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USING DIRAC'S PRINCIPLE OF SUP ERPOSITION OF EVOLUTION OF TIME IN RESTORING DETERMINACY IN QUANTUM MECHANICS

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ABSTRACT

A foundational problem of quantum mechanics is that measurement of quantum systems seems to affect them in an indeterminate manner. This indeterminacy has been the bane of interpretation of quantum mechanics. The Dirac's principle of superposition is a collection of all possible measurement outcomes of a quantum system. It is proposed here that for a linear evolution of time, there will be four possible motions in time and they are the motion from past time, the motion backward to past time, the motion forward in future time and the motion backward from future time. It follows then that since the laws of physics are usually based on evolution of physical systems with time, this special case which is referred to as the Dirac's principle of superposition for evolution of time (DPSET) predicts the four possible processes of the dynamic evolution of a system. Our formulation of the DPSET resulted into a subtle geometrization of time which in principle is applicable to quantum systems in all dimensions, thereby restoring the determinacy in quantum measurement. The DPSET is then successfully used to naturally account for the electron hopping in the beryllium atom resulting in both ionic and covalent bonding as well as neutron decay and decay by electron capture.

INTRODUCTION

One of the challenging problems of quantum mechanics from the origin of its formation is the measurement problem. The reason is that measurements of quantum systems affect them in an indeterminate manner and therefore lead to randomness in observations (Wheeler and Zurek, 1983; Cohen-Tannoudji, 2006; Volz et. al., 2011). This is the origin of the fundamental randomness implied by quantum mechanics in contrast to the determinism basis of physics as even depicted by the Schrödinger's wave equation which predicts a perfectly deterministic time evolution of the wave function. However, only abstract probabilities are involved in Schrödinger determinism and are confirmed only in the statistics of large numbers of identically prepared experiments. Randomness is assumed to enter only whenever a measurement is made because the wavefunction is assumed to 'collapsed' into one of the possible states of the system (Bassi and Ghirardi, 2000; Dumitru, 2014; Modak and Sudarsky, 2018). This is the kernel of the Copenhagen interpretation whose most remarkable shortcoming is the lack of determinism. This wavefunction 'collapse' is caused by the mindsets of the experimentalists in the opinion of the proponents of the consciousness theory of interpretation of quantum mechanics (McQueen, 2017). But even with the consciously induced wave function 'collapse', the theory did not also give rise to determinism. Interestingly, both aforementioned interpretations do not violate the Dirac's principle of superposition which is a collection of all possible measurement outcomes. However, unlike the Copenhagen interpretation of quantum mechanics, the Dirac's principle of superposition does not say that a system can be in two or more states at the same time, but that there is a non-zero probability of finding it in one of these state should it be measured (HariDass, 2013).This is clearly in line with the conjecture of the many world interpretation that all possible measurement outcomes really existed representing an actual "world" (or "universe") independent of each other before and after the measurement process (Barrett, 2009; Gribbin, 2020). Therefore what one observed as an experimental outcome is a real existing universe in the universal wavefunction. Then as postulated in the decoherence theory, the measuring process will determine the observed existing universe. Thus there is determinism in both the many world and decoherence interpretations. If one may use intuition, then one can re-interprete both theories as follows: though a quantum system has several ways to be configured into a state, the actual initial state and its evolution into final state are determined by the measuring process.

Initial state ⟶ *Measuring process* ⟶ *Final state*

It is pertinent to quickly emphasize the difference between measurement as act and the process of implementing this act which is the measuring process. The postulation here is that if we can configure a quantum state so that we know its past, then we can choose its future and how to evolve it precisely into that future as well as back into the past. This conjecture is in line with the assertion Einstein once made that whether one observed a thing or not depends on the theory which one used (Akpojotor and Echenim, 2010).

It is common knowledge that every dynamic systemevolves with time. If we follow our intuition, then there will be four possible evolutions in time observed at time *t = 0* and they are the evolution from past time, the evolution backward to past time, the evolution forward in future time and the evolution backward from future time. Every physical state is represented by a family of vectors in a Hilbert space symbolically denoted by the Dirac ket$|\ \rangle$. Therefore, if we denote t_p^f as the time for the evolution from past time, t_p^b as the time for the evolution backward to past time, t_f^f as the time for the evolution forward in future time and t_f^b as the time for evolution backward from future time, then we can represent the state for each of the four possible evolutions as

$$|\varphi_1(t_p^f)\rangle,\ |\varphi_2(t_p^b)\rangle,\ |\varphi_3(t_f^f)\rangle,\ |\varphi_4(t_f^b)\rangle \tag{1}$$

We can refer to this special case as the Dirac's principle of superposition for evolution of time (DPSET). The important observation here is that the measurement outcome of a quantum system will only result in only one of these possible evolutions and this is commonly referred to as the wavefunction which could be a linear combination of all four motions now 'collapsing' into just one state. This is often misconstrued that before measurement, a quantum system can and will occupy multiple states right up until the instant that it is measured and the measurement process will yield only one state (Bassi and Ghirardi, 2000; Cowan and Tumulka, 2014a; Cowan and Tumulka, 2015). Physically, it implies it is the measurement process that determines the wavefunction. Therefore, if we know the initial condition and have idea of all the possible outcomes, then we can theoretically predict this new state of the system even before measurement is done (Cowan and Tumulka, 2014b: Cowan and Tumulka, 2015) and thereby restoring determinacy in physics. This is the purpose of the study here. In the next section, we will demonstrate the DPSET using a hypothetical quantum particle in a linear motion. Thereafter, we will demonstrate how the dynamic evolution of the electron in a hypothetical atom as envisaged in the DPSET can be used to account for the chemical reactions and decay processes. This precursory demonstration will enable using the DPSET to naturally account for the bonding and decay processes of the beryllium atom.

BASIC DYNAMIC EVOLUTION OF A QUANTUM PARTICLE

It is a common knowledge today that time evolution of physical systems depend on the reference frame which can be designated by the space-time coordinates. Thus for a particle, the essence of measuring its position is to establish the distance along a particular direction between the particle and a reference point with respect to time (Harrell, 2016). In other words, the spatial dynamics of a particle in a given frame depends on the time evolution.This can be represented by the Hamiltonian, H which gives the rate at which a particle has amplitude to go from one position to another. In quantum mechanics, predictions are made by way of calculating expectation values of observables, which take the form of Hermitian operators (Bolduc et. al. 2016). The condition that H should be Hermitian demands that the amplitude to move backward to past time is the complex conjugate of the amplitude to move forward from past time and it is given by the time differential equation:

$$i\frac{d\psi}{dt} = \alpha^*\left|\varphi_2(t_p^b)\right\rangle + \alpha\left|\varphi_1(t_p^f)\right\rangle \qquad (2)$$

where α is the amplitude which if properly chosen and the ψ considered linearly, then Eq. (2) can be expressed as the Schrodinger equation for free particle of mass *m*:

$$i\hbar\frac{d\psi(x,t)}{dt} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2}. \qquad (3)$$

Now if the particle is to be described as moving to future time, then the condition that H is Hermitian demands again that the amplitude to move backward from future time is the complex conjugate of the amplitude to move forward to future time and it is given by the time differential equation

$$i\frac{d\psi}{dt} = \beta^*\left|\varphi_4(t_f^b)\right\rangle + B\left|\varphi_3(t_f^f)\right\rangle \qquad (4)$$

where β is the amplitude which if properly chosen and the ψ considered linearly, then Eq.(4) can be expressed again as the Schrodinger equation for the free particle given by Eq. (3).

It is obvious from this simple analysis that Eqs. (2) and(4) collectively represent all four possible motions of a free particle. Now each of these possible motions can only be instigated by the 'force' acting on them that provides a potential energy *V* which determines the particle direction so that the two equations can be combined to yield a general expression:

$$i\hbar\frac{d\psi(x,t)}{dt}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2}+V(x,t)\psi(x,t) \quad . \qquad (5)$$

Eq.(5) is obviously the usual time dependent Schrodinger equation (TDSE) which is the dynamical law that governs wave mechanics which is the branch of quantum mechanics without spin or relativistic effects. The equation is made in the Schrödinger Picture, where a system's state is held to evolve with time and observables are fixed. This is why the *V* will now determine the two possible motions for both past and future times and is clearly illustrated by a quantum particle of energy *E* in a linear motion with a potential *V*: the application of the stationary form of Eq. (5) which is the time independent Schrodinger equation (TIDSE), to this particle will yield the usual four possible motions for it which when observed at the position $x = 0$ has a one to one correspondence with Eq.(1) (Pereyra, 2012):

$$|\varphi_1(x)\rangle = A_1e^{ik_px}, \quad |\varphi_2(x)\rangle = A_2e^{-ik_px}, \quad |\varphi_3(x)\rangle = A_3e^{ik_fx}, \quad |\varphi_4(x)\rangle = A_4e^{-ik_fx}. \qquad (6)$$

where $|\varphi_1(x)\rangle = A_1e^{ik_px}$ is the incident wave travelling from $x = -\infty$ to $x = 0$ (that is moving forward from past time), $|\varphi_2(x)\rangle = A_2e^{-ik_px}$ is the reflected wave travelling from $x = 0$ to $x = -\infty$ (that is moving backward to past time), $|\varphi_3(x)\rangle = A_3e^{ik_fx}$ is the transmitted wave travelling from $x = 0$ to $x = \infty$ (that is moving forward to future time) and $|\varphi_4(x)\rangle = A_4e^{-ik_fx}$ is the reflected transmitted wave travelling from $x = 0$ to $x = \infty$ (that is moving backward from future time). It is now a basic knowledge that the actual physical motion to be observed depends on the *E* of the particle in relation to the *V*, that is, if $E \geq V$ or $E <$ V (Akpojotor *et. al.* 2010). Thus by knowing the values of the *E* and *V*, it is possible to predict the motion hence the state of the particle (Branden and Joachan, 1989).

The physical interpretation of Eqs.(1) and (6) is that while the latter are the four possible stationary states of the motion of the particle, their evolutions with time are respectively given by the former. Thus to see how the stationary states propagate, that is their evolution with time, we have to combine the Eqs.(1) and (6):

$$|\varphi_1(x)\varphi_1(t_p^f)\rangle = A_1e^{ik_px}\varphi_1(t_p^f), \qquad |\varphi_2(x)\varphi_2(t_p^b)\rangle = A_2e^{-ik_px}\varphi_2(t_p^b)\ ,$$

$$|\varphi_3(x)\varphi_3(t_f^f)\rangle = A_3e^{ik_fx}\varphi_3(t_f^f), \qquad |\varphi_4(x)\varphi_4(t_f^b)\rangle = A_4e^{-ik_fx}\varphi_4(t_f^b)\,. \qquad (7)$$

Interestingly, Eq(7) can be obtained as the common solutions of the TDSE in Eq.(5) by the method of separation of variable if a potential that depends only on position is considered and the energy is definite (Branden and Joachan, 1989):

$$\psi_1(x,t_p^f) = A_1e^{ik_px}e^{\frac{-iEt_p^f}{\hbar}}, \psi_2(x,t_p^b) = A_2e^{ik_px}e^{\frac{-iEt_p^b}{\hbar}}\ ,$$

$$\psi_3(x,t_f^f) = A_3e^{ik_fx}e^{\frac{-iEt_f^f}{\hbar}},\ \psi_4(x,t_f^b) = A_4e^{ik_fx}e^{\frac{-iEt_f^b}{\hbar}}\ . \qquad (8)$$

It is straightforward to see that at t = 0, we recover Eq.(6) thereby reaffirming that the stationary states before the time evolution is $\psi(x,0)$ and both the past and future states is $\psi(x,t)$. Therefore in the DPSET, t = 0 will be the starting point. It is pertinent to quickly point out that the t = 0 does not mean the past times are on the negative part of the non-feasible time inequality line because in physics, the *t* generally means a time interval instead of a clock reading. Thus there is no self adjoint time operator in quantum mechanics (Deo and Satpathi, 2019). However, the time evolution can be expressed via an operator $U(t)$:

$$\psi(x,t) = U(t)\psi_n(x,0) \tag{9}$$

where $U(t) = e^{-iHt/h}$ so that the time evolution is unitary:

$$U^+(t) = U^{-1}(t) \text{ and } U(-t) = U^+(t). \tag{10}$$

These important properties reflect an important feature of time evolution in quantum mechanics: the time evolution is invariant under time reversal and conserve the norm. Thus the t = 0 means just the initial time before the evolution from the initial state. So as earlier designated and now depicted in Figure 1, $t_p^b > t = 0$ is the evolution time from the initial state ($t = t_2$) backward to past state ($t_1 > 0 = t_2$), $t_p^f > t = 0$ is the evolution time from the initial state ($t = t_1$) forward from a past state ($t_2 > 0 = t_1$), $t_f^f > t = 0$ is the evolution time from the initial state ($t = t_2$) forward to a future state ($t_3 > 0 = t_2$) and $t_f^b > t = 0$ is the evolution time from the initial state ($t = t_3$) backward from a future state ($t_2 > 0 = t$). We quickly point out that this formulation of the predictability of time evolution and its one to one correspondence with the linear motion of the four possible motions of a one dimensional (1D) quantum particle is a subtle geometrical interpretation of time (Olkhov, 2007a). This implies that the dynamics of the quantum particle can be can theoretically predicted not merely as abstract probabilities as in the formation of the Schrodinger equation. Therefore, in principle, the formulation here can be extended to other dimensions which will then enable the application of the DPSET to be used to theoretically predict the measurements of quantum systems.

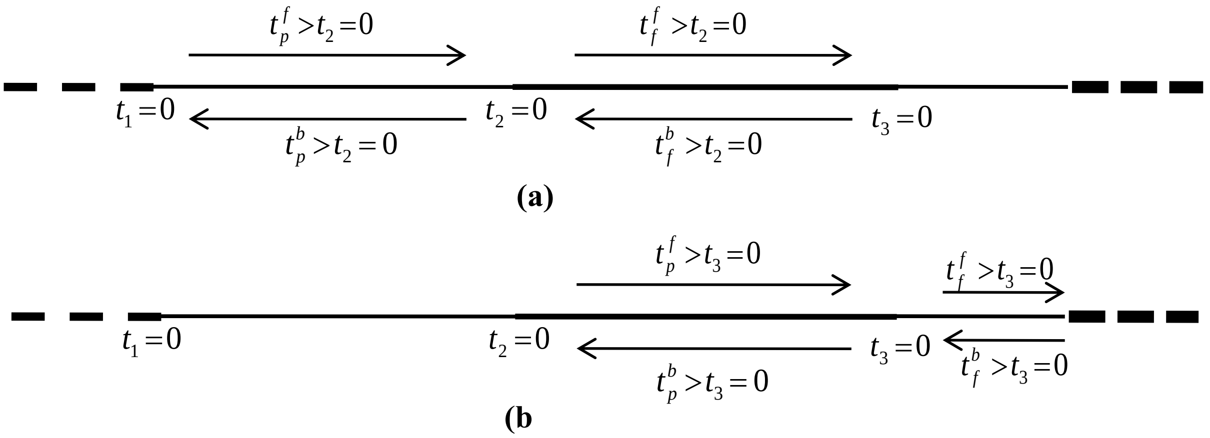

Figure 1. The time evolution of possible motion from the initial state at (a) $t_2 = 0$ and (b) $t_3 = 0$.

ELECTRON HOPPING IN AN ATOM

One of the earliest seemly indeterminate quantum system behavior is the hopping of electron from energy level to another which is also known as quantum jump. The still unanswered question is: at one point does the electron decide to make a quantum jump? This lack of determinacy of electron hopping so irked Schrodinger that he was quoted as telling Bohr "You surely must understand, Bohr, that the whole idea of quantum jumps necessarily leads to nonsense... If we are going to have to put up with these damn quantum jumps, I am sorry that I ever had anything to do with quantum theory." It was while seeking a means to completely stop this line of thinking that he postulated his famous hypothetic Schrodinger cat experiment. It is therefore our thinking that the simplest multiple particle physical system to demonstrate these four motions of time is the hopping of an electron in a hypothetical multi-electron atom as depicted in Figure 2. Though the historical perception that each electron orbited the nucleus of an atom in a separate layer often referred to as shells is changing as they are considered to hover in specific areas of the atom (Goodisman, 2012), the valence shell is still used to describe electron availability and hopping (Volz et. al., 2011).

In Figure 2, both atoms in (a) and (b) have a central nucleus and then a ground state level, a middle orbit and valence shell n = 1, n = 2 and n = 3 respectively. It is common knowledge that an electron in the inner shell or the valence shell can either absorb or release energy in the form of a photon. As it is shown in Figure 2a, based on the DPSET, an electron on the atomic shell n = 2 which losses energy will move backward into past time by hopping to the ground state n = 1 just as an electron in the ground state n = 1 gaining energy will move forward from past time by being excited to n = 2. Similarly an electron on the atomic shell n = 3 which absorbs energy will move forward to future time depending on the atomic configuration while if our atom needs an electron to complete its outmost shell, then it will receive an electron that has move backward from future time. So future time here means beyond the atomic shells either beyond the outer shell or beyond the groundstate shell. Therefore, if this our atom has one more electron say than the number required to form a closed shell and has to react with an atom that has one less electron say than the number required to form a closed shell, then there will be forward to future time hopping of the electron from our atom to the second atom to form ionic bonding. On the contrary, if the second atom has one more electron than the number required to form a closed shell while our atom has one less electron than the number required to form a closed shell, then there will be backward from future time hopping of the electron from the second atom to our atom to form ionic bonding. However, if our atom and the second atom have similar electronegativities (Manning, 2009), then the electrons in them will make restricted motion forward in future time thereby forming the various types of covalent bonding. The restricted motion here means the electron only moves part of itself into the future time just as the receiving atom only experience partial backward from future time hopping of the electron from the second atom. Therefore, depending on how much of this part of the electron is restrictively moved into the future time, the resulting covalent bonding could be nonpolar covalent (close to 50:50 sharing)

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or polar covalent (somewhere in between) (Kotz et. al., 2006).

As in the case of our atom in Figure 2a, an electron on the atomic shell n = 2 in our atom in Figure 2b which losses energy will move backward in time by hopping to the ground state n = 1 just as an electron in the ground state n = 1 gaining energy will move forward in time by being excited to n = 2. Based on the DPSET, an electron on the ground state which losses energy can move backward in future time by hopping to the nucleus where it will combine with a proton to form a neutron. Contrariwise, it is also expected that when a neutron in the nucleus disintegrates into a proton and an electron, the latter will move forward in future time by hopping into the ground state.

Application of the DPSET to the Beryllium Atom

For brevity, we will now demonstrate how to use the electron hopping in an atom in line with the DPSET to account for ionic bonding, covalent bonding, decay by electron capture and neutron decay using beryllium as our main atom. The choice of the beryllium is because it is one unique atom that can form both ionic and covalent bonding. According to the Pauling's scale, if the electronegativity difference between two atoms forming a bond is more than 1.7, then ionic bonds are formed and if the electronegativity difference between two atoms forming a bond is less than 1.7, then covalent bonds are formed. Further, the stable Be usually contains 4 protons and 5 neutrons in its nucleus (this is not considered a very large difference). However, there exists a lighter isotope of Be which contains 4 protons and only 3 neutrons which gives a total mass of 7 amu so that this extra proton makes it unstable (that is now the initial state) and therefore can undergo decay by electron capture. There is also a heavier isotope of Be which contains 4 protons and 6 neutrons which gives a total mass of 10 amu so that this extra neutron makes it unstable (that is now the initial state) and therefore undergo neutron decay to stabilize.

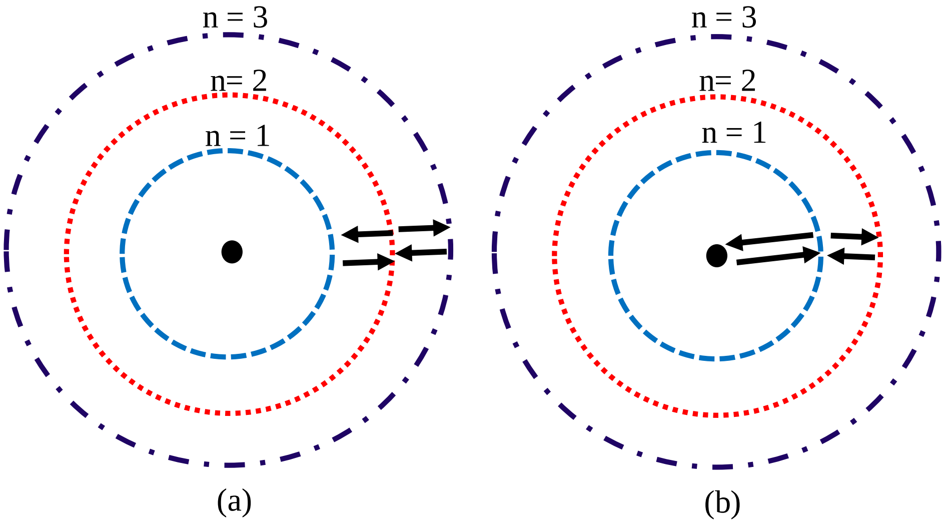


Figure 2 (colour online): (a) The electron hopping in four directions of time within the shells predicting ionic bonding and covalent bonding (b) The electron hopping in four directions of time within the shells and the nucleus predicting decay by electron capture and neutron decay.

The ionic bonding

To illustrate the ionic bonding, consider the ionic compound beryllium oxide, BeO formed from reaction of Be and O. The electronegativity of Be and O are 1.57 and 3.44 respectively. The difference in the electronegativity is 1.87 hence BeO is an ionic compound. As depicted in Figure 3, the beryllium atom has two electrons in shell n = 1 and two electrons in its valence shell n = 2 and therefore can give out this two electrons to form a closed shell. In otherwords, it can be doubly ionized and has the symbol Be^{2+}. The oxygen atom has two electrons in shell n = 1 and six electrons in its outermost shell n = 2 and therefore needs two electrons to form a closed shell. It can also be doubly ionized and has the symbol O^{2-}. It follows then that to produce the BeO, the Be atom loses its two valence electrons by hopping forward in future time to O atom which see these two electrons as moving backward from future time.

$$2Be + O^{2} \longrightarrow 2Be^{2+} + 2O^{2-} \longrightarrow 2BeO$$

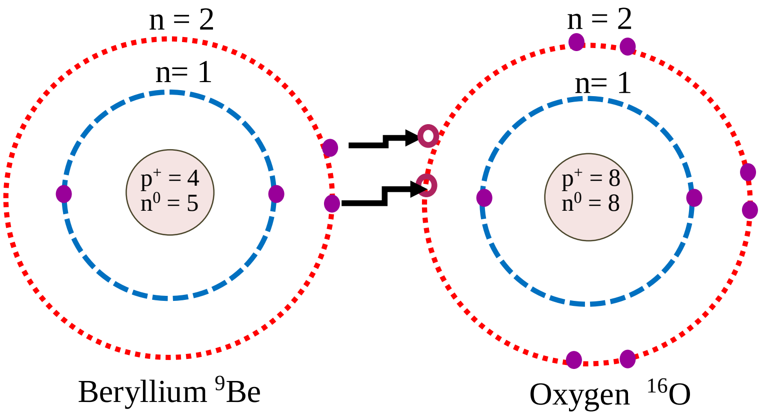


Figure 3 (colour online): The formation of the ionic compound, beryllium oxide, BeOfrom reaction of beryllium and oxygen.

The Covalent bonding

The electronegativity of Cl is 3.16. This makes its difference in the electronegativity with Be to be 1.59 which is less than 1.70 hence $BeCl_2$ is a covalentcompound.TheCl atom has 17 electrons meaning it needs just an electron to complete its outer shell. Since the Be atom has two electrons in the outermost shell but unlike the O, the Cl cannot pull them out of the Be shell, there will only be restricted hopping:the Be two electrons in the outermost shellwill make this restricted hopping forward in future time to the two Cl atoms which see these two electrons as restricted hopping backward from future time while the outermost electron of each of the Cl atoms will also make this restricted hopping forward in future time to the Be atom which see these two electrons as restricted hopping backward from future time.

$$Be + Cl_2 = Be^{2+} + Cl_2^{-} = BeCl_2$$

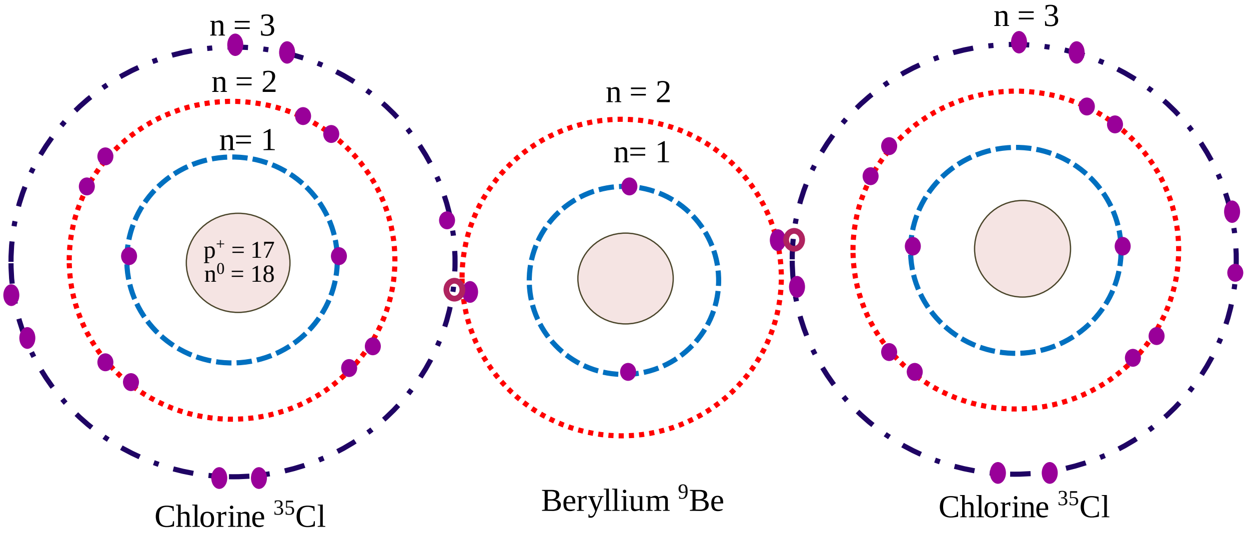
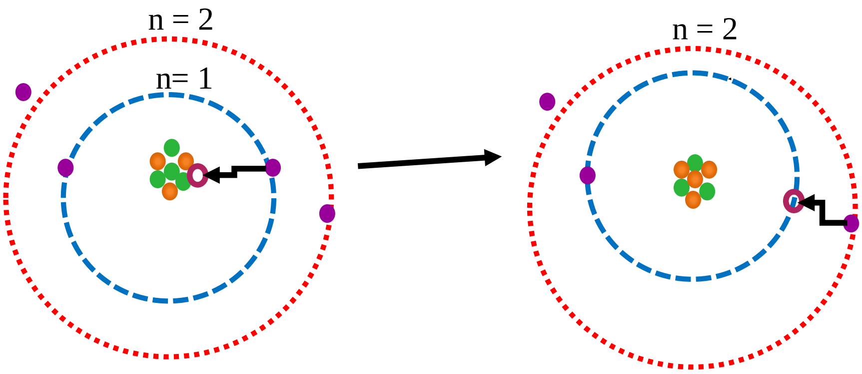


Figure 4 (colour online): Theformation of the covalent compound, beryllium Chloride, $BeCl_2$from reaction of beryllium and chlorine.

The Decay by Electron Capture

The lighter isotope of the beryllium ^{7}Bedecays into ^{7}Li through electron capture. Here an electron in the ground state shell of the ^{7}Be moves back from future time into the nucleus where it is captured by proton and becomes a neutron. This resulting new ^{7}Li has the same atomic mass unit as ^{7}Bebut one less proton which stabilizes the element. Further, the vacancy created by the captured electron will be occupied by an electron hopping backward in time from n = 2 to n = 1. This now leaves the Li with one electron in its outermost shell hence it become singly ionized Li^+

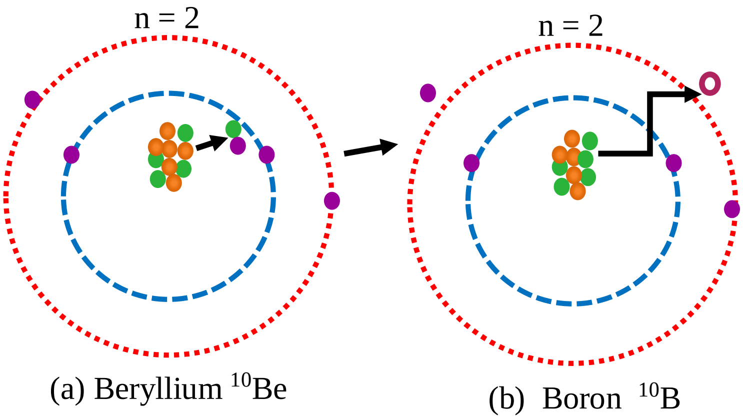


(a) Beryllium ^{7}Be (b) Lithium ^{7}Li

Figure 5: (colour online) The decay process is that the nucleus of (a) Beryllium ^{7}Be which is unstable because it has more protons (green) than the neutrons (light brown) will capture an electron (purple) and decay into Lithium ^{7}Li .

The Neutron Decay

The heavier isotope of the beryllium ^{10}Be decays into boron ^{10}B through neutron decay. Here one of the 6 neutrons in the nucleus splits into a proton and an electron. Since there is no place for the electron to occupy in the nucleus, it will then move forward into future time to the ground state shell which see this electron as moving foreward from past time. This resulting new ^{10}B has the same atomic mass unit as ^{10}Be but the same number of protons and neutrons which stabilizes the element.



(a) Beryllium ^{10}Be (b) Boron ^{10}B

Figure 6: (colour online) The neutron decay process is that the nucleus of (a) Beryllium ^{10}Be which is unstable because of its 4 protons (green) and 6 neutrons (light brown) which gives it a total mass of 10 amu so that this extra neutron makes it unstable and will therefore split into a proton and an electron (purple) and the resulting new atom (b) boron ^{10}B now has 3 electrons in its outermost shell.

CONCLUSION

We have shown that the Dirac's principle of superposition for evolution of time holds that when a quantum system is caused to evolve from its initial time, it must be one of the four possible outcomes: t_p^f as the time for the evolution from past time, t_p^b as the time for the evolution backward to past time, t_f^f as the time for the evolution forward in future time and t_f^b as the time for evolution backward from future time. This clearly established the linearity of time and therefore a one to one correspondence with the linear motion of the four possible motions of a one dimensional (1D) quantum particle. This observation which can be considered as a subtle way to demonstrate the geometrical interpretation of time (Olkhov, 2007a) so that it can be extended to other dimensions for the geometrization of quantum systems (Olkhov, 2007b), is a clear demonstration that using the DPSET, we can theoretically predict the outcome of a quantum measurement. Therefore, we have resolved the problem of indeterminacy in quantum mechanics using the DPSET so that the correct interpretation of quantum mechanics can now be formulated.

REFERENCES

Akpojotor G. E. and Echenim M. W. (2010) A pedagogical study of the Oyibo's grand unification theorem with realization of some standard equations. Journal of Nigerian Association of Mathematical Physics 17, 447 - 456. e-print *arXiv:1307.1098*

Akpojotor G., Ehwerhemuepha L, Echenim M. and Akpojotor F. (2010) Modeling and visualization of some physics phenomena with Python simulations. African Journal of Physics 3, 94-118.

Bassi A. and Ghirardi G.C. (2000)A general argument against the universal validity of the superposition principle. Physics Letters A, 275: 373-381.

Barrett J.A. (2009) Many Worlds Interpretation of Quantum Mechanics. In: Greenberger D., Hentschel K., Weinert F. (eds) Compendium of Quantum Physics. Springer, Berlin, Heidelberg, pp 363-367.

Bolduc E., Gariepy G. and Leach J. (2016) Direct measurement of large-scale quantum states via expectation values of non-Hermitian matrices.Nature Communications 7,10439.

Branden B.H and Joachan C.J. (1989) Introduction to Quantum Mechanics, Longman Scientific & Technical Limited, Essex (England)

Cohen-Tannoudji C. (2006) Quantum Mechanics (2 volumes).Wiley,New York

Cowan C.W. and Tumulka R. (2014a) Epistemology of wave function collapse in quantum physics. The British Journal for the Philosophy of Science, p.axu038. e-print arXiv:1307.0827.

Cowan C. W. and Tumulka R. (2014b)Can one detect whether a wave function has collapsed? J. Phys. A: Math. Theor. 47,195303; e-print arXiv:1307.0810

Cowan C. W. and Tumulka R. (2015) Detecting wave function collapse without prior knowledge.Journal of Mathematical Physics 56, 082103.

Deo P. S. and Satpathi U. (2019) Transmitting a signal in negative time.Results in Physics
12, 1506-1512.

Dumitru S. (2014) Views about the “Oxford Questions”. Wave Function Collapse and Schrödinger’s Cat: Are They Real Scientific Topics or Plain Fictions?.PROGRESS, 10, p.111.

Gribbin J (2020) The Many-Worlds Theory, Explained. THE MIT PRESS READER. Accessed on 27/08/2020 from https://thereader.mitpress.mit.edu/the-many-worlds-theory/

Goodisman J. (2012) Contemporary Quantum Chemistry: An Introduction. Plenum Press, New York.

HariDass N.D (2013) The Superposition Principle in Quantum Mechanics - did the rock enter the foundation surreptitiously?arXiv:1311.4275v1

Harrell L. E. (2016) Wavefunction collapse through backaction of counting weakly interacting Photons. Journal of Mathematical Physics 57, 032104.

Kotz J. C., Treichel P. M. and Weaver G. C. (2006) Chemistry and Chemical Reactivity (Vol 2, 6th Ed), Thomson Brooks/Cole, Belmont (USA).

McQueen K. J. (2017) Does Consciousness Cause Quantum Collapse? Philosophy Now. Accessed on 27/08/2020 from https://philosophynow.org/issues/121/Does_Consciousness_Cause_Quantum_Collapse

Manning P. (2009) Chemical Bonds. Infobase Publishing, New York.

Modak S. K. and Sudarsky D. (2018) Collapse of the wavefunction, the information paradox and backreaction. Eur. Phys. J. C 78, 556

Monroe C., Meekhof D. M., King B. E. and Wineland D. J. (1996) A “A Schrodinger Cat” Superposition State of Atom. Science 272 1131 – 1336.

Olkhov, O. A. (2017a) Geometrical Interpretation of Time and New Models of the Space-Time. Journal of

High Energy Physics, Gravitation and Cosmology 3, 564-571.

Olkhov, O.A. (2007b) Geometrization of Quantum Mechanics. Journal of Physics: Conference Series, 67, 012037-012041.

Pereyra P. (2012) Fundamentals of Quantum Physics.Springer-Verlag Berlin Heidelberg

Volz J., Gehr R., Dubois G., Esteve J. and Reichel J. (2011) Measuring the internal state of a single atom without energy exchange. Nature 475, 210-213e-print arXiv:1106.1854v1

Wheeler J. A. and Zurek W. H. (1983)Quantum Theory and Measurement.Princeton University Press, Princeton, New Jersey.