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MULTIVARIATE GARCH MODELLING OF VOLATILITY OF NIGERIAN STOCK MARKET AND SOME ECONOMIC INDICES.

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ABSTRACT

Volatility and co-volatility modelling among financial series is an important aspect in financial time series. Multivariate generalized autoregressive conditional heteroscedasticity (MGARCH) models, model the variances and co-variances among financial data. The GARCH model has been applied in modelling volatility of various univariate time series data but limited work has been done in the application of multivariate GARCH models in modelling multivariate time series data. This study is aimed at applying the Multivariate GARCH model on the co-volatility of Nigerian stock market, USD/Naira exchange rate and inflation rate using the Baba-Engle-Kraft-Kroner (BEKK) model for estimation. The best fit model for the series is the BEKK (1, 2) model in terms of minimum model evaluation tools.The variance-covariance models shows that volatilities of exchange rate and inflation rate influence volatility of stock price returns. It also shows that shock in stock prices greatly affect the volatility of stock prices.

Keywords: Baba-Engle-Kraft-Kroner model, Exchange rate, Inflation rate, Stock prices, Univariate GARCH,

INTRODUCTION

Stock Market is a public place where facilities are provided to investors to be able to buy and sell shares, bonds etc. Stock market is very important in the economy of any country. Stock market volatility is the degree of variation in the stock price returns over time. Volatility measures the risk of the price of an investment.

Volatility of returns in stock market has a number of negative effects on performance of an economy. A volatile stock market weakens consumer confidence and drives down consumer spending (Porteba, 2000). It affects business investment because it conveys a rise in risk of equity investment (Arestis et al, 2001; Mala and Reddy, 2007). Issues of volatility in stock market behaviour are of importance as they shed light on the data generating process of the returns (Hongyu and Zhichao, 2006).

Since volatility is not directly observable, modeling, measuring and forecasting of volatility are important aspects in financial econometrics. MGARCH (multivariate generalized autoregressive conditional heteroscedasticity model) is one of the most important models for volatility. It allows us to specify a dynamic process for the whole time varying variance-covariance matrix of the time series.

It is widely accepted that financial data are interrelated. Knowing how the markets are interrelated is of great importance in finance. The univariate model, models the conditional variance of each series independently of all other series. The covariances between series are of interest, as well as the variances of the individual series. MGARCH models explain how the covariance and correlations among financial data move over time. This study is aimed at the application of Multivariate GARCH model in modelling the Volatility of Nigerian Stock Market Returns in relation to US Dollar-Naira exchange rate and inflation rate.

LITERATURE REVIEW

The initial advancement in modelling volatility was Engle (1982), where it was introduced that conditional heteroskedasticity can be modelled using the autoregressive conditional heteroskedasticity (ARCH) model. Bollerslev (1986) and Taylor (1986) individually generalised the ARCH model to make it more realistic, which was called the Generalised ARCH (GARCH).

The ARCH/GARCH models were successful in capturing facts about time series especially financial series under study such as volatility clustering and time varying volatility. However the model does not take into consideration interaction between two or more time series. This brought about the generalization of the univariate GARCH models to the multivariate versions by Bollerslev, Engle and Wooldridge (1988).

The MGARCH (Multivariate Generalised autoregressive conditional heteroskedasticity) was developed in order to meet the need of examining several assets at a time (i.e. does volatility of one asset influence the other?). In this case the correlations and covariances are also put to consideration. Bollerslev, et al (1988) consider a multivariate extension of the GARCH (P, Q).

Bollerslev, et al (1988) first propose the direct generalization of the univariate GARCH model to multivariate GARCH model namely the Vector-Garch model, for modelling the conditional variance covariance matrix (Ht). Implementation of the model was difficult, since the number of parameters is very large and it is difficult to impose the positive definiteness on the (Ht) model.

Bollerslev, et al (1988) introduced a simplified version known as the Diagonal Vector-Garch model. This model reduced the number of parameters and made it easier to derive the conditions to guarantee the positive definiteness of variance covariance matrix. But this model cannot capture the relationship between the variances and covariances.

Engle and Kroner (1995) proposed a restricted version of Vector-Garch model which is known as the Baba-Engle-Kraft-Kroner (BEKK) model. BEKK model is constructed with the property that the conditional variance covariance matrix is positive definite. The model still had the problem of a large number of parameters.

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Minovic (2007) applied MGARCH (restricted BEKK, DVEC and CCC) models to the analysis of the Serbian financial market. Bivariate and trivariate time series models were applied. It was found that conditional covariances exhibited significant changes over time for both stocks (Hemofarm and Energoprojekt) and the index (BELEX15). So, the correlation between log returns of stocks and index was very unstable over time in Serbian frontier markets. She showed that restricted version of BEKK, DVEC and CCC models with reduced number of parameters could give fairly accurate results.

Song (2009) applied the multivariate GARCH to the daily data of the four Greater China region stock markets, namely Hong Kong, Shanghai, Shenzhen, and Singapore to investigate the volatility and shocks spill over behaviour and also to establish the market linkage among the four markets. The result showed that volatility spill over between Shanghai and Shenzhen is obvious and correlation contagion is detected. Conditional variance and conditional correlations are time varying and dynamic.

Belasri and Ellaia (2017) compared the BEKK and Dynamic Conditional Correlation (DCC) models to examine which model performs better in modelling variance covariance matrices. They applied the models to 10 years of daily Moroccan stock prices. The results show that BEKK model performs better than DCC in modeling variance covariance matrices. Based on the Nigerian context, literature has shown that the application of the Multivariate GARCH models in modelling time series data has been neglected.

METHODOLOGY

The Baba-Engle-Kroner-Kraft (BEKK) model

The BEKK model models the variance covariance matrix H_t directly and it ensures positive definiteness of H_t.
The BEKK (p,q) model is given as

$$H_t = CC' + \sum_{j=1}^{p}\sum_{i=1}^{k} A'_{ij}\epsilon_{(t-j)}\,\epsilon'_{t-j}A_{ij} + \sum_{j=1}^{p}\sum_{i=1}^{k} B'_{ij}H_{t-j}B_{ij} \qquad (3.1)$$

where $H_t = var_{t-1}\,(y_t)$ denoting the (nxn) conditional covariance matrix; A_{ij}, B_{ij} denotes an NxN parameter matrices; C an NxN lower triangular matrix; N=3 (the number of series).

The BEKK model requires $[N(N+1)/2] + N^2(p+q)$ parameters to be estimated.

Example, for BEKK(1,1) with N=3, 24 parameters are estimated.

Considering the first order model

$$H\_\{t\} = CC' + A\epsilon_{t-1}A' + BH_{t-1}B' \qquad (3.2)$$

Diagnostic Checking

In estimating MGARCH models, it is necessary to check whether the data exhibit the presence of ARCH effect and also to test for stationarity of the data. This is to ensure appropriate choice of the estimation technique (MGARCH) for the data. A series is said to be stationary if the mean and variance remain constant over time. A non-stationary series is characterized by a unit root.

Model Order Selection

For the order selection of our models, we use the AIC values. Model with the lowest values is preferred, the formulas of these criteria is:

$$AIC = -2 \times LLF + 2 \times m \qquad (3.3)$$

where, LLF is the log likelihood function and m is the number of parameters estimated in the model.

RESULTS

Data

Data employed in this study are monthly observations on Nigerian stock prices, USD-Naira exchange rate and inflation rate. Data series start from Jan, 2003 to Dec, 2016 providing a total of 168 observations for each series. The data were retrieved from the central of Nigeria statistical bulletin downloadable from www.cenbank.org.

The returns data were derived from the price data using the expression:

$$R_t = \ln\frac{P_t}{P_{t-1}} \qquad (3.4)$$

where R_t is the return at time t; P_t is the price at time t; P_{t-1} is the lagged price and ln is the natural logarithm, the natural logarithm is used to the data on the same scale for easy computation.

Plot of Monthly series

Figures below plot monthly series and its returns. It can be seen from the figures that the series volatilities don't keep constant over time, also high returns tend to be followed by high returns and low returns tend to be close with low returns, this shows the volatility clustering property.

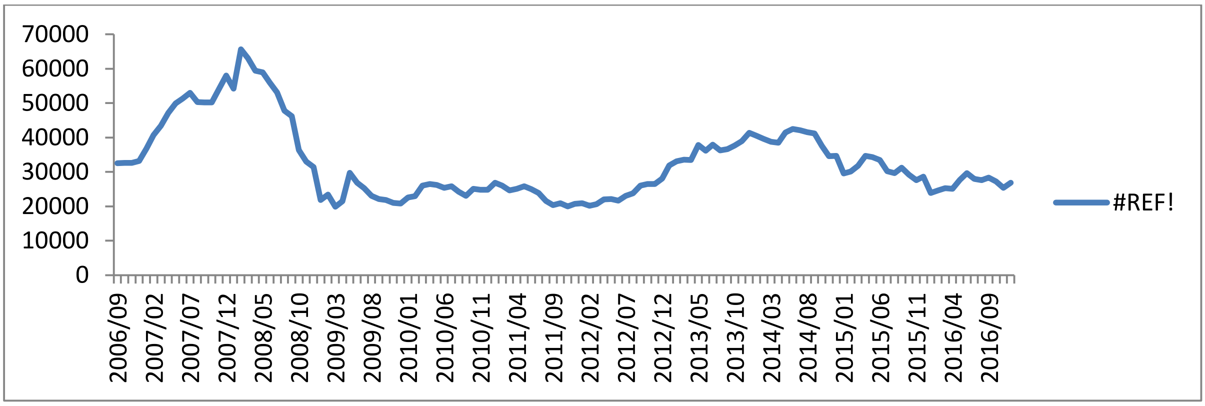

Figure 4.1: Stock Prices

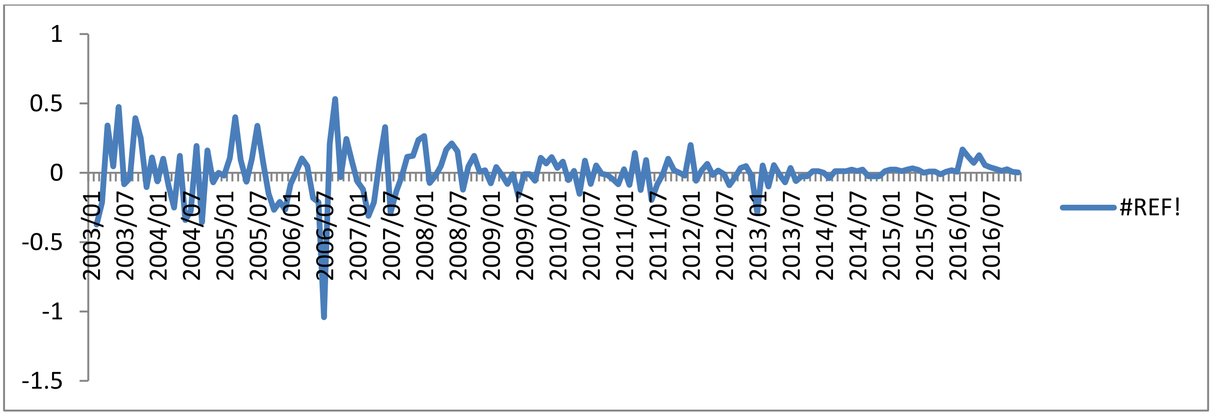

Figure 4.2: Monthly Return of Stock Prices

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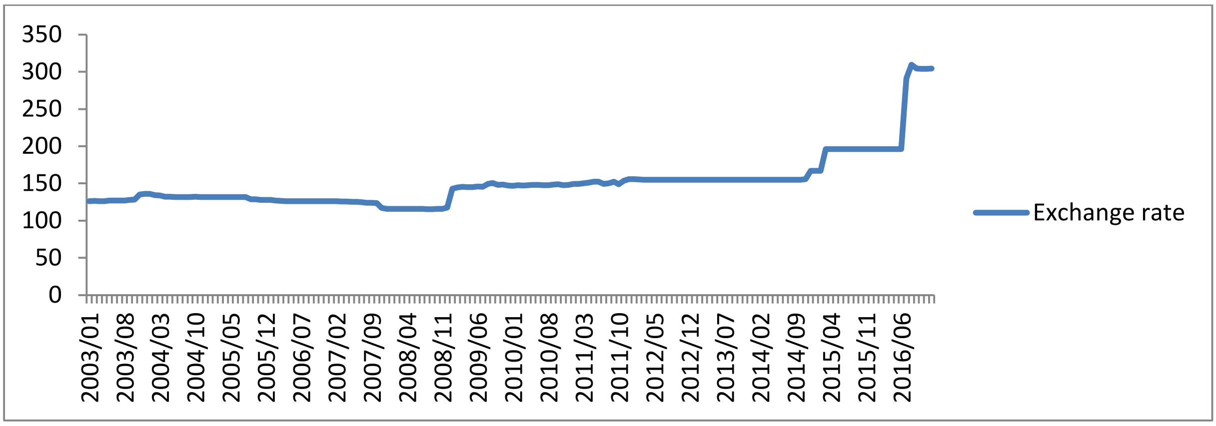

Figure 4.3: USD-Naira Exchange Rate

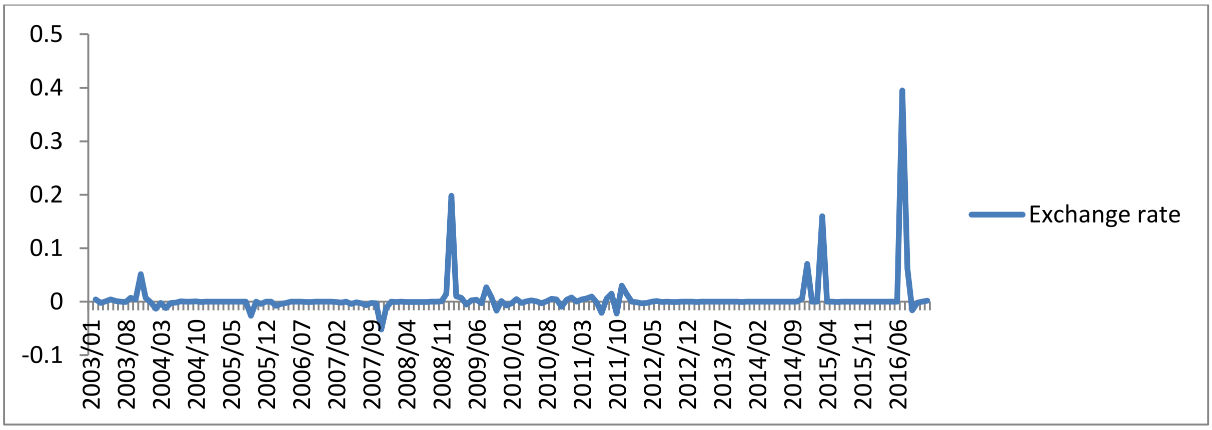

Figure 4.4: Monthly Return of USD-Naira Exchange Rate

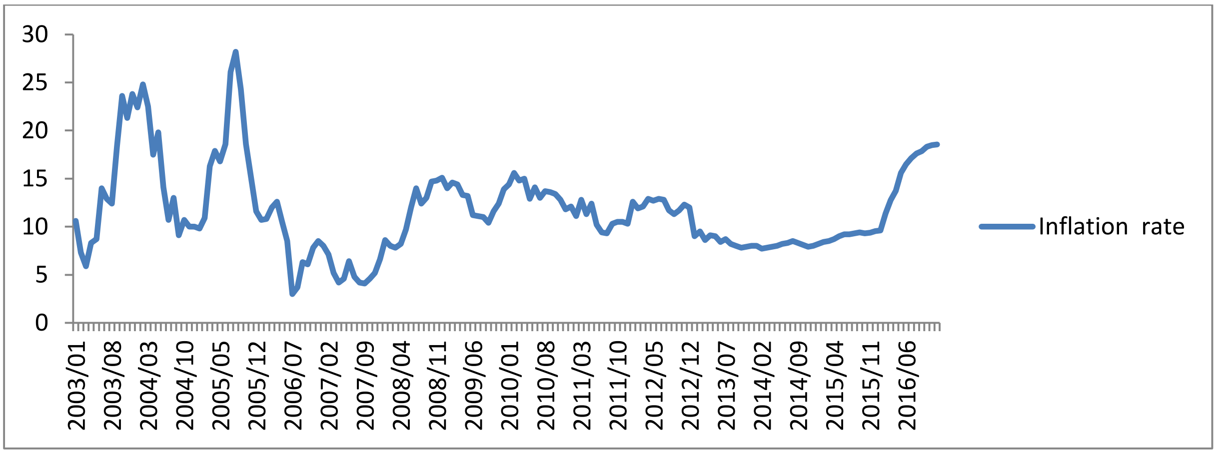

Figure 4.5: Inflation Rate

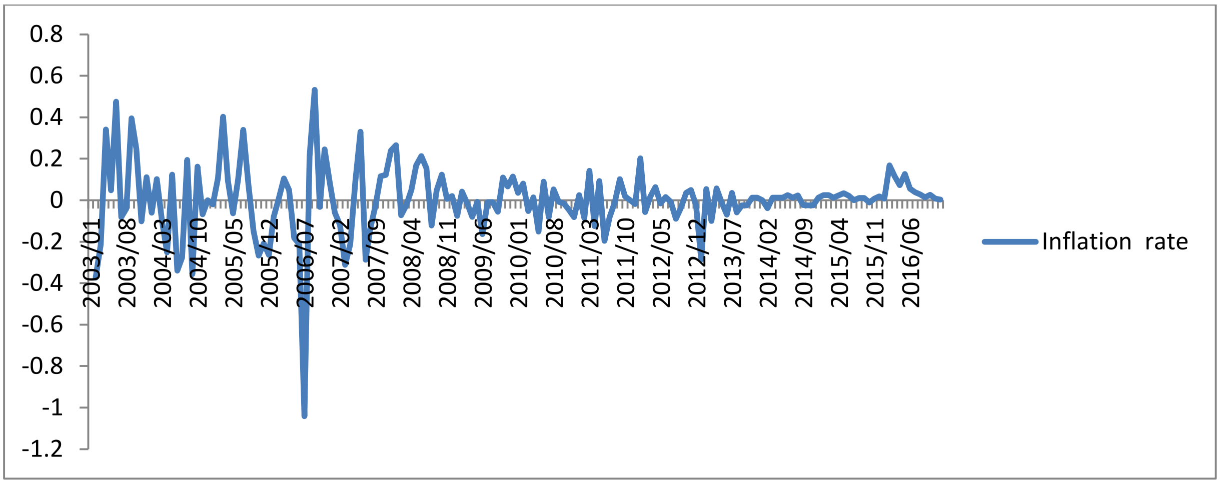

Figure 4.5: Monthly Return of Inflation Rate

Descriptive Statistics

Table 1: Descriptive Statistics of Monthly returns

Statistics	Stock Market	Exchange rate	Inflation rate
Mean	0.004213	0.0052885	0.003351
Standard Deviation	0.07550956	0.03775212	0.1652674
Skewness	-0.4415196	7.907114	-1.205231
Kurtosis	4.688346	71.9377	9.538312
Minimum	-0.365883	-0.0518052	-1.041454
Maximum	0.323516	0.3952086	0.532217

The P-Values of the Jarque-Berra normality test is 0.05.

Table 1 shows the summary and description of our data. The skewness value was found to be negative for both stock prices and inflation rate, but positive for exchange rate. There is greater tendency for stock prices and inflation rate to fall while exchange rate to rise. However, the skewness of the series are different from zero indicating that they are asymmetric distributions. Kurtosis calculated for our series presented in Table 1 is large exceeding the benchmark value of three and suggestive of leptokurtic distribution, indicating that they have fat tailed distributions. The P-Values of the Jarque-Berra normality test are all less than the significant value (in this case it is = **0.05**) which rejects the hypothesis of normality of the monthly returns.

Diagnostic Checking

Table 2: Test for Stationarity and Conditional Heteroscedasticity

Test	ADF unit root Test		ARCH Test	
	Test Stat.	p-value	Test Stat.	p-value
Stock prices	-2.5775	0.3352	922.0757	<0.002
Exchange rate	1.1074	0.99	515.285	<0.002
Inflation rate	1.5223	0.13	356.411	<0.002

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The stationarity of the series was tested using the ADF unit root test. From the result, p-values are greater than 0.05 the null hypothesis which states that
H_0: the series contains unit root is not rejected.
The result of the ARCH test shows that the series contains conditional heteroscedasticity.

Table 3: Model Order selection

Model	AIC
BEKK (1,1)	-781.9219
BEKK (1,2)	**-867.5902**
BEKK (2,1)	-786.4891
BEKK (2,2)	-863.3054

As shown in table 3, BEKK (1, 2) model is the best fit for our data. The bold figure in the table indicates the lowest values, the AIC criteria confirms our model order choice.

Multivariate GARCH Models Estimation

Table 4: BEKK (1, 2) coefficients estimation

Co-efficient	**Estimation**	**Standard error**	**T - statistics**
C(1,1)	0.05072264**	0.004903527	10.34411
C(2,1)	0.0003428859**	0.001202231	0.285208
C(2,2)	0.0000971982	0.001193314	0.081452
C(3,1)	-0.007777283**	0.01333731	-0.58312
C(3,2)	0.001544528	0.01484814	0.010402
C(3,3)	-0.00004518225	0.01447379	-0.00312
A(1,1)	-0.56425449**	0.07779089	-7.25348
A(1,2)	0.02466602**	0.016831937	1.46543
A(1,3)	0.03279441	0.1871212	0.17526
A(2,1)	-1.14241079**	0.25943038	-4.403535
A(2,2)	0.29767984**	0.032101275	9.273147
A(2,3)	-0.30182855	0.3904563	-0.77302
A(3,1)	-0.04342454**	0.05538679	-0.78402
A(3,2)	-0.00119439	0.008928917	-0.13377
A(3,3)	0.64769737**	0.2662136	2.43299
B1(1,1)	-0.03035066	0.16128904	-0.18818
B1(1,2)	0.0045112649**	0.014585069	0.30931
B1(1,3)	0.4977754**	0.2007810	2.479195
B1(2,1)	0.04458476	0.16496189	0.270273
B1(2,2)	-0.0669054268**	0.062308912	-1.073769
B1(2,3)	0.1328642**	0.1178441	1.127457
B1(3,1)	0.03660476**	0.07964607	0.459593
B1(3,2)	-0.0002574133	0.007029744	-0.036618

B1(3,3)	-0.9297877**	0.4526067	-2.054295
B2(1,1)	0.1884312**	0.07979407	2.361469
B2(1,2)	0.35759662**	0.01026523	34.835714
B2(1,3)	0.038211886**	0.08449575	0.452234
B2(2,1)	-0.1035707**	0.20862212	-0.496451
B2(2,2)	0.07177028**	0.03548773	2.022397
B2(2,3)	0.003055909	0.09461017	0.0323
B2(3,1)	-0.1819334**	0.04878600	-3.72921
B2(3,2)	0.09086803**	0.02745452	3.309766
B2(3,3)	0.031271963**	0.08154137	0.38351

This table shows the result of the maximum likelihood estimation in terms of the full BEKK (1, 2) model. In the above table,
C (i, j) means the model parameter of the matrix C's i-th row j-th column.
A (i, j) means the model parameter of the matrix A's i-th row j-th column.
B (i, j) denotes the model parameter of the matrix B's i-th row j-th column.
** denotes the statistical significance of the parameter estimates at 5% level.

The BEKK model is given as

$$h_{11} = 0.003 + 0.318\epsilon^2{}_{1,t-1} - 0.014\epsilon_{1,t-1}\epsilon_{2,t-1} - 0.037\epsilon_{1,t-1}\epsilon_{3,t-1} + 0.002\epsilon_{2,t-1}\epsilon_{3,t-1} + 0.0006\epsilon^2{}_{2,t-1} + 0.001\epsilon^2{}_{3,t-1} + 0.036h_{11,t-1} + 0.134h_{12,t-1} - 0.016h_{13,t-1} + 0.032h_{23,t-1} + 0.0128h_{22,t-1} + 0.249h_{33,t-1} \quad (4.1)$$

$$h_{22} = 0.00000013 + 0.305\epsilon^2{}_{1,t-1} - 0.68\epsilon_{1,t-1}\epsilon_{2,t-1} + 0.69\epsilon_{1,t-1}\epsilon_{3,t-1} - 0.18\epsilon_{2,t-1}\epsilon_{3,t-1} + 0.089\epsilon^2{}_{2,t-1} + 0.091\epsilon^2{}_{3,t-1} + 0.0134h_{11,t-1} - 0.009h_{12,t-1} + 0.011h_{13,t-1} - 0.006h_{23,t-1} + 0.008h_{22,t-1} + 0.866h_{33,t-1} \quad (4.2)$$

$$h_{33} = 0.000063 + 0.002\epsilon^2{}_{1,t-1} - 0.0001\epsilon_{1,t-1}\epsilon_{2,t-1} + 0.056\epsilon_{1,t-1}\epsilon_{3,t-1} - 0.002\epsilon_{2,t-1}\epsilon_{3,t-1} + 0.000001\epsilon^2{}_{2,t-1} + 0.42\epsilon^2{}_{3,t-1} + 0.034h_{11,t-1} - 0.033h_{12,t-1} - 0.04h_{13,t-1} + 0.006h_{23,t-1} + 0.008h_{22,t-1} + 0.866h_{33,t-1} \quad (4.3)$$

$$h_{12} = 0.00002 + 0.645\epsilon^2{}_{1,t-1} - 0.196\epsilon_{1,t-1}\epsilon_{2,t-1} + 0.133\epsilon_{1,t-1}\epsilon_{3,t-1} + 0.002\epsilon_{2,t-1}\epsilon_{3,t-1} + 0.133\epsilon^2{}_{2,t-1} - 0.01\epsilon^2{}_{3,t-1} - 0.024h_{11,t-1} - 0.02h_{12,t-1} - 0.015h_{13,t-1} - 0.029h_{23,t-1} + 0.025h_{22,t-1} + 0.066h_{33,t-1} \quad (4.4)$$

$$h_{13} = -0.0004 + 0.025\epsilon^2{}_{1,t-1} - 0.0004\epsilon_{1,t-1}\epsilon_{2,t-1} - 0.367\epsilon_{1,t-1}\epsilon_{3,t-1} + 0.016\epsilon_{2,t-1}\epsilon_{3,t-1} - 0.00003\epsilon^2{}_{2,t-1} + 0.021\epsilon^2{}_{3,t-1} - 0.035h_{11,t-1} - 0.048h_{12,t-1} - 0.0136h_{13,t-1} + 0.01h_{23,t-1} + 0.033h_{22,t-1} - 0.462h_{33,t-1} \quad (4.5)$$

$$h_{23} = -0.000003 + 0.05\epsilon^2{}_{1,t-1} - 0.012\epsilon_{1,t-1}\epsilon_{2,t-1} - 0.727\epsilon_{1,t-1}\epsilon_{3,t-1} + 0.193\epsilon_{2,t-1}\epsilon_{3,t-1} - 0.0004\epsilon^2{}_{2,t-1} - 0.196\epsilon^2{}_{3,t-1} + 0.005h_{11,t-1} - 0.006h_{12,t-1} - 0.04h_{13,t-1} + 0.065h_{23,t-1} + 0.007h_{22,t-1} + 0.0001h_{33,t-1} \quad (4.6)$$

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Variance Equation of Stock market
From equation (4.1), 0.003 is given as the intercept. All the squared coefficients in the variance equation positively affect the next month's stock price return volatility. The negative A(1,1)A(2,1) of -0.014 means that a shock to the stock price negatively affects the next month’s stock return volatility.

The positive B1(11)B1(12)+ B2(11)B2(12) of 0.1344 means that an increase of the previous month's stock return and exchange rate covariance weakly increases volatility of stock return.

The negative B1(11) B1(13)+ B2(11) B2(13) of -0.0159 means that an increase of the previous month's stock return and inflation rate covariance weakly decrease volatility of stock return.

Variance Equation of Exchange Rate
From equation (4.2), The positive B1(21)B1(22)+ B2(21)B2(22) of 0.0094 means that an increase of the stock return and exchange rate covariance very weakly increase next month's exchange rate.

The negative B1(22) B1(23) + B2(22) B2(23) of -0.0174 means that an increase of the exchange rate and inflation rate covariance weakly decrease next month's exchange rate.

Variance Equation of Inflation rate
From equation (4.3), The negative B1(31)B1(33)+ B2(31)B2(33) of -0.0331 means that an increase of the stock return and inflation rate covariance very weakly decrease next month's inflation rate.

The positive B1(32)B1(33)+ B2(32)B2(33) of 0.0061 means that an increase of the exchange rate and inflation rate covariance very weakly increase next month's inflation rate.

Covariance Equation between Stock Market and Exchange Rate
From equation (4.4), we have 0.00002 as our intercept. The squared coefficient (0.645) shows that a shock to stock prices affects the Covariance between stock price and exchange rate.

The previous month's variance of stock prices, covariance between stock price and exchange rate negatively affects the covariance of stock price and inflation rates.

Covariance Equation between Stock Market and Inflation Rate
From equation (4.5), we have -0.0004 as our intercept. The squared coefficient (0.025) shows that a shock to stock prices positively affects the Co-variance between stock price and inflation rate.

The previous month's variance of inflation rate positively affects covariance between stock price and exchange rate. The variance of stock prices, Inflation rate, covariance between stock prices and exchange rate negatively affects the covariance of stock price and inflation rates.

CONCLUSION
This study applied one of the most widely used multivariate GARCH model which is the BEKK model to modelling the volatility of the Nigerian stock prices, USD-Naira exchange rate and inflation rate. However, the parameters of the model increases rapidly with the order of the model and the number of series considered.

Model order selection test which is the AIC showed that BEKK (1,2) model is the model that best fit the volatility of our series. The Lagrange Multiplier test also above show that there is a strong evidence of conditional heteroscedasticity for all our series since the p-value= 0 for all our series. This ensured the appropriate choice of the estimation technique (MGARCH) for our model. The model showed that shocks in the stock prices positively affects the volatility of Stock return. Also the volatility of exchange and inflation rate has effect on volatility of Stock return. In order to be able to find out the forecast performance of this model further research is required. It is also important to examine the relationship between stock prices and other economic indices such as oil prices, interest rate and so on.

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