THE EFFECT OF INTERACTION OF LARGE AMPLITUDE WAVE ON SEA WITH ITS APPLICATION

I.O. Ejinkonye

Department of Mathematics and Computer Science,
Western Delta University, Oghara, Delta State, Nigeria.
E-mail: ejinkonye.ifeoma@yahoo.com Phone: +2348060297345

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ABSTRACT

In this paper we attempt to analyse the higher order deterministic wave equations. This study will concerns on the third order term of the same Stoke's expansion for wave profile and velocity potential. The wave steepness parameter will be introduced in the expansion. This is preferred because, steepness parameter estimates the vertical asymmetry of the sea-surface elevation. It also enhances the convergence of the Stoke's expansion. Approximate rogue wave solutions of the equation are presented and discussed. Numerical implications on wave crest height and trough depth will be better understood.

Key words: deterministic wave equations, rogue waves, wave profile

INTRODUCTION

The rogue waves might be caused by many factors, such as the energy focusing due to the seabed geometry, wind-wave interaction, wave-current interaction, modulation instability, e.t.c. which have been discussed and reviewed by other researchers (Kharif et al., 2009; Haver, 2005).

The most distinguished feature of rogue wave is its transience, which means that it can happen and disappear very rapidly (Kharif et. al.,2009). Therefore, studies need be carried out in time domain to explore the physics of rogue waves.

In this consideration and at the initial state of wave development in deep water, short wave group are at the front of long wave group (wave packet). As time involves, the longer wave group with higher group velocity will overtake the shorter one. Consequently, the longer wave group will extract energy from the shorter group, thus, grow in size. The effectiveness of this mechanism had been justified not only by observation but by numerical simulation and by laboratory experimental modeling (Boccoti, 1981, 1982, 2008; Fedele and Arena, 2006)

Bearing the above in mine, Boccoti (1981, 2008) showed that in a Gaussian sea, if a high wave develops at some point in time and space; with a high probability, then a well defined quasi-deterministic wave group generates extreme wave profile (maximum or minimum structure in wave profile). Boccoti, (1989) provided the quasi-determinism theory; this is a form of generalised Fourier series for wave group. Applying Stoke's expansion, the quasi-determinism wave group was extended to the second order by Fedele (2006).

In this study, the theorem will be extended to third order in Stoke's expansion. Numerical implications on wave crest height and trough depth will be better

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understood. This study will concern the third order term of the same Stoke's expansion for wave profile and velocity potential. The wave steepness parameter will be introduced in the expansion. This is preferred because steepness parameter estimates the vertical asymmetry of the sea-surface elevation. It also enhances the convergence of the Stoke's expansion.

Deterministic wave group

The quasi-determinism theory provides the method of evaluating the subsequent wave crest $\eta(x+X,t+T)$ when a wave group with high crest occurs at the point x and time t (Arena and Fedele 2005). The theory is based on the superposition of large number of quasi-monochromatic waves forming the groups with randomly distributed phases; thus resulting in rogue wave with high crest and deep trough.

Review of the basic quasi-determinism theorem.

Following Fedele and Arena (2003) a high wave crest with height H_c at the point x_o and at time t_o occurs with elevation $\eta(x_0,t_0)$. With the probabilityapproaching unity, the subsequent elevation $\eta(x_0+X,t_0+T)$ as $H_c/\sigma \to \infty$ (σ =standard deviation calculated from discrete wave records of wave elevation time series) is obtained by Fedele and Arena (2003) in the deterministic form.

$$\overline{\eta}(x_0+X,t_0+T)=\frac{\psi(X,T,x_0)}{\psi(0,0,x_0)}H_c \qquad (1)$$

where

$$\psi(X,T,x_0)=\frac{1}{T_c}\int_0^{T_c}\eta(x_0,t_0)\eta(x_0+X,t_0+T)dt_0$$

$$\psi(0,0,x_0)=\frac{1}{T_c}\int_0^{T_c}\eta^2(x_0,t_0)dt_0 \qquad (3)$$

$\frac{H_c}{\sigma}\to\infty$ implies that wave height H_c is large compared with mean wave height. Corresponding to equation (1), the deterministic velocity potential ϕ at depth z is given by

$$\overline{\phi}(x_0+X,z,t_0+T,)=\frac{\Phi(X,z,T;x_0)}{\psi(0,0;x)}H_c \qquad (4)$$

as $\frac{H_c}{\sigma}\to\infty$

$$\Phi(X,z,T,x_0)=\frac{1}{T_c}\int_0^{T_c}\eta(x_0,t)\phi(x_0+X,z,t+T)dt_0 \qquad (5)$$

equation (5) is inverse relationship to equation (4)

T_c = period of the dominant wave group forming the system.

The Integral Representations

If $E(\omega)$ is the frequency amplitude spectrum calculated from the wave record, H_c is the elevation of the high wave crest at $x=x_0$ and time $t=t_0$, then alternative representation of equation (1) in the form of half range Fourier transformation is given by

$$\eta(x,t)=\frac{H_c}{\sigma^2}\int_0^{\infty}E(\omega)\cos(kx-\omega t)d\omega \qquad (6)$$

$$E(\omega)=\frac{2}{\pi}\int_0^{\infty}\eta(t)\cos\omega t dt \qquad (7)$$

at fixed point

$$\sigma^2=\int_0^{\infty}E(\omega)d\omega \qquad \mathbf{(8)}$$

$$\phi(x,z,t)=\frac{gH_c}{\sigma^2}\int_0^{\infty}\frac{E(\omega)}{\omega}e^{-kz}\sin(kx-\omega t)d\omega \qquad (9)$$

In deep water $k=\frac{\omega^2}{g}$ (10)

Equation (10) is derived from the deep water form of dispersion relation $\omega^2=kg$

To (2) second order in Stokes expansion (Fedele and Arena 2003) provides that the free surface displacement and the velocity potential for the long crested deep-water waves are given respectively as follows;

$$\eta(x,t)=\eta_1+\lambda\eta_2=\sum_{n=1}^{N}a_n\cos\psi_n+\frac{\lambda}{4}\sum_{n=1}^{N}\sum_{m=1}^{M}a_na_m\{(k_n+k_m)\cos(\psi_n+\psi_m)+|k_n-k_m|\cos(\psi_n-\psi_m)\} \quad (11)$$

$$\phi(x,t)=\phi_1+\lambda\phi_2=g\sum_{n=1}^{N}a_n\omega_n^{-1}\exp(k_nz)\sin\psi_n+$$

$$\lambda\sum_{n=1}^{N}\sum_{m=1}^{M}a_na_m\omega_n\exp[z|k_n-k_m|][(k_n+k_m)\sin(\psi_n+\psi_m)+|k_n-k_m|\sin(\psi_n-\psi_m)] \quad (12)$$

$$where\ \psi_n=k_nx-\omega_nt+\varepsilon_n \quad (13)$$

the coefficients a_m, a_n are amplitudes for mode components usually specified, k is the wave number and in deep water $k=\frac{\omega^2}{g}, \lambda$ is the wave steepness parameter $(0.1 \le \lambda \le 0.3)$

On the generalized Fourier series

Following Rahman 2003, a smooth function of x, $f(x)$ is considered. $f(x)$ is smooth in the range $0<x<l,\ l=\frac{\pi}{k}$. k is a typical wave n umber. The half range Fourier sine series representation of $f(x)$ is as follows

$$f(x)=A_1\sin kx+A_2\sin 2kx+A_3\sin 3kx+\ldots$$
$$=A_1\sin kx+A_2\sin(k+k)x+A_3\sin(k+k+k)x+\ldots \quad (14)$$
$$f(x)=A_1\sin kx+A_2\sin(k+k)x+A_3\sin(2k+k)x+\ldots \quad (15)$$

Equation (2) allows for the combination of identical group with same wave characteristics. Introduction of the parameters i and j specifies the groups distinctly.

Thus,

$$\bar{f}(x)=A_i\sin k_ix+B_{ij}\sin(k_i+k_j)x+C_{ij}\sin(2k_i+k_j)x+\ldots \quad (16)$$

To accommodate a large number of group evolution

$$\bar{f}(x)=\sum_{i=1}^{N}A_i\sin k_ix+\sum_{i}^{N}\sum_{j}^{M}B_{ij}\sin(k_i+k_j)x+\sum_{i}^{N}\sum_{j}^{M}C_{ij}\sin(2k_i+k_j)x+\ldots \quad (17)$$

Each single mode in the entire group satisfies the hyperbolic equation of the form

$$\frac{d\bar{f}}{dx^2}+k_n^2\bar{f}=0 \quad (18)$$

Where k_n varies with the wave group combination. The effects of the intercrossing of the monochromatic components in the group modify equation (17) to give

$$\bar{f}(x)=\sum_{i=1}^{N}A_i\sin k_ix+\sum_{i}^{N}\sum_{j}^{M}B_{ij}[\sin(k_i+k_j)x+\sin(k_i-k_j)x]$$
$$+\sum_{i}^{N}\sum_{j}^{M}C_{ij}[\sin(2k_i+k_j)x+\sin(2k_i-k_j)x]+\ldots \quad (19)$$

Application to time involving wave form

Equation (19) describes a group of spatial involving hyperbolic systems. However, in geophysical environment, events are space – time dependent. In this consideration equation (19) takes the form

$$\bar{f}(x,t)=\sum_{i=1}^{N}A_i\sin(k_ix+\omega_it)+\sum_{i}^{N}\sum_{j}^{M}B_{ij}[\sin[(k_i+k_j)x+(\omega_i+\omega_j)t]+\sin[(k_i-k_j)x+(\omega_i-\omega_j)t]]$$
$$+\sum_{i}^{N}\sum_{j}^{M}C_{ij}[\sin[(2k_i+k_j)x+(2\omega_i+\omega_j)t]+\sin[(2k_i-k_j)x+(2\omega_i-\omega_j)t]]+\ldots \quad (20)$$

Consider the spatial gradient $\frac{\partial\bar{f}(x,t)}{\partial x}$ thus equation (20) takes the form

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$$\frac{\partial \bar{f}(x,t)}{\partial x} = \sum_{I=1}^{N} A_i k_i \cos(k_i x + \omega_j t)x +$$
$$\sum_{i=1}^{N}\sum_{j=1}^{M} \{B_{ij}(k_i + k_j)\cos(k_i + k_j)x + (\omega_i + \omega_j)t + |k_i - k_j|\cos(k_i - k_j)x + (\omega_i - \omega_j)t\} +$$
$$+\sum_{i=1}^{N}\sum_{j=1}^{M} \{C_{ij}(2k_i + k_j)\cos(2k_i + k_j)x + (2\omega_i + \omega_j)t + |2k_i - k_j|\cos(2k_i - k_j)x + (2\omega_i - \omega_j)t\} + \ldots - (21)$$

We define,

$$\psi_i = k_i x + \omega_i t, \quad \psi_j = k_j x + \omega_j t$$
$$\psi_i + \psi_j = (k_i + k_j)x + (\omega_i + \omega_j)t$$
$$\psi_i - \psi_j = (k_i - k_j)x + (\omega_i - \omega_j)t$$
$$2\psi_i + \psi_j = (2k_i + k_j)x + (2\omega_i + \omega_j)t$$
$$2\psi_i - \psi_j = (2k_i - k_j)x + (2\omega_i - \omega_j)t$$

If we substitute the values into (8)

$$\frac{\partial \bar{f}(x,t)}{\partial x} = \sum_{I=1}^{N} A_i k_i \cos\psi_i + \sum_{i=1}^{N}\sum_{j=1}^{M} \{B_{ij}(k_i + k_j)\cos(\psi_i + \psi_j) + |k_i - k_j|\cos(\psi_i - \psi_j)\} +$$
$$+\sum_{i=1}^{N}\sum_{j=1}^{M} \{C_{ij}(2k_i - k_j)\cos(2\psi_i + \psi_j) + |2k_i - k_j|\cos(2\psi_i - \psi_j)\} + \qquad (22)$$

If we let $a_i = A_i k_i$ $B_{ij} = \frac{\lambda}{4} a_i a_j$ $C_{ij} = \frac{\lambda}{8} a_i^2 a_j$

The above definition follows structural observed characteristics of rogue wave event. The deterministic profile $\eta(x,t)$ of rogue wave event is thus provided by

$$\eta(x,t) = \sum_{i=1}^{N} a_i \cos\psi_i + \frac{\lambda}{4}\sum_{i=1}^{N}\sum_{j=1}^{M} a_i a_j \{(k_i + k_j)\cos(\psi_i + \psi_j) + |k_i - k_j|\cos(\psi_i - \psi_j)\} +$$
$$\frac{\lambda^2}{8}\sum_{i=1}^{N}\sum_{j=1}^{M} a_i^2 a_j \{(2k_i + k_j)\cos(2\psi_i + \psi_j) + |2k_i - k_j|\cos(2\psi_i - \psi_j)\} \qquad (23)$$

Equation (23) embodies three orders. The first order contains a single group which was studied by Boccotti (1982), the second order which consists of two wave groups was studied by Arena and Fedele (2005). The disparity between their combined results and observed datae was significant enough to arouse my curiosity.

Thus, I studied the third order which consists of three groups. Of these, two are assumed to be identical. This assumption is physically realistic because, the wave event is usually observed to be uni-directional.

THE THIRD ORDER EXPANSION

To a third order in Stoke's expansion and, for the free surface displacement and the velocity potential in the event of the long crested deep water waves group, we have

$$\eta(x,t) = \eta_1 + \lambda\eta_2 + \lambda^2\eta_3 + O(\lambda^3) =$$
$$\sum_{n=1}^{N} a_n \cos\psi_n + \frac{\lambda}{4}\sum_{n=1}^{N}\sum_{m=1}^{M} a_n a_m \{(k_n + k_m)\cos(\psi_n + \psi_m) + |k_n - k_m|\cos(\psi_n - \psi_m)\} +$$
$$\frac{\lambda^2}{8}\sum_{n=1}^{N}\sum_{m=1}^{M} a_n^2 a_m \{(2k_n + k_m)\cos(2\psi_n + \psi_m) + |2k_n - k_m|\cos(2\psi_n - \psi_m)\} + O(\lambda^3) = h_0 + h_1 + h_2 + h_3 \qquad (24)$$

$$\phi(x,t) = \phi_1 + \lambda\phi_2 + \lambda^2\phi_3 + O(\lambda^3) = g\sum_{n=1}^{N} a_n\omega_n^{-1}\exp(k_n z)\sin\omega_n + \lambda\sum_{n=1}^{N}\sum_{m=1}^{M} a_n a_m \omega_n \exp[z(k_n - k_m)]\sin(\psi_n - \psi_m) +$$
$$\lambda^2\sum_{n=1}^{N}\sum_{m=1}^{M} a_n^2 a_m \omega_m^2 \exp[z|2k_n - k_m|][(2k_n + k_m)\sin(2\psi_n + \psi_m) + |2k_n - k_m|\sin(2\psi_n - \psi_m)] + O(\lambda^3) \quad (25)$$

$0.1 < \lambda < 0.3$

Equations (24) and (25) above model asymptotically a system of interacting large amplitude mode in the long wave (n, m) system with probability of occurrence approaching 1, forming a large crested wave.

The third order term in the expansion is expected to involve three summation conventions. However, it is assumed that in the evolution of wave group, some of the modes are identical with same amplitude components, wave numbers and frequencies. In this case, only two summation conventions are required and this makes analytical manipulation quite clearer and less complicated. Identical representation had been applied with success in the calculations involving double frequency micro-seismic waves and related earth tremors, so induced in the layered earth (Okeke 1985) and (Okeke and Asor 1998)

THIRD ORDER SPECTRAL REPRESENTATION.

$$when\ \ \psi_n = \psi_m = 0 \quad (26)$$

$$\eta_3(0,0) = \frac{1}{8}\sum_{nm}^{N} a_n^2 a_m [(2k_n + k_m) + |2k_n - k_m|] \quad (27)$$

Take $s_w[\ ]$ as the smooth frequency spectral operator. Then, we deduce the following representations, $s_\omega[a_n^2] = E_1(2\omega_1)$, $s_\omega[a_m] = E_2(2\omega_2)$; then, the following third order continuous representation apply;

$$h_3 = \frac{H_c^3}{8\sigma^6}\int_0^\infty\int_0^\infty E_1(2\omega_1)E_2(\omega_2)[(2k_1 + k_2) + |2k_2 - k_1|]d\omega_1 d\omega_2. \quad (28)$$

h_3 is the third order term for observed wave elevation.

If H is the height of the high wave crest at $x_0 = t_0 = 0$ when $\frac{H_c}{\sigma} \to \infty$, then

$$h = H + H_c^2 + \frac{H_c^3}{8\sigma^6}\int_0^\infty\int_0^\infty E_1(2\omega_1)E_2(\omega_2)[(2k_1 + k_2) + |2k_2 - k_1|]d\omega_1 d\omega_2 \quad (29)$$

h = wave elevation to third order.

The free surface wave elevation when exceptionally high wave crest occurs at (x_0, t_0) is generally represented in the form.

$$\eta(x,t) = \eta_1 + \lambda\eta_2 + \lambda^2\frac{H_c^3}{8\sigma^6}\int_0^\infty\int_0^\infty E_1(2\omega_1)E_2(\omega_2)[(2k_1 + k_2)\cos(2\psi_1 + \psi_2) + |2k_2 - k_1|\cos(2\psi_1 - \psi_2)]d\omega_1 d\omega_2 \quad (30)$$

equation (30) is Stoke's expansion for wave group (Boccotti, 2000) but now extended to third order in this study using Kinsman (1965) method in this study.

Where $\eta_1(x,t)$ is stated in equation (6) and

$$\eta_2(x,t) = \frac{H_c^2}{2\sigma^2}\int_0^\infty\int_0^\infty E_1(\omega_1)E_2(\omega_2)[(k_1 + k_2)\cos(\psi_1 + \psi_2) + |k_2 - k_1|\cos(\psi_1 - \psi_2)]d\omega_1 d\omega_2 \quad (31)$$

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(see Fedele 2006)

$$\sigma^2 = \int_0^\infty E(\omega)d\omega \qquad (32)$$

σ^2 is still the variance calculated from $\eta_n(t)$ in time domain.
Similarly the third order velocity potential in continuous from is derived as

$$\phi_3(X,T;z) = \frac{H_c^3}{\sigma^6}\int_0^\infty\int_0^\infty E_1(2\omega_1)E_2(\omega_2)[\exp|2k_1-k_2|z](2k_1+k_2)\sin(2\psi_1+\psi_2)+|2k_2-k_1|\sin(2\psi_1-\psi_2)d\omega_1 d\omega_2 \qquad (33)$$

where k_1 and k_2 are parameters identified with wave numbers of the dominant wave group.
$\phi_2(X,T;z)$ was derived by Fedele and Arena (2003) and stated as;

$$\phi_2(X,T;z) = \frac{H_c^2}{\sigma^4}\int_0^\infty\int_0^\infty E_1(\omega_1)E_2(\omega_2)[\exp|k_1-k_2|z](k_1+k_2)\sin(\psi_1+\psi_2)+|k_1-k_2|\sin(\psi_1-\psi_2)d\omega_1 d\omega_2 \qquad (34)$$

and

$$\phi_1(X,T;z) = \frac{gH_c}{\sigma^2}\int_0^\infty E(\omega)\omega^{-1}\exp(kz)\sin\psi d\omega \qquad (35)$$

Thus,

$$\phi(X,T;z) = \phi_1(X,T;z) + \lambda\phi_2(X,T;z) + \lambda^2\phi_3(X,T;z) + O(\lambda^3) \qquad (36)$$

NUMERICAL CALCULATIONS

JONSWAP (Joint North Sea Wave Project) spectrum is given in (Fedele and Arena 2003, Rahman 2003) as

$$E(w\omega_p) = \alpha g^2\omega_p^{-5}E_j(w) \qquad (37)$$

where α *is the Phillips parameter,* $\omega_p = \frac{2\pi}{T_p}$ *the peak frequency, and* $w = \frac{2\pi}{T}$, *T is wave period, thus,*

$$E(w) = \omega^{-5}\exp\left(\frac{-1.25}{\omega^4}\right)\exp\left[InX_1^{3.3}\exp\left(\frac{(\omega-1)^2}{2X_2}\right)\right] \qquad (38)$$

is the non-dimensional spectrum, for a mean JONSWAP spectrum, we $X_1 = 3.3,\ X_2 = 0.08$ (Fedele andArena 2003). Thus, assume that the spectrum is narrow banded with w_p as the peak frequency; k_p the dominant wave number, then in terms of non-dimensional variables we obtain;

$$w_i = \frac{\omega_i}{w_p},\quad \bar{k} = \frac{k_i}{k_p},\ (p=1,2)$$

$$T_p = \frac{2\pi}{w_p},\quad L_p = \frac{2\pi}{k_p},\quad \bar{X} = \frac{X}{L_p}$$

$$\bar{T} = \frac{T}{T_P},\quad \bar{\psi}_i = k_i X - \bar{\omega}T \qquad (39)$$

In the subsequent calculations, we introduce the following mathematical expectations

$$E(\eta(t)) = \int_{-\infty}^{\infty}\eta(t)p(t)dt \qquad (40)$$

$\eta(t)$ is the profile of the sea-surface time series p(t) is the related probability density both in time domain. The non-dimensional form of the Rayleigh distribution ($p_R\eta(t)$) is calculated from

$$p_R(\eta(t)) = \frac{\pi\eta(t)}{2\mu_0^2}\exp\left[-\frac{\pi}{4}\left(\frac{\eta(t)}{\mu_0}\right)^2\right],\qquad t>0. \qquad (41)$$

(Rahman 1994) $\mu_0 = \eta_{rms}$ = root mean square wave crest height.

$$\mu_0 = \eta_{rms} = \left(\frac{4\tau_a}{\pi} \int_0^{\frac{\pi}{\tau}} \cos^2 \in tdt \right)^{1/2} = \eta_0 \sqrt{2} \qquad (42)$$

η_0 = mean wave height. $\tau_a = \frac{2\pi}{\Delta\sigma}$ = modulation wave period, $\Delta\sigma$ = spectral band width in the frequency/amplitude spectrum

Generally,

$$\eta_p = \frac{\sqrt{2}}{\pi} \eta_{rms} \sin\left(p\pi/3 \right) \qquad (43)$$

$\eta_{1/3}$ is the significant wave height, it is computed from

$$\eta_{1/3} = \sqrt{2}/\pi \sin\left(\pi/6\right) \eta_{rms} \qquad (44)$$

For rogue wave event, the global accepted value for $\eta_{1/3}$ is 10m. Thus,

$$\mu_0 = \eta_{rms} = \frac{220}{21\sqrt{6}} m \qquad (45)$$

The frequency spectrum is derived in this case from (using equation 40)

$$E(\omega) = \frac{2}{\pi} \int_0^\infty p(\eta(T)) \cos \omega T dT \qquad (46)$$

The non-dimensional frequency $w_j^2, \ j = 1,2$

$$\eta(X,T) = \frac{H_c}{\sigma^2} \int_0^\infty E(w) \cos \psi dw +$$

$$\frac{H_c^2}{4\sigma_a^4} \lambda \frac{w_p^2}{g} \int_0^\infty \int_0^\infty E_j(w_1) E_j(w_2) \left[\left(w_1^2 + w_2^2\right) \cos(\psi_1 + \psi_2) + \left| w_1^2 - w_2^2 \right| \cos(\psi_1 - \psi_2) \right] dw_1 dw_2 +$$

$$\lambda^2 \frac{H_c w_p^2}{\sigma_a^6 g} \int_0^\infty \int_0^\infty E_j(2w_1) E_j(w_2) \left\{ \left(4w_1^2 + w_2^2\right) \cos(2\psi_1 + \psi_2) + \left| 4w_1^2 - w_2^2 \right| \cos(2\psi_1 - \psi_2) \right\} dw_1 dw_2 =$$

$$\eta = \eta_1 + \lambda \eta_2 + \lambda^2 \eta_3 \qquad (47)$$

$$p(\eta) = r/N$$

numerically, r is a proper subset of N (r = 0,1,2,...,N).

$E_j(w)$ is computed from the recorded wave profile when reduced to the non-dimensional form. It represents the frequency amplitude spectrum of the sea-surface profile.

The recorded wave profile datae were supplied by a collegue Prof. E.O. Okeke.

THE PROBABILITY OF EXCEEDENCE DERIVED TO THE THIRD ORDER

From the previous analysis (equation 22) the crest elevation h to third order in non-dimensional form is give now by

$$h = h_1 + \lambda h_2 + \lambda^2 h_3 + \ldots \qquad (48)$$

where

$$h_1 = H_c, \qquad h_2 = \phi_2 \frac{H_c^2}{\sigma}, \qquad h_3 = \phi_3 \frac{H_c^3}{8\sigma^6}$$

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$$and\ \phi_1 = \frac{k_p}{\sigma_w^2}\int_0^\infty w_1 E(w_1)dw_1 \qquad (49)$$

$$\phi_2 = \frac{k_p\sigma}{4\sigma_w^4}\int_0^\infty\int_0^\infty E_a(w_1)E_a(w_2)\left[\left(w_1^2+w_2^2\right)-\left|w_1^2-w_2^2\right|\right]dw_1dw_2 \qquad (50)$$

$$\phi_3 = \frac{k_p^2\sigma^2}{8\sigma_w^6}\int_0^\infty\int_0^\infty E_a(2w_1)E_a(2w_2)\left[\left(4w_1^2+w_2^2\right)-\left|4w_1^2-w_2^2\right|\right]dw_1dw_2 \qquad (51)$$

$$\sigma_w = \frac{\sigma^2}{\beta} \qquad (52)$$

In terms of third order, as previously deduced by Arena and Fedele (2003) first and second order expansion, we obtained

$$\beta = \left[1+\frac{k^2\sigma^2}{8\sigma_w^2}\int_0^\infty\int_0^\infty E_w(2w_1)E_w(w_2)\left[4w_1^2+w_2^2\right]dw_1dw_2\right]^{-\frac{1}{2}} \qquad (53)$$

The notation $E_a(2w_1) = E_a^2(w_1)$, the superposition of two interacting component of quasi- monochromatic waves with equal frequency.

Thus, crest height $\xi_h = h/\sigma_w$, to give

$$\xi_h = 1/\sigma_w\left[h_1+\lambda h_2+\lambda^2 h_3+\ldots\right] \qquad (54)$$

Thus, the probability density of exceedence relating to wave with absolute maximum crest height $H_c < \infty$ (i.e. measurable height) is obtained from parameter z_0 where

$$\phi_3\beta z_0^2 + \beta z_0 - H = 0 \qquad (55)$$

$$p(\xi_h > H) = \ell^{=\frac{z_0}{2}}$$

$$z_0 = 1/2\phi_1^2\left[-1+\left(1+\frac{4\phi_1^2 H}{\beta}\right)^{\frac{1}{2}}\right] \qquad (56)$$

The expression is a function of two parameters ϕ_3 and β.

Hence $p(\xi_h > H)$ depends on the order of the Stoke's expansion of these parameters.

RESULTS

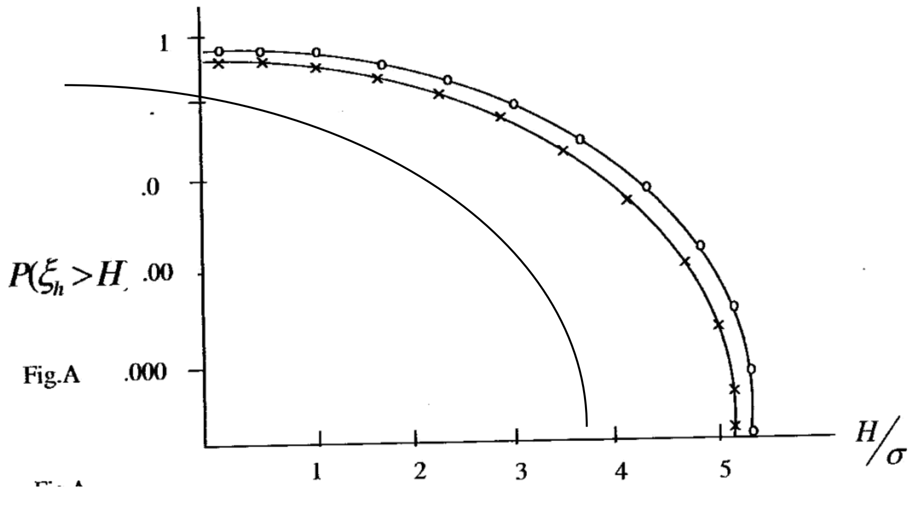


Fig.A

Figure A The exceedence probability density for the linear wave group η_1, the second order term η_2 and the third order term η_3

ooooo….. the exceedence probability density to third order.

xxxxx…..the exceedence probability density to second order.

_________ the exceedence probability density first order

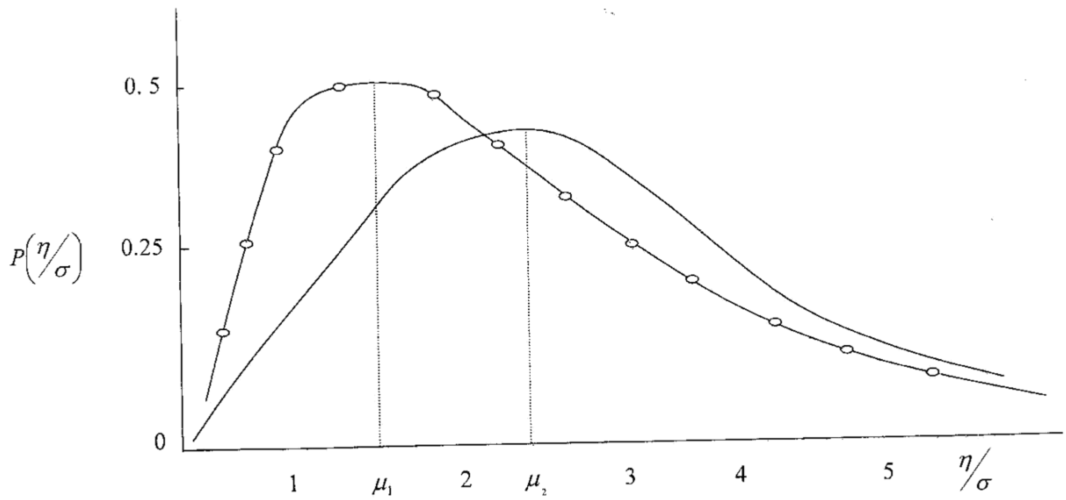
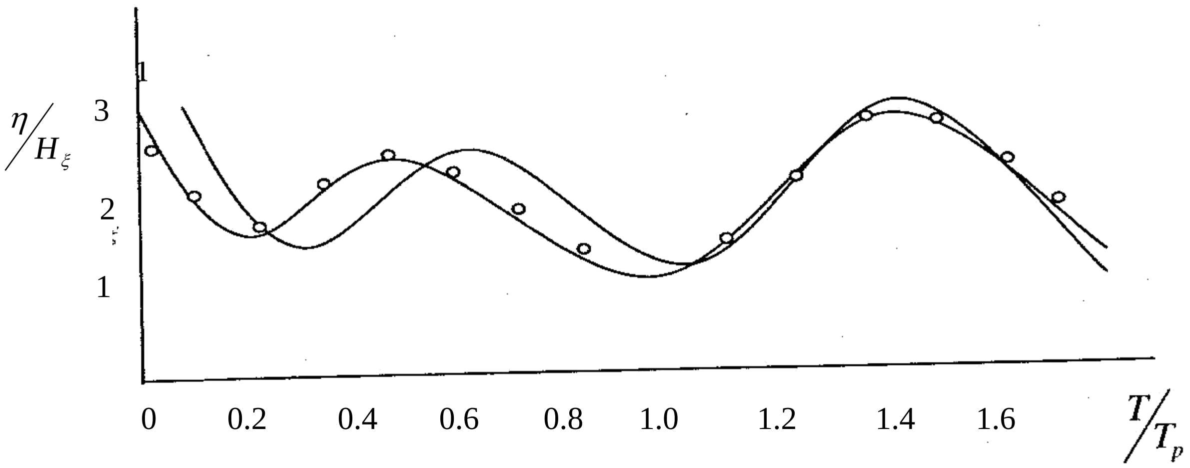


Figure B Comparison between Rayleigh distribution and the third order term η_3

ooo Rayleigh probability density with mean $\overline{\mu}_1$

___________ Rayleigh probability density with mean $\overline{\mu}_2$ generalised to third order in $\eta(t)$



FigureC The free surface displacement $\eta = \eta_1 + \eta_2 + \eta_3$ and the second order $\eta = \eta_1 + \eta_2$

ooooo…. $\eta = \eta_1 + \eta_2 + \eta_3$

________ $\eta = \eta_1 + \eta_2$

$T_p = 10\,\text{sec}\,ond$

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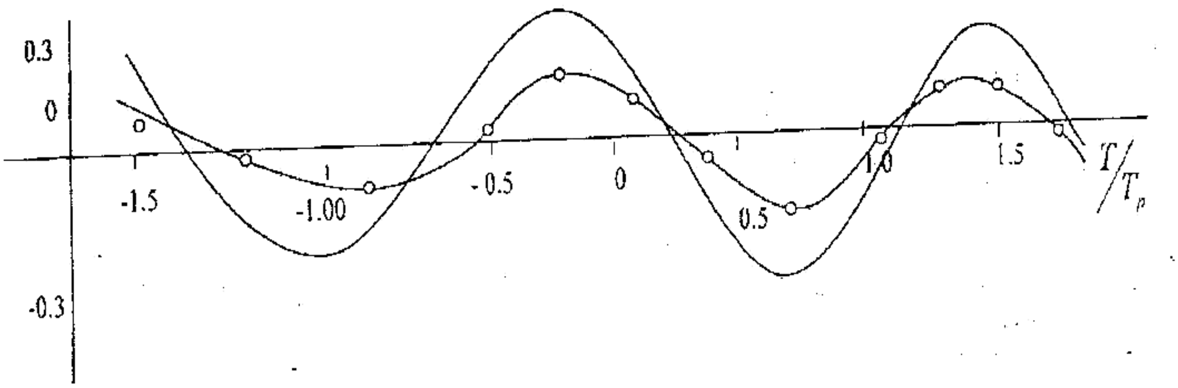

Figure DThe linear wave group η_1 and the free surface displacement $\eta = \eta_1 + \eta_2 + \eta_3$ a
ooooo....$\eta = \eta_1 + \eta_2 + \eta_3$
________ $\eta = \eta_1$

DISCUSSION

Figure A and Figure B appear to suggest that prediction efficiency of the third order model is slightly higher. For practical purposes, this difference is quite significant. In this regard, numerical calculation Figure A suggests a factor of 8.5% increase in probability prediction. In figure B, the third order form has higher global mean than that calculated from Rayleigh

η_c = extreme crest wave height
η_T = extreme trough wave depth
H_C = global standard crest height (22m)
H_T = global standard trough depth (19m)
$\lambda = 0.3$

Table 1

	First order term	Second order term	Third order term	Forth and higher
$\frac{\eta_c}{H_c}$	0.65	0.22	0.12	0.001
$\frac{\eta_T}{H_T}$	0.61	0.23	0.14	0.002

The contribution of various terms in wave group Stokes expansion is depicted. The dominance of the first is clearly shown. The data seem to suggest that after the third, the contribution from the higher order terms is negligibly small.

By using generalized Stokes expansion and Rayleigh distribution spectrum, both $\eta_2 and \eta_3$ appear to be higher in amplitude, η_3 contributing quite significantly.

Applying the Stoke's expansion theorem, the quasi-determinism wave group was extended to the third order in Stoke's expansion. The third order spectral components that give an extreme crest were derived. They are solutions of a well-defined constrained optimization problem. By means of the theory of quasi-determinism of Boccotti, the probability of exceedance of the wave crest is then obtained. The numerical calculations using JONSWAP showed that for fixed values of the linear wave steepness λ, the parameter reaches a maximum. The linear crest amplitude H increases due to the third order nonlinear interaction among the free harmonics.

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