Scientia Africana, Vol. 20 (No. 2), August, 2021. Pp 61-72 https://dx.doi.org/10.4314/sa.v20i2.6
 ISSN 1118 – 1931

INVERSE AKASH DISTRIBUTION AND ITS APPLICATIONS

E. W. Okereke[1], S. N. Gideon[2] and J. Ohakwe[3]

[1,3]Department of Statistics, Michael Okpara University of Agriculture, Umudike
([1]emmastat5000@yahoo.co.uk, [3]ohakwejohnson@gmail.com)
[2] Department of Statistics, Abia State Polytechnic, Aba (dearngos2012@gmail.com)

Received: 17-03-2021
Accepted: 12-05-2021

ABSTRACT

A new one-parameter distribution named inverse Akash distribution, for modelling lifetime data, has been introduced. Important statistical properties of the proposed distribution such as the density function, hazard rate function, survival function, stochastic ordering, entropy measure, stress-strength reliability and the maximum likelihood estimation of the parameter of the distribution have been discussed. Two real data sets were employed in illustrating the usefulness of the new distribution. Comparatively, the inverse Akash distribution provided better fits to the data than each of the inverse exponential distribution and inverse Lindley distribution.

Keywords: Akash distribution, lifetime data, model selection criteria, stress-strength reliability, upside-down bathtub shape

INTRODUCTION

Modelling lifetime data is crucial in many fields including medicine, engineering, insurance and finance, amongst others (Shanker, 2015). Exponential, Akash (Shanker, 2015), Lindley (Lindley, 1958), gamma, lognormal and Weibull distributions and their generalizations are some of the continuous distributions used to model lifetime data. Notably, the exponential, Lindley and the Weibull distributions are more popular than the gamma and the lognormal distributions because the survival functions of the later cannot be expressed in closed forms. Though each of exponential, Lindley distribution and Akash distribution have one parameter, the Lindley and Akash distributions have one advantage over the exponential distribution. The exponential distribution has constant hazard rate function whereas the Lindley and Akash distributions have monotonically decreasing hazard rate functions (Shanker, 2015), making the Lindley and Akash distributions applicable in some cases where the exponential distribution is not useful.

In his study, Shanker (2015) compared the goodness of fits of the Akash distribution, exponential distribution and Lindley distribution and concluded that the Akash distribution can be the most suitable distribution for data among the three distributions. The probability density function (pdf) and the cumulative distribution function (cdf) of Akash distribution are respectively given by Shanker (2015) as

$$f\left(y;\theta\right)\frac{\theta^3}{\theta^2+2}\left(1+y^2\right)e^{-\theta y},\ \ \mathrm{y}>0,\theta>0 \tag{1}$$

and

$$F\left(y;\theta\right)=1-\left[1+\frac{\theta y\left(\theta y+2\right)}{\theta^2+2}\right]e^{-\theta y},\quad y>0,\theta>0 \qquad (2)$$

The density in Eq. (1) is a two-component mixture of an exponential distribution with scale parameter θ and a gamma distribution with shape parameter 3 and scale parameter θ with their mixing proportions $\frac{\theta^2}{\theta^2+2}$ and $\frac{2}{\theta^2+2}$ respectively.

Though the Akash distribution has been found suitable for modelling some data sets, it still has a major limitation. In fact, it is incapable of having non-monotonic hazard rates. Consequently, authors have made considerable efforts to extend the Akash distribution in order to have relatively more flexible distributions. In particular, Shanker (2016) introduced the quasi Akash distribution and discussed its properties. Shanker and Shaukla (2016) proposed a weighted Akash distribution. Poisson Akash distribution was propounded by Shanker (2017) for the analysis of discrete data. Shanker *et al.* (2018) developed the generalised Akash distribution. Using the exponentiation method, Okereke and Uwaeme (2018) used the exponential technique to obtain the exponentiated Akash distribution. Eyo *et al.* (2019) introduced the weighted quasi Akash distribution. Additionally, the zero-truncated discrete Akash and power size biased two-parameter Akash found by Sium and Shanker (2020) and Alhyasat *et al*. (2000) respectively, are also among distributions in the literature that are related to the Akash distribution ,

From the foregoing, it is certain that not much has been done to introduce an extension of the Akash distribution that can have upside-down bathtub shaped hazard rates. Given a positive continuous random variable Y with a known distribution, one of the known methods of deriving more flexible distributions is to determine the pdf of $X = Y^{-1}$. This results in the inverse distribution of Y. The advantages of inverse distributions are obvious. First, they are as parsimonious as their corresponding parent distribution since they do not require extra parameters (Eliwa *et al*., 2018). Second, they can have upside-down bathtub shaped hazard rates (Lee *et al*., 2017; Eliwa *et al*., 2018).

Considering the above mentioned desirable qualities of inverted distributions, we are motivated to introduce a new inverse distribution called the inverse Akash distribution and derive its mathematical properties. The rest of the paper is organised as follows. In Section 2, we derive the probability density function (pdf), cumulative distribution function (cdf), survival function and hazard rate function of the distribution. Plots of both the pdf and hazard rate function are given in this section so as to illustrate the possible shapes of the two functions. Other properties of the distribution such as stochastic ordering, entropy and stress-strength reliability of the distribution are discussed in Section 3. Real data application is presented in Section 4 while the paper is concluded in Section 5.

Scientia Africana, Vol. 20 (No. 2), August, 2021. Pp 61-72 https://dx.doi.org/10.4314/sa.v20i2.6
© Faculty of Science, University of Port Harcourt, Printed in Nigeria *ISSN 1118 – 1931*

THE INVERSE AKASH DISTRIBUTION

Proposition 1. If a random variable Y follows Akash distribution AD(θ), then the random variable $X = \frac{1}{Y}$ has inverse Akash distribution with scale parameter θ and its probability pdf and cdf are respectively given by

$$f(x;\theta)\frac{\theta^3}{\theta^2+2}\left(\frac{1+x^2}{x^4}\right)e^{-\frac{\theta}{x}},\ x>0,\theta>0 \tag{3}$$

and

$$F(x;\theta)=\left[1+\left(\frac{\theta+2x}{\theta^2+2}\right)\frac{\theta}{x^2}\right]e^{-\frac{\theta}{x}},\quad \mathrm{x}>0,\theta>0. \tag{4}$$

Proof.

If Y follows Akash distribution with parameter θ, the pdf of Y becomes

$$g(y;\theta)=\frac{\theta^3}{\theta^2+2}\left(1+y^2\right)e^{-\theta y}.$$

Let $x=y^{-1}$. $\frac{\mathrm{d}y}{\mathrm{d}x}=-x^{-2}$. Thus, the pdf of Y is given by

$$f(x)=g\left(x^{-1}\right)\left|\frac{\mathrm{d}y}{\mathrm{d}x}\right|$$

$$=\frac{\theta^3}{\theta^2+2}\left(\frac{1}{x}\right)\left(1+\left(\frac{1}{x}\right)^2\right)e^{-\frac{\theta}{x}}$$

$$=\frac{\theta^3}{\theta^2+2}\left(\frac{1+x^2}{x^4}\right)e^{-\frac{\theta}{x}},x>0,\theta>0.$$

The cdf for the inverse Akash distribution can be expressed as

$$F(x,\theta)=\theta^3\left(\theta^2+2\right)^{-1}\int_0^x t^{-4}\left(1+t^2\right)e^{-\frac{\theta}{t}}\mathrm{d}t$$

$$=\theta^3\left(\theta^2+2\right)^{-1}\left[\int_0^x t^{-4}e^{-\frac{\theta}{t}}\mathrm{d}t+\int_0^x t^{-2}e^{-\frac{\theta}{t}}\mathrm{d}t\right]$$

Let $w=\theta t^{-1}$. $t=\theta w^{-1}$ and $\mathrm{d}t=-\theta w^{-2}\mathrm{dw}$.

Applying the technique of integration by parts, we haxe

$$F(x,\theta)=\theta^3\left(\theta^2+2\right)^{-1}\left[\theta^{-3}\int_{\theta x^{-1}}^{\infty}w^2e^{-w}\mathrm{d}w+\theta^{-1}\int_{\theta x^{-1}}^{\infty}e^{-w}\mathrm{d}w\right]$$

$$=\theta^3\left(\theta^2+2\right)^{-1}\theta^{-3}\left(\left(\frac{\theta}{x}\right)^2e^{-\frac{\theta}{x}}+2\left(\left(\frac{\theta}{x}\right)e^{-\frac{\theta}{x}}+e^{-\frac{\theta}{x}}\right)+\theta^2e^{-\frac{\theta}{x}}\right)$$

$$=\theta^3\left(\theta^2+2\right)^{-1}\theta^{-3}\left(\left(\frac{\theta}{x}\right)^2+2\left(\frac{\theta}{x}\right)+\theta^2+2\right)e^{-\frac{\theta}{x}}$$

$$=\left(1+\left(\frac{\theta+2x}{\theta^2+2}\right)\frac{\theta}{x^2}\right)e^{-\frac{\theta}{x}}.$$

We denote the inverse Akash distribution in Eq. (3) by IAD(θ) and show the shapes of its pdf for different values of θ in Figure 1.

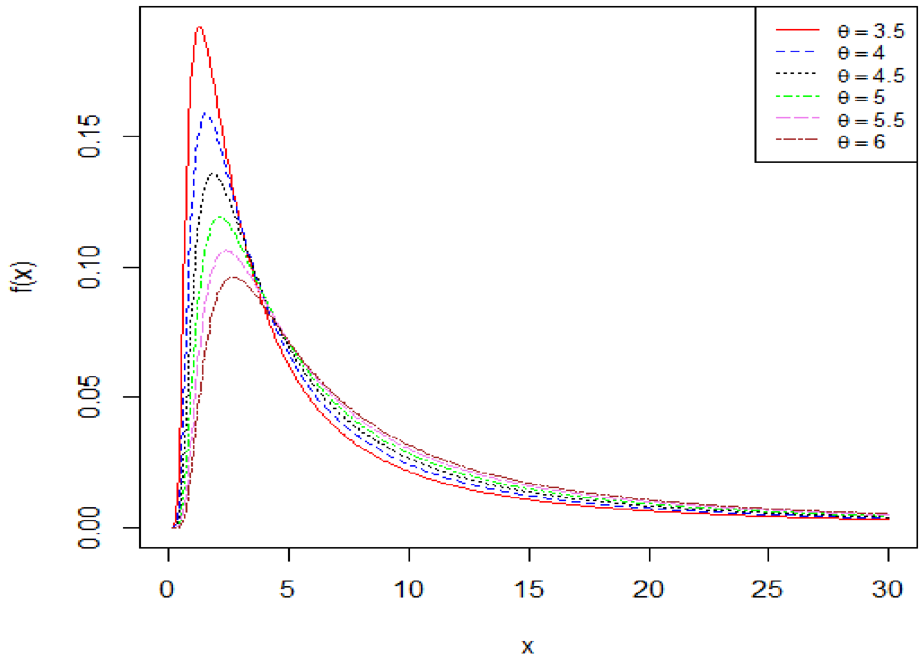

Figure 1. Pdf of $IAD(\theta)$ for different values of $\boldsymbol{\theta}$.

Figure 1 indicates that the density function of IAD(θ) is uni-modal in x.

The survival function of $IAD(\theta)$ is given by

$$s(x;\theta)=1-F(x;\theta)=1-\left[1+\left(\frac{\theta+2x}{\theta^2+2}\right)\frac{\theta}{x^2}\right]e^{-\frac{\theta}{x}},\quad x>0.\theta>0 \tag{5}$$

Thus, the hazard rate function is given as:

Scientia Africana, Vol. 20 (No. 2), August, 2021. Pp 61-72 https://dx.doi.org/10.4314/sa.v20i2.6
© *Faculty of Science, University of Port Harcourt, Printed in Nigeria* ISSN 1118 – 1931

$$h(x;\theta)=\frac{f(x,\theta)}{s(x;\theta)}=\frac{\theta^3\left(1+x^2\right)}{x^2\left[x^2\left(\theta^2+2\right)\left(e^{\frac{\theta}{x}}-1\right)-\theta(\theta+2x)\right]}$$

(6)

The various shapes of hazard rate function of IAD(θ) for different values of θ are given in Figures 2.

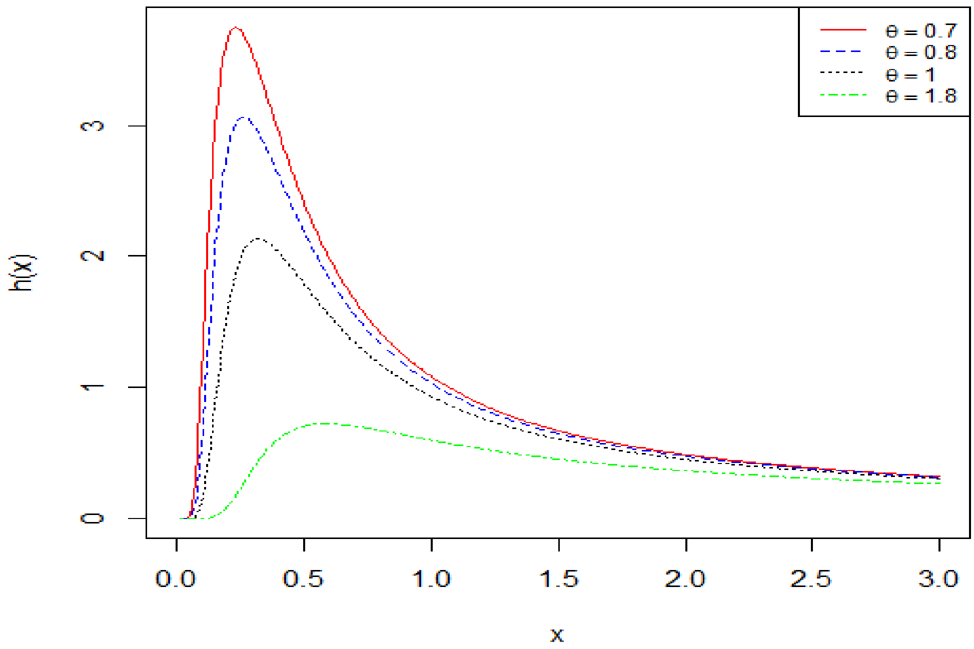

Figure 2. Hazard rate functions of $IAD(\theta)$ for different values of $\boldsymbol{\theta}$.

From Figure 2, it can be observe that the hazard rate function of $IAD(\theta)$ has upside-down bathtub shape for different values of the parameter θ.

OTHER PROPERTIES OF *IAD*

In this section we discuss the stochastic ordering, entropy measure and the stress-strength reliability in inverse Akash distribution.

Stochastic Ordering

Stochastic ordering of positive continuous random variables is an important tool for judging their comparative behaviour. According to Shanker (2015), a random variable X is said to be smaller than a random variable Y in the;

i. Stochastic order $(X \leq_{st} Y)$ if $F_X(x) \geq F_Y(x)$ for all x.
ii. Hazard rate order $(X \leq_{hr} Y)$ if $h_X(x) \geq h_Y(x)$ for all x.
iii. Mean residual life order $(X \leq_{mrl} Y)$ if $m_X(x) \leq m_Y(x)$ for all x.

iv. Likelihood ratio order $(X \leq_{lr} Y)$ if $\left(\frac{f_X(x)}{f_Y(x)}\right)$ decreases in x.

The following results due to Shaked *et al*. (1994) are well known for establishing stochastic ordering of distributions

$$(X \leq_{lr} Y) \Rightarrow (X \leq_{hr} Y) \Rightarrow (X \leq_{mrl} Y)$$
$$\Downarrow$$
$$(X \leq_{st} Y)$$

The inverse Akash distributions are ordered with respect to the strongest likelihood ratio ordering as shown in Theorem 1.

Theorem 1: Let $X \sim IAD(\theta_1)$ and $Y \sim IAD(\theta_2)$.

If $\theta_1 < \theta_2$, then $(X \leq_{lr} Y)$ and hence $(X \leq_{hr} Y), (X \leq_{mrl} Y)$ and $(X \leq_{st} Y)$

Proof.

We have

$$\frac{f_X(x)}{f_Y(x)} = \frac{\theta_1^3(\theta_2^2+2)e^{-\frac{\theta_1}{x}}}{\theta_2^3(\theta_1^2+2)e^{-\frac{\theta_2}{x}}}$$

$$= \frac{\theta_1^3(\theta_2^2+2)}{\theta_2^3(\theta_1^2+2)} e^{-\frac{(\theta_1-\theta_2)}{x}}; x > 0$$

(7)

$$\log\frac{f_X(x)}{f_Y(x)} = \log\left[\frac{\theta_1^3(\theta_2^2+2)}{\theta_2^3(\theta_1^2+2)}\right] - \left[\frac{(\theta_1-\theta_2)}{x}\right]$$

This gives

$$\frac{d}{dx}\log\frac{f_X(x)}{f_Y(x)} = \frac{(\theta_1-\theta_2)}{x^2}$$

(8)

Thus, for $\theta_1 < \theta_2$, $\frac{d}{dx}\log\frac{f_X(x)}{f_Y(x)} > 0$. This means that $X \leq_{lr} Y$ and hence $X \leq_{hr} Y$, $X \leq_{mrl} Y$ and $X \leq_{st} Y$.

Entropy Measure

Entropy of a random variable X is a measure of variation of uncertainty. A popular entropy measure is Renyi entropy Renyi (1961). If X is a continuous random variable having probability density function$f(.)$, then Renyi entropy is defined as

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\{\int f^{\gamma}(x)\,dx\},$$

where $\gamma > 0$ *and* $\gamma \neq 1$.

For the Inverse Akash distribution, the Renyi entropy measure is defined by;

Scientia Africana, Vol. 20 (No. 2), August, 2021. Pp 61-72 https://dx.doi.org/10.4314/sa.v20i2.6
© Faculty of Science, University of Port Harcourt, Printed in Nigeria ISSN 1118 – 1931

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\int_0^{\infty}\frac{\theta^{3\gamma}}{(\theta^2+2)^{\gamma}}\left[\frac{(1+x^2)^{\gamma}}{x^{4\gamma}}\right]e^{-\frac{\theta\gamma}{x}}dx$$

We know that

$$(1+z)^j = \sum_{j=0}^{\infty}\binom{\gamma}{j}z^j \text{ and } \int_0^{\infty}e^{-\frac{b}{x}}x^{-a-1}dx = \frac{\Gamma(a)}{b^a}.$$

Therefore,

$$T_R(\gamma) = \frac{1}{1-\gamma}\log\left[\frac{\theta^{3\gamma}}{(\theta^2+2)^{\gamma}}\sum_{j=0}^{\infty}\binom{\gamma}{j}\int_0^{\infty}\frac{e^{-\frac{\theta\gamma}{x}}}{x^{4\gamma-2j}}dx\right]$$

$$= \frac{1}{1-\gamma}\log\left[\frac{\theta^{3\gamma}}{(\theta^2+2)^{\gamma}}\sum_{j=0}^{\infty}\binom{\gamma}{j}\frac{\Gamma(4\gamma-2j-1)}{(\theta\gamma)^{4\gamma-2j-1}}\right] \quad (9)$$

Stress-Strength Reliability and Maximum Likelihood Estimation

Let Y and X be independent stress and strength random variables that follow Inverse Akash distribution with parameter θ_1 *and* θ_2 respectively. Then, the stress-strength reliability (R) is defined as

$$R = P[Y<X] = \int_0^{\infty}P[Y<X \mid X=x]\, f_x(x)dx = \int_0^{\infty}f(x,\theta_1)\; F(x,\theta_2)dx$$

$$= \int_0^{\infty}\left(\left[1+\left(\frac{\theta_2+2x}{\theta_2^2+2}\right)\frac{\theta_2}{x^2}\right]e^{-\frac{\theta_2}{x}}\right)\frac{\theta_1^3}{\theta_1^2+2}\left(\frac{1+x^2}{x^4}\right)e^{-\frac{\theta_1}{x}}dx$$

$$= \frac{\theta_1^3}{\theta_1^2+2}\int_0^{\infty}\left(\frac{1+x^2}{x^4}\right)e^{-\frac{(\theta_1+\theta_2)}{x}}dx + \frac{\theta_1^3\theta_2}{(\theta_1^2+2)(\theta_2^2+2)}\int_0^{\theta}\left(\frac{1+x^2}{x^4}\right)\left(\frac{\theta_2+2x}{x^2}\right)e^{-\frac{(\theta_1+\theta_2)}{x}}dx$$

$$= \frac{\theta_1^3}{\theta_1^2+2}\left[\int_0^{\infty}x^{-4}e^{-\frac{(\theta_1+\theta_2)}{x}}dx + \int_0^{\infty}x^{-2}e^{-\frac{(\theta_1+\theta_2)}{x}}dx\right] + \frac{\theta_1^3\theta_2}{(\theta_1^2+2)(\theta_2^2+2)}\int_0^{\infty}\left(\frac{\theta_2+\theta_2x^2+2x+2x^3}{x^6}\right)e^{-\frac{(\theta_1+\theta_2)}{x}}dx$$

Consequently, we get the expression for the stress-strength reliability as

$$R = \left(\frac{\theta_1^3}{\theta_1^2+2}\right)\left[\frac{\Gamma(3)}{(\theta_1+\theta_2)^3}+\frac{\Gamma(1)}{(\theta_1+\theta_2)}\right] + \frac{\theta_1^3\theta_2}{(\theta_1^2+2)(\theta_2^2+2)}\left[\theta_2\left(\frac{\Gamma(5)}{(\theta_1+\theta_2)^5}+\frac{\Gamma(3)}{(\theta_1+\theta_2)^3}\right) + 2\left(\frac{\Gamma(4)}{(\theta_1+\theta_2)^4}+\frac{\Gamma(2)}{(\theta_1+\theta_2)^2}\right)\right]$$

$$= \frac{\theta_1^3\{[2+(\theta_1+\theta_2)^2][(\theta_2^2+2)(\theta_1+\theta_2)^2]+\theta_2[24\theta_2+2\theta_2(\theta_1+\theta_2)^2+12(\theta_1+\theta_2)+2(\theta_1+\theta_2)^3]\}}{(\theta_1^2+2)(\theta_2^2+2)(\theta_1+\theta_2)^5}$$

(10)

Since R is the stress-strength reliability function with parameters θ_1 and θ_2, we need to obtain the maximum likelihood estimators (MLEs) of θ_1 and θ_2 to compute the maximum likelihood estimation R using the invariance property of the maximum likelihood estimators.

Suppose $X_1, X_2, \dots, X_n$ is a Strength randon variable sample from Inverse Akash distribution (θ_1) and $Y_1, Y_2, \dots, Y_m$is a Stress random sample from Inverse Akash distribution (θ_2). Thus, the likelihood function based on the observed sample is given by;

$$L\left(\theta_1, \theta_2 / \underline{x}, \underline{y}\right) = \frac{\theta_1^{3n}\theta_2^{3m}}{(\theta_1^2+2)^n(\theta_2^2+2)^m} \prod_{i=1}^{n}\left(\frac{1+x_i^2}{x_i^4}\right) \prod_{j=1}^{m}\left(\frac{1+y_j^2}{y_j^4}\right) e^{-(\theta_1 S_1+\theta_2 S_2)},$$

(11)

where, $S_1 = \sum_{i=1}^{n}\frac{1}{x_i}$, $\quad S_2 = \sum_{j=1}^{m}\frac{1}{y_j}$

The log-Likelihood function is given by

$$\log L(\theta_1,\theta_2) = 3n\log\theta_1 + 3m\log\theta_2 - n\log(\theta_1^2+2) - m\log(\theta_2^2+2) - \theta_1 S_1 - \theta_2 S_2 + \sum_{i=1}^{n}\log\left(\frac{1+x_i^2}{x_i^4}\right) + \sum_{j=1}^{m}\log\left(\frac{1+y_j^2}{y_j^4}\right)$$

(12)

Consider the partial derivatives in Eq. (13) and Eq. (14)

$$\frac{\partial \log L(\theta_1,\theta_2)}{\partial\theta_1} = \frac{3n}{\theta_1} - \frac{2\theta_1 n}{(\theta_1^2+2)} - S_1$$

(13)

$$\frac{\partial \log L(\theta_1,\theta_2)}{\partial\theta_2} = \frac{3m}{\theta_2} - \frac{2\theta_2 m}{(\theta_2^2+2)} - S_2$$

(14)

From Eq. (14) and Eq. (15), we obtain the maximum likelihood equations

$$\theta_1^3 S_1 - \theta_1^2 n + 2\theta_1 S_1 - 6n = 0$$

(15)

$$\theta_2^3 S_2 - \theta_2^2 m + 2\theta_2 S_2 - 6m = 0$$

(16)

The MLEs $\hat{\theta}_k$ of θ_k for $k = 1,2$ can be obtained by solving the cubic equations in Eq. (15) and Eq. (16). The roots of the equations can be obtained using R software. Again, by the invariance property of the MLEs, the maximum likelihood estimator $\hat{R}_{mle}$ of R can be obtained by substituting $\hat{\theta}_k$ in place of θ_k for k = 1, 2. Hence,

$$\hat{R}_{mle} = \frac{\theta_1^3\{[2+(\theta_1+\theta_2)^2][(\theta_2^2+2)(\theta_1+\theta_2)^2]+\theta_2[24\theta_2+2\theta_2(\theta_1+\theta_2)^2+12(\theta_1+\theta_2)+2(\theta_1+\theta_2)^3]\}}{(\theta_1^2+2)(\theta_2^2+2)(\theta_1+\theta_2)^5}\Bigg|_{\theta_k=\hat{\theta}_k, k=1,2} . \quad (17)$$

Scientia Africana, Vol. 20 (No. 2), August, 2021. Pp 61-72 https://dx.doi.org/10.4314/sa.v20i2.6
© *Faculty of Science, University of Port Harcourt, Printed in Nigeria* ISSN 1118 – 1931

REAL DATA APPLICATION

Two real life data sets are used to demonstrate the application of *IAD*.

Fits of the new model to the data are compared with those of the inverse exponential distribution (*IED*) and inverse Lindley distribution (*ILD*). The data sets given in Appendix, represent the survival times of two groups of patients suffering from Head and Neck cancer disease. The patients in one group were treated using radiotherapy (RT) (Data X) whereas the patients belonging to the other group were treated using a combined radiotherapy and chemotherapy (RT+CT) (Data Y).

Firstly, we obtained the estimates and the standard errors (SE) of the estimates of the parameters of the IAD, IED and ILD) as shown in Table 1

Table1. Estimates of the Parameters of the Models Fitted to Data X and Data Y and their Corresponding Standard Errors

	Distributions	Parameter Estimate	SE
Data X	*IAD*	59.2532	7.7671
	IED	59.1225	7.7632
	ILD	60.0883	7.7649
Data Y	*IAD*	76.7452	11.5580
	IED	76.7006	11.5631
	ILD	77.6754	11.5649

Table 1 shows the proposed *IAD* performs better in the analysis of the two survival data sets than the *IED* and the *ILD*.

Secondly, we compare fits of the distributions using Akaike information criterion (AIC), Bayesian information (BIC), Kologorov-Smirnov statistic (K-S), Cramer-Von Mises Criterion (W*) and Anderson Darling statistic (A*) as given in Table 2.

Table 2. Goodness of Fit Statistics for the Distributions Fitted to Data X and Data Y

	Distribution	-L	AIC	BIC	K-S	W*	A*
Data X	*IAD*	385.6517	773.3034	775.3638	0.30430	1.18450	5.65980
	IED	385.6871	773.3742	775.4346	0.30480	1.18960	5.68220
	ILD	385.7031	773.4062	775.4666	0.30480	1.19040	5.68630
Data Y	*IAD*	279.5750	561.1500	562.9342	0.08881	0.07589	0.49308
	IED	279.5773	561.1546	562.9388	0.08884	0.07592	0.49308
	ILD	279.5784	561.1568	562.9409	0.08888	0.07596	0.49308

Observe from Table 2 that the *IAD* produced the smallest value of each of AIC, BIC, K-S, W*and A* as compared to the *IED* and the *ILD*. Thus, among the three models under consideration, the *IAD* provides the best fit to each of the two data sets.

With the data, Eq. (13), Eq. (15), Eq. (16) and Eq. (17), we obtain the maximum likelihood estimates of θ_1, θ_2 and R as shown in Table 3.

Table 3. The Maximum Likelihood Estimates of θ_1, θ_2 and R

Parameter	MLE
θ_1	0.0042
θ_2	76.7472
R	4.4294×10^{-6}

CONCLUSION

The inverse Akash distribution (*IAD*) for modelling lifetime data has been proposed in this study. Its statistical properties such as the shape characteristics of density, survival function, hazard rate function, stochastic ordering are obtained. Specifically, the pdf of the distribution can be unimodal while the hazard rate function of the distribution has upside-down bathtub shape. Further, expressions for the entropy measure and stress-strength reliability of the proposed distribution have been derived. The method of maximum likelihood estimation has also been discussed for estimating its parameter. On the basis of two data sets, the fits of the new model are compared with those of the inverse exponential and inverse Lindley distributions. Consequently, the results obtained using some model selection criteria indicate that among the three models, the inverse Akash distribution is the best fitting model to the data. Therefore, the newly introduced distribution can be a better alternative to a number of well-known distributions.

REFERENCES

Alhyasat, K., Kamarulzaman, I., Al-Omari, A. I. and Bakar, M. A. A. (2020). Power size biased two-parameter Akash distribution. *Statistics in Transition*, 21(3), 73-91.

Eliwa, M. S., El-Morshedy, M. and Ibrahim, M. (2018). Inverse Gompertz distribution: Properties and different estimation methods with application to complete and censored data. *Annals of Data Science*, https://doi.org/ 10.1007/s40745-018-0173-0.

Eyo, T., Shanker, R., Shukla, K.K and Leonidal, A. A (2019). Weighted quasi Akash distribution: Properties and applications. *American Journal of Mathematics and Statistics*, 9(1), 30-43.

Lee, S., Noh, Y. and Chung, Y. (2017). Inverted exponentiated Weibull distribution with applications to lifetime data. *Communications for Statistical Applications and Methods*, 24(3), 227-240.

Lindley, D. V. (1958).Fiducial distributions and Bayes theorem, *Journal of the Royal Statistical Society*, 20, 102–107.

Okereke, E. W. and Uwaeme O.R (2018) Exponentiated Akash distribution and its application. *Journal of the Nigerian Statistical Association*, 30, 1-13.

Renyi, A. (1961). Measures of entropy and information proceedings of the 4th

Scientia Africana, Vol. 20 (No. 2), August, 2021. Pp 61-72 https://dx.doi.org/10.4314/sa.v20i2.6
© Faculty of Science, University of Port Harcourt, Printed in Nigeria ISSN 1118 – 1931

Berkeley Symposium on Mathematical statistics and probability, University of California Press, Berkeley.

Shaked, M. and Shanth, J.G. K. (1994). *Stochastic orders and their applications*, Boston: Academic Press.

Shanker, R. (2015). Akash distribution and its applications. *International Journal of Probability and Statistics,* 4(3), 65-75.

Shanker, R. (2016): A Quasi Akash Distribution, *Assam Statistical Review*, 30(1), 135-160

Shanker R. (2017). The Discrete Poisson-Akash Distribution. *International Journal of Probability and Statistics*, 6(1), 1–10.

Shanker R, and Shukla K.K. (2016). Weighted Akash distribution and its Application to model lifetime data. *International Journal of Statistics* 39 (2), 1138-1147.

Shanker, R., Shukla, K. K, Shanker R., and Singh, A.P. (2018). A generalised Akash distribution. *Biometrics and Biostatistics International Journal*, 7(1):1 -10

Sium, S. and Shanker, R. (2020). A zero-truncated discrete Akash distribution with properties and applications. *Hungarian Statistical Review*, 3(2), 12-25.

APPENDIX

Data X:
6.53,7,10.42,14.48,16.10,22.70,34,41.55,42,45.28,49.40,53.62,63,64,83,84,91,108,112,129,133,133,139,140,140,146,149,154,157,160,160,165,146,149,154,157,160,160,165,173,176,218,225,241,248,273,277,297,405,417,420,440,523,583,594,1101,1146,1417

Data Y:
12.20,23.56,23.74,25.87,31.98,37,41.35,47.38,55.46,58.36,63.47,68.46,78.26,74.47,81.43,84,92,94,110,112,119,127,130,133,140,146,155,159,173,179,194,195,209,249,281,319,339,432,469,519,633,725,817,1776