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EFFECTS OF CHEMICAL REACTION AND HEAT SOURCE ON MHD OSCILLATORY VISCOELASTIC FLOW IN A CHANNEL FILLED WITH POROUS MEDIUM

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ABSTRACT

The partial differential equations with the boundary conditions of effects of chemical reaction and heat source on MHD oscillatory viscoelastic flow in a channel were formulated based on assumptions and already existing model. The partial differential equations were transformed to dimensionless equations using suitable variables. Analytical solution was obtained for the dimensionless equations. With aids of Matlab, graphs were plotted and a table was generated to study the necessary parameters present in the flow. Increase in heat source resulted to increase in velocity of the flow while increase in chemical reaction led to decrease in velocity of the flow. The rate of heat transfer at both wall increases as heat source increases.

Keywords: Oscillatory, MHD, Viscoelastic, chemical reaction, heat source and porous medium.

INTRODUCTION

A chemical reaction can be defined as the rearrangement of atoms of the reactants to create a new substance known as the product. The substances that goes into chemical reactions are called reactants while the ones produced at the end of the reactions are known as products. Chemical reactions takes place in numerous engineering and biological fluid flow. A heat source is an object that produces or radiates heat. The study of heat generation/absorption effects in fluid flows is necessary in several fluid problems that undergoes exothermic or endothermic chemical reactions. Magnetohydrodynamics (MHD) is the dynamical study of electrically conducting fluid under the influence of magnetic field. Oscillatory flow can be described as a flow in which motion under consideration have a dominant frequency that can be maintained by an oscillating boundary conditions or self-oscillation of the flow. The ability of fluid to have property that exhibit both viscous and elastic characteristics when undergoing deformation is known as viscoelasticity. A medium is said to be porous when it allows the passage of gas or liquid through it's interstices and such medium is also called permeable medium. The permeability of a medium is very important in fluid flow.

Authors have investigated work relating to effects of chemical reaction and heat source on magnetohydrodynamics (MHD) oscillatory viscoelastic flow in a channel filled with porous medium. Lawanya et. al. (2019) discuss the effects of heat and mass transfer on oscillatory flow with couple stress in a wavy channel in the presence of heat source and chemical reaction. Closed form solution was assumed to solve the coupled dimensionless governing equations which was used for analysis. Narayana et. al. (2015) studied chemical reaction and heat source effects on MHD oscillatory flow in an irregular channel using analytical solution for analysis.Olajuwon and Oahimire (2014) studied the effects of Hall current and thermal radiation on heat and mass transfer of unsteady MHDflow of a viscoelasticmicro polar fluid through a porous medium using analytical solution for analysis. Ali and Asghar (2014) investigated two – dimensional oscillatory flow inside a rectangular channel for Jeffrey fluid with small suction. The viscoelastic behavior of non-Newtonian fluids subjected to time harmonic oscillation was studied with the analytical solution obtained for the governing equations. Masuda and Tagawa (2019) investigated quasi-periodic oscillating flows in a channel with suddenly expanded section. Two-dimensional numerical simulation was carried out for an oscillatory flow between parallel flat plates having a suddenly expanded section. The governing equations were discretized and solvednumerically for analysis.Tabakova et. al. (2020) considered the oscillatory of carrean fluid in a channel at different womersley and carrean numbers. Kalpana and Vijaya (2019) studied the effects of suction/injection on unsteady MHD oscillatory second grade fluid flow in a vertical channel with non-uniform wall temperature. Closed form solution was obtained for the dimensionless governing equations which was used for the analysis of the pertinent parameters present in the flow. Haciogulu and Narayanan(2016) investigated effects on species separation by two-dimensional laminar flow arising in a rectangular channel. Saleem et. al. (2020) studied oscillation and radiation effects on MHD casson fluid model within an asymmetric wavy channel. The governing equations were handled analytically by choosing the group theoretical method. Sharma and Dubewar (2019) considered MHD flow between two parallel infinite plate and used finite difference method to obtain solution for analysis. Makinde and Mhone (2005) investigated heat transfer to MHD oscillatory flow in a channel filled with porous medium. Close-form analytical solution was constructed for the problem which was used for analysis. Choudhury and Das(2012) studied the combined effect of a transverse magnetic field and radiative heat transfer on unsteady flow of optically thin viscoelastic fluid through a channel filled with saturated porous medium and non-uniform wall temperature. We extend the work of Choudhury and Das (2012) by incorporating concentration equation with chemical reaction and heat source term to study effects of chemical reaction and heat source on MHD oscillatory viscoelastic flow in a channel filled with porous medium which has not be studied by any author to the best of our knowledge.

MATHEMATICAL FORMULATION

We considered an optically thin fluid flow in a channel under the influences of an externally applied magnetic field. It is assume that the fluid has small electrical conductivity and the electromagnetic force produce is very small. The fluid flow is between two parallel walls at $y' = 0$ and $y' = a.$ The x-axis is taken along the centre of the channel while y- axis is taken to be perpendicular to it as shown in figure 1.

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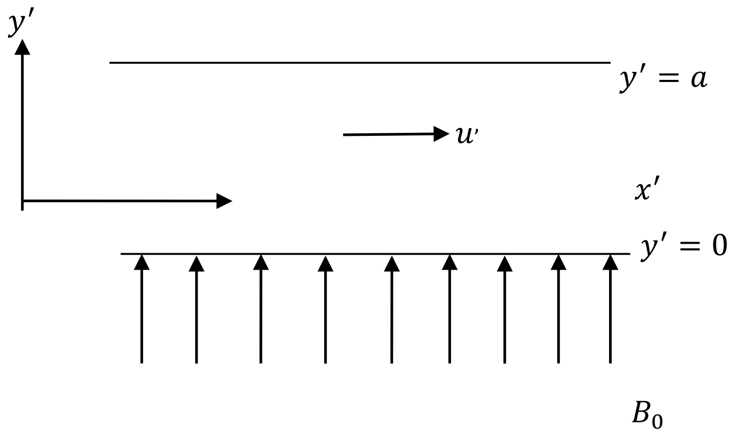

Fig. 1

Assuming Boussineq incompressible fluid model and extending the work of Choudhury and Das(2012) , the governing equations are

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial \rho'}{\partial x'} + v_1 \frac{\partial^2 u'}{\partial y'^2} + v_2' \frac{\partial^3 u'}{\partial y'^2 \partial t'} - \frac{v_1}{K} u' - \frac{\sigma B_0^2}{\rho} u' + g\beta_{t'}(T - T_0) + gB_c(C - C_0) \quad (1)$$

$$\frac{\partial T}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} + \frac{Q_0(T - T_0)}{\rho C_p} \quad (2)$$

$$\frac{\partial c}{\partial t'} = D\frac{\partial^2 C}{\partial y'^2} + K_c(C - C_0) \quad (3)$$

With the boundary condition given by

$$u' = 0,\ T = T_0 + (T_w - T_0)e^{iw't'}, C = C_0 + (C_w - C_0)e^{iw't'} at y = a \quad (4)$$

$$u' = 0, T = T_0, C = C_0 at y' = 0 \quad (5)$$

While $u'$is the axial velocity, $(x'y')$ is the space co-ordinates, t' is time, w' is frequency of oscillation, T is the temperature of fluid, C is the mass concentration of the fluid, ρ is the fluid density,$p'$is the pressure, g is gravitational force,$v_i = {}^{\mu_i}/_{\rho}\ (i = 1,2)$ is dynamic viscosity, K is thepermeability of the porous medium, σ is the conductivity of the medium. $B_0 = (\mu_e, H_o)$ is the electromagnetic induction where μ_e is the magnetic permeability and H_o is the intensity of magnetic field,B_t is the coefficient of volumetric thermal expansion, $B_c$is the coefficient of volumetric mass expansion, $k$is the thermal conductivity, C_p is the specific heat at constant pressure, q_r is the radiative heat flux,Q_0 is the constant heat flux, K_c is the rate of chemical reaction,C_w, $C_{0,}T_w$, $T_0 and\ a$ are concentration at the wall, concentration far from wall, temperature at wall, temperature far from wall and distant between the walls respectively.

Following Cogley et. al(1968),for optically thin fluid with low density, radiative heat flux is

$$\frac{\partial q_r}{\partial y'} = 4\alpha^2(T_0 - T) \quad (6)$$

where α is the mean radiation absorption coefficient

We now use the following non-dimensional variables for transformation;

$$y = \frac{y'}{a}, x = \frac{x'}{a}, u = \frac{u'}{U}, t = \frac{t'U}{a}, p = \frac{ap'}{\rho v_1 U}, \theta = \frac{T - T_0}{T_w - T_0}, \emptyset = \frac{C - C_0}{C_w - C_0}, w = \frac{w'a}{U}, \quad (7)$$

Applying equation(7) for transformation, yield the following dimensionless equation.

$$R_e \frac{\partial u}{\partial t} = \frac{-\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + V_0 \frac{\partial^2 T}{\partial y^2 \partial t} - \{S^2 + H^2\}u + Gr\theta + G_c\emptyset \quad (8)$$

$$P_e \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + R^2\theta + H_s\theta \quad (9)$$

$$s_c \frac{\partial \emptyset}{\partial t} = \frac{\partial^2 \emptyset}{\partial y^2} - \mathrm{C_r}\emptyset \quad (10)$$

With the dimensionless boundary conditions:

$$u = 0, \theta = e^{iwt}, \emptyset = e^{iwt} \; at \; y = 1 \quad (11)$$

$$u = 0\,, \theta = 0, \emptyset = 0 \; at \; y = 0 \quad (12)$$

Where $R_e = \frac{Ua}{v_1}, V_0 = \frac{v_{1U}}{v_2 a}, S^2 = \frac{1}{Da}, Da = \frac{K}{a^2}, H^2 = \frac{\sigma B_0^2 a^2}{\rho v_1}, G_r = \frac{gB_t(T_w - T_0)a^2}{v_1 U}, G_c = \frac{g\beta_c(T_w - T_0)a^2}{v_1 U}$

$p_e = \frac{Ua\rho C_p}{k}$, $R^2 = \frac{4\alpha^2 a^2}{\rho v_1}, S_c = \frac{Ua}{D}, H_s = \frac{a^2 Q_0}{K}, C_r = \frac{K_c a^2}{D}$are Renolds number($Re$), viscoelastic parameter(Vo), porous medium shape parameter(S), Darcy number(Da), Hartmann number(H), Grashof number(Gr), modified Grashof number(Gc), peclet number(Pe), radiation parameter(R), Schmidth(Sc) number, heat source parameter(H_s) and chemical reaction (C_r)respectively.

METHOD OF SOLUTION

Inorder to solve the dimensionless governing equations for purely oscillatory flow, let

$\frac{\partial P}{\partial x}$ =Ze^{iwt}(where Z is a constant, Nirmala et al. (2018)) (13)

$$u(y,t) = u_0(y)e^{iwt} \quad (14)$$

$$\theta(y,t) = \theta_0(y)e^{iwt} \quad (15)$$

$$\emptyset(y,t) = \emptyset_0(y)e^{iwt} \quad (16)$$

Substituting equation (13) – (16) into (8)-(12), we have

$$\mathrm{M}u_0''(y) - Nu_0(y) = -Z - G_r\theta_0(y) - G_c\emptyset_0(y) \quad (17)$$

$$\theta_0''(y) + Q\theta_0(y) = 0 \quad (18)$$

$$\emptyset_0''(y) - (iws_c + C_r)\emptyset_0(y) = 0 \quad (19)$$

With the boundary conditions;

$$u_0(y) = 0, \theta_0(y) = 1, \; \emptyset_0(y) = 1 \; at \; y = 1 \quad (20)$$

$$u_0(y) = 0, \theta_0(y) = 0, \emptyset_0(y) = 0 \; at \; y = 0 \quad (21)$$

Where $M = (1 + iwV_0), N = iwR_e + \{S^2 + H^2\}$ and $Q = R^2 - iwP_e + H_s$

Solving (17)-(19) with (20) and (21), yield:

$$u(y,t) = \left(B_1 e^{m3y} + B_2 e^{m4y} + B_3 + B_5 \sin\left(\sqrt{Q}y\right) + B_6 e^{m1y} + B_7 e^{m2y}\right)e^{iwt} \quad (22)$$

$$\theta(y,t) = \frac{\sin(\sqrt{Q}y)}{\sin(\sqrt{Q})} e^{iwt} \quad (23)$$

$$\emptyset(y,t) = (A_1 e^{m1y} + A_2 e^{m2y})e^{iwt}(25)$$

Where $m_1 = \sqrt{iwSc + C_r}$

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$m_2 = -\sqrt{iwSc + C_r}$

$m_3 = \sqrt{N/_M}$

$m_4 = -\sqrt{N/_M}$

$A_1 = \frac{1}{e^{m1} - e^{m2}}$

$A_2 = -A_1$

$B_3 = \frac{Z}{N}$

$B_4 = 0$

$B_5 = \frac{Gr}{(MQ-N)\sin(\sqrt{Q})}$

$B_6 = \frac{-G_c A_1}{Mm_1^2 - N}$

$B_7 = \frac{-G_c A_2}{Mm_2^2 - N}$

$B_1 = \frac{(B_3+B_6+B_7)e^{m4} - B_3 - (B_5 \sin(\sqrt{Q}) + B_6 e^{m1} + B_7 e^{m2})}{e^{m3} - e^{m4}}$

$B_2 = -(B_1 + B_3 + B_6 + B_7)$

The rate of heat and mass transfer across the channel at the upper wall are:

$N_u = -\frac{\partial\theta}{\partial y} = -\frac{\sqrt{Q}\cos(\sqrt{Q})}{\sin(\sqrt{Q})} e^{iwt} \quad (at\ y = 1)$

$S_h = -\frac{\partial\emptyset}{\partial y} = -(m_1 A_1 e^{m1y} + m_2 A_2 e^{m2y})e^{iwt} \quad (at\ y = 1)$

The rate of heat and mass transfer across the channel at the lower wall are

$N_u = -\frac{\partial\theta}{\partial y} = -\frac{\sqrt{Q}}{\sin(\sqrt{Q})} e^{iwt} \quad (at\ y = 0)$

$S_h = -\frac{\partial\emptyset}{\partial y} = -(m_1 A_1 + m_2 A_2)e^{iwt}\ (at\ y = 0)$

RESULTS AND DISCUSSION

Numerical evaluation of the analytical solution of chemical reaction and heat source to MHD oscillatory viscoelastic flow in a channel filled with porous medium was performed. The results are presented in graphs and in a table. This was done to know the influences of important parameters in the flow. In this study, we choose $t = 0.1$ and $w = 1$ while other parameters are varied over range.

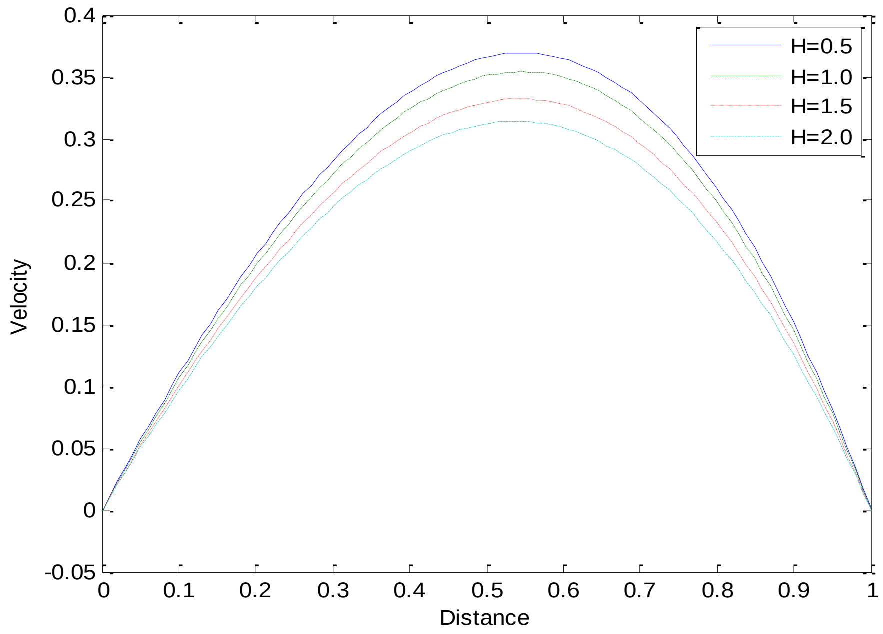

Fig.2: Velocity profile for different values of Hartmann number.

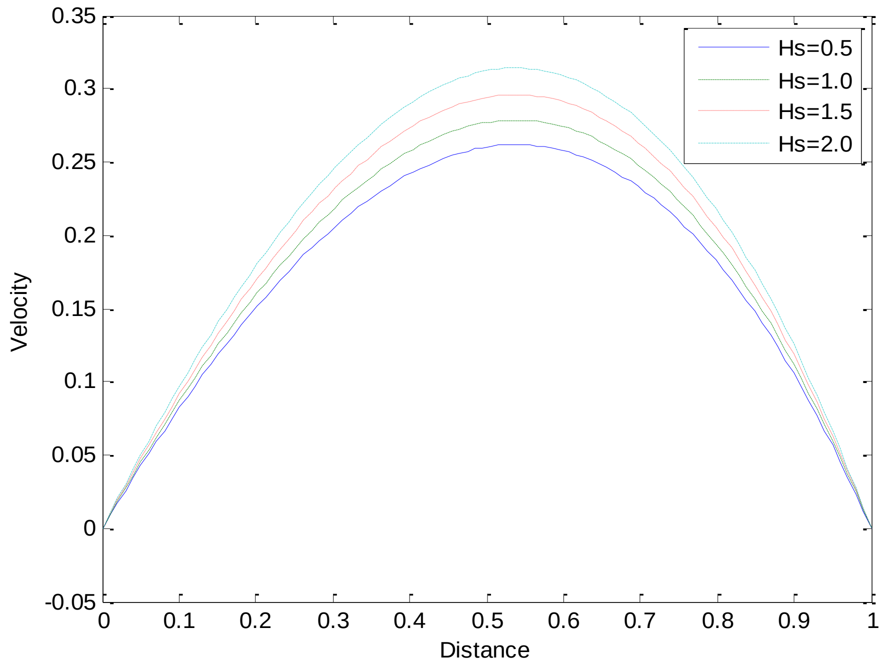

Fig.3: Velocity profile for different values of heat source parameter.

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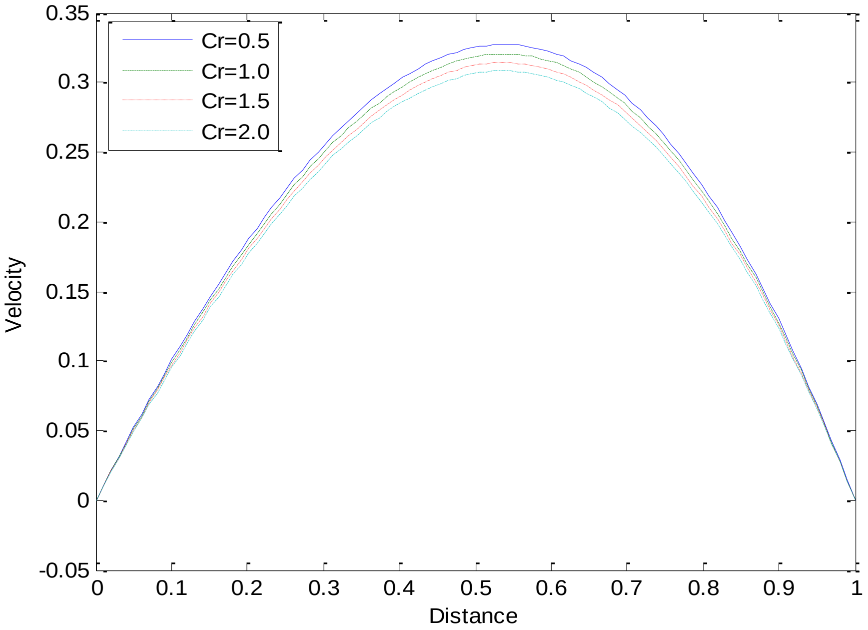

Fig.4: Velocity for different values of chemical reaction parameter

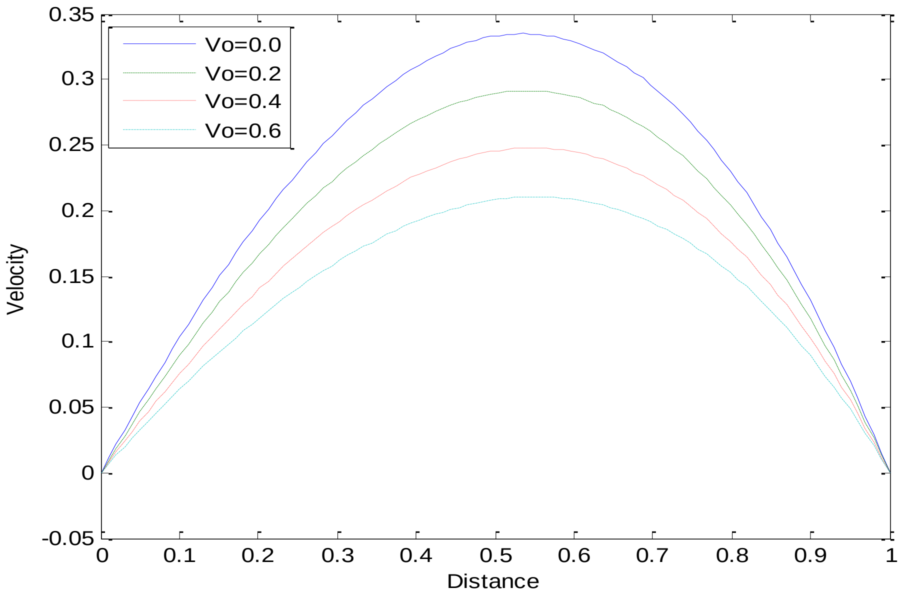

Fig.5: Velocity profile for different values of viscoelastic parameter.

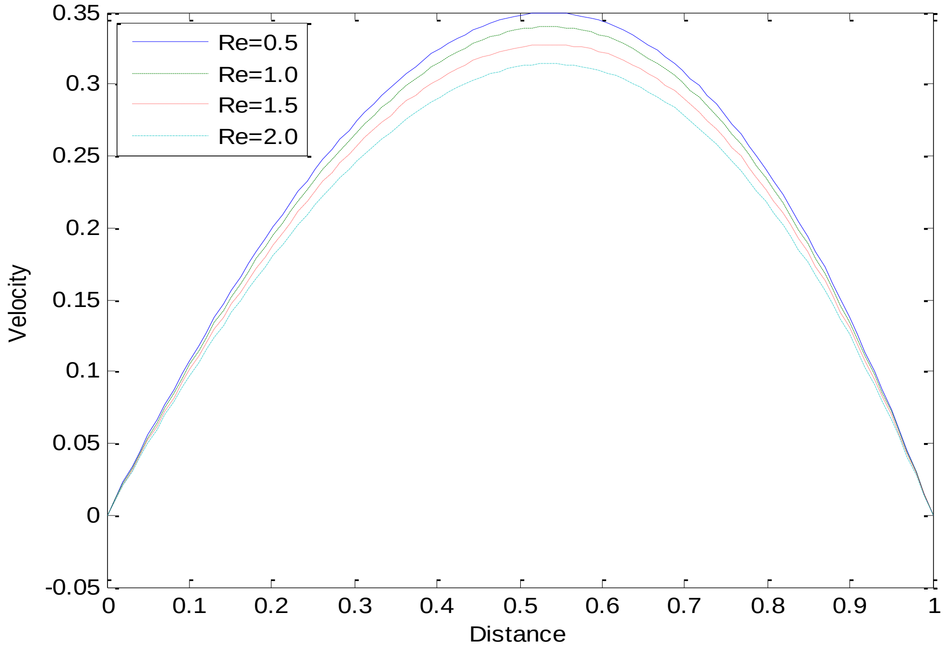

Fig.6: Velocity profile for different values of Reynolds number

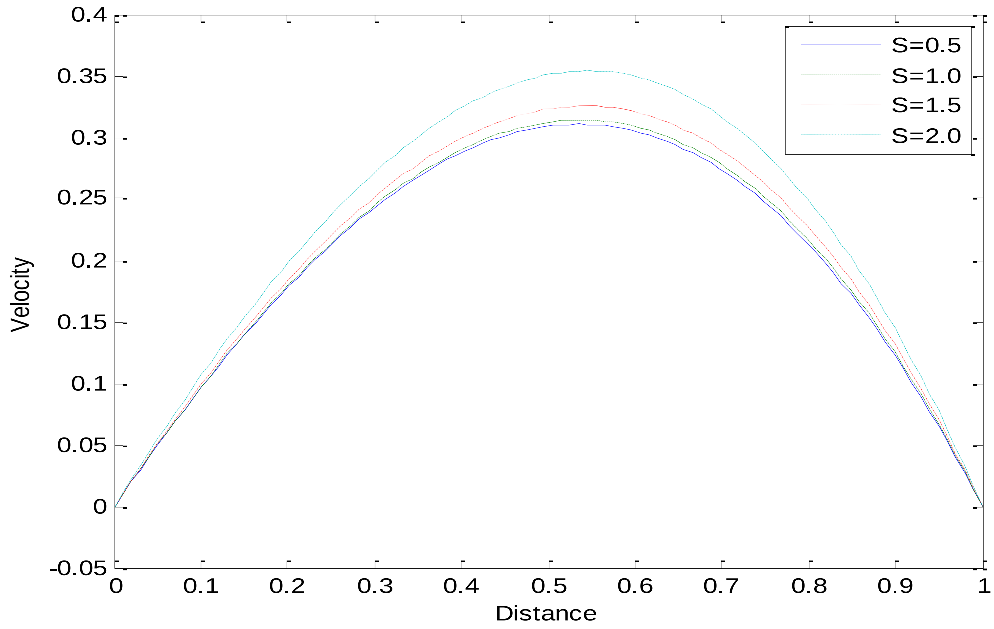

Fig.7: Velocity profile for different values of porous medium shape parameter.

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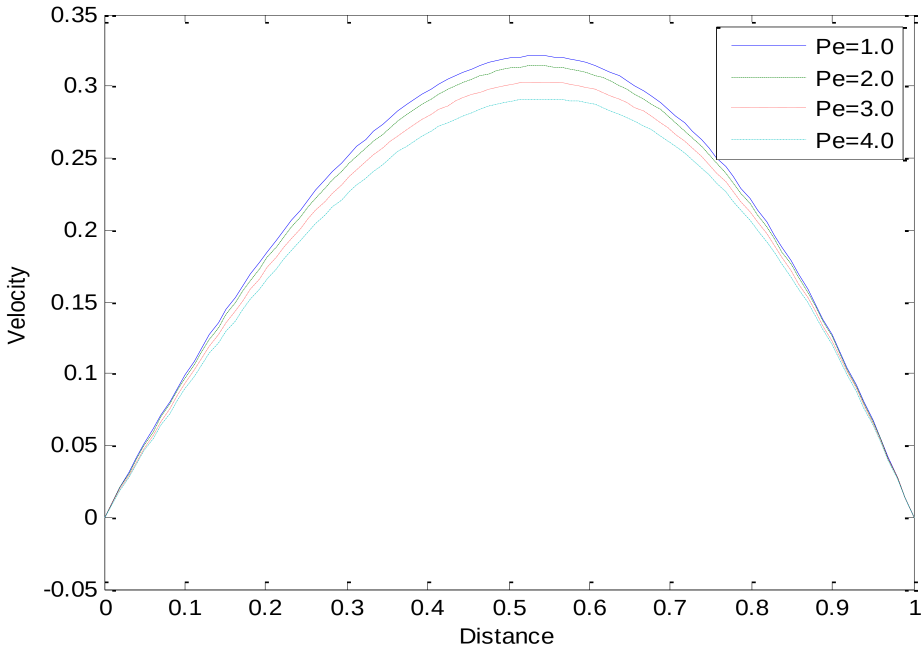

Fig.8: Velocity profile for different values of peclet number

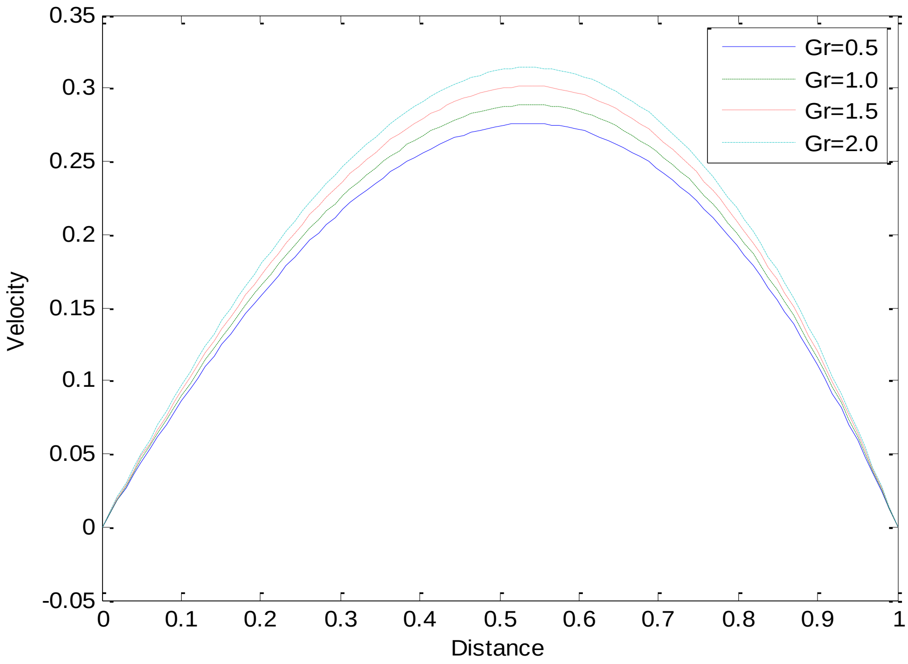

Fig.9: Velocity profile for different values of Grashof number.

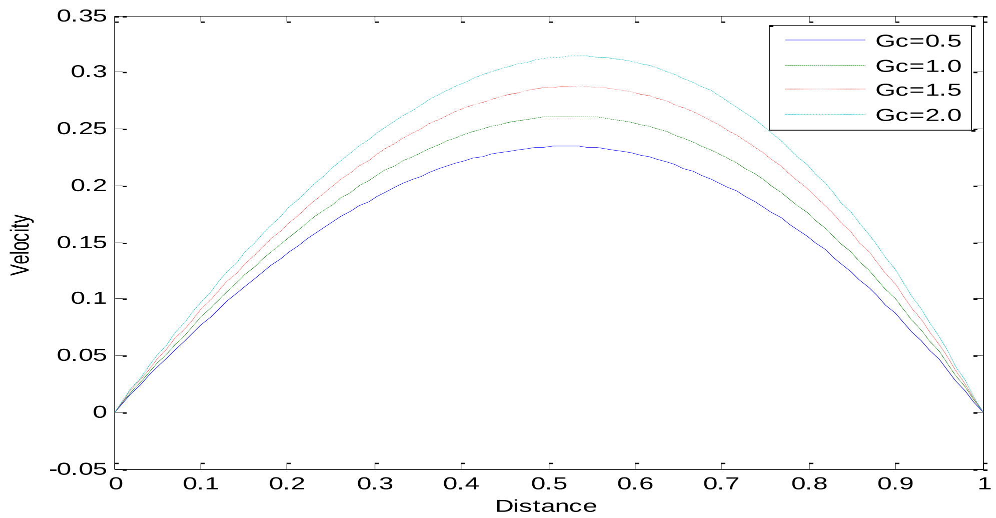

Fig.10: Velocity profile for different values of modified Grashof number

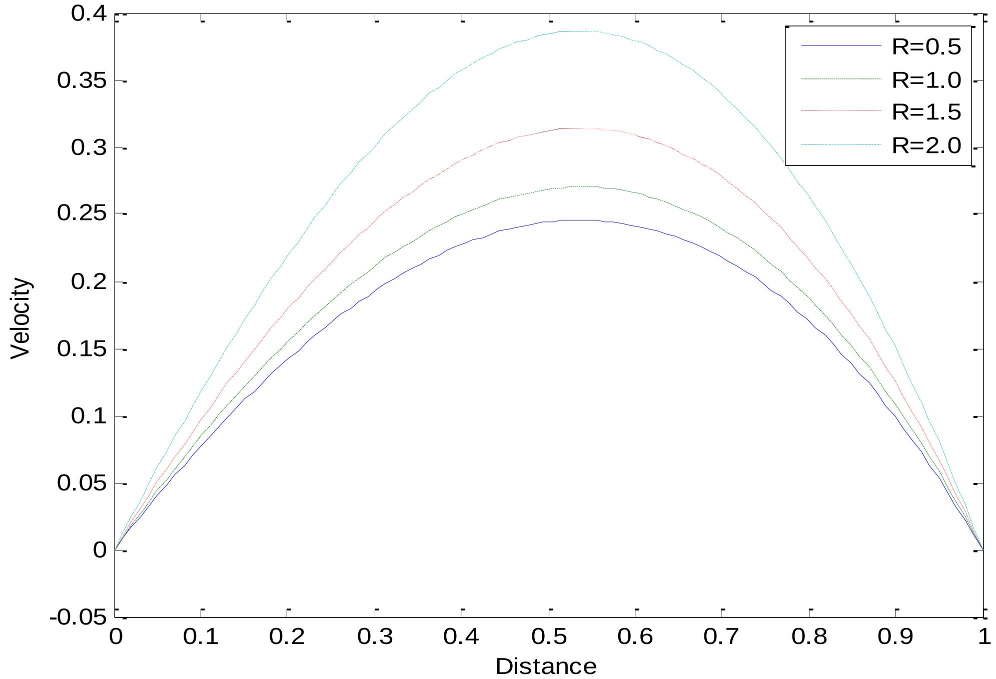

Fig.11: Velocity profile for different values of radiation parameter

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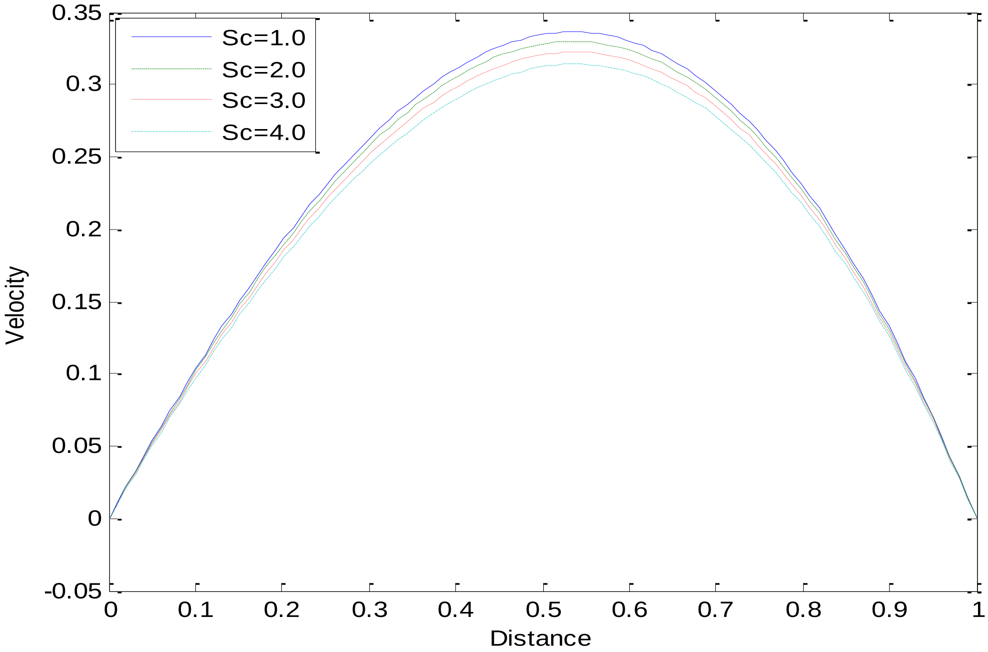

Fig.12: Velocity profile for different values of Schmidt number.

TABLE :

R	Pe	Hs	$-\theta(1)$	$-\theta(0)$
0.5	2.0	2.0	0.1886	-1.3906
1.0	2.0	2.0	0.1540	-1.5966
1.5	2.0	2.0	0.8343	-2.0405
2.0	0.5	2.0	2.9289	-3.7984
2.0	1.0	2.0	2.7590	-3.6214
2.0	1.5	2.0	2.4665	-3.3240
2.0	2.0	0.5	0.9905	-2.1479
2.0	2.0	1.0	1.3263	-2.3849
2.0	2.0	1.5	1.6956	-2.6530
Sc	Cr		$-\emptyset(1)$	$-\emptyset(0)$
1.0	2.0		-1.5708	-0.7271
2.0	2.0		-1.5835	-0.7060
3.0	2.0		-1.6200	-0.6660
2.0	0.5		-1.2272	-0.8200
2.0	1.0		-1.3640	-0.7641
2.0	1.5		-1.4947	-0.7129

Figure 2 to figure 12 which are parabolic in nature and satisfied prescribed boundary condition, demonstrates the effects of varied parameter over range on velocity distribution of the fluid flow across the channel. Fig.2 shows that increase in Hartmann number led to decrease in velocity which is not unexpected since transverse magnetic field gives rise to resistive force that slow down the motion of the fluid, fig.3 displays that the velocity increases as heat source parameter increases which is not surprising because the higher the heat transfer the higher the velocity,fig.4 illustrates that the effects of increase in chemical reaction is to decrease the velocity distribution across the channel, fig.5 and fig.6 demonstrates that velocity decreases as the viscoelastic parameter/Reynolds number increases, fig.7 depicts that increase in porous shape parameter led to increase in velocity, fig.8 displays that velocity decreases as Peclet number increases, the effects of increasing Grashof number/modified Grashof number is to increase the velocity as illustrated by fig.9 and fig.10 , fig.11 shows that the velocity increases as radiation parameter increases which is expected because the bond holding the components of the fluid particles can easily be broken when the intensity of heat generated through thermal radiation is increased and fig.12 depicts that velocity decreases as Schmidt number increases. The table illustrates the effects of varied parameters on Nuselt number and Sherwood number at upper wall and lower wall respectively; increase in radiation parameter increases the Nuselt number at upper wall while the reverse is the case at the lower wall, increase in Peclet number decreases the Nuselt number at both upper and lower wall, The Nuselt number at both upper and lower wall increases as heat source parameter increases, increase in Schmidt number decreases the Sherwood number at upper wall while it increases it at lower wall and the effects of increasing chemical reaction parameter is to decrease Sherwood number at upper wall but increases it at the lower wall.

CONCLUSION

The problem of chemical reaction and heat source on MHD viscoelastic flow through a channel filled with porous medium was studied using analytical solution. The results are discussed through graphs and a table. The following conclusion can be seen from the results.

1) Increase in heat source resulted to increase in velocity of the flow
2) Increase in chemical reaction led to decrease in velocity of the flow
3) The rate of heat transfer at both upper wall and lower wall increases as heat source increases
4) Increase in the rate of chemical reaction led to decrease in the rate of mass transfer at the upper wall while the reverse is the case at lower wall

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