APPLICATION OF STEPPING STONE METHOD FOR AN OPTIMAL SOLUTION TO A TRANSPORTATION PROBLEM

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ABSTRACT

The purpose of this study was to determine the optimal routes to apply in the distribution of Big Treat products so as to improve the profit obtained by the company. In this work, a transportation problem of the distribution of Big Treat bread (Happy Chef) was solved using the transportation tableau approach. The initial basic feasible solution was obtained using the Vogel's approximation method and the stepping stone method was used to test and solve for optimality. From the data which was used to solve the six weeks distribution of bread to respective districts, the optimal cost of transportation obtained is \$10,294,850 and the result was confirmed using TORA, a statistical software. The optimal allocations are;

 $X_{11} = 9950$

 $X_{13} = 12600$

 $X_{14} = 10283$

 $X_{21} = 8033$

 $X_{22} = 2178$

 $X_{25} = 4500$

Key words: Optimal Solution, Stepping Stone Method, Feasible Solution, Slack Variable, Vogel's Approximation Method.

INTRODUCTION

Companies that deal with distributions of their goods and services are always faced with the Transportation problem. The Transportation Problem is a special type of linear programming that deals with finding the optimal transportation schedule that minimizes the total cost of transporting goods and services from supply points to demand centers. It can be expressed in different forms like a network model, integer programming model, linear programming model or transportation tableau due to the different transportation routes from several supply points to several demand centers. Jude & Vitus (2014) and (2014)applied Salami the vogel's approximation method in solving the Transportation problem. Chaudhuri et al (2013), Aggrawal & Gupta (2014) and Chakraborty et al (2014) used Trapezoidal fussy numbers methods in different transportation problems while others like saravanan (2005), Jalilzadeh & Hamedani (2014) applied the linear & integer programming as well as combined quadratic programming and optimization methods. Most recently, Jain &

Sood (2015) developed the maximum difference method for finding the initial basic feasible solution, while Sharma et al (2016) extended the maximin zero suffix method to quadratic transportation problem with the goal of finding the minimum possible allocation cost. In applying the different methods, certain assumptions are taken into consideration. They include:

- The total quantity available and demanded are equal
- The products distributed are homogenous
- The demand enters will accept products from any supply point
- All transportation costs are known.

The different methods have been compared to determine the one that gives the optimal result for instance Saravanan examined real life transportation problem involving two iron ore mines (supply) and three steel plants (demand) using the linear and integer programming method and the result suggested that the integer programming model produces a more minimized cost compared to the linear programming model. Salami (2014)compared three methods; North-west corner method, Least cost method and Vogel's approximation method using data from 7-Up Bottling Company Plc. Ilorin, Nigeria. He concluded that the three methods yielded the same result and recommended that any of the methods can be used by the company to obtain minimum transportation cost. Joshi (2013) using numerical examples illustrated the use of the North-west corner method, the least cost method, Vogel's approximation method and MODI method giving their algorithms. He concluded that the MODI method is the less complex and computed the optimal solution faster than the other methods. Furthermore, Jain & Sood (2015) proposed a new method of finding the initial basic feasible solution for solving transportation problems. They compared their method with Vogel's approximation method using some numerical examples and their results shows that their method was better than Vogel's approximation in most cases.

In this paper, the transportation problem of Big Treat Plc. Port Harcourt, one of the leading bakeries in the eastern part of Nigeria is solved using the transportation tableau approach. In other to obtain optimality, the transportation problem must be balanced or made balanced. The initial basic feasible solution is then obtained using the Vogel's approximation method while the optimal solution is gotten using the Stepping stone method.

The rest of the paper is organized as follows: section 2 examines the methodology involved in a transportation problem. Section 3 presents the data and the analysis while the last section concludes the paper and makes recommendation.

METHODOLOGY

Mathematical Formulation of the Transportation Problem

Let's assume that we are to transport goods from a certain number of warehouses (say m warehouses) to several retail outlets (say n retail outlets) and that the total supply of goods from the warehouses is a and the total number of goods demanded by the retail outlets is b. Assuming that we also know the cost of transporting goods between the various warehouses and retail outlets, then we can say that;

 a_i denotes the total supply of goods from warehouse i, where i = 1, 2, ..., m.

 b_j denotes the total demand of goods from retail outlet j, where j = 1, 2, ..., n.

 C_{ij} denotes the unit transportation cost from warehouse i to retail outlet j.

 X_{ij} denotes the quantity of goods transported from warehouse i to retail outlet j.

Hence our objective function which is usually a minimization problem is,

Minimize
$$\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$
, Let $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$
Our objective function becomes: Min $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$

We then state our constraints which our objective function must be subjected to. Constraints are conditions which the supply and demand forces must adhere to. The constraints are as follows:

$$\sum_{j=1}^{n} X_{ij} = a_i \qquad \text{(Supply constraint)}$$

$$\sum_{i=1}^{m} X_{ij} = b_{j} \quad \text{(Demand constraint)}$$

$$X_{ii} \ge 0$$
 (Non-negative constraint)

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$
 (Balance constraint)

The general mathematical representation of the transportation problem is:

Min
$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

subject to:

$$\sum_{j=1}^{n} X_{ij} = a_{i} \qquad i = 1, 2, ..., m$$

$$\sum_{i=1}^{m} X_{ij} = b_{j} \qquad j = 1, 2, ..., n$$

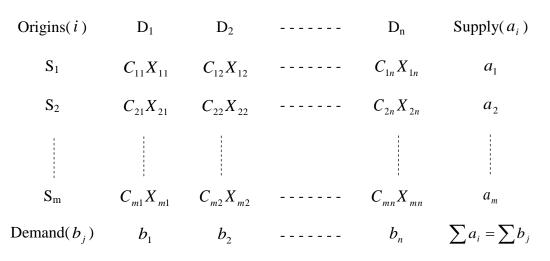
$$X_{ij} \ge 0$$

Transportation Tableau

The transportation problem can also be written in a tabular form as what is generally called the transportation tableau.

Table 1: A transportation Tableau

Destinations(j)



The transportation problem can also be viewed as a network problem and can be represented in form of a network model with m supply nodes and n demand nodes.

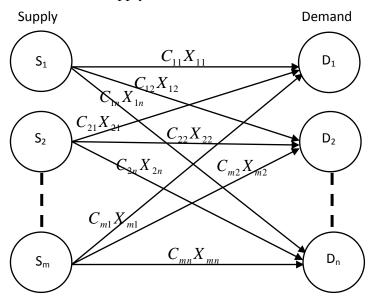


Figure 1: Network representation of a transportation problem

Algorithm for Solving Transportation Problem Using the Transportation Tableau Approach

Step 1:Formulate the transportation tableau.

Step 2:Ensure that the tableau is balanced else make it balanced.

Step 3:Obtain the initial basic feasible solution.

Step 4:Test for optimality of the initial basic feasible solution.

Step 5:If the initial basic feasible solution is not optimal, update the solution until it becomes optimal.

Step 6:Compute the total transportation cost.

Balanced and Unbalanced Transportation Problem

Naturally, not all transportation problems are balanced, i.e. the total number of goods demanded does not equal the total supplied $(\sum a_i \neq \sum b_j)$ and they are called unbalanced transportation problem. To correct this defection, the use of dummy

variables was employed. There are two cases of unbalanced transportation problem:

1. $\sum a_i > \sum b_j$, i.e. the total supply of goods exceeds the total demand.

In this case, dummy variables are added to the demand constraint and summed up to $\sum a_i - \sum b_j$ so as to fill in the gap created by the difference between the demand and the supply.

2. $\sum a_i < \sum b_j$, i.e. the total demand exceeds the total supply of goods.

The same solution in the first case is applied here. The only difference is that the dummy variables are applied to the supply constraints.

Initial Basic Feasible Solution

The initial basic feasible solution of an $m \times n$ transportation problem is a possible optimal solution whose total number of allocations is equal to m + n - 1. There are three generally known methods of obtaining the initial basic feasible solution:

- 1. North-west corner method
- 2. Minimum/Least cost method
- 3. Vogel's approximation method

Degeneracy

Sometimes, when we think we have obtained our initial basic feasible solution, we come to realize that the solution we've obtained is not the initial basic feasible solution in the sense that the total number of allocations is not equal to m+n-1. This situation is usually referred to as degeneracy and a solution that possess this trait is known as a degenerate basic feasible solution. To solve the problem of degeneracy, create an artificially occupied cell by placing zero in one of the unoccupied cells and then treat that cell as if it were occupied.

METHOD OF TESTING FOR AND OBTAINING OPTIMALITY

After a non-degenerate feasible solution has been obtained, it is tested to determine if it is optimal or not. If it is not optimal, then it is made optimal by the use of several methods such as:

- 1. Modified distribution (MODI) method
- 2. Stepping stone method

These methods yield the same result. An optimal solution is one where there is no other set of transportation routes that will further reduce the transportation cost. To obtain an optimal solution, successive improvements are made to the initial basic feasible solution until no further decrease in the transportation cost is possible.

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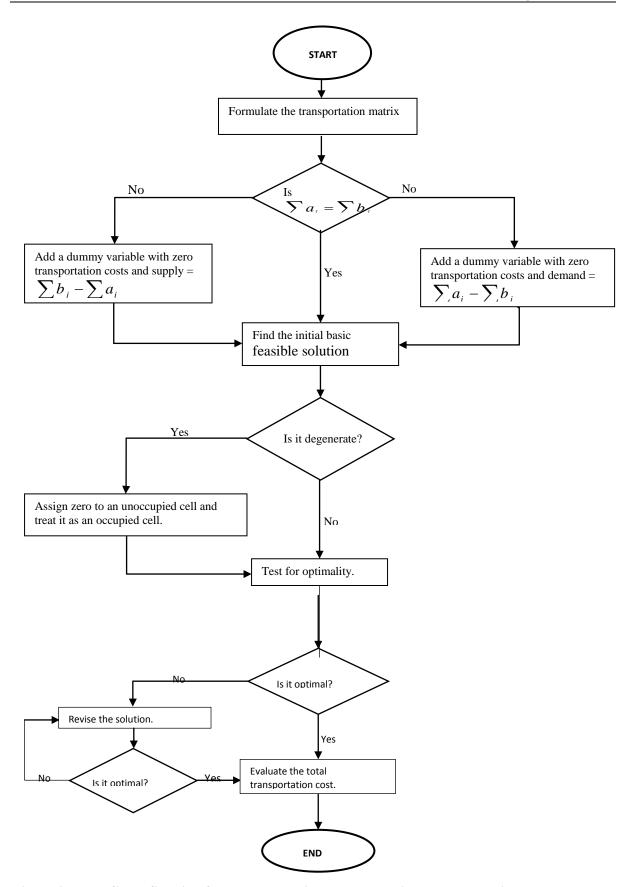


Figure 2: Flow Chart Solution for Transportation Problem Using Transportation Tableau

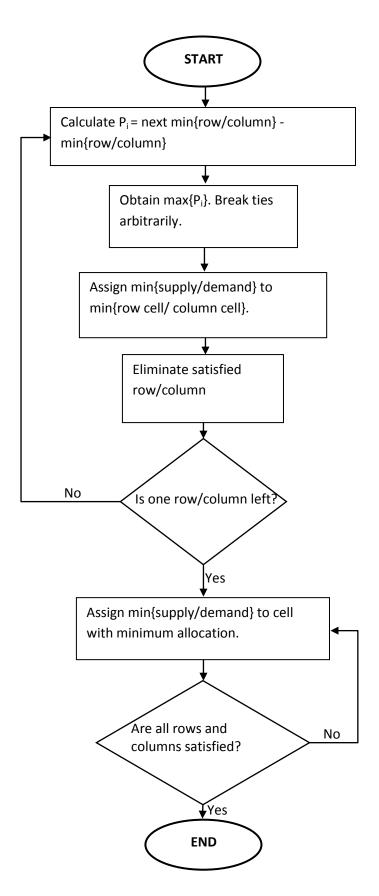


Figure 3: Flow Chart of Vogel's Approximation Method

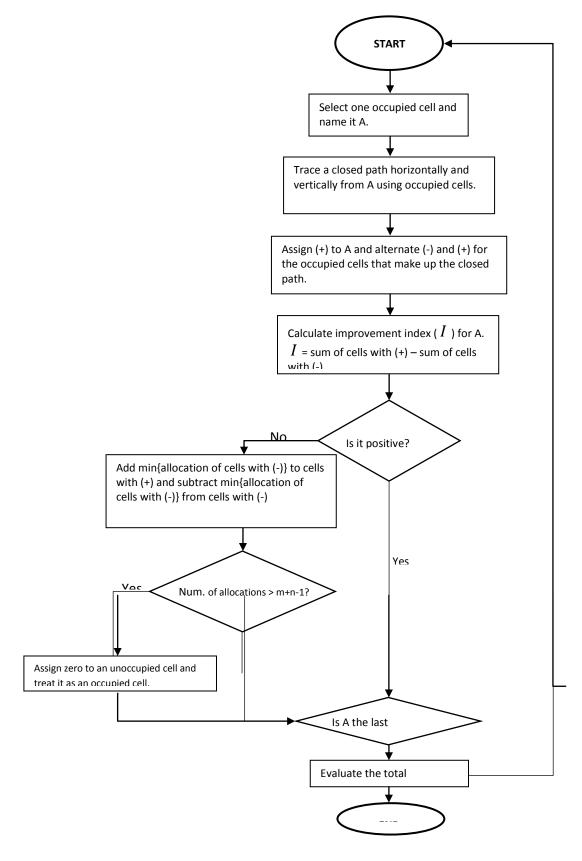


Figure 4: Flow Chart of Stepping Stone Method

DATA COLLECTION AND ANALYSIS

The data collected for the purpose of this work, was obtained from Big Treat Plc. Port Harcourt and it is based on the distribution of only bread (bread is not the only product of the company) to major Big Treat distribution sites in Port Harcourt. Some of the major distribution sites are Artillery, Mgbuoba, Rumuokoro and Rumuola. The products are distributed from Big Treat bakeries situated at Air Force junction and Rukpoku road. The data collected was obtained through verbal interview and information documented which was received from the bakery managers.

Data collected comprises of;

- > Number of supply centers
- Number of demand centers
- Average quantities demanded weekly by the different districts
- Average quantities supplied weekly by each of the bakeries

➤ Unit cost of distribution from the bakeries to the different districts

Big Treat Port Harcourt produces bread on a daily basis and they supply based on the request from the different districts. The data collected is based on the average weekly distribution of Big Treat's Bread for six weeks, starting from August 10th, 2015 to September 20th, 2015.

Transportation Tableau Representation of the Problem

For the purpose of computation and tabular representation of the problem,

Let D_1 represent Artillery

D₂ represent Mgbuoba

D₃ represent Rumuokoro

D₄ represent Rumuola

S₁ represent Big Treat, Air force Jct

S₂ represent Big Treat, Rukpoku

The transportations tableau representation of the problem is as follows;

	D_1	D_2	D_3	D_4	Supply
S_1	50	200	200	100	32833
S_2	100	250	300	200	34316
Demand	17983	21783	12600	10283	

Data Analysis

In this section, our focus would be to examine a practical application of the solution to transportation problem via transportation tableau approach, using Vogel's approximation method to obtain the initial basic feasible solution and the stopping stone method to test for and obtain optimality.

Balancing the Transportation Problem

Total supply:
$$\sum a_i = 32833 + 34316 = 67149$$

Total demand: $\sum b_j = 17983 + 21783 + 12600 + 10283 = 62649$

Total supply is greater than total demand, i.e. $\sum a_i > \sum b_j$ therefore we would add a dummy column (D₅) and a demand of $\sum a_i - \sum b_j$.

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	D_1	D_2	D_3	D_4	D_5	Supply
S_1	50	200	200	100	0	32833
S_2	100	250	300	200	0	34316
Demand	17983	21783	12600	10283	4500	67149

Solving for Initial Basic Feasible Solution (Ibfs) Using Vam Tableau 1

	\mathbf{D}_1	D_2	D_3	D_4	D_5	Supply	Penalty
S_1	50	200	200	100 (10283)	0	32833	50 - 0 = 50
S_2	100	250	300	200	0	34316	100 - 0 = 100
Demand	17983	21783	12600	10283	4500	67149	
Penalty	50	50	100	(100)	0		•

0 = 100

Highest penalty = 100Selected row\column = D_4 Least cost in $D_4=100$ Min $\{10283, 32833\} = 10283$ Satisfied row\column= D₄

Tableau 2

	D_1	D_2	D_3	D_5	Supply	Penalty
S_1	50	200	200 (12600)	0	22550	50 - 0 = 50
S_2	100	250	300	0	34316	100 - 0 = 100
Demand	17983	21783	12600	4500	56866	
Penalty	50	50	(100)	0		•

Highest penalty = 100Selected row/column = D_3 Least cost in $D_3 = 200$ Min $\{12600, 22550\} = 12600$ Satisfied row/column= D₃

Tableau 3

	D_1	D_2	D_5	Supply	Penalty
S_1	50	200	0	9950	50 - 0 = 50
S_2	100	250	0 (4500)	34316	$100 - 0 \neq 100$
Demand	17983	21783	4500	56866	
Penalty	50	50	0		

Highest penalty = 100Selected row/column = S_2 Least cost in $S_2 = 0$ Min $\{4500, 34316\} = 4500$ Satisfied row/column= S₂

Tableau 4

	D_1	D_2	Supply	_ Penalty
S_1	50 (9950)	200	9950	150
S_2	100	250	29816	(150)
Demand	17983	21783	39766	
Penalty	50	50		

Highest penalty = 150 Selected row/column = S_1 Least cost in S_1 = 50 Min (17983, 9950) = 9950 Satisfied row/column= S_1

Tableau 5

	D_1	D_2	Supply
S_2	100 (8033)	250 (21783)	29816
Demand	8033	21783	29816

Final tableau

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	50 (9950)	200	200 (12600)	100 (10283)	0	32833
S_2	100 (8033)	250 (21783)	300	200	0 (4500)	34316
Demand	17983	21783	12600	10283	4500	67149

Number of allocations m+n-1=5+2-1=6

IBFS = 50 (9950) + 200 (12600) + 100 (10283) + 100 (8033) + 250 (21783) + 0 (4500)

IBFS = 10, 294, 850

Testing for optimality using stepping stone method

	\mathbf{D}_1	D_2	D_3	D_4	D_5	Supply
S_1	50 (9950)	200	200 (12600)	100 (10283)	0	32833
S_2	100 (8033)	250 (21783)	300	200	0 (4500)	34316
Demand	17983	21783	12600	10283	4500	67149

To test for optimality using the stepping stone method, the improvement index, I is calculated for each unoccupied cell.

Considering route S₁D₂

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	50 (9950)	200 +	200 (12600)	100 (10283)	0	32833
S_2	100 (8033) +	250 (21783)	300	200	0 (4500)	34316
Demand	17983	21783	12600	10283	4500	67149

$$I(S_1D_2) = +200 - 250 + 100 - 50 = 0$$

Considering route S₁D₅

S_1
S_2
Demand

D_1	D_2	D_3	D_4	D_5	Supply
50 (9950)	200	200 (12600)	100 (10283)	0 +	32833
100 (8033) +	250 (21783)	300	200	0 (4500)	34316
17983	21783	12600	10283	4500	67149

$$I(S_1D_5) = +0 - 0 + 100 - 50 = +50$$

Considering route S₂D₃

~
S_1
S_2
Demand

D_1	D_2	D_3	D_4	D_5	Supply
50 (9950) +	200	200 (12600)	100 (10283)	0	32833
100 (8033)	250 (21783)	300 +	200	0 (4500)	34316
17983	21783	12600	10283	4500	67149

$$I (S_2D_3) = +300 - 100 + 50 - 200 = +50$$

Considering route S₂D₄

S_1
S_2
Demand

D_1	D_2	D_3	D_4	D_5	Supply
50 (9950) +	200	200 (12600)	100 (10283)	0	32833
100 (8033)	250 (21783)	300	200 +	0 (4500)	34316
17983	21783	12600	10283	4500	67149

$$I (S_2D_4) = +200 - 100 + 50 - 100 = +50$$

The improvement index calculated for the unoccupied cells are all non-negative and it implies that no improvement of the transportation tableau is required. Therefore the initial basic feasible solution obtained is an optimal solution.

Optimal solution = 10,294,850

Hence.

Min Z = 50 (9950) + 200 (12600) + 100(10283) + 100 (8033) + 250 (21783) + 0 (4500)

Min Z = 10, 294, 850

Alternative Optimal Solution

An alternative optimal solution is a solution that can be used in place of the originally obtained solution without incurring further cost and giving the same result. It is possible for a transportation problem to have other optimal solutions (alternative optimal solutions) that can be used in place of the initially obtained optimal solution. In a transportation problem, an alternative optimal solution is identified when the calculated improvement index for an unoccupied cell is zero. This implies that it is possible to design alternative transportation routes with the same total transportation cost by using the unoccupied cell.

While calculating the improvement indices for the unoccupied cells, it was observed that the improvement index for S_1D_2 is zero. This implies that with the use of S_1D_2 , an alternative transportation route can be designed.

Considering route S₁D₂

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	50 (9950)	200 +	200	100 (10283)	0	32833
			(12600)			
S_2	100 (8033)	250 (21783)	300	200	0 (4500)	34316
	+					
Demand	17983	21783	12600	10283	4500	67149

 $I (S_1D_2) = +200 - 250 + 100 - 50 = 0$

To obtain the alternative optimal solution, the minimum allocation in the cells with (-) is added to the allocations in the cells with (+) and subtracted from allocations in the cells with (-).

Min $\{9950, 21783\} = 9950$

Tableau with alternative allocations

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	50	200 (9950)	200 (12600)	100 (10283)	0	32833
S_2	100 (17983)	250 (11833)	300	200	0 (4500)	34316
Demand	17983	21783	12600	10283	4500	67149

Alternative optimal solution = 200 (9950) + 200 (12600) + 100 (10283) + 100 (17983) + 250 (11833) + 0 (4500) = 10, 294, 850

APPENDIX A

BIG TREAT PLC. AVERAGE WEEKLY SUPPLY **BRANCHES** WEEK 1 WEEK 2 WEEK 3 WEEK 4 WEEK 5 WEEK 6 **AVERAGE** BIG TREAT, AIR FORCE JCT 35000 42000 30000 29500 30000 30500 32833 BIG TREAT, RUKPOKU 30000 30100 35000 35000 40200 35600 34316 **TOTAL** 65000 72100 65000 64500 70200 66100

AVERAGE WEEKLY DEMAND

DISTRICTS	WEEK 1	WEEK 2	WEEK 3	WEEK 4	WEEK 5	WEEK 6	AVERAGE
ARTILLERY	19000	18500	15600	17200	19500	18100	17983
MGBUOBA	24100	25600	20000	17500	21000	22500	21783
RUMUOKORO	12000	13400	12200	11500	13000	13500	12600
RUMUOLA	10000	9500	11000	10500	10500	10200	10283
TOTAL	65100	67000	58800	56700	64000	64300	

UNIT COST OF SUPPLY

DISTRICTS	BIG TREAT, AIR FORCE JCT	BIG TREAT, RUKPOKU
ARTILLERY	50	100
MGBUOBA	200	250
RUMUOKORO	200	300
RUMUOLA	100	200

APPENDIX B

Python Program for Vogel's Approximation Method

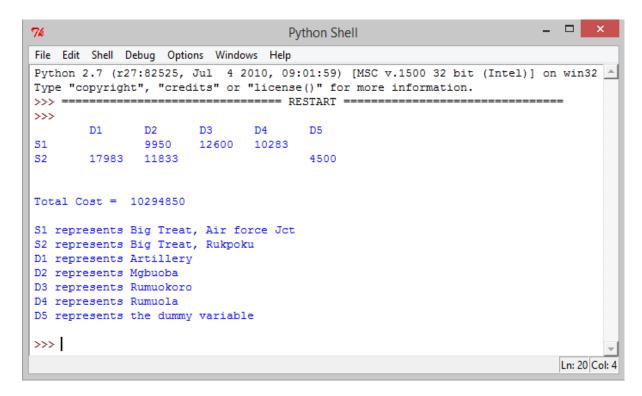
#Program for solving initial basic feasible solution using VAM

from collections import defaultdict

```
costs = {'S1': {'D1': 50, 'D2': 200, 'D3': 200,
'D4': 100, 'D5': 0},
      'S2': {'D1': 100, 'D2': 250, 'D3': 300,
'D4': 200, 'D5': 0}}
demand = {'D1': 17983, 'D2': 21783, 'D3':
12600, 'D4': 10283, 'D5': 4500}
cols = sorted(demand.iterkeys())
supply = {'S1': 32833, 'S2': 34316}
res = dict((k, defaultdict(int)) for k in costs)
g = \{\}
for x in supply:
  g[x]
                    sorted(costs[x].iterkeys(),
key=lambda g: costs[x][g])
for x in demand:
                      sorted(costs.iterkeys(),
  g[x]
key=lambda g: costs[g][x])
while g:
  d = \{ \}
  for x in demand:
                      (costs[g[x][1]][x]
     d[x]
              =
costs[g[x][0]][x]) if len(g[x]) > 1 else
costs[g[x][0]][x]
  s = \{ \}
  for x in supply:
     s[x]
                      (costs[x][g[x][1]]
costs[x][g[x][0]]) if len(g[x]) > 1 else
costs[x][g[x][0]]
  f = max(d, key=lambda n: d[n])
  t = max(s, key=lambda n: s[n])
  t, f = (f, g[f][0]) \text{ if } d[f] > s[t] \text{ else } (g[t][0],
t)
  v = min(supply[f], demand[t])
```

```
res[f][t] += v
  demand[t] = v
  if demand[t] == 0:
     for k, n in supply.iteritems():
       if n != 0:
          g[k].remove(t)
     del g[t]
     del demand[t]
  supply[f] = v
  if supply[f] == 0:
     for k, n in demand.iteritems():
       if n != 0:
          g[k].remove(f)
     del g[f]
     del supply[f]
for n in cols:
  print "\t", n,
print
cost = 0
for g in sorted(costs):
  print g, "\t",
  for n in cols:
     y = res[g][n]
     if y != 0:
       print y,
     cost += y * costs[g][n]
     print "\t",
  print
print "\n\nTotal Cost = ", cost
print"
S1 represents Big Treat, Air force Jct
S2 represents Big Treat, Rukpoku
D1 represents Artillery
D2 represents Mgbuoba
D3 represents Rumuokoro
D4 represents Rumuola
D5 represents the dummy variable
```

PYTHON PROGRAM OUTPUT



DISCUSSION

In this work, a transportation problem of the distribution of Big Treat bread (Happy Chef) was solved using the transportation tableau approach. The initial basic feasible solution was obtained using the Vogel's approximation method and the stepping stone method was used to test and solve for optimality. From the data which was used to solve the six weeks distribution of bread to respective districts, the optimal cost of transportation obtained is \$\frac{1}{2}\$10,294,850 and the result was confirmed using TORA, a statistical software. The optimal allocations are;

$$X_{11} = 9950$$

 $X_{13} = 12600$

 $X_{14} = 10283$

 $X_{21} = 8033$

 $X_{22} = 2178$

 $X_{25} = 4500$

The objective function is;

The aim of this work is to obtain the appropriate means of transporting bread form Big Treat Plc. Port Harcourt to four major districts in Port Harcourt so as to minimize the total cost of transportation. To minimize the total transportation cost, the management of Big Treat Plc. Port Harcourt should adhere to the following transportation schedule.

TRANSPORTATION SCHEDULE

Transport 9950 loaves of bread from Air force junction to Artillery.

Transport 12600 loaves of bread from Air force junction to Rumuokoro.

Transport 10283 loaves of bread from Air force junction to Rumuola.

Transport 8033 loaves of bread from Rukpoku to Artillery.

Transport 21783 loaves of bread from Rukpoku to Mgbuoba.

Alternative Transportation Schedule

Transport 9950 loaves of bread from Airforce junction to Mgbuoba.

Transport 12600 loaves of bread from Airforce junction to Rumuokoro.

Transport 10283 loaves of bread from Airforce junction to Rumuola.

Transport 17983 loaves of bread from Rukpoku to Artillery.

Transport 11833 loaves of bread from Rukpoku to Mgbuoba.

This work employed mathematical and solve statistical techniques to the transportation problem of distribution of bread and make optimal decisions. The software used for the computation and solving of the transportation problem can be used by the industry for easy and accurate decision making. From the computations and solution, to minimize the transportation cost, improve profit and render good services, the industry should distribute their products as stated below.

Optimal Transportation Schedule with Cost of Transportation

Big Treat's branches	Districts	No of loaves of bread	Transportation cost (₹)
Airforce junction	Artillery	9950	497500
Airforce junction	Rumuokoro	12600	2520000
Airforce junction	Rumuola	10283	1028300
Rukpoku	Artillery	8033	803300
Rukpoku	Mgbuoba	21783	5445750
Total	-	-	10,294,850

Alternative optimal transportation schedule with cost of transportation

Big Treat's branches	Districts	No of loaves of bread	Transportation cost (N)
Airforce junction	Mgbuoba	9950	1990000
Airforce junction	Rumuokoro	12600	2520000
Airforce junction	Rumuola	10283	1028300
Rukpoku	Artillery	17983	1798300
Rukpoku	Mgbuoba	11833	2958250
Total	-	-	10,294,850

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