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NUMERICAL MODELLING OF THE IMPACT OF THE RECRUITMENT RATES ON THE STABILITY OF TWO INTERACTING COMPETITIVE DYNAMICAL SYSTEMS

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ABSTRACT

*The impact of the changes in the capital recruitment for two dominant competitive interacting political parties on the type of stability has been an open problem. The method adopted in quantifying this impact is called numerical simulation approach. Full stability is achieved when the value of the initialcapital recruitment values from party **S**to party **T,** $\tau_1 = 0.0236$ and the value of the initialcapital recruitment values from party **T**to party **S,** $\tau_2 = 0.0087$ while the capital recruitment rates are undergoing some perturbations thatare likely to be affected or influenced by political campaigns and manifestoes. However, this scenario can be altered by loss of full stability when the value of the initial capital recruitment values from party **S** to **T** $\tau_1 = 0.0250$ and the initial capital recruitment values from party **T** to party **S**, $\tau_2 = 0.2000$. The result of this novel study is presented and discussed*

Keywords: Stability, degeneracy, dominant political parties, competitive interaction.

INTRODUCTION

The important ingredients of democracy are competitive elections. It is essential to note that capital recruitment rate of political parties are not new in themathematical literature. (Khan, 2000;Ekaka and Gambari, 2015; Bazuaye, 2019;Calderon etal., 2005). Bazuaye (2017) considered the computational simulation of the impact of system perturbation on the stabilization of the growth of two political Parties. In the paper, the dynamical system is perturbed with different degrees of perturbations. The impact of the effect of the capital recruitment rate on the type of stability has not been reported in the literature and hence, the desire to go into this study. While the capital recruitment rate of the political party defines the growth of the party over the period of existence of the party, the stability of any two dynamical systems can be either said to be stable or unstable depending on the result of the solution trajectories.

MATHEMATICAL MODEL

Arvind (2012), Poitoriusand Utterback, (1997), consider U total population whichis assumed to be constant. U is divided into groups; (i) The electorates P (ii) members of political party **S** (iii) Members of political party **T**. while the per capital recruitment rate of party S is $\gamma_1 = \ell_1\gamma_1$. So, the individual P may decide to join party S at a rate $\gamma_1(S/U)$

Similarly, let the per capital recruitment rate γ_2 of party T as $\gamma_2 = \ell_2\gamma_2$, where

ℓ_2 is the mean number of contacts of members of party T The electorates P may decide to join party T at a rate $\gamma_2(T/U)$ In addition, let τ_1 and τ_2 be the capital recruitment from party S to party T and from party T to party S respectively. So, member of party S leaves the party at the rate $\gamma_1 S(T/U)$ and join party T at the same rate. Also, members of party T leave the party at the rate $\gamma_2 T(S/U)$ and join party S at the same rate. We assumed that individual enters the voting class at the attainment age of 18 years and at the rate of φU where φ is the rate at which individual enter or leave the voting system, with φP , φS and φT represents those that leave the political system from each class respectively, as a result of incapacitation or death. Also, it is also assumed that each electorate can either join party S or T on the basis of political interest or any other personal conviction. Let ℓ_1 be the average number of contacts of members of political party S with electorates per unit time and τ_1 be the probability of convincement per contact on electorate with a member of party S, while the per capital recruitment rate of party S is $\gamma_1 = \ell_1 \tau_1$. So, the individual P may decide to join party Q at a rate $\gamma_1(S/U)$

So, the modeling equation which is dynamical in nature as given by Arvind (2012) model:

$$\frac{dP}{dt} = \varphi U - \tau_1 P\frac{S}{U} - \tau_2 P\frac{T}{U} - \varphi P$$

$$\frac{dS}{dt} = \gamma_1 P\frac{S}{U} - \tau_1 Q\frac{T}{U} - \tau_2 T\frac{S}{U} - \varphi S \qquad (1)$$

$$\frac{dT}{dt} = \gamma_2 P\frac{T}{U} + \tau_1 S\frac{T}{U} - \tau_2 T\frac{S}{U} - \varphi T$$

With the conditions the initial conditions $P(0) > 0$, $S(0) \geq 0$, $T(0) \geq 0$.

Here, the model parameter starting values γ_1 of party $S = 0.0417$ and $S = 0.0278$. In addition, let $\tau_1 = 0.0236$ and $\tau_2 = 0.0087$ be the initial capital recruitment values from party **S**to party **T**and from party **T**to party **S**respectively.

METHOD OF SOLUTION

The model parameter values of the two capital recruitment rates were varied in conjunction with a change in the value of the capital recruitments from party **S**to party **T**(τ_1) and from party **T**to party **S**(τ_2) in order to investigate the nature of the stability. The results of this investigation are presented and discussed below.

For clarity, we introduced the following notations. The capital recruitment values from party **S**to party **T** and the capital recruitment values from party **T**to party **S** are γ_1 and γ_2 respectively**.** Their corresponding eigenvalues are denoted as λ_1 and λ_2 respectively. The nature or type of stability is denoted as **NAS**

RESULTS

This present analysis deals essentially on the numerical simulation in the context of competitive interaction between two dominant political parties. It has been discovered that there are cases of stable systems and in other cases degenerate type of stability due to the perturbations of the initial capital recruitment values from party **S** to **T** and the initial capital recruitment values from party t to party S.We have also established the fact that the choice of the model parameter starting values is very significant to the type of stability the system will posses. This is a key contribution which has not been seen anywhere else and we expect it to contribute to other types of numerical analysis on modeling of two dominant dynamical systems.

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DISCUSSIONS

It can be observed that full stability is achieved when the value of the initialcapital recruitment values from party **S**to party **T,** $\tau_1 = 0.0236$ and the value of the initialcapital recruitments values from party **T**to party **S,** $\tau_2 = 0.0087$ while the capital recruitment rates are undergoing some perturbations which are likely to be affected or influenced by political campaigns and manifestoes. However, this scenario can be altered when the value of the initial capital recruitment values from party **S** to **T** $\tau_1 = 0.0250$ and the initial capital recruitment values from party **T** to party **S,** $\tau_2 = 0.2000$**.** As seen in Table 3 below.

Table 1:The influence of the capital recruitment rate on the stability of two dominant competitive dynamical systems when the initial capital recruitment values from party Sto party T, $\tau_1 = 0.0236$ **and the initial capital recruitment values from party T**to party **S,** $\tau_2 = 0.0087$.

Eg	γ_1	γ_2	$1-norm$	$2-norm$	$3-norm$	$\infty-norm$	λ_1	λ_2	NAS
1	0.0417	0.0278	0.4179	0.1077	0.0709	0.0394	-0.0394	-0.0327	Stable
2	0.0442	0.0303	0.4167	0.1074	0.0708	0.0394	-0.0394	-0.0325	Stable
3	0.0467	0.0328	0.4156	0.1071	0.0706	0.0393	-0.0393	-0.0324	Stable
4	0.0492	0.0353	0.4144	0.1068	0.0704	0.0392	-0.0392	-0.0322	Stable
5	0.0517	0.0378	0.4133	0.1065	0.0702	0.0391	-0.0391	-0.0321	Stable
6	0.0542	0.0403	0.4122	0.1063	0.0701	0.0390	-0.0390	-0.0319	Stable
7	0.0567	0.0428	0.4110	0.1060	0.0699	0.0389	-0.0389	-0.0318	Stable
8	0.0592	0.0453	0.4098	0.1057	0.0697	0.0388	-0.0388	-0.0316	Stable
9	0.0617	0.0478	0.4087	0.1054	0.0695	0.0387	-0.0387	-0.0315	Stable
10	0.0641	0.0503	0.4076	0.1051	0.0693	0.0386	-0.0386	-0.0313	Stable

Table 1 above clearly shows that the system is stable in line with the choice of the values of the model parameters.

The same argument holds in Table 2 below, as full stability is achieved.

Table 2:The influence of the capital recruitment rate on the stability of two dominant competitive dynamical systems when the initial capital recruitment values from party Sto party T, $\tau_1 = 0.0250$ **and the initial capital recruitment values from party T**to party **S,** $\tau_2 = 0.0050$.

Eg	γ_1	γ_2	$1-norm$	$2-norm$	$3-norm$	$\infty-norm$	λ_1	λ_2	NAS
1	0.0417	0.0278	0.4197	0.1081	0.0712	0.0396	-0.0396	-0.0335	Stable
2	0.0442	0.0303	0.4186	0.1078	0.0710	0.0395	-0.0394	-0.0325	Stable
3	0.0467	0.0328	0.4174	0.1075	0.0709	0.0394	-0.0394	-0.0332	Stable
4	0.0492	0.0353	0.4162	0.1073	0.0707	0.0393	-0.0392	-0.0322	Stable
5	0.0517	0.0378	0.4151	0.1070	0.0705	0.0392	-0.0392	-0.0329	Stable
6	0.0542	0.0403	0.4139	0.1067	0.0703	0.0391	-0.0391	-0.0328	Stable
7	0.0567	0.0428	0.4128	0.1064	0.0701	0.0390	-0.0390	-0.0326	Stable
8	0.0592	0.0453	0.4116	0.1061	0.0699	0.0389	-0.0389	-0.0325	Stable
9	0.0617	0.0478	0.4104	0.1058	0.0697	0.0388	-0.0388	-0.0323	Stable
10	0.0641	0.0503	0.4093	0.1055	0.0696	0.0387	-0.0387	-0.0322	Stable

The stability can be lost with a wrong choice of the values of the model parameters. This can be seen in Table 3 below.

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Table 3:The influence of the capital recruitment rate on the stability of two dominant competitive dynamical systems when the initial capital recruitment values from party Sto party T, $\tau_1 = 0.0250$ **and the initial capital recruitment values from party T**to party **S,** $\tau_2 = 0.2000$.

Eg	γ_1	γ_2	$1-norm$	$2-norm$	$3-norm$	$\infty-norm$	λ_1	λ_2	NAS
1	0.0417	0.0278	0.4878	0.1278	0.0850	0.0488	-0.0488	0.0061	Degenerate
2	0.0442	0.0303	0.4868	0.1275	0.0848	0.0488	-0.0488	0.0063	Degenerate
3	0.0467	0.0328	0.4857	0.1273	0.0847	0.0487	-0.0487	0.0066	Degenerate
4	0.0492	0.0353	0.4847	0.1270	0.0845	0.0486	-0.0486	0.0068	Degenerate
5	0.0517	0.0378	0.4836	0.1267	0.0844	0.0485	-0.0485	0.0071	Degenerate
6	0.0542	0.0403	0.4826	0.1265	0.0842	0.0485	-0.0485	0.0074	Degenerate
7	0.0567	0.0428	0.4815	0.1262	0.0840	0.0484	-0.0484	0.0076	Degenerate
8	0.0592	0.0453	0.4805	0.1260	0.0839	0.0483	-0.0483	0.0079	Degenerate
9	0.0617	0.0478	0.4794	0.1257	0.0837	0.0482	-0.0482	0.0081	Degenerate
10	0.0641	0.0503	0.4784	0.1255	0.0836	0.0482	-0.0482	0.0084	Degenerate

From Table2, it can be observed that an increase in the initial capital recruitment values from party **T**to party **S** to enhance the growth of party **S** during period of membership drive can generate a dominant incidence of a degenerate that does not have physical meaning in terms of stability. The numerical simulation analysis showedthat the dominant steady state solution can be totally lost in a scenario when the value of the initial capital recruitment values from party **S**to party **T,** is the same while the capital recruitment rates are undergoing some perturbations which are likely to be affected or influenced by political campaigns and manifestoes. This negative scenario can be corrected to a full stability status when the value of the initial capital recruitment values from party **S**to **T** **(**τ_1**)** is $\tau_1 = 0.2000$ and the initial capital recruitment values from party **T** to party **S,** $\tau_2 = 0.0087$**.** The corresponding solutions of this investigation are shown in Table 4 below

Table 4:The influence of the capital recruitment rate on the stability of two dominant competitive dynamical systems when the initial capital recruitment values from party Sto party T, $\tau_1 = 0.2000$ **and the initial capital recruitment values from party T**to party **S,** $\tau_2 = 0.0087$.

Eg	γ_1	γ_2	$1-norm$	$2-norm$	$3-norm$	$\infty-norm$	λ_1	λ_2	NAS
1	0.0417	0.0278	0.7641	0.1935	0.1261	0.0672	-0.0672	-0.0333	Stable
2	0.0442	0.0303	0.7635	0.1933	0.1261	0.0672	-0.0672	-0.0331	Stable
3	0.0467	0.0328	0.7629	0.1932	0.1260	0.0671	-0.0671	-0.0330	Stable
4	0.0492	0.0353	0.7623	0.1931	0.1259	0.0671	-0.0671	-0.0328	Stable
5	0.0517	0.0378	0.7617	0.1929	0.1258	0.0671	-0.0671	-0.0327	Stable
6	0.0542	0.0403	0.7610	0.1928	0.1257	0.0670	-0.0670	-0.0325	Stable
7	0.0567	0.0428	0.7604	0.1926	0.1256	0.0670	-0.0670	-0.0324	Stable
8	0.0592	0.0453	0.7598	0.1925	0.1255	0.0670	-0.0670	-0.0322	Stable
9	0.0617	0.0478	0.7592	0.1924	0.1255	0.0670	-0.0670	-0.0320	Stable
10	0.0641	0.0503	0.7586	0.1922	0.1254	0.0669	-0.0669	-0.0319	Stable

In Table 4 above, we have discovered a dominant stable steady state solution when the value of the initial capital recruitment values from party **S** to **T (**τ_1**)** is 0.2000 and the initial capital recruitment values from party **T** to party **S,** 0.0087**.** The appropriate choices of **the** initial capital recruitment values from party **S** to **T (**τ_1**)** and the initial capital recruitment values from party **T** to party **S** (τ_1) has brought about stable system which is desirable for any numerical scheme.

This present analysis deals essentially on the numerical simulation. In the study, we have found cases of stable systems and in other cases degenerate type of stability due to the perturbations of the initial capital recruitment values from party **S** to **T** and the initial capital recruitment values from party t to party S.We have also established the fact that the choice of the model parameter starting values is very significant to the type of stability the system will posses. This is a key contribution which has not been reported before and we expect it to contribute to other types of numerical

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analysis on modeling of two dominant dynamical systems.

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