Scientia Africana, Vol. 13 (No.1), June 2014. Pp 36-42
 ISSN 1118 - 1931

THE CRITICAL MAGNETIC FIELDS OF HEAVY FERMIONS SUPERCONDUCTORS

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Received: 30-09-13
Accepted: 28-01-14

ABSTRACT

The effects on the upper critical and lower critical fields of a system with two almost degenerate order parameters were considered. Within the first order perturbations, we write the two order parameters as linear combinations of the states |0> and |2>. The upper critical field is close to $T_c = T_5$ and occurs below a certain temperature T'. It can easily be seen that sharp change of HC_2 between the two solutions exists in all orders of perturbation because there is no finite matrix element between the two states $(\eta, \eta_3) = (|0>, 0)$ and $(\eta, \eta_3) = (0, |0>)$ in any higher order of perturbation in the coupling term. This is different if the Magnetic field is pointing along some arbitrary direction.

INTRODUCTION

We now consider effects on the upper critical and lower critical fields for a system with two almost degenerate order parameters. Let us first consider the upper critical field Hc_2. Such an investigation has recently been carried out including the order parameters of two representations, by Joynt, R (1990). Several other groups have also considered the problem of a single representation whose degeneracy is lifted by the presence of a magnetic ordering. We have to extend our free-energy expression by including the gradient terms.

$$
\begin{aligned}
f = {} & A_i(T)|\eta|^2 + \beta|\eta|^4 + A_5(T)(|\eta_1|^2 + |\eta_2|^2 + |\eta_2|^2 + |\eta_3|^2) \\
& + \beta_1(+|\eta_1|^2 + |\eta_2|^2 + |\eta_3|^2) \\
& + \beta_2\big(+|\eta_1|^4 + +|\eta_2|^4 + |\eta_3|^4 + 2|\eta_1|^2|\eta_2|^2 cos(2\phi_1 + 2\phi_2) \\
& + 2|\eta_2|^2|\eta_3|^2 \, cos(2\phi_2 + 2\phi_3) + 2|\eta_3|^2 - |\eta_1|^2 \, cos(2\phi_3 + 2\phi_1)\big) \\
& + \beta_3(|\eta_1|^2|\eta_2|^2|\eta_3|^2 + |\eta_3|^2|\eta_1|^2) + \theta_1|\eta_1|^2(|\eta_1|^2 + |\eta_3|^2) \\
& + \theta_2|\eta_1|^2[(|\eta_1|^2\cos(2\phi_1 + 2\phi) + (|\eta_2|^2\cos(2\phi_2 + 2\phi) \\
& + (|\eta_3|^2\cos(2\phi_3 + 2\phi)] \\
& + \theta_2|\eta_1||\eta_{21}||\eta_3|[cos(\phi_3 - \phi_2)cos(\phi_1 - \phi) + (\phi_2 - \phi)cos(\phi_3 - \phi) \\
& + (\phi_3 - \phi)cos(\phi_2 - \phi)]
\end{aligned}
$$

The coupling terms can easily be derived by the decomposition of a Kronecker product $\Gamma_1^* \otimes\Gamma_5\otimes\Gamma_4^* \otimes\Gamma_4$ + c. c ($=\Gamma_1\otimes\Gamma_2\otimes 2\Gamma_3\otimes 3\Gamma_4\otimes 4\Gamma_5$), where Γ_4 is the representation of the gradient $D= \nabla - 2eA/c$. Only one term can be found in this example.

$$K\left[(D_x\eta)^* (D_y\eta_3 + D_z\eta_2) + (D_y\eta)^* (D_z\eta_1 + D_x\eta_3) + (D_z\eta)^* (D_x\eta_2 + D_y\eta_1) + c.\ c\right] \quad \text{--- 1.1}$$

As an example, let us consider the critical field along one of the main axes, say the zaxis. By neglecting D_z and setting $H\pm = q(D_x + \imath D_y)/\sqrt{2}$ and $\eta\pm =(\eta i \pm i\eta_2)/\sqrt{2}$ (q^2= c/2eH, we obtain the linaerized Ginzburg-Landau equations

$$K_1 (H_+ H_- + H_- H_+)\eta + k(H_+^2 + H_-^2)\eta_3 = -A_1(T)q^2\eta \quad 1.2$$

$$K_2 (H_+ H_- + H_- H_+)\eta_3 + k(H_+^2 + H_-^2)\eta= -A_5(T)q^2\eta_3,$$

Which are completely decoupled from the other two equations for η_+ and η_-. These latter two equations have their solution leads to a linear temperature dependence of the critical field.

$$Hc_2^{(1)} (T)= \frac{cA_5(T)}{eC(K_1', K_2', K_3', K_4',)} \quad 1.\ 3$$

where$C(K_1', K_2', K_3', K_4',)$ is a constant depending on K_1', and is obtained from the lowest elgenvalue of an infinite matrix.

A more interesting problem is connected with the $\eta - \eta_3$ equation system, where the coupling term also enters. These equations, moreover, lead to the problem of finding the lowest elgenvalue in an infinite dimensional system. However, a goal insight into the properties of the solution can be obtained if we treat the problem in a perturbative way, assuming that the coupling term is very small ($k \ldots K_1', K_2'$) Joynt, R(1990).

Starting with the zeroth order, we find two solutions (let us assume $T_5> T_1$), which correspond to $\eta =|0\rangle$ and $\eta_1 =|0\rangle$, respectively. These leads to the occupation number representation.

$$Hc_2^{(0)} (T)= - \frac{c\,A_5 (T)}{2e\,K_2^1} \quad 1.\ 4$$

$$Hc_2^{(0)} (T)= - \frac{cA_1 (T)}{2e\,K_1} \quad 1.\ 5$$

Where $Hc_2^{(0)}$ represents the upper critical field (the lowest eigenvalue) immediately below T_5. If $K_1< K_2$, there is a crossing point of the $Hc_2^{(0)}$ and $Hc_2^{(0)}$ line at same T' defined by $A_1(T^1)K_2^1= A_5(T^1)K_1$. Below T', $Hc_2^{(0)}$ is the critical field.

Going to first-order parameter, we write the two order parameters as linear combinations of the states $|0\rangle$ and $|2\rangle$. Diagonalizing the matrix in this subspace, we obtain corrections to our former solutions [$(\eta, \eta_3) =(a_0|0\rangle, b_2|2\rangle)$ and $(\eta, \eta_3)= (a_2, |2\rangle, b_o |0\rangle$ respectively],

Scientia Africana, Vol. 13 (No.1), June 2014. Pp 36-42
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$$Hc_2^{(1)} = \frac{c}{e} A_1 A_5 \left[\{(5k_1A_5 - K'_2A_1)^2 + \overline{8}kA_1A_5\}^{1/2} - 5k_1A_5 - K'_2A_1 \right]^{-1} > Hc_2^{(0)},$$

$$Hc_2^{(1)} = \frac{c}{e} A_1 A_5 \left[\{(k_1A_5 - 5K'_2A_1)^2 + \overline{8}kA_1A_5\}^{1/2} - k_1A_5 - 5K'_2A_1 \right]^{-1} > Hc_2^{(0)}, \qquad 1.6$$

where $Hc_2^{(1)}$ is the upper critical field close to $T_c = T_5$ and $Hc_2^{(1)}$ occurs below a certain temperature T^1. It can easily be seen that sharp change of slope of Hc_2 between the two solutions exists in all orders of perturbation, because there is no finite matrix element between the two states $(\eta, \eta_3) = (|0\rangle, 0)$ and $(\eta, \eta_3) = (0\ |0\rangle$ in any higher order of perturbation in the coupling term. This is different if the magnetic field is pointing along some arbitrary direction. Then all four components of the order parameter $(\eta, \eta_1, \eta_2, \eta_3,)$ coupled. In such a case a slope change in the critical field is mostly smooth.

We have three typical situations

(a) $K_2' > C(K_1', K_2', K_3', K_4')$. The critical field goes linear with the possibility of a change to Hc_2^1 as in (equation 1.5). if $K_1 < C$ (fig 1a); otherwise, see (fig 1b).

(b) $K_2' < C(K_1', K_2', K_3', K_4')$, K_1', the critical field Hc_2 as in (equation 1. 5) without any Kink (fig. 1b).

(c) $K_1 < K_2' < C\ (K_1', K_2', K_3', K_4')$, the critical field has a kink, as discussed above (fig. 1a).

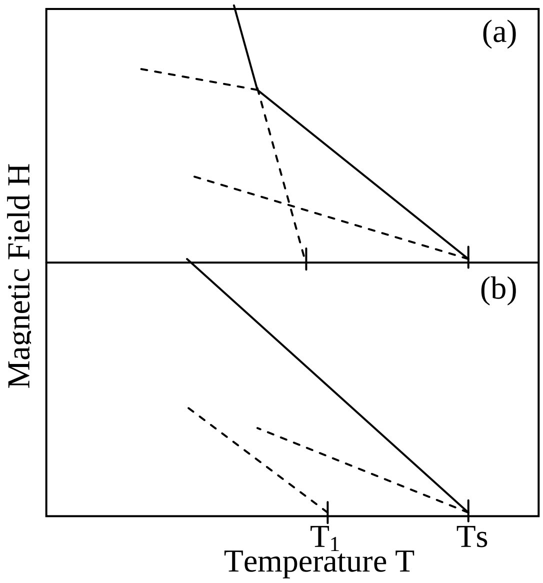

Fig. 1a & b: Possible behaviours of the upper critical field H in a superconductor with two almost degenerate order parameters situation (a) a crossing of the lowest Landau levels leads to a kink and a change of the high-field superconducting state, situation (b) no crossing occurs.

We assume for our discussion that the form of this □$_5$ order parameter is not charged for any temperature. For $T_5>T_1$ we would expect that immediately below T_5 the □$_5$ state would appear with

$$|\eta|^2 = \frac{-A_5(T)}{6(\beta_1^1 + \beta_2^1) + 2\beta_3^1}$$

However, this is prohibited by the θ_3 term, which leads to an admixture of the σ_1 order parameter even if T_1 is very small compared with T_5. So we find for T close to T_5.

$$|\eta| = \frac{\theta_3}{2A_1(T)}|\bar{\eta}|^3,$$

With the relative phase

$$\acute{Ø} - Ø = \begin{cases} 0, \theta_3 < 0, \\ \pi, \theta_3 < 0, \end{cases}$$

The □$_1$ component increases proportionally in $|T - T_5|^{½}$, that is a "driven" order parameter. This combined representation (CR) state conserves time –reversal symmetry and its fourfold degenerate [D_{3d} (□$_1$)].

According to the conditions for an admixture of another representation mentioned above, the actual □$_5$ state can mix with the □$_3$ representation, since it has the symmetry D_{3d} (□$_1$), comparable with □$_1$. The combined representation (CR) state maintains the symmetry of the originally classified state (Monien et al, 1986a, 1986b, Wojtanowski and Wolfle, 1986).

For lower temperatures an additional second-order phase transition can appear. The only symmetry that can be broken in our restricted free energy is time reversal symmetry, by a charge of the relative phase $\acute{Ø}$ - Ø. Obviously, this is favourable only if $\theta_2 > 0$, since both $\acute{Ø}$ - Ø = 0 and = □ minimize the θ_2 term for $\theta_2 > 0$.

Differentiating the free energy with respect to the relative phase, we obtain the extremum condition.

$$\mathrm{Sin}\,(\acute{Ø} - Ø)[4\theta_2]\eta[\cos(\acute{Ø} - Ø) + \theta_3]\,|\eta|] = 0$$

The expression in brackets gives a temperature – dependent solution for $\acute{Ø}$ - Ø only if $|\theta_3|\ \eta| \not 4\theta_2|\eta||\leq|$.

Thus a continuous transition from a state with $\acute{Ø}$ - Ø = 0 or □ takes place at the temperature T_o with $|\theta_3|\eta(T_o)| = 4\theta_2[\eta(T_o)]$.

Obviously, for $\theta_2 > 0$ no such transition is possible.

Finally, we mention the possibility of a phase transition with decreasing field when the fourth order terms in the free energy become important and favour a state with

Scientia Africana, Vol. 13 (No.1), June 2014. Pp 36-42
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other symmetry than that induced by the magnetic field. This would, for example be the case if we assumed situation (a) and the coefficient β_1, with the condition ($4\beta_2 < \beta_3$, $\beta_3 > 0$). At high fields a state appears with two finite components of the Γ_3 order parameter (time- reversal-breaking), whereas for low fields a one component state and, depending on the temperature and field, a finite Γ, order parameter component is more favourable.

We turn now to the lower critical field Hc_1, which is more closely related to the zero-field behaviour of the system. The effect of an additional phase transition on this quantity is of special interest, since it allows a direct observation of an additional phase transition, as we shall show here, and will be compared with experimental data. Kumar, P, and Wolfle, (1987), Langner, A. D, et al (1988), Hess, D, W., et al (1989), Sigrist, M, et al (1989).

RESULTS AND DISCUSSION

The limit of a London penetration depth is very large compared with the coherence length of the order-parameter, the main contribution to the line energy of a vortex comes from the magnetic field and the kinetic energy stored in the circulating super current.

Abrikosov, A.A, et al (1963). However, it is essential to take into account that the London penetration depth is not a scalar, but a tensor quantity in an unconventional superconductor. Thus the London equation has the general form

$$\nabla x [\wedge^2 (\nabla x H)] + H = 0, \qquad 1.7$$

where the tensor $\wedge^2$ is defined as $\wedge^2 = c^2 p^{\wedge -1}/8\pi e^2$ with $\hat{p}$ as the superfluid tensor defined by the expression for the diamagnetic current ($J_{dia} = 2e^2pA/c^2$). The equation for the field around a vortex is obtained from (equation 1. 6) by replacing the right-hard zero by $\phi_o n\delta$ (n x r) (where n is the direction of the external field and ϕ_o is a flux quantum). If the applied field (n) is parallel to one of the main axes of $\wedge^2$, the vortex line will also be parallel to n. For an arbitrary n, however, these directions need not coincide, as discussed in detail by Balatzkii, A, V, et al (1986).

For this phase the tensor p has the rather simple form

$$\hat{P} = k_1(\hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z}) |\eta|^2 + [k_1' \hat{x}\hat{x} + k_2' (\hat{y}\hat{y} + \hat{z}\hat{z})] |\eta_1|^2 \qquad 1.8$$

with $\hat{\imath}\hat{j}$ denoting the tensor element $p_{\iota j}$. For this example the crystal axis is the main axis of the tensor, because there are no coupling terms between the order-parameters components.

We choose n parallel to such as axix. Then the field calculated from the modified (equation 1.7) is

$$H = n\frac{\phi_o}{2\pi\lambda_o\lambda}K_o(\sqrt{x_o^2/\lambda_o^2 + x_\beta^2/\lambda_\beta^2})\quad 1.9$$

where $x_{o(\beta)}$ denote the directions perpendicular to n having the corresponding London penetration depths $\lambda_{o\ (\beta)}$. (K_o is a modified Bessel function).

This form becomes very simple if we choose n parallel to the x axis, because

$$\lambda_o^2 - \lambda_\beta^2 = \lambda^2 = \frac{c^2}{8\pi e^2 k_1}\frac{1}{|\eta|^2 + k_2^1|\eta_1|^2}$$

1.10

leads to a completely axial vortex. The line energy is obtained in general from

$$\varepsilon = \frac{1}{8\pi}\int dx_o dx_\beta\,[H^2 + (\nabla \times H)\wedge^2(\nabla \times H)],$$

1. 11

where the integration is restricted to the region $\sqrt{(x_o/\xi_o)^2 + (x_\beta/\xi_\beta)^2} \rangle$ 1. Evaluating this integral in the usual way (see, for example, De Gennes, P. G, (1966), we find (n || x)

$$Hc_1 = \frac{4\pi\varepsilon}{\phi_o} = \frac{\phi_o}{8\pi\lambda^2}\ \text{Ink}\quad 1.12$$

with the Ginzurg-Landau Parameter $K = \lambda/\xi$ (for this case ξ also is constant in the y – z direction).

Now let us consider the change of Hc_1 at the transition from the high temperature phase $D_{4h}(\Gamma_4)$ to the lower temperature phase $D_{4h}(\Gamma_1 \otimes \Gamma_4)$, using the equation for λ^2 and k, we obtain a sharp change in the slope of Hc_1, since λ decreasing due to the additional contribution of the Γ_1 order parameter to the super fluid density. The Ginzburg-Landau parameter drops rapidly from a constant value in the high temperature phase down to a lower, almost constant value Sigrist, M, et al (1989).

Comparing the two slopes $Hc_1^1 = (dHc_1/dT)$, above and below the second transition at T_1'', we find

$$\frac{Hc_1(T_1^1 - \delta)}{Hc_1(T_1^1 + \delta)} = \frac{\lambda^1(T_1^1 - \delta)}{\lambda^1(T_1^1 + \delta)}\left|1 - \frac{1}{\text{Ink}}\right| \frac{1}{\text{Ink}} > 1\quad 1.13$$

where $\lambda^1 = d\lambda/dT$ and δ is an infinitesimal numbers. This ratio is larger than 1 in the large K limit where Ink >1 (k taken at T_1'), if the London penetration depth is decreasing faster below the additional transition as T_1' than above Hess, D, W., et al (1989). Comparing the ratio $\lambda'(T_1' - \delta) / \lambda'(T_1' + \delta)$ with the one of the specific heat $C(T_1' - \delta)/ C(T_1' + \delta)$ we find that this condition is usually satisfied if the discontinuity of the specific heat ΔC is

Scientia Africana, Vol. 13 (No.1), June 2014. Pp 36-42
 ISSN 1118 - 1931

positive provided that all coefficients k in the tensor p are of the same order of magnitude. This qualitative behaviour is in agreement with experimental results.

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