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EFFECTS OF VISCOUS DISSIPATION AND JOULE HEATING OF MAGNETOHYDRODYNAMICS (MHD) CONVECTIVE NANO FLUID HEAT TRANSFER OVER A FLAT POROUS PLATE

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ABSTRACT

In the present investigation, we studied the effects of Viscous Dissipation and Joule Heating of magnetohydrodynamic (MHD) convective heat transfer over a flat porous plate. The governing partial differential equations is expressed into a nonlinear partial differential equation, using a suitable similarity transformation. A semi analytical method of Homotopy Parturbation was applied to solve the equation and the obtained numerical solution for the velocity, temperature and other parameters of the nano fluid are discussed and represented graphically. Also, the effect of other parameters on the velocity and temperature profiles are also presented.

Keywords: Viscous Dissipation, Joule heating, Hartmann's number, Temperature, Heat transfer.

INTRODUCTION

MHD flow with convective heat transfer over a flat porous plate is of great importance as it is connected to many engineering problems, its characteristics, play major role in industrial processes such as food processing, polymer manufacturing, geological processes in fluid contained in various bodies, plastic material production, paints production, preventive coating and many others. On the other hands, fluid flow through porous media has become an important topic because of the recovery of crude oil from the pores of the reservoir rock. Also, there has been several interesting studies on heat transfer and Joule parameter effect of such flow in a porous media due to its magnetic effect. (Makinde et al., 1998; Kumar et al., 2017), studied characteristic of Joule heating and viscous dissipation on three dimensional flow of Oldroyd B nanofluid with thermal radiation using Runge Kutta Fehlberg fourth – fifth order through shooting method and concluded from their graph that at various B (Deborah numbers), E_c number, there was an increase in temperature $\theta(\eta)$ for higher values of the parameter R, E_{cj}, E_{cr}. A reduction in the interface heat transfer increases the temperature profile for the joint effect of joule and viscous heating. (David et al., 2013), studied Nonlinear MHD boundary layer flow of a liquid metal with heat transfer over a porous stretching surface with nonlinear radiation effects, with the used of Fourth order Runge Kutta shooting method along with the Nachtsheim Swigert iteration. Findings, shown that the effect of thermal radiation reduces the temperature and that temperature increases with the increase in surface temperature parameter. Also, the velocity exponent parameter decreases the velocity and increases the magnitude the skin friction coefficient for both suction and injection, while the thermal boundary layer thickness decreases with increasing prandtl number among others.

Jhankal (2017) studied application of homotopy perturbation method for MHD free convection of water at 4℃ through porous medium bounded by a moving vertical plate. Finding, shows that as the magnetic parameter increases, the velocity profile decreases in the flow region, while the temperature decreases with an increase in the suction parameter. Srinivas et al., (2018) looked at, Joule heat effects on unsteady MHD flow over a stretching sheet with viscous dissipation and heat source using numerical implicit finite difference. It was discovered that radiative parameter R increases the magnitude Nusselt number $-\theta''(0)$, and decreased the velocity profile. Nnenna (2017), did a study on convective heat and mass transfer of MHD flow over a flat plate using HPM, it was discovered that increase in the presence of magnetic field parameter had an inhibitive effect on the fluid flow by decreasing it's velocity and increasing the temperature. Venkata et al., (2019) studied Exact and Numerical Solutions for MHD Boundary Layer Flow of Casson Fluid over a Stretching Sheet with the used of Runge Kutta Forth order shooting method. Their conclusion is that velocity profile decreases along with the magnetic and fluid parameter increments. Also, velocity profile is diminished due to the presence of magnetic parameter for the case of suction while it is opposite for the case of injection. Hunegnaw (2021), examined unsteady boundary layer flow of Williamson nanofluids over a heated permeable stretching sheet embedded in a porous medium in the presence of viscous dissipation using fourth order Runge Kutta. He noticed that increase in magnetic parameter decreases the velocity profile and increase the temperatures profile. It was also observed that the temperature distribution improved significantly with increase in the thermal radiation parameter, which led to a reduction in the thermal boundary layer.

The present work is aimed at examining how convective heat transfer of a nano fluid is influenced by viscous dissipation and joule heating when it is over a flat porous plate.

Mathematical and physical formulation

A steady incompressible two dimensional flow of a fluid that conduct electricity over a flat porous plate lying parallel to the upward vertical X axis is considered. The fluid is assume to flow with a uniform velocity in the direction parallel to the plate. A transverse magnetic field B_o is applied in the Y direction.

To obtain the equation of flow, we made use of the Maxwell equation, Ohm's law, the equation of continuity, the momentum equation with J×B body force and the energy equation with viscous and porosity terms.

$$\nabla \times H = J$$

$$\nabla \cdot \mathrm{B} = 0$$

$$\nabla . \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad 1$$

$$\nabla . \vec{J} = 0$$

Ohm's laws

$$\vec{J} = \sigma\left(\vec{E} + \vec{U} \times \vec{B}\right) \qquad 2$$

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Continuity equation

$$\frac{\partial p}{\partial t} + U(\nabla . U) = 0 \qquad 3$$

Momentum equation

$$\rho\left(\frac{\partial \vec{U}}{\partial t} + (\vec{U}\cdot\nabla)\vec{U}\right) = -\nabla P - \rho\nabla\varphi + \nabla\vec{T} + J\times B + \frac{\mu}{K}u \qquad 4$$

Energy equation

$$\rho C_\rho \frac{DT}{Dt} = Q\nabla\cdot\vec{U} + \mu\left(\frac{\partial U}{\partial y}\right)^2 + K_T\nabla_T^2 + \vec{E}\,.\vec{J} \qquad 5$$

Where

K_T = Thermal conductivity

U = Specific internal energy

φ = gravitational potential

$\frac{\mu}{K}u$= porosity term.

Using the assumptions and simplifications, the dimensional governing equations of continuity, momentum and energy are now written as:

$$u\frac{\partial U}{\partial X} + v\frac{\partial U}{\partial y} = 0. \qquad 5$$

$$u\frac{\partial U}{\partial x} + v\frac{\partial U}{\partial y} = \frac{-1}{\rho}\frac{dp}{dx} + \mu\frac{\partial^2 U}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma B_O^2}{\rho}u - \frac{\mu}{K}u \qquad 6$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 U}{\partial y^2} + \frac{Q_O}{\rho C_\rho}(T - T_\infty) + \frac{\sigma H^2}{\rho C_\rho}u^2 + \frac{\mu}{\rho C_\rho}\left(\frac{\partial U}{\partial y}\right)^2 \qquad 7$$

and boundary conditions

$$V(x,0) = U(x,0) = 0 \qquad T(x,0) = T_w \text{ at } \eta = 0 \qquad 8$$

$$U(x,\infty) = U_\infty \qquad T(x,\infty) \to T_w \text{ at } \eta \to \infty$$

Then,

$$\nabla\cdot T = \mu_f(\nabla^2\vec{U}) + \left(£ + \frac{\mu}{3}\right)\nabla(\nabla\cdot\vec{U}) \qquad 9$$

Dimensionless variables are defined as,

$$\left.\begin{array}{l} \eta = y\sqrt{\frac{U_\infty}{\mu x}}, \quad \varphi = \sqrt{\mu x U_\infty}\, f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad U = \frac{\partial\varphi}{\partial y} \\ V = -\frac{\partial\varphi}{\partial x}, \quad \gamma = \frac{xQ_O}{U\rho C_\rho}, \quad P_r = \frac{\mu}{\alpha}, \quad M^2 = \frac{\sigma B_{OX}^2}{\rho U}, \\ G_t = \frac{g\beta(T_w - T_\infty)}{U_\infty^2}x, \quad J = \frac{\sigma H^2 U_\infty x}{\rho C_\rho(T_{w-T_\infty})}, \quad E_C = \frac{U_\infty^2}{C_\rho(T_{w-T_\infty})} \end{array}\right\} \qquad 10$$

Using equation 10 in equations 6 and 7, and the boundary conditions in equation 8, we have;

$$f''' + G_t\theta + \frac{ff''}{2} + (M^2 + P_r\gamma)f' = 0 \qquad 11$$

$$\theta'' + \frac{p_r f\theta'}{2} + P_r\gamma\theta + P_r J f'^2 + P_r E_C f''^2 = 0 \qquad 12$$

And the transformed boundary conditions became

$\theta(0) = 1 \qquad \theta(\infty) = 0, \qquad$ f= (0) $\qquad f'(0) = 0$ 13

$f'(\infty) = 1$

Where prime denote differentiation with respect to η

$\gamma =$ Heat generation parameter
$G_t =$ Thermal Grashof number
M = Hartmann number
J = Joule parameter
E_c = Viscous dissipation
P_r = Prandtl number

Analysis

Using Homotopy Perturbation Method following the homotopy technique of He (1999) i.e;

$$H(u,p) = [L(v) - L(u_o)] + \mathrm{p}[lA(u) + f(r)] \qquad 14$$

Equation (11) and (12) are simplfied as;

$$H(u,p) = (1-p)\left[f''' - f'''_0\right] + \mathrm{p}\left[f'''_0 + G_t\theta + \frac{1ff''}{2} + (M^2 + P_r\gamma)f'\right] = 0 \qquad 15$$

$$H(u,p) = (1-p)\left[\theta'' - \theta''_0\right] + \mathrm{p}\left[\theta''_0 + \frac{1P_rf\theta'}{2} + P_r\gamma\theta + P_rJf'^2 + P_rE_cf''^2\right] = 0 \qquad 16$$

Expanding equation (15) and (16) we have;

$$p^0f_0''' + p^1f_1''' + p^2f_2''' + p^3f_3''' = -p\Big[G_t(\textstyle\sum\theta) + \frac{1}{2}(p^0f_0 + p^1f_1 + p^2f_2 + p^3f_3)(p^0f_0'' + p^1f_1'' + p^2f_2'' + p^3f_3'') + (M^2 + P_r\gamma)(p^0f_0' + p^1f_1' + p^2f_2' + p^3f_3')\Big] \qquad 17$$

$$p^0\theta_0'' + p^1\theta_1'' + p^2\theta_2'' + p^3\theta_3'' = -p\Big[\frac{1}{2}P_r\textstyle\sum F\,(p^0\theta_0' + p^1\theta_1' + p^2\theta_2' + p^3\theta_3') + {}_r\gamma(p^0\theta_0 + p^1\theta_1 + p^2\theta_2 + p^3\theta_3) + P_rJ\sum F'^2 + P_rE_c\sum F''^2\Big] \qquad 18$$

Equate powers of $p's$ in equations (17) and (18) we have;

$$\left.\begin{aligned}
&p^0 \quad f_0''' = 0\\
&p^1 \quad f_1''' = -\left[G_t(\textstyle\sum\theta) + \frac{1}{2}f_0f_0'' + (M^2 + P_r\gamma)f_0'\right]\\
&p^2 \quad f_2''' = -\left[\frac{1}{2}(f_0f_1'' + f_1f_0'') + (M^2 + P_r\gamma)f_1'\right]\\
&p^3 \quad f_3''' = -\left[\frac{1}{2}(f_0f_2'' + f_1f_1'') + (M^2 + P_r\gamma)f_2'\right]\\
&p^0 \quad \theta_0''\\
&p^1 \quad \theta_1'' = -\left[\frac{1}{2}P_r\theta_0'\textstyle\sum F + P_r\gamma\theta_0 + P_rJ\sum F'^2 + P_rE_c\sum F''^2\right]\\
&p^2 \quad f_2'' = -\left[\frac{1}{2}P_r\theta_1'\textstyle\sum F + P_r\gamma\theta_1\right]\\
&p^3 \quad \theta_3'' = -\left[\frac{1}{2}P_r\theta_2'\textstyle\sum F + P_r\gamma\theta_2\right]
\end{aligned}\right\} \qquad 19$$

Thus, the solution of equation (11) and (12) are expressed as:

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$$\left.\begin{aligned} F &= F_O + pF_1 + p^2F_2 + p^3F_3 = \sum_{I=0}^{\infty} f_i \\ \theta &= \theta_O + p\theta_1 + p^2\theta_2 + p^3\theta_3 = \sum_{I=0}^{\infty} \theta_i \end{aligned}\right\} \quad (20)$$

RESULT AND DISCUSSION

The governing equations (11) and (12) subject to the boundary condition equation (13) are solved numerically with the used of HPM. The numerical values of the velocity, temperature coefficient were obtained for different values of the parameters G_t = 5,10,15,20. $E_c = 0.1$, 0.5, 1, 1.5. J= 0.1, 0.3, 0.5, 0.7, M = 0.5, 0.6, 0.7, 0.8. γ = 0.1, 0.5, 1.0, 1.5 and P_r = 0.71 using MATLAB. Detailed numerical results for the velocity, temperature and other parameters obtained are explained graphically in figures respectively. It is observed that the Hartmann's number M decreases the velocity profile of the nano fluid as observed in figure 1. This is due to the rate of heat transfer at the surface in the presence of the magnetic Hartmann number M, thus, heat transfer rate at the surface decreases as M increases, showing positive agreement with Swarnalathamma (2018). Figure 2, show that the heat generation parameter γ caused an increase in the temperature profile of the fluid. Figure 3 and Figure 4, show increasments in the velocity profile and temperature as a result of thermal Grashoff and heat generation parameters respectively. Furthermore, while the temperature profile has remarkable increases due to Joule heating in Figure 5, the same was found to have gradual increase due to viscous dissipation as noticed in figure 6. This is due to the viscous action that tend to inhibit the free flow of the fluid.

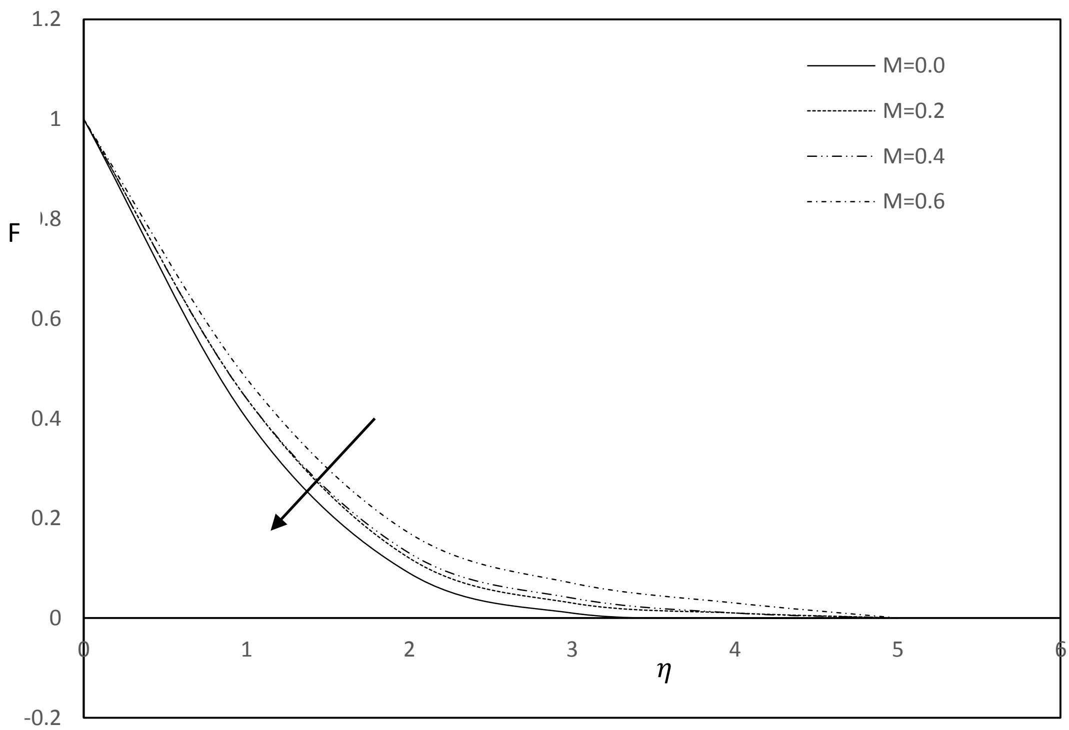

Figure 1: Effect of Hartmann number M on velocity profile of the nano fluid with M= 0.0, 0.2,0.4, 0.6 at fixed value of G_t = 5 and $\gamma = 0.1$

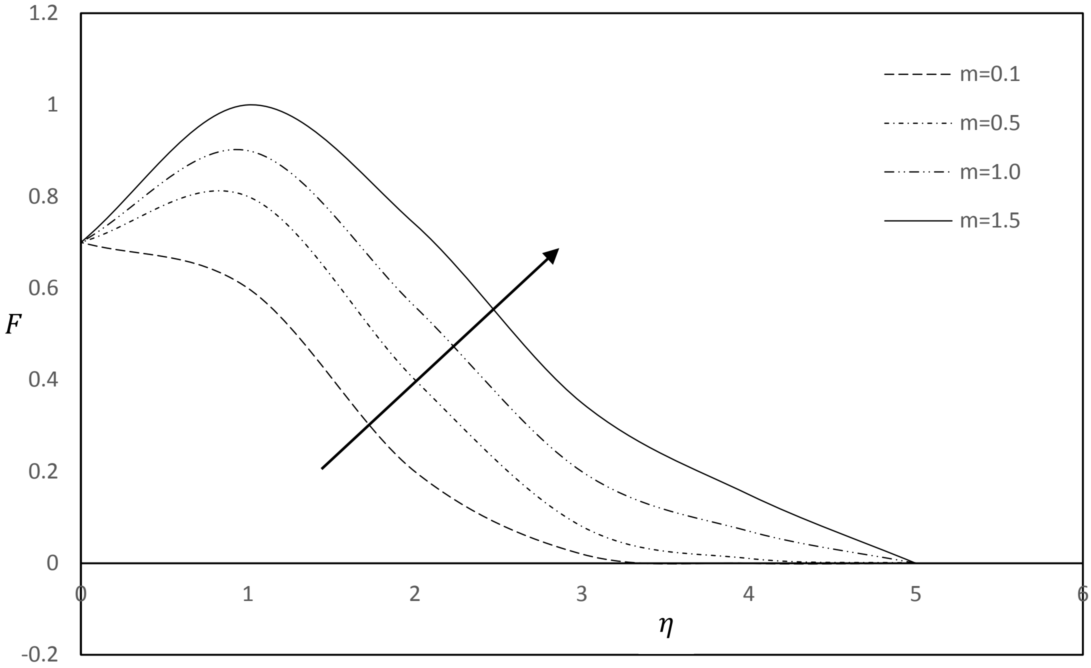

Figure 2: Effect of Heat generation parameter γ on velocity profile with γ= 0.1,0.5, 1.0,1,5. At fixed value of G_t = 5 and M$=0\,.5$

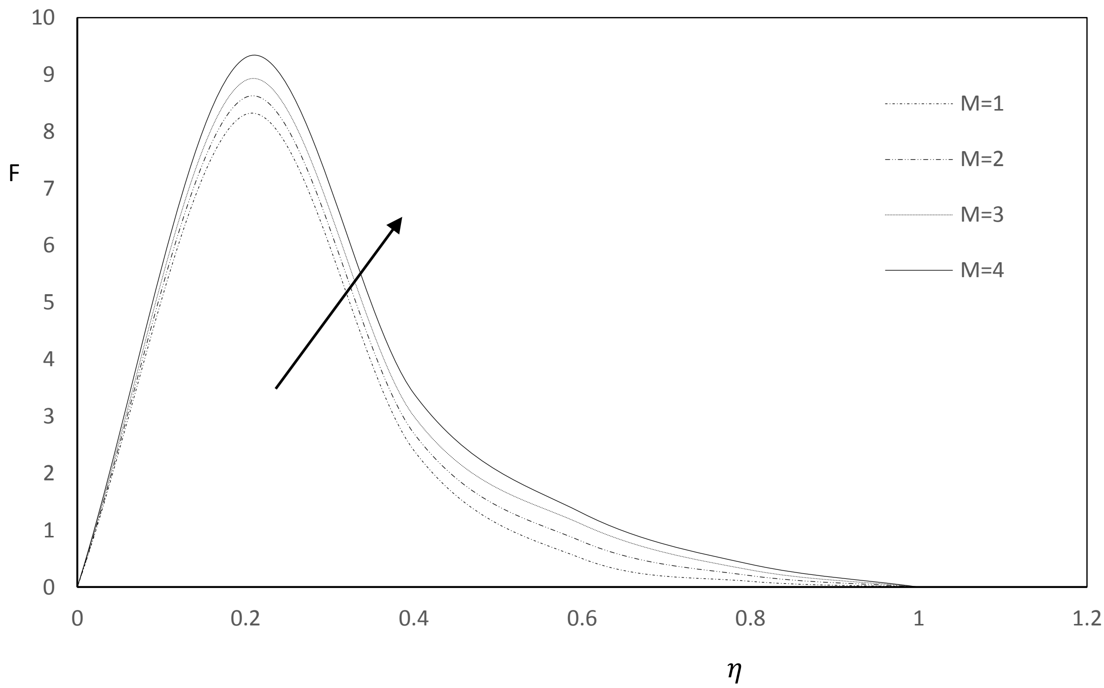

Figure 3: Effect of Thermal Grashoff number parameter G_t on velocity profile at M$=0\,.5, \gamma=0.1$

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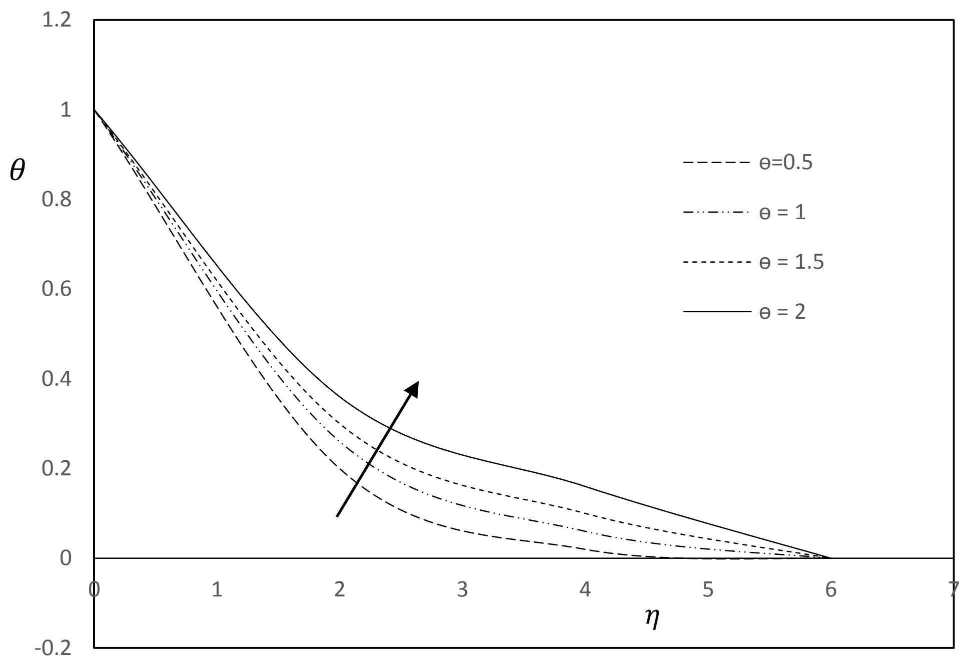

Figure 4: Effect of Heat generation parameter γ on temperature with value of E_c = 0.1 and J$=$ 0 .3

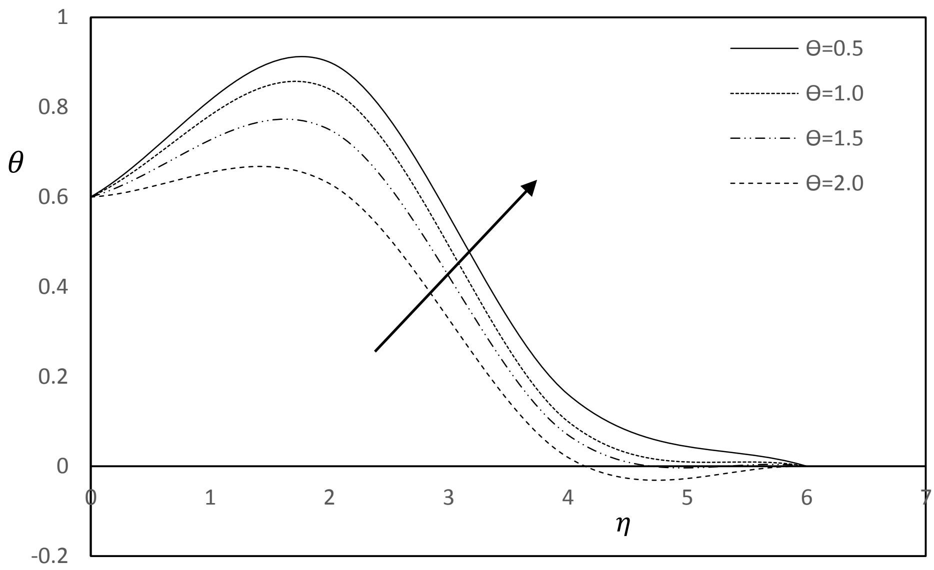

Figure 5: Effect of Joule Heat parameter J on temperature with E_c = 0.1 and $\gamma = 0$ .1

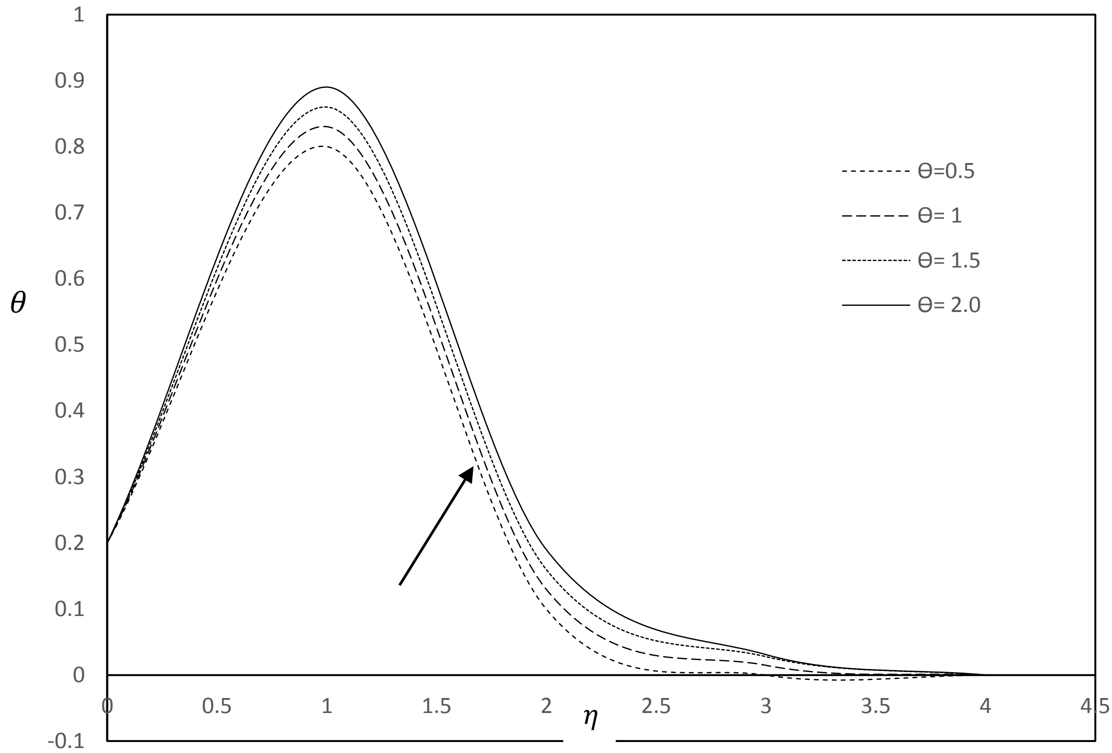

Figure 6: Effect of Viscous dissipation parameter E_c on temperature with value of J = 0.3 and $\gamma =$ 0.1

CONCLUSION

As the magnetic parameter increases, the velocity profile of the nano fluid decreases in the flow region. Joule heating affects the temperature in the conducting MHD fluid, the temperature of the fluid increased. Increase in the viscous dissipation parameter results in an increase in the nano fluid temperature. An increase in heat source parameter results in an increase in the temperature profile. An increase in the thermal Grashof increases the temperature profile of the fluid.

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