 , MAPE, χ^2 , and MABE) for evaluating the effectiveness of these NEMs across all five locations taken into consideration. The statistical-based metrics for gauging estimation efficiency were taken up to 9 decimal places but presented in the tables to three or four decimal places for a thorough comparison of all NEMs. These tables show that of the 10 NEMs for estimating Wbl parameters, EMJ and EML have the same estimation efficiency since they have been estimated using the same values for k and c , respectively, in each of the five locations. On a

scale from 1 to 9, each NEM has been rated according to its performance (with 1 being the best efficient and 9 as the least efficient). According to Table 3, AMM and MOMAB performed best and worst, respectively, using RMSE and R² as performance metrics while EVM performed best and MOMAB performed worse with χ^2 and MAPE as prediction effectiveness metrics. In terms of MABE, MMLM is the best while MOMAB continues to be the least effective. Other NEMs have also been ranked halfway between these two extremes.

Table 3: Statistical comparison of numerical methods for Maiduguri.

	MLM	MMLM	EMJ	EML	AMM	PCM	MQM	PWMB PM	MOMA B	EVM
<i>k</i>	4.12	4.37	4.46	4.46	4.50	3.99	3.87	4.47	2.31	4.09
<i>C</i>	6.01	6.02	6.02	6.02	6.02	5.86	5.63	6.04	6.20	6.05
RMSE	0.0298	0.0291	0.0295	0.0295	0.0297	0.0317	0.0410	0.0298	0.0794	0.0307
Ranking	4	1	2	2	3	7	8	5	9	6
R ²	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9997	0.9999
Ranking	4	1	2	2	3	7	8	5	9	6
χ^2	0.0450	0.0432	0.0449	0.0449	0.0461	0.0587	0.1204	0.0445	0.3392	0.0464
Ranking	4	1	3	3	5	7	8	2	9	6
MAPE	37.717	34.516	34.145	34.145	33.998	47.289	67.267	32.894	119.77	38.721
Ranking	5	4	3	3	2	7	8	1	9	6
MABE	0.0245	0.0253	0.0259	0.0259	0.0261	0.0261	0.0350	0.0256	0.0621	0.0246
Ranking	1	3	5	5	6	7	8	4	9	2

Table 4: Statistical comparison of numerical methods for Jos.

	MLM	MMLM	EMJ	EML	AMM	PCM	MQM	PWMB PM	MOMA B	EVM
<i>k</i>	5.63	6.15	6.52	6.52	6.58	7.61	7.08	6.89	3.19	5.71
<i>C</i>	12.41	12.45	12.48	12.48	12.48	12.2	12.16	12.48	12.99	12.58
RMSE	0.0294	0.0260	0.0252	0.0252	0.0250	0.0186	0.0161	0.0247	0.0643	0.0320
Ranking	7	6	5	5	4	2	1	3	9	8
R ²	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99996	0.99999
Ranking	3500	4925	5240	5240	5296	7395	8050	5399	8881	2295
χ^2	0.08665	0.08887	0.11236	0.11236	0.11934	2.41458	0.82773	0.17559	0.46763	0.09439
Ranking	2032	8297	3315	3315	812	9695	6345	8252	4817	7009
MAPE	38.6406	33.4456	31.0142	31.0142	31.0675	29.6502	30.1254	31.1795	74.6192	39.5666
Ranking	0725	8467	3691	3691	6905	9226	3494	4362	0678	326
MABE	0.02548	0.02236	0.02123	0.02123	0.02133	0.01614	0.01441	0.02179	0.05413	0.02783
Ranking	2249	0498	6171	6171	1801	7508	1028	8674	6353	3531
Ranking	7	6	3	3	4	2	1	5	9	8

According to RMSE, R² and χ^2 correspondingly, Table 4 show that MMLM is listed as the most efficient NEM whereas MOMAB is regarded as the least efficient NEM. While PWMBPM and MLM are regarded as the most effective in terms of MAPE and MABE, respectively, MOMAB continues to hold the position of least effective NEM. According to Table 5, MQM is the most efficient NEM approach while MOMAB is the least effective one in terms of RMSE and R², respectively. Regarding χ^2 , MLM exhibited the best performance and MOMAB the lowest

performance with the least numerical value. Additionally, in Table 5, the PCM and MQM performed best using MAPE and MABE as accuracy gauging metrics, respectively whereas MOMAB continued to hold the poorest NEM performer position. Furthermore, it can be observed in Table 6 that MOMAB continued to exhibit the least effective performing NEM in terms of RMSE, R^2 , and MABE while the PCM displayed the best performance. The most effective model is found to be MQM while the PWMBPM and MOMAB exhibited the worst performance in terms of χ^2 and MAPE, respectively. Because the computed Wbl parameters for EMJ, EML, and AMM were the same, they demonstrated the best performance in Table 7, but MOMAB continued to perform the poorest in terms of RMSE and R^2 metrics, respectively. Additionally, EVM and MMLM showed the best performance in terms of χ^2 , MAPE and MABE, respectively, with MOMAB being the least effective.

Table 5: Statistical comparison of numerical methods for Abuja.

	MLM	MMLM	EMJ	EML	AMM	PCM	MQM	PWMB PM	MOMA B	EVM
k	4.76	4.99	5.95	5.95	6.01	6.77	5.97	6.34	2.18	5.27
c	5.23	5.25	5.25	5.25	5.25	4.99	5.03	5.27	5.50	5.29
RMSE	0.0704	0.0719	0.0692	0.0692	0.0696	0.0362	0.0406	0.0755	0.0774	0.0711
Ranking	5	6	3	3	4	1	2	8	9	7
R^2	0.9997	0.9997	0.9997	0.9997	0.9998	0.9999	0.9999	0.9997	0.9997	0.9998
Ranking	5	7	3	3	4	1	2	8	9	6
χ^2	0.1091	0.1123	0.1667	0.1667	0.2148	0.2619	0.0416	1.5390	0.4461	0.1082
Ranking	3	4	5	5	6	7	1	9	8	2
MAPE	110.77	104.93	88.155	88.155	78.418	47.316	34.884	49.757	117.51	97.036
Ranking	8	7	5	5	4	2	1	3	9	6
MABE	0.0385	0.0439	0.0476	0.0476	0.0478	0.0258	0.0280	0.0520	0.0533	0.0443
Ranking	3	4	6	6	7	1	2	8	9	5

Table 6: Statistical comparison of numerical methods for Akure.

	MLM	MMLM	EMJ	EML	AMM	PCM	MQM	PWMB PM	MOMA B	EVM
k	3.01	3.18	3.04	3.04	3.04	2.33	2.36	2.99	1.84	2.96
c	3.86	3.88	3.88	3.88	3.88	3.95	3.83	3.87	3.90	3.89
RMSE	0.0190	0.0184	0.0177	0.0177	0.0177	0.0455	0.0449	0.0187	0.0732	0.0182
Ranking	5	3	1	1	1	7	6	4	8	2
R^2	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998	0.9999	0.9995	0.9999
Ranking	5	3	1	1	1	7	6	4	8	2
χ^2	0.0180	0.0210	0.0162	0.0162	0.0162	0.0775	0.0745	0.0168	0.2096	0.0154
Ranking	4	5	2	2	2	7	6	3	8	1
MAPE	12.734	10.425	12.654	12.654	12.654	49.649	42.644	13.720	71.443	15.506
Ranking	3	1	2	2	2	7	6	4	8	5
MABE	0.0163	0.0138	0.0153	0.0153	0.0153	0.0372	0.0376	0.0163	0.0624	0.0162
Ranking	5	1	2	2	2	6	7	4	8	3

As the statistics-based metrics (RMSE, R^2 , χ^2 , MAPE, and MABE) for ranking estimation changed, it is observed that the efficiency of the NEMs also changed. For example, in Table 3, AMM was determined to be the most efficient NEM concerning RMSE and R^2 metrics, but seventh

for χ^2 , sixth for MAPE, and third for MABE. As a result, a location-based overall rating of these NEMs is necessary, and this ranking was based on an average of the individual rankings shown in Table 8. It can be observed in Table 8 that the best NEMs for estimating the Wbl parameters for Kano, Maiduguri, Jos, Abuja, and Akure locations were MMLM, MMLM, MQM, MQM, and EMJ/EML/AMM, respectively. The results obtained in this study through the use of conventional NEMs (MMLM, EMJ, and EML) for Wbl parameter estimation are found to agree with previous studies (Kang *et al.*, 2018; Teyabeen *et al.*, 2017; Li *et al.*, 2020). Also, this present study showed that the use of rare NEMs such as MQM and AMM outside the orthodox NEMs is effective in predicting the Wbl parameters of wind speed with good performance metric values.

Table 7: Overall ranking of Weibull parameters estimation

NEMs	Kano	Maiduguri	Jos	Abuja	Akure
MLM	7	4	7	4	4
MMLM	1	1	6	7	2
EMJ	3	2	3	3	1
EML	3	2	3	3	1
AMM	2	5	4	5	1
PCM	8	7	2	2	6
MQM	4	8	1	1	5
PWMBPM	6	3	5	8	3
MOMAB	9	9	9	9	7
EVM	5	6	8	6	2

Wind characteristics of selected locations

The locations wind speed characteristics are shown in Table 8. It can be inferred that the V_{ms} varied from 3.47 m/s (lowest) in the Akure site to 11.63 m/s (highest) in Jos as the k varied from 3.04 (least) in Akure to 7.08 (most) in Jos while the c varied from 3.88 m/s (lowest) in Akure to 12.16 m/s (highest) in Jos. This implies a direct relationship between the V_{ms} and the c and k . Also, the V_{mps} increased from 3.40 m/s in Akure to 11.90 m/s in Jos as the $V_{e\max}$ rose from 4.58 m/s in Akure to 12.59 m/s in Jos. The P_{WPD} was also found to augment from 36.45 W/m² (in Akure) to 1000.06 W/m² (in Jos), all at a hub height of 10 m. Because the wind power density at a hub height of 10 m is greater than 400 W/m², the outcomes further demonstrate that the research locations in Kano and Jos are suitable for grid integration (Okakwu *et al.*, 2023). Maiduguri is only appropriate for a stand-alone application since the wind power density there is >100 W/m², however, the locations of Abuja and Akure were not suitable for generating wind energy because the wind power density in these locations is <100 W/m² (Ahmed, 2013).

Table 8: Wind speed characteristics of selected locations

Location	V_{ms}	k	c	V_{mps}	$V_{e\max}$	P_{WPD}
Kano	9.39	4.93	10.19	9.73	10.92	593.91
Maiduguri	5.49	4.39	6.02	5.68	6.56	123.98
Jos	11.63	7.08	12.16	11.90	12.59	1000.06
Abuja	4.87	5.97	5.03	4.88	5.28	70.78
Akure	3.47	3.04	3.88	3.40	4.58	36.45

CONCLUSION

This work has conducted a thorough comparative evaluation of ten NEMs for estimating the Wbl parameters of the wind speed in five locations (Kano, Maiduguri, Jos, Abuja, and Akure) in Nigeria. Eleven years of daily wind speed data for these locations were sourced and used in this study. The k and c parameters range from 1.84 to 7.61 and 3.83 to 12.99 respectively, with the least values recorded for the Kano location and the maximum values observed for the Jos location. According to the findings, the best methods for estimating the Wbl parameters for the Kano, Maiduguri, Jos, Abuja, and Akure locations were MMLM, MMLM, MQM, MQM, and EMJ, EML, and AMM, respectively, as MOMAB remained the least performing NEM for all the locations. Also, the results showed that the V_{ms} varied from 3.47 m/s (lowest) in the Akure site to 11.63 m/s (highest) in Jos. Also, the V_{mps} increased from 3.40 m/s in Akure to 11.90 m/s in Jos as the $V_{e\max}$ rose from 4.58 m/s in Akure to 12.59 m/s in Jos. Additionally, the P_{WPD} was found to augment from 36.45 W/m² (in Akure) to 1000.06 W/m² (in Jos). The results of the wind speed characteristics revealed that Jos was the best location for installing wind turbines while Kano was an excellent place for integrating the grid. Also, the Maiduguri location was determined to be suitable for a stand-alone application while Abuja and Akure were considered to be unsuitable for wind energy applications.

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